Smith-Purcell radiation from surface waves

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Radiation in periodic structures

- Radiation from charged particles in periodic structures has a number of remarkable properties and is widely used in various regions of science and technology

- Generation of the electromagnetic radiation in various wavelength ranges by beams of charged particles

- Determination of the characteristics of emitting particles by using the properties of the radiation field

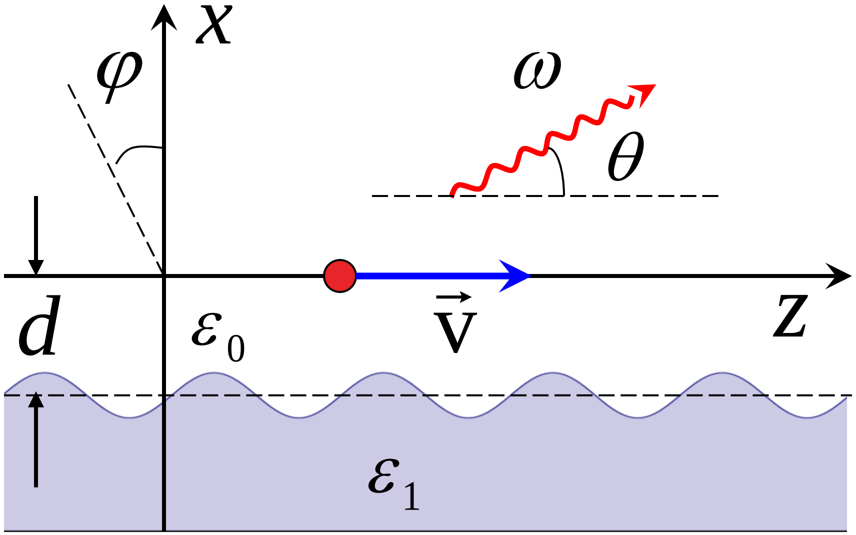
Examples

•Transition radiation from a charge traversing a stack of plates or moving in a medium with periodically varying dielectric permittivity

•Smith-Purcell radiation, which arises when charged particles are in flight near a diffraction grating

•Smith-Purcell radiation is one of the main mechanisms for the generation of electromagnetic waves in the millimeter and submillimeter wavelength range

Geometry of the problem and the spectrum



Equation of the interface

$$x = x_0(z,t) = -d + f(k_0 z \mp \omega_0 t)$$

From the system invariance under the transformation

$$t \to t + \frac{\lambda_0}{v \mp v_0}, \quad z \to z + \frac{\lambda_0 v}{v \mp v_0}$$

$$\omega = \frac{m(k_0 v \mp \omega_0)}{1 - \beta\sqrt{\varepsilon_\alpha}\cos\theta}, \quad \beta = \frac{v}{c}$$

Waves emitted from two neighboring humps of the surface wave are in phase

Radiation intensity (methods used)

- For the evaluation of the radiation intensity in the Smith-Purcell effect various approximate methods were used (see A. P. Potylitsyn, Radiation of electrons in periodic structures (2009))
- For the problem under consideration we have used two independent approximate methods

(i) Small permittivity changes ($\left|\varepsilon_1 - \varepsilon_0\right| << \varepsilon_0$)

(ii) Small amplitude wave

- High-amplitude surface waves are excited with nano-second laser pulses

Radiation intensity: First method

- Spectral-angular density of the radiation intensity for a given *m* in the case $f(x) = a \sin x$

$$\frac{dW_m}{d\omega d\Omega} = \frac{q^2}{8\pi c^2}\left(1-\frac{\varepsilon_1}{\varepsilon_0}\right)^2 \omega b \left|\frac{1}{u\sigma} J_m(u\omega a/v)\right|^2 e^{-2\omega d \mathrm{Re}\sigma/v}$$
$$\times \delta\left(\cos\theta - \frac{1}{\beta_0} + m\frac{k_0 v \mp \omega_0}{\omega\beta_0}\right), \quad \beta_0 = v\sqrt{\varepsilon_0}/c$$

$$\sigma = \sqrt{\left(1-\beta_0^2\right)\omega_1^2/\omega^2 + \beta_0^2 \sin^2\theta \sin^2\varphi},$$
$$u = \beta_0 \sin\theta\cos\varphi + i\sigma, \ \omega_1 = \omega \pm m\omega_0,$$
$$b = |\sigma|^2 + \beta_0^2 \sin^2\theta \sin^2\varphi + \left(\beta_0^2 - 1\right)^2 \omega_1^2/\omega^2$$
$$- \left|\left(\beta_0^2 - 1\right)\omega_1 \cos\theta/\omega - \beta_0 \sin^2\theta \sin^2\varphi + i\sigma \sin\theta\cos\varphi\right|^2$$

- There is no radiation radiation for $v = \omega_0/k_0$

Radiation intensity: Limiting case

For $v, \omega_0/k_0 \ll c$ one has $\omega = m|k_0 v \mp \omega_0|$

$$\frac{dW_m}{d\Omega} = \frac{q^2 v^3 k_0^3}{8\pi c^3}\sqrt{\varepsilon_0}\left(1 - \frac{\varepsilon_1}{\varepsilon_0}\right)^2 \left(1 \mp \frac{\omega_0}{k_0 v}\right)^4$$

$$\times\left(1 + \sin^2\theta \sin^2\varphi\right) m^2 I_m(mak_0) e^{-2mk_0 d}$$

Radiation intensity: Spectral-angular distribution

Spectral-angular distribution of the radiation intensity

$$\boxed{\frac{dW_m}{d\omega d\Omega}=\frac{2q^2\omega^3}{\pi c^2 v^2}\left(1-\frac{\varepsilon_1}{\varepsilon_0}\right)^2\frac{|f_m|^2\sin^2\theta\cos^2\varphi}{\exp(2\omega\sigma d/v)}a(\theta,\varphi)\delta\left(\cos\theta-\frac{1}{\beta\sqrt{\varepsilon_0}}+\frac{mk_0c}{\omega\sqrt{\varepsilon_0}}\right)}$$

Notations: $f_m=\frac{1}{2\pi}\int_{-\pi}^{\pi}f(u)e^{-imu}du$, $\sigma=\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}$

$$a(\theta,\varphi)=\frac{\sigma^2A_1+\sin^2\theta\sin^2\varphi\,A_2+\varepsilon_0\beta^2\sin^4\theta\sin^4\varphi\,A_3}{(\sigma_1^2+\varepsilon_1\sigma^2/\varepsilon_0)(\sigma_2+\varepsilon_1\sin\theta\cos\varphi/\varepsilon_0)^2}$$

$$\sigma_1=\sqrt{\beta^2(\varepsilon_1-\varepsilon_0\sin^2\theta\sin^2\varphi)-1},\quad \sigma_2=\sqrt{\varepsilon_1/\varepsilon_0-1+\sin^2\theta\cos^2\varphi}$$

$$A_1=[\sigma_1\sigma_2\cos\theta-\varepsilon_1(1-\sin^2\theta\cos^2\varphi)/\varepsilon_0]^2+\sin^2\theta[\sigma_1(\sigma_2\cos\varphi+\sin\theta\sin^2\varphi)-\varepsilon_1\cos\theta\cos\varphi/\varepsilon_0]^2,$$

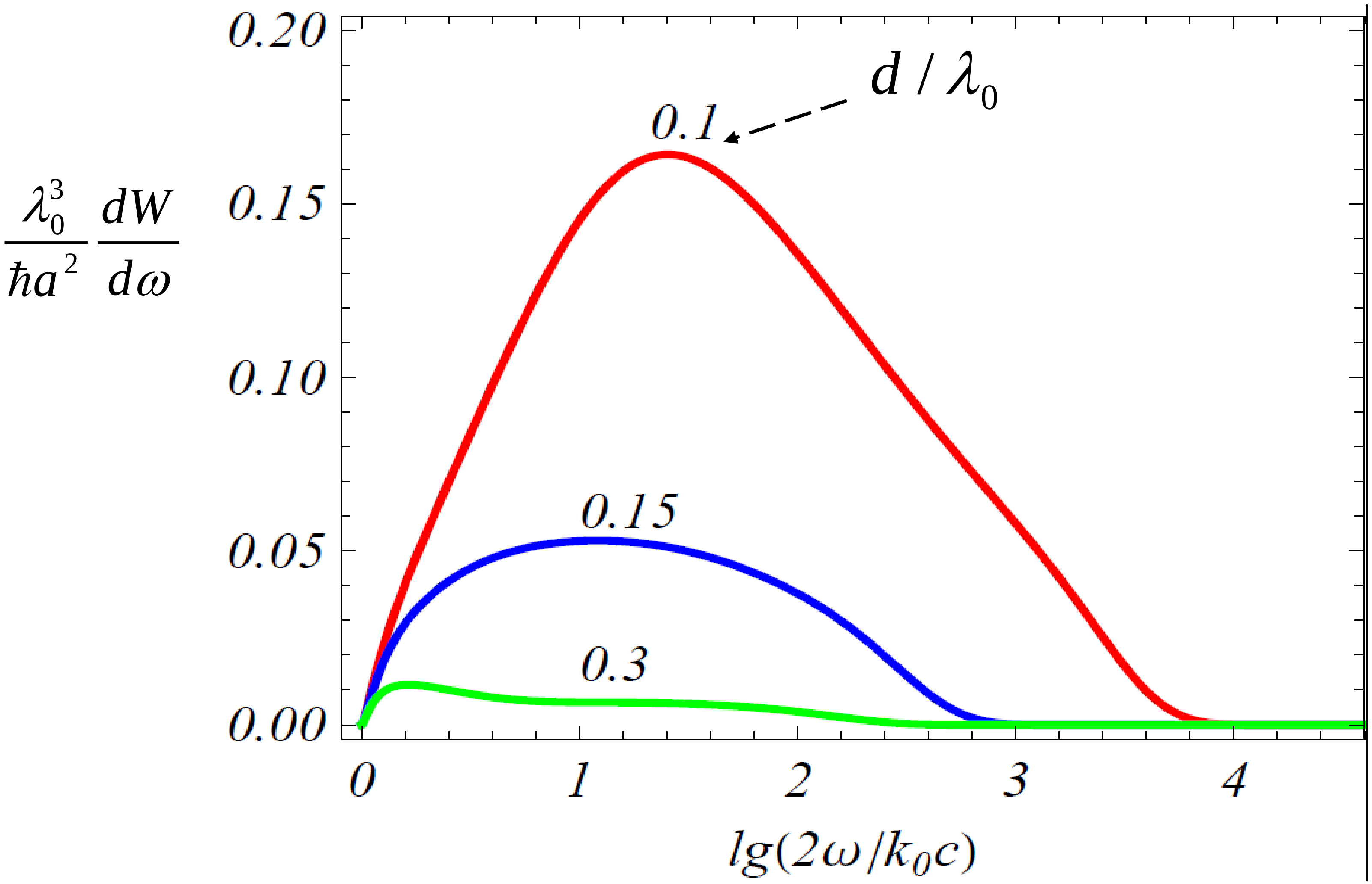
$$A_2=\sigma^2(\sigma_1\cos\theta+\varepsilon_1\sin\theta\cos\varphi/\varepsilon_0)^2+\varepsilon_0\beta^2(\sigma_2\sin\theta\cos\varphi+\beta\sqrt{\varepsilon_0}\cos\theta\sin^2\theta\sin^2\varphi+\cos^2\theta)^2,$$

$$A_3=\sigma_2^2(1-\beta\sqrt{\varepsilon_0}\cos\theta)^2+[\cos\theta+\beta\sqrt{\varepsilon_0}\sin\theta(\sigma_2\cos\varphi+\sin\theta\sin^2\varphi)]^2$$

Radiation intensity (numerical results)

Sinusoidal surface wave $f(x) = a \sin x$

Electron energy = 100 MeV, $\varepsilon_0 = 1, \ \varepsilon_1 = 2.9$



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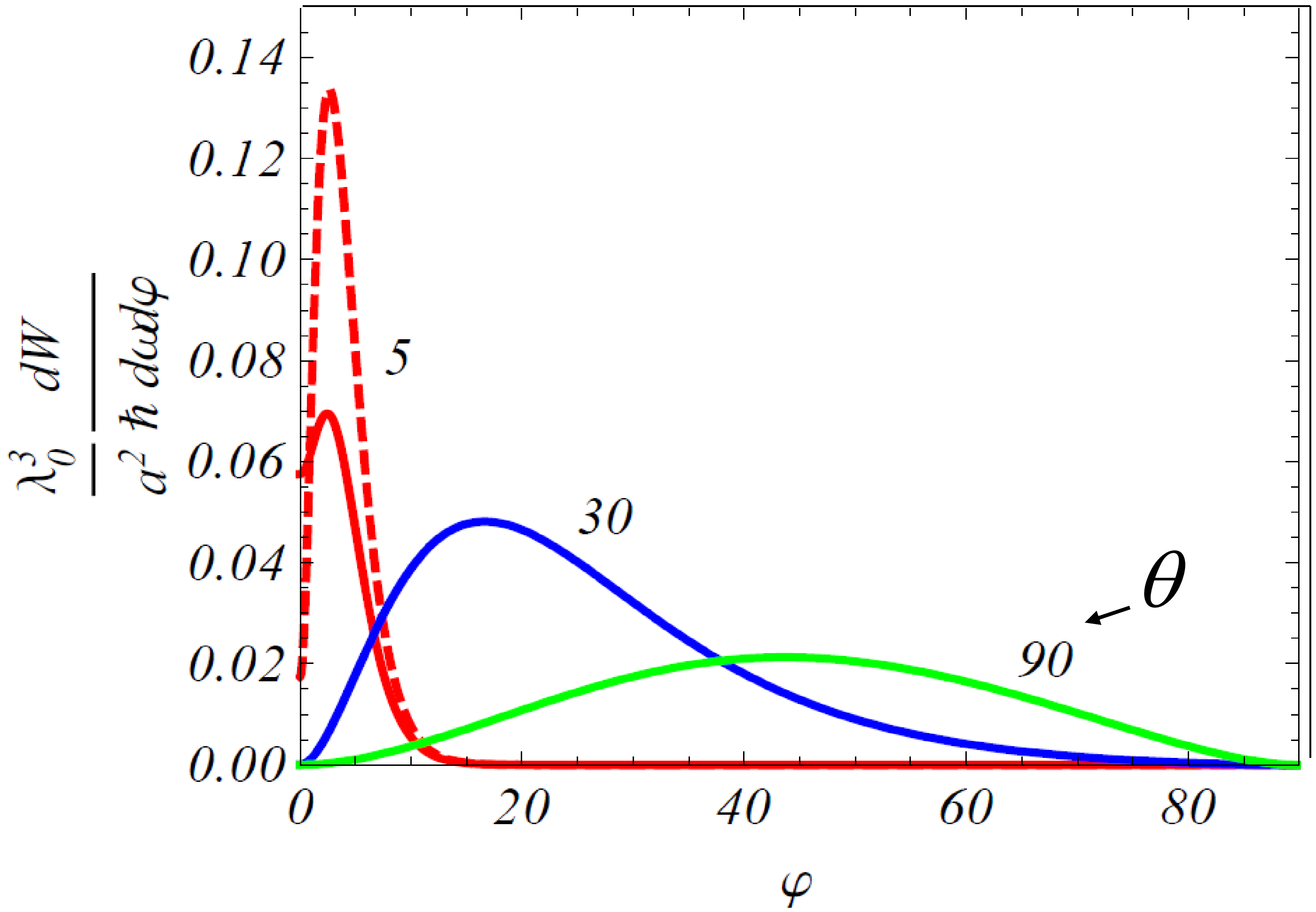
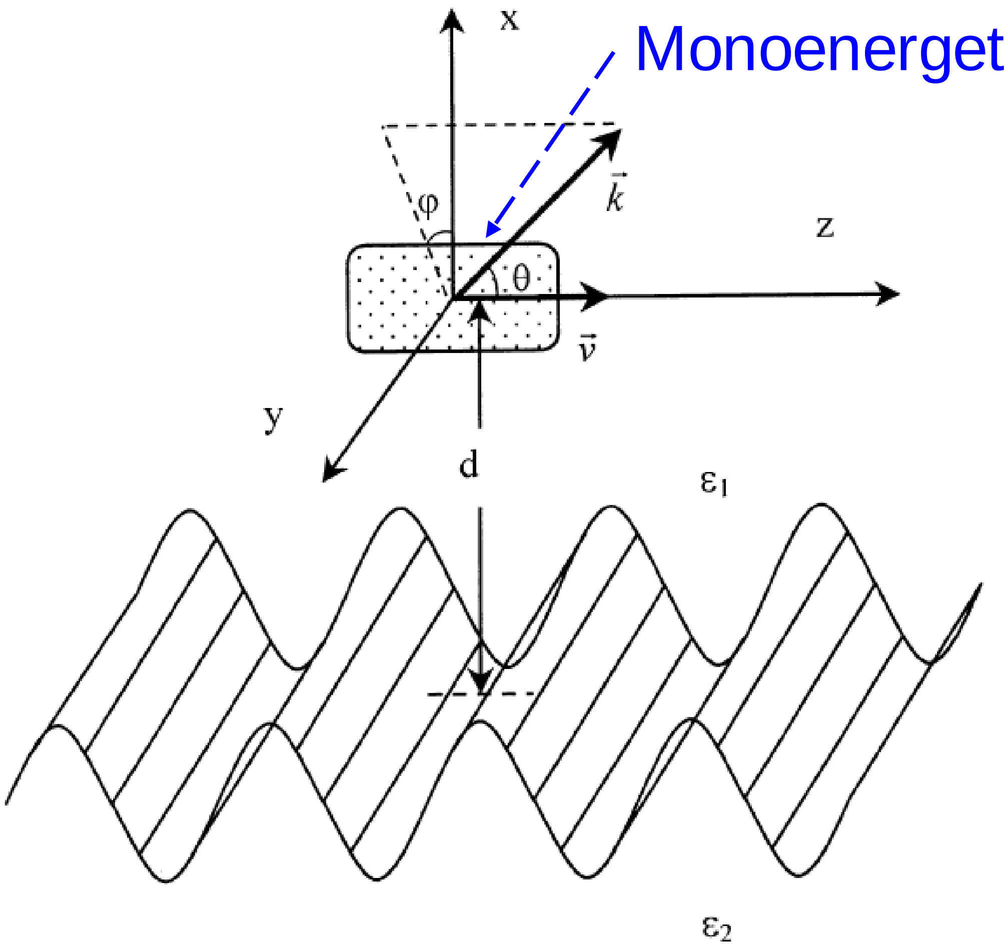


Radiation from an electron bunch

Geometry of the problem

Monoenergetic bunch of N particles



Radiation intensity

Spectral density of the radiation energy flux in the medium α for a given m

corresponding function for the radiation of a single charge

$$\vec{P}_m^{(N)}(\omega) = \vec{P}_m^{(1)}(\omega) S_N,$$

$$S_N = \left| \sum_{j=1}^{N} \exp\left(\mathrm{i} g_0(\omega_1) X_j - \mathrm{i} k_y Y_j - \mathrm{i} \frac{\omega_1}{v} Z_j \right) \right|^2$$

$$g_0 = \mathrm{i} \frac{\omega}{v} \sigma, \; \sigma = \left[\left(1 - \beta^2 \varepsilon_1\right) \frac{\omega_1^2}{\omega^2} + \beta^2 \varepsilon_\alpha \sin^2\theta \sin^2\varphi \right]^{1/2}, \; \omega_1 = \omega \pm m\omega_0$$

$\vec{R}_j = (X_j, Y_j, Z_j)$ ⇦ Position of the *j* th particle in the bunch at the initial moment

Bunch form factor

Averaging over the positions of a particle in the bunch

$$\langle \vec{P}_m^{(N)} \rangle = \langle S_N \rangle \vec{P}_m^{(1)},$$

contribution of coherent effects

$$\langle S_N \rangle = Nh + N(N-1)\left|h_x h_y h_z\right|^2$$

$$h = \left\langle \exp\left(-\frac{2\omega}{v} X \, Re\, \sigma\right)\right\rangle,$$

$|h_l|^2$ bunch form factors in corresponding directions

$$h_l = \langle \exp(\mathrm{i}K_l l)\rangle, \quad l = x, y, z,$$

$$K_x = -\mathrm{i}\frac{\omega}{v}\sigma, \quad K_y = k_y = \frac{\omega}{c}\sqrt{\varepsilon_0}\sin\theta\sin\varphi,$$

$$K_z = \frac{\omega_1}{v},$$

Conventionally it is assumed that the coherent radiation is produced at wavelengths equal and longer than the electron bunch length

Coherent effects (Gaussian bunch)

Gaussian distribution $$f_l = \frac{1}{\sqrt{2\pi} b_l} \exp\left(-\frac{l^2}{2b_l^2} \right), \quad l = x, y, z,$$

Form factor
$$\langle S_N \rangle = N \exp\left(\frac{2\omega^2}{v^2} (Re\,\sigma)^2 b_x^2 \right) \left[1 + (N-1) \right.$$
$$\left. \times \exp\left(-\frac{\omega^2}{v^2} |\sigma|^2 b_x^2 - k_y^2 b_y^2 - \frac{\omega_1^2 b_z^2}{v^2} \right) \right]$$

For a relativistic bunch the relative contribution of coherent effects for the radiation with $\sin\theta \sin\varphi \lesssim \gamma^{-1}$

$$N \exp\left\{ -(2\pi b_x/\lambda\gamma)^2 - (2\pi/\lambda)^2 (b_z^2 + b_y^2 \sin^2\theta \sin^2\varphi) \right\}$$

Transverse form factor is strongly anisotropic

Coherent effects (non-Gaussian bunch)

- Due to various beam manipulations the bunch shape can be highly non-Gaussian
- For non-Gaussian bunches the form factor for the short wavelengths may decrease as power-law instead of being exponential

Asymmetric Gaussian bunch

- Example: asymmetric Gaussian bunch

N.A.Korkhmazian, L.A.Gevorgian, M.L.Petrosyan, Zhur. Tekh. Fiz. 47 (1977) 1583

$$f(z) = \frac{2}{\sqrt{2\pi}(1+p)b_l}\left[\exp\left(-\frac{l^2}{2p^2b_l^2}\right)\theta(-l) + \exp\left(-\frac{l^2}{2b_l^2}\right)\theta(l)\right]$$

- Form factor: $\langle S_N \rangle = N \exp\left(\frac{2\omega^2}{v^2}\sigma^2 b_x^2\right)\left[1 + (N-1)\right.$

$$\left. \times \exp\left(-\frac{\omega^2}{v^2}\sigma^2 b_x^2 - k_y^2 b_y^2\right)|F(\omega_1/v)|^2\right]$$

$$F(u) = \frac{1}{p+1}\left\{e^{-t^2} + pe^{-p^2t^2} - \frac{2i}{\sqrt{\pi}}[W(t) - pW(pt)]\right\}, \quad t = \frac{ub_l}{\sqrt{2}}$$

$$W(t) = \int_0^t \exp(l^2 - t^2)\,dl$$

$$\boxed{F(u) \sim i\sqrt{\frac{2}{\pi}\frac{1-p}{u^3 b_l^3 p^2}}\,, \quad pt \gg 1}$$

- Even for weakly asymmetrical bunch $N|F|^2 \sim N(v/\omega_1 b_z)^6$
radiation is coherent if $b_z \lesssim \lambda N^{1/6}/2\pi$

Other non-Gaussian bunches

- Rectangular bunch having exponentially decreasing asymmetric tails

$$f(z, a_l, b_l, l_0) = \frac{1}{4l_0}\left[\tanh\left(\frac{l+l_0}{a_l}\right) - \tanh\left(\frac{l-l_0}{b_l}\right)\right]$$

- Superposition of two Gaussian functions

$$f(l) = \frac{\exp(-l^2/2a_l^2) + \exp[\alpha - (l-l_1)^2/2b_l^2]}{\sqrt{2\pi}(a_l + e^{\alpha}b_l)}$$

Conclusion

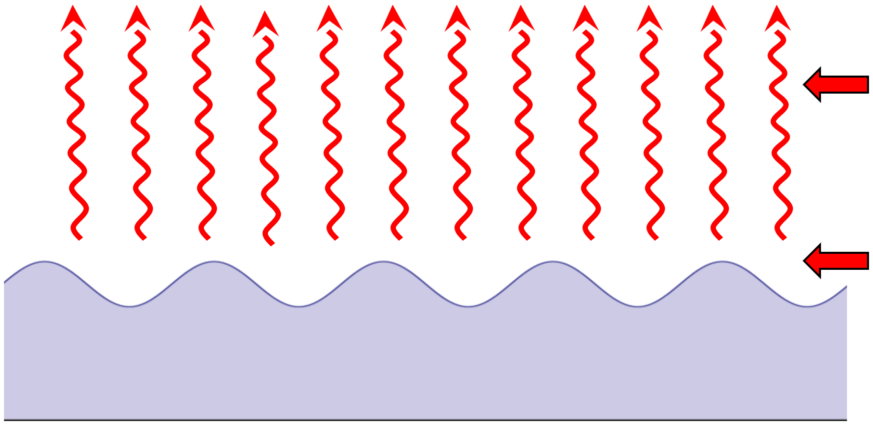
•We investigate the radiation from an electron bunch of arbitrary structure flying over the surface wave excited in plane interface between media with different dielectric constants

•Radiation from a bunch can be partially coherent in the range of wavelengths much shorter than the characteristic longitudinal size of the bunch

•Main contribution to the radiation intensity comes from the parts of the bunch with large derivatives of the distribution function

•For short wavelengths the relative contribution of coherent effects decreases as a power-law instead of exponentially decreasing

Quantum radiation from surface waves



Quantum radiation due to the dynamical Casimir effect

Surface wave

Quantum radiation arises due to the interaction of dynamical boundary with the quantum fluctuations of the vacuum

Vacuum fluctuations in quantum field theory

- Among the most important consequences of quantum field theory is the presence of non-trivial properties of the vacuum state
- Vacuum is a state of a quantum field with zero number of particles

Particle number operator

$$\hat{n}|0\rangle = 0$$

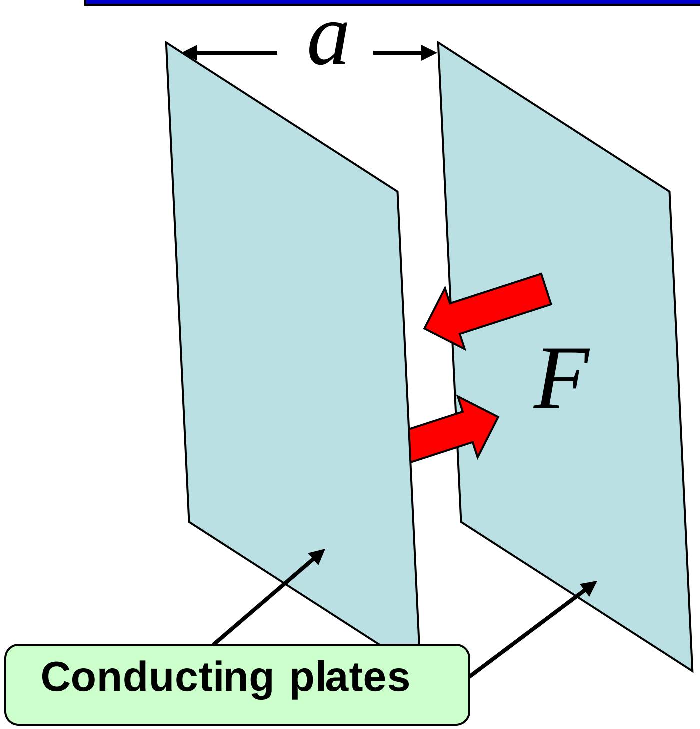
- Particle number and field operators do not commute

$$[\hat{n}, \hat{\varphi}] \neq 0$$

- In the vacuum state the field fluctuates

Vacuum or zero-point fluctuations

The Casimir effect as a macroscopic manifestation of the vacuum fluctuations



The Casimir effect (Casimir, 1948):

Two conducting neutral parallel plates in the vacuum attract by the force per unit surface

$$F = \frac{\pi^2 \hbar c}{240\, a^4}$$

The plates modify the spectrum of the electromagnetic field vacuum fluctuations → The vacuum energy is changed

$$\Delta E(a) = \sum \hbar\omega(a)/2 - \sum \hbar\omega/2 \quad \Rightarrow \quad F = -\frac{\partial}{\partial a}\Delta E(a)$$

Vacuum energy in the presence of plates

Vacuum energy in the absence of plates

Dynamical Casimir effect

- Boundaries and boundary conditions are static

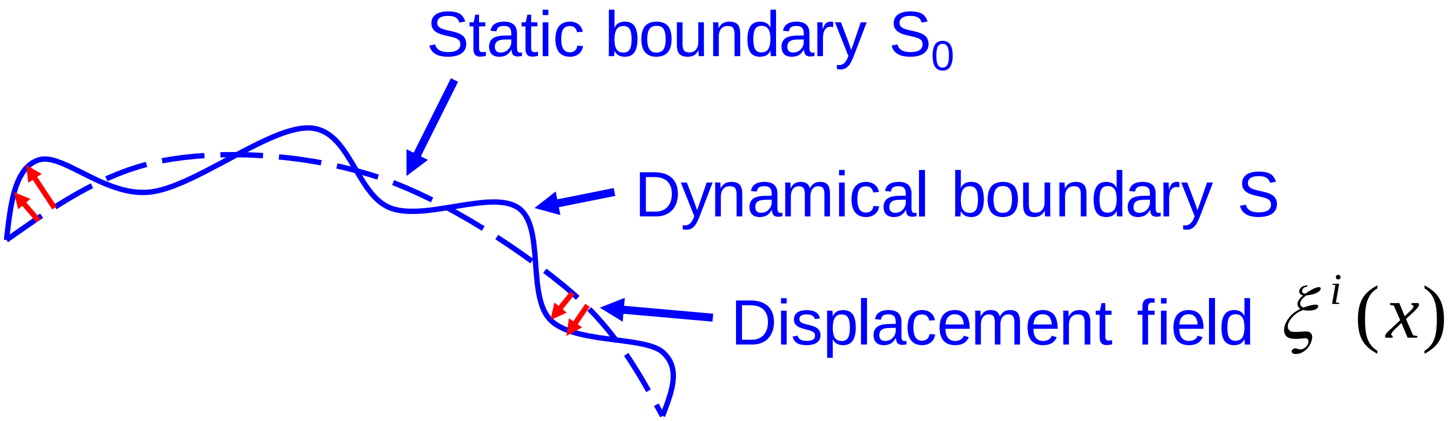
  Static Casimir effect

- Geometrical configuration and boundary conditions depend on time

  Dynamical Casimir effect

- Manifestations of dynamic behavior
  - Dependence of the force on time
  - Creation of particles from vacuum by a moving boundary

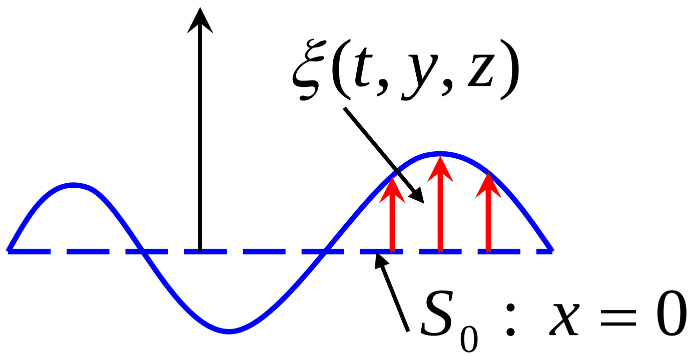
General problem



Model: Scalar field with Dirichlet boundary condition

$$\left(\Box + m^2\right)\varphi(x) = 0, \quad \varphi(x)|_S = 0$$

Number of radiated quanta



$$\xi(\omega,\vec{k}_\perp) = \int_{-\infty}^{+\infty} \xi(t,y,z)\, e^{i\vec{k}_\perp \vec{x}_\perp - i\omega t}\, dt\,dy\,dz$$

Number of the radiated quanta

$$n(\vec{k}) = \frac{k_1^2}{16\pi^6\omega}\int_0^\infty \frac{k_1'^2}{\omega'} \left|\xi(\omega+\omega',\vec{k}_\perp + \vec{k}_\perp{}')\right| d\vec{k}',\ \ \omega = \sqrt{\vec{k}^2 + m^2},$$

Special case: $\xi = \xi(t)$

Number of the radiated quanta:

$$n(\vec{k}) = \frac{k_1^2}{4\pi^4\omega}\int_0^\infty \left|\xi(\omega+\omega')\right| \sqrt{\omega'^2 - k_\perp^2}\, d\omega'$$

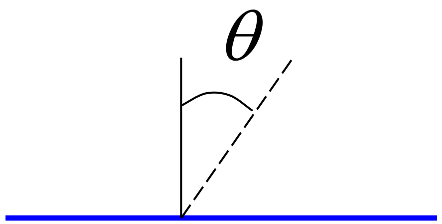
Radiated energy:

$$E = \frac{1}{720\pi^2}\int_{-\infty}^{+\infty} \left|\xi'''(t)\right|^2 dt$$

Harmonic oscillations of boundary

Consider $\xi(t) = \xi_0 \cos(\omega_0 t)$

Spectral-angular density of the number of radiated quanta

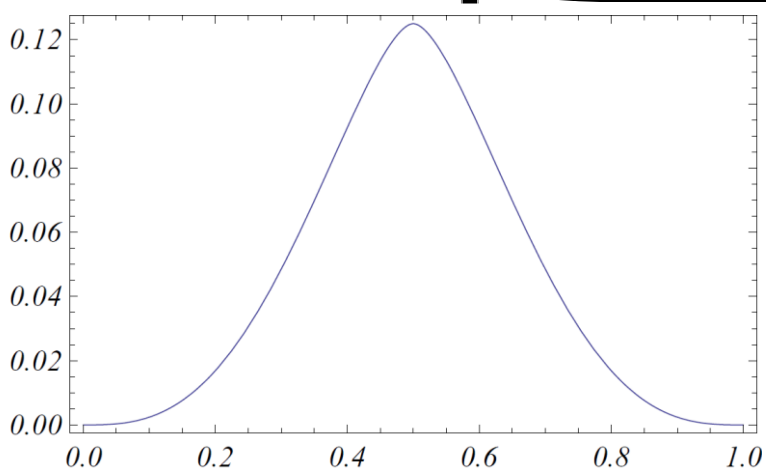


$$\frac{dn}{d\omega d\Omega} = \frac{\xi_0^2}{8\pi^3} \cos^2\theta \left(\omega_0^2 - 2\omega_0\omega + \omega^2\cos^2\theta\right)^{1/2}$$

$$\omega_0 - \omega \geq \omega\left|\sin\theta\right|, \quad -\pi/2 \leq \theta \leq \pi/2$$

Spectral density ($u = \omega/\omega_0$)

$$n(\omega) = \frac{\xi_0^2\omega_0^4}{32\pi^2}\left[\underbrace{u(1-u)^3 + u^3(1-u) + \frac{1}{2}(1-2u)^2\ln|1-2u|}\right]$$



Total number of radiated quanta and total energy

$$N = \frac{\xi_0^2\omega_0^5}{720\pi^2c^4}, \quad E = \frac{\hbar\xi_0^2\omega_0^6}{1440\pi^2c^4}$$

Standing surface wave

Surface wave excited on the strip $0 \le z \le l,\ -\infty < y < \infty$

$$\xi(t) = \xi_0 \cos(\omega_0 t)\sin(k_0 z), \quad k_0 = \pi n/l, \quad n = 1, 2, \ldots$$

Number of the radiated quanta per unit time and per unit length along the axis *y*

$$\boxed{n(\mathbf{k}) = \frac{k_1^2 \xi_0^2}{8\pi^4 \omega} \int_{u_1}^{u_2} du \frac{1 - (-1)^n \cos nu}{(1 - u^2)^2} \sqrt{(u - u_1)(u_2 - u)}}$$

$$k_0 u_{2,1} = k_3 \pm \sqrt{(\omega_0 - \omega)^2 - k_2^2}$$

$$\omega = \sqrt{k_1^2 + k_2^2 + k_3^2} \le \omega_0, \quad |k_2| \le \omega_0 - \omega$$

Thank you!