Hydrodynamic response to initial state fluctuations

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AA-collisions

- Initial particle/energy production, followed by
- Hydrodynamic evolution, followed by
- Freeze-out/Hadron cascade

- Goal is to determine QGP properties: EoS, transport coefficients ...
- Large(st) uncertainty in determining e.g. shear viscosity is initial state for hydrodynamic evolution

Complicated (and necessary) problem is to determine initial conditions simultaneously with QGP properties

Hydrodynamics

Conservation laws

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu n_i^\mu = 0$$

$$T^{\mu\nu} = eu^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

IS equations for viscous parts of $T^{\mu\nu}$

e.g. shear viscosity:

$$\tau_\pi \frac{d}{d\tau}\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} + \cdots$$

To solve this set of equations we need

- Equation of state $p = p(e)$ and $T = T(e)$
- Initial condition $T^{\mu\nu}(\tau_0, \mathbf{x})$
- Transport coefficients, e.g. shear viscosity $\eta(T)$, relaxation time $\tau_\pi(T)$, ...

Hydrodynamics $\longrightarrow$ particles (Freeze-out)

- Cooper-Frye freeze-out for particle i = calculate number of particles crossing the freeze-out hypersurface

$$E\frac{dN}{d^3\mathbf{p}} = \frac{g_i}{(2\pi)^3}\int d\sigma^\mu p_\mu f_i(\mathbf{p}, x)$$

$+$ decays

Hadron spectra

$$\longrightarrow \frac{dN_h}{dydp_T^2 d\phi}$$

$\sim$ 300 hadronic states: takes most of the computing time

Characterizing p_T-spectra

Fourier decomposition: w.r.t. event plane

$$\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}p_T^2\mathrm{d}\phi} = \frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}p_T^2}\left[1 + 2v_1(p_T)\cos(\phi - \psi_1) + 2v_2(p_T)\cos\left[2(\phi - \psi_2)\right] + \cdots\right]$$

$$\psi_n = (1/n)\arctan\left(\langle p_T \sin n\phi\rangle / \langle p_T \cos n\phi\rangle\right)$$

- $v_n(p_T)$, $\psi_n(p_T)$, dN/dy, ... characterize single event

Ensemble of events: Full characterization

- Averages: $\langle v_n\rangle$, $\langle \psi_n\rangle$, ...
- Probability distributions: $\mathcal{P}(v_n)$, $\mathcal{P}(\psi_n)$, ...
- Correlations: $\langle v_n, v_m\rangle$, $\langle \psi_n, \psi_m\rangle$, ...

Characterizing initial state

Eccentricity (for energy density ε)

$$\epsilon_{m,n} = -\frac{\int \mathrm{d}x\mathrm{d}y\; r^m \cos\left[n\left(\phi - \Psi_{m,n}\right)\right] \varepsilon\left(x,y,\tau_0\right)}{\int \mathrm{d}x\mathrm{d}y\; r^m \varepsilon\left(x,y,\tau_0\right)}$$

$$\Psi_{m,n} = \frac{1}{n} \arctan \frac{\int \mathrm{d}x\mathrm{d}y\; r^m \sin\left(n\phi\right) \varepsilon\left(x,y,\tau_0\right)}{\int \mathrm{d}x\mathrm{d}y\; r^m \cos\left(n\phi\right) \varepsilon\left(x,y,\tau_0\right)} + \pi/n$$

- $e_{m,n}$, $\Psi_{m,n}$ characterize single event (initial energy density)

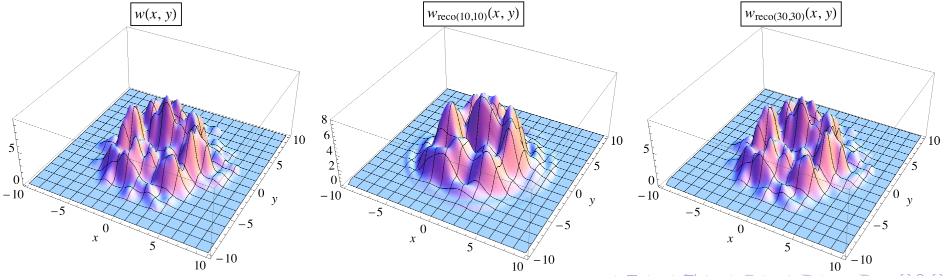
Ensemble of events (initial conditions): Full characterization

- Averages: $\langle e_{m,n}\rangle$, $\langle \Psi_{m,n}\rangle$, ...
- Probability distributions: $\mathcal{P}(e_{m,n})$, $\mathcal{P}(\Psi_{m,n})$
- Correlations: $\left\langle e_{m,n}, e_{m',n'}\right\rangle$, $\left\langle \Psi_{m,n}, \Psi_{m',n'}\right\rangle$, ...

Bessel-Fourier expansion: S. Floerchinger, U. Wiedemann: arXiv:1307.7611 [hep-ph]

$$w_l^{(m)} = \frac{2}{R^2\left[J_{m+1}(k_l^{(m)}R)\right]^2}\int_0^R dr\, r\, w^{(m)}(r)\, J_m\left(k_l^{(m)} r\right)$$

$$w_{\mathrm{reco}(N_m,N_l)}(r,\phi) = \sum_{l=1}^{N_l} w_l^{(m=0)}\, J_0\left(z_l^{(0)} r/R\right) + 2\sum_{m=1}^{N_m}\sum_{l=1}^{N_l} |w_l^{(m)}| \cos\left[m\left(\phi-\varphi_l^{(m)}\right)\right]\, J_m\left(z_l^{(m)} r/R\right)$$



Connecting initial condition to hadron spectra

Ensemble of events (initial conditions): Full characterization

- Averages: $\langle e_{m,n}\rangle$, $\langle \Psi_{m,n}\rangle$, ...
- Probability distributions: $\mathcal{P}(e_{m,n})$, $\mathcal{P}(\Psi_{m,n})$
- Correlations: $\left\langle e_{m,n}, e_{m',n'}\right\rangle$, $\left\langle \Psi_{m,n}, \Psi_{m',n'}\right\rangle$, ...

↓ Hydrodynamic response (EoS, η/s, $T_{\rm dec}$, ...)

Ensemble of events (spectra): Full characterization

- Averages: $\langle v_n\rangle$, $\langle \psi_n\rangle$, ...
- Probability distributions: $\mathcal{P}(v_n)$, $\mathcal{P}(\psi_n)$, ...
- Correlations: $\langle v_n, v_m\rangle$, $\langle \psi_n, \psi_m\rangle$

- Problem is that typically the connections depend on the full details of the space-time evolution...

e_n–v_n correlation

Correlation coefficient

How are v_n's and ϵ_n's related?

- $\langle v_n \rangle \propto \epsilon_n$
- $v_n \propto \epsilon_n$

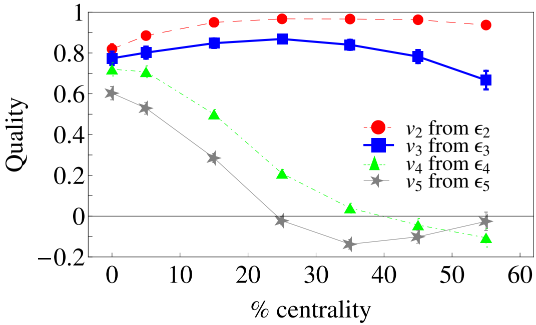
Measure by linear correlation coefficient:

$$c\,(a,b) = \left\langle \frac{\left(a - \langle a\rangle_{\rm ev}\right)\left(b - \langle b\rangle_{\rm ev}\right)}{\sigma_a \sigma_b} \right\rangle_{\rm ev}$$

- $c = 0$ no (linear) correlation
- $c = 1(-1)$ fully (anti-)correlated

$e_n - v_n$ correlation

F. G. Gardim, F. Grassi, M. Luzum and J. -Y. Ollitrault, Nucl. Phys. A904-905 **2013**, 503c (2013)



- v_2 and v_3 strongly correlated to the corresponding eccentricities
- Higher harmonics: no correlation (except in central collisions)

Monte-Carlo Glauber

$$s(x,y) = W \sum_{i=1}^{N_{\rm part,bin}} \exp\left\{-\left[(x-x_i)^2+(y-y_i)^2\right]/\left(2\sigma^2\right)\right\}$$

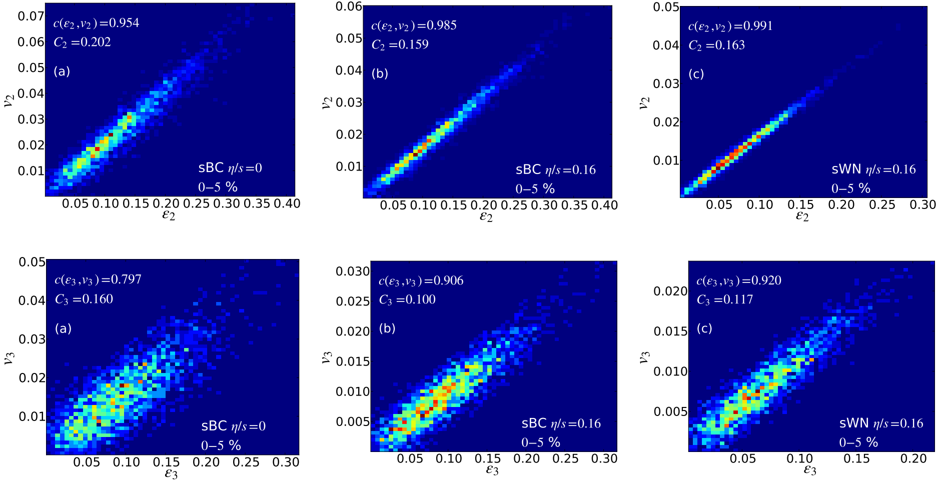
- (x_i, y_i) position of wounded nucleon or binary collision
- W normalization constant
- Centrality selection according to N_{bin} or N_{part}

p_T-spectra for each event $\left.\frac{dN}{dydp_T^2d\phi}\right|_{ev}$

$v_{2/3}$ vs $\epsilon_{2/3}$ ($0-5\%$ centrality class)

2d histogram $\sim$ 2000 hydro events per case

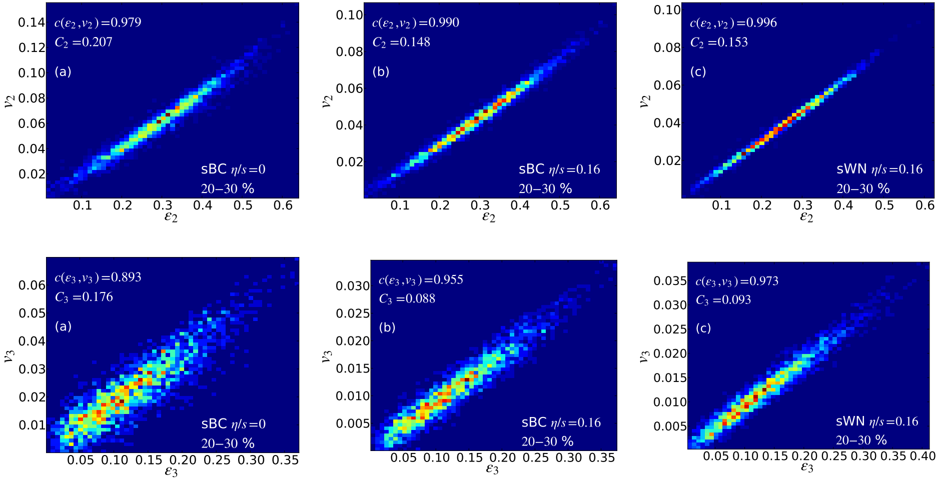
sBC $\eta/s = 0$ sBC $\eta/s = 0.16$ sWN $\eta/s = 0.16$



Both v_2 and v_3 correlated to corresponding eccentricities (v_2 event-by-event)

$v_{2/3}$ vs $\epsilon_{2/3}$ $(20 - 30\%$ centrality class)

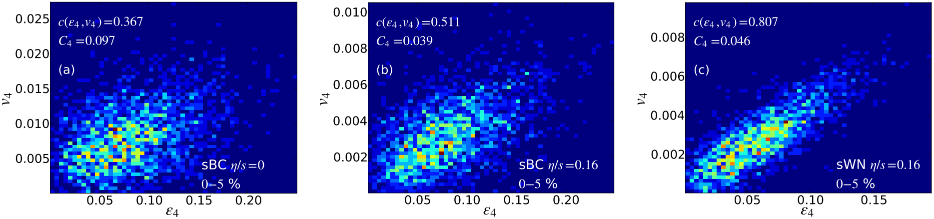
sBC $\eta/s = 0$ | sBC $\eta/s = 0.16$ | sWN $\eta/s = 0.16$



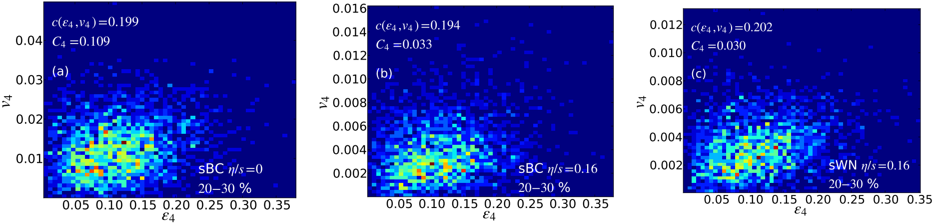
Even better correlation

v_4 vs ϵ_4

$0-5$ %



$20-30$ %



Weak correlation in central collision, no correlation in peripheral collisions

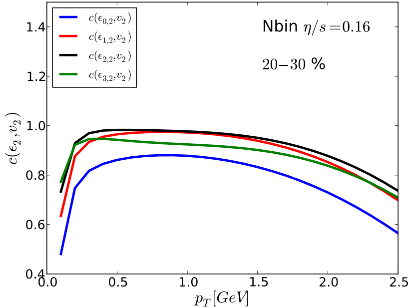
$e_{m,n} - v_n(p_T)$ correlation

Do $v_n(p_T)$ correlate with different $\epsilon_{m,n}$ at different p_T's ($\epsilon_{m,n}$ with different r^m weight)

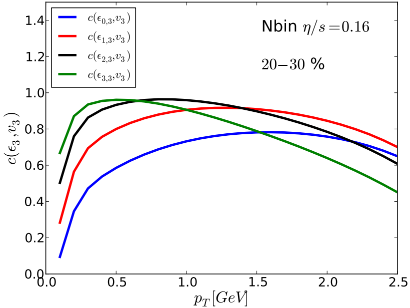
- $v_2(p_T)$: not really.
- $v_3(p_T)$: kind of, but rather weak difference

$\longrightarrow$ No straightforward access to r-dependence of initial density profiles

$c(v_2, e_2)$:



$c(v_3, e_3)$:



Bessel-Fourier expansion?

v_n distributions

v_n distributions: ATLAS measurement. G. Aad *et al.* [ATLAS Collaboration], arXiv:1305.2942 [hep-ex]

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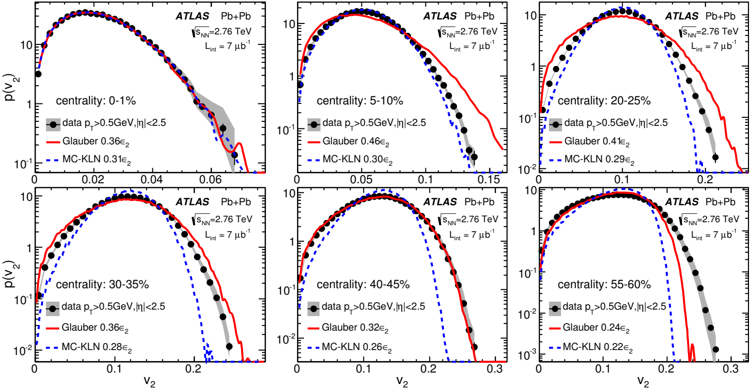
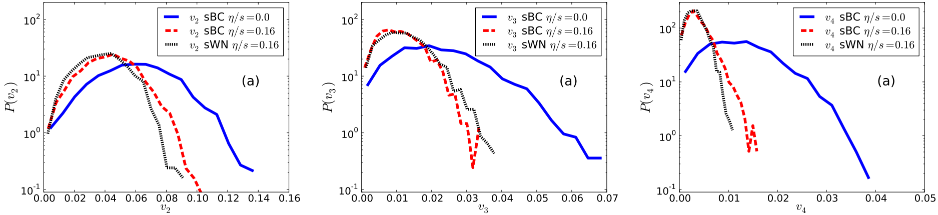


v_n distributions

- Full description of heavy-ion collisions should also get distributions of v_n correct (not only event averaged value)
- Each case gives different probability distribution

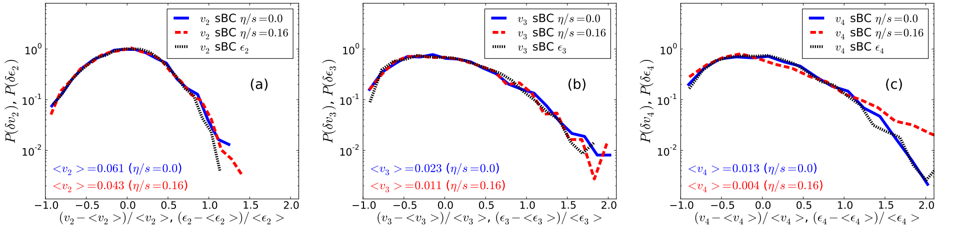


v_n distibutions $(20 - 30\ \%$ centrality$)$

Each type of initial state and η/s gives different distribution of v_n's, but if we introduce scaled variable

$$\delta v_n = \frac{v_n - \langle v_n \rangle_{\rm ev}}{\langle v_n \rangle_{\rm ev}}, \qquad \text{and} \qquad \delta \epsilon_n = \frac{\epsilon_n - \langle \epsilon_n \rangle_{\rm ev}}{\langle \epsilon_n \rangle_{\rm ev}}.$$

Then...

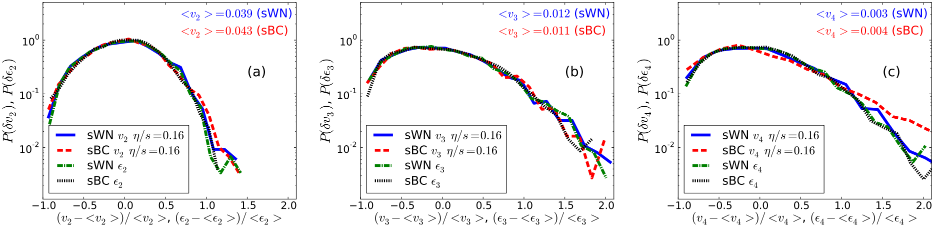


- **Probability distributions of δv_2 and δv_3 independent of hydrodynamic evolution, and follow the corresponding δe_n distribution.**

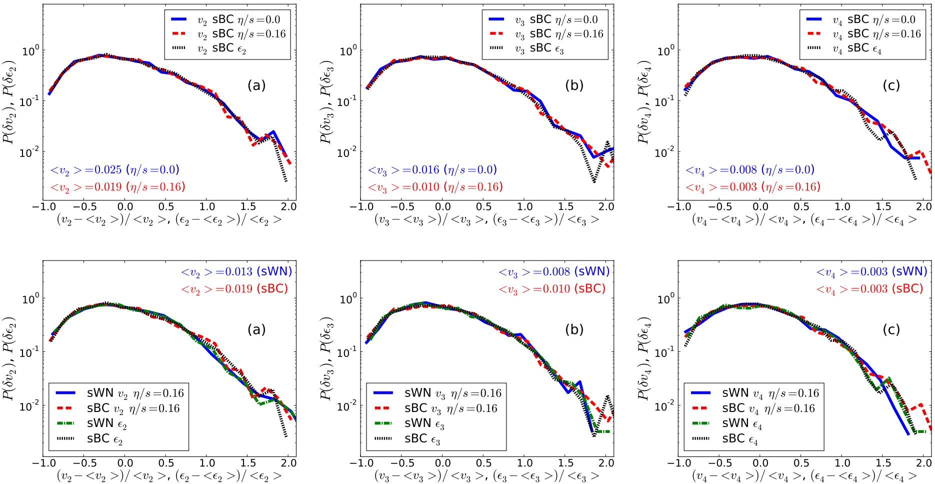
HN, Denicol, Holopainen and Huovinen, Phys. Rev. C 87, 054901 (2013), arXiv:1212.1008 [nucl-th]

v_n distibutions (20 – 30 % centrality)

Same result if we change between the Glauber variants (sBC or sWN):



v_n distibutions (central collisions)



v_2 distibutions

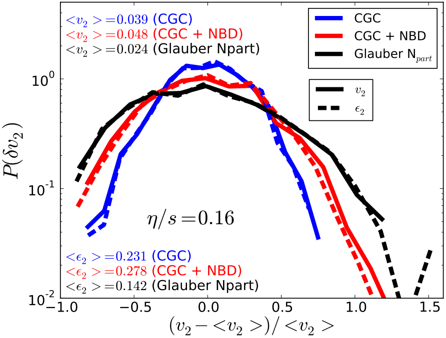
$$\mathcal{P}(\delta v_2) = \mathcal{P}(\delta \epsilon_2)$$
$$\mathcal{P}(\delta v_3) = \mathcal{P}(\delta \epsilon_3)$$

(probes initial condition directly)

and what is this good for...

v_n distributions: MC-KLN (CGC) vs MC-Glauber

Two different Glauber limits give the same distribution, but ...



- For example different variations of MC-KLN model give clearly different δv_2 distributions

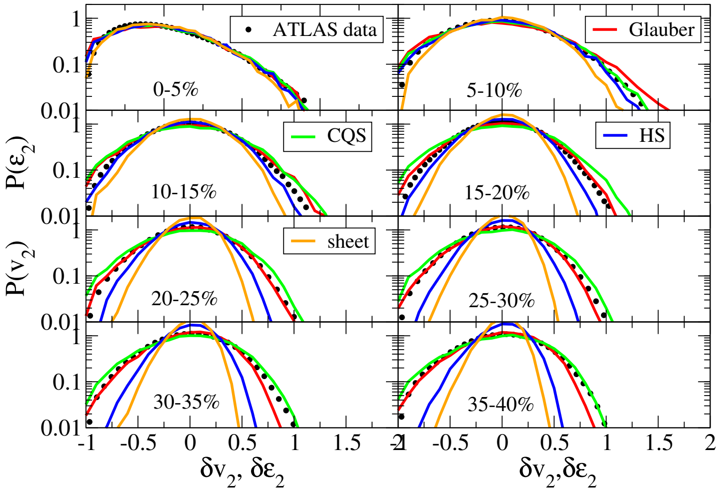
- **δv_2 distributions can distinguish between the initial state models (without doing a single hydrodynamic calculation)**

Initial conditions from A. Dumitru

Can we save the Glauber model...

v_n distibutions: Tuning MC-Glauber model

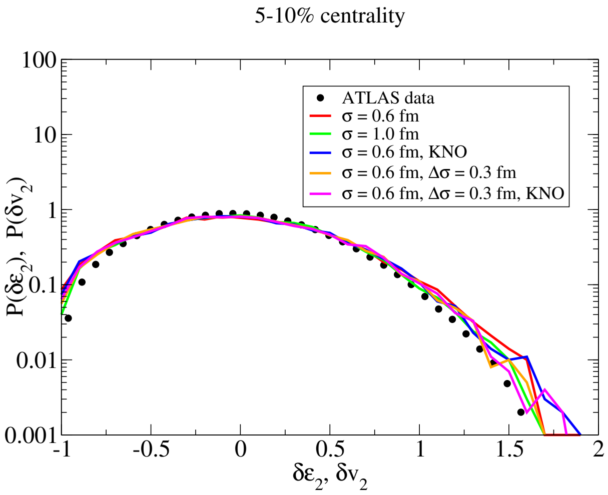
Different distributions of nucleons (or quarks): CQS = constitutient quark scaling, HS = hard sphere



Thorsten Renk and HN

v_n distibutions: Tuning MC-Glauber model

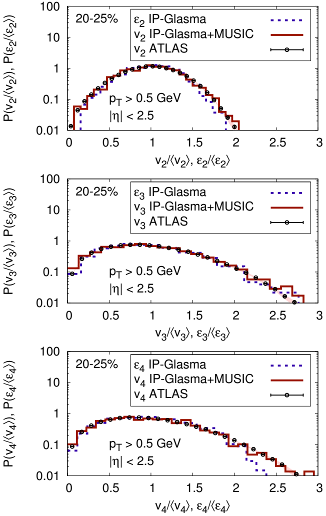
Change the width of the Gaussian peaks, multiplicity fluctuations (normalization),. . .



Thorsten Renk and HN

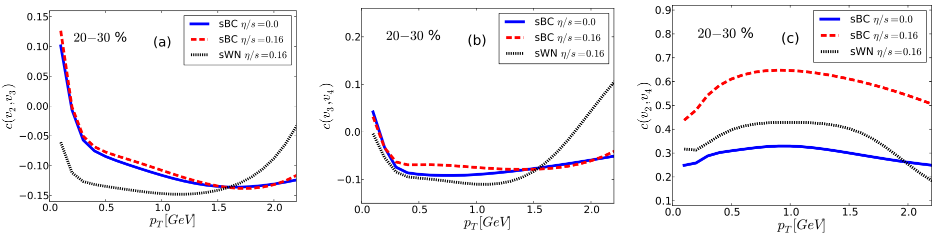
v_n distibutions: IP-Glasma

C. Gale, S. Jeon, B. Schenke, P. Tribedy and R. Venugopalan, Phys. Rev. Lett. **110**, 012302 (2013)



(v_i, v_j) correlations

$c(v_2, v_3)$ $c(v_3, v_4)$ $c(v_2, v_4)$



	$c(\epsilon_2, \epsilon_3)$	$c(v_2, v_3)$	$c(\epsilon_2, \epsilon_4)$	$c(v_2, v_4)$	$c(\epsilon_3, \epsilon_4)$	$c(v_3, v_4)$
sBC $\eta/s = 0.0$	-0.09	-0.11	0.26	0.32	-0.03	-0.11
sBC $\eta/s = 0.16$	-0.09	-0.11	0.25	0.63	-0.03	-0.09
sWN $\eta/s = 0.16$	-0.15	-0.14	-0.04	0.42	0.03	-0.11

Conclusions

- v_2 and v_3 linearly correlated to corresponding eccentricities (v_2 event-by-event)
- Correlation between v_4 and e_4 depends strongly on the details of fluid dynamics
- Distributions $P(\delta v_2)$ follow directly from the initial conditions (details of the fluid dynamical evolution do not matter)
- Clear constraints for the initial condition (without doing a single hydro run)
- (v_i, v_j) correlations provide additional information, but not as easy as the distributions