Introduction - generalized parton distributions, parton - parton correlations

MPI in pp scattering - parton parton correlations are necessary

MPI in pA scattering: forward dipion production

“MPI - antishadowing effect”

Small x kinematics
Important characteristic of high energy collisions is the impact parameter of collision. Well defined since angular momentum is conserved and $L = bp$

Different intensity of interactions for small and large impact parameters

Peripheral pp collisions

Central pp collisions

Small b ➞ large overlap of partons

Large probability of multiparton, soft/hard interactions

Using realistic transverse parton distributions is critical for genuine understanding of final states (underlying events,...) in pp though it does not enter into cross sections of inclusive dijet & inclusive MPI
Geometry of pp collision with production of dijet in the transverse plane

Diagonal Generalized Parton distribution -

For hard collision

\[ \vec{\rho}_1 + \vec{b} - \vec{\rho}_2 \propto \frac{1}{p_{tjet}} \sim 0 \]

\[
\sigma_h \propto \int d^2b d^2 \rho_1 d^2 \rho_2 \delta(\rho_1 + b - \rho_2) f_1(x_1, \rho_1) f_2(x_2, \rho_2) \sigma_{2\to2} \\
= \int d^2 \rho_1 d^2 \rho_2 f_1(x_1, \rho_1) f_2(x_2, \rho_2) \sigma_{2\to2} = f_1(x_1) f_2(x_2) \sigma_{2\to2}
\]

For inclusive cross section at high virtuality transverse structure does not matter - convolution of parton densities

\[ \downarrow \]

However critical for understanding global structure of inelastic events
Two pairs of partons can collide in a single collision

Naive geometric picture - two independent parton - parton collisions --- rate depends on the nucleon size only. However this assumes lack of parton - parton correlations. Pairs collide at relative transverse distance $\sim 0.5$ fm.
If pQCD works at LHC at $p_t > 4 \text{ GeV/c}$ - a generic inelastic event should contain many minidijets:

$$\text{jet multiplicity} = \frac{\text{(inclusive jet } \sigma)}{\sigma_{\text{inel (nondiffr)}} (pp)}$$

10 dijets with $p_t > 4 \text{ GeV/c}$ per nondiffractive event !!!
Understanding of MPI is necessary for realistic modeling of pp collisions. **Nuclei - higher local transverse density** → enhancement of MPI in pA collisions (MS & Treleani 95 &02) → are MPI even more important in pA?

**Key question -- how strongly are the parton - parton correlations**

\[ f(x_1, x_2, \vec{\rho}_1 - \vec{\rho}_2) = \int d^2\rho_1 d^2\rho_2 \delta(\vec{\rho}_1 - \vec{\rho}_2) f(x_1, \vec{\rho}_1) f(x_2, \vec{\rho}_2) \]

Assumed in all MC models
For gluons transverse spread of partons can be determined from analysis of onium production:

\[ A(\gamma^* + p \to "Onium" + p) \propto G(x_1, x_1 - x, t) \]

\[ G(x, x, t) = \int d^2 \rho e^{-i \vec{\Delta}_\perp \cdot \vec{\rho}} G(x, \rho) \]

transverse spatial distribution of gluons (Q dependent)

\[ G(x, t, Q_0^2) = 1/(m_g^2 - t)^2, m_g^2(x = 0.01) \sim 1.1 GeV^2 \]

transverse size < e.m. size

\[ \int d^2 \rho G(x, \rho) = G(x) \]

total gluon density
Experimentally one measures the ratio

$$\frac{d\sigma(p+p\to jet_1+jet_2+jet_3+\gamma)}{d\Omega_{1,2,3,4}} \cdot \frac{d\sigma(p+p\to jet_3+\gamma)}{d\Omega_{3,4}} = \frac{f(x_1, x_3)f(x_2, x_4)}{\sigma_{eff}f(x_1)f(x_2)f(x_3)f(x_4)}$$

where $f(x_1, x_3), f(x_2, x_4)$ are longitudinal light-cone double parton densities and $\sigma_{eff}$ is the "transverse correlation area". One selects kinematics where $2 \to 4$ (LT two partons into four partons) contribution is small.

CDF observed the effect in a restricted $x$-range: two balanced jets, and jet + photon and found

$$\sigma_{eff} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$$

No dependence of $\sigma_{eff}$ on $x_i$ was observed.

A naive expectation (based on $r_N=0.8 \text{ fm}$) is $\sigma_{eff} \sim 55 \text{ mb}$ indicating high degree of correlations between partons in the nucleon in the transverse plane - next few more technical slides

Similar results from D0.
Challenge to explain the data since in independent parton approximation

\[ \sigma_{\text{eff}} = \frac{28\pi}{m_g^2} \sim 32 \text{ mb.} \]

MPI rate a factor of two smaller than experiment !!!

LF, MS, Weiss 03
To reproduce $\sigma_{\text{eff}}$ in the independent parton approximation one needs $m_g^2 \sim 2\text{GeV}^2$ (PYTHIA).

Realistic model of MPI should include a factor $\sim 2$ contribution of correlations.

Area occupied by gluons is at least a factor of two smaller than experiment.

$m_g^2 \sim 2\text{ GeV}^2$ leads to $t$-dependence which is too weak to reproduce $J/\psi$ exclusive photoproduction.
Blok, Dokshitzer, LF, MS (BDFS) 11-12 derived geometric results from the first principles and developed pQCD theory of MPI. We also discovered new pQCD mechanism of MPI due to pQCD evolution.

\[
\Delta \text{- transverse disbalance between in and out partons}
\]

\[
d\sigma_4 = \int \frac{d^2 \Delta}{(2\pi)^2} \int dx_1 \int dx_2 \int dx_3 \int dx_4 \times D_a(x_1, x_2, p_{1}^2, p_{2}^2, \Delta) D_b(x_3, x_4, p_{1}^2, p_{2}^2, -\Delta) \times \frac{d\sigma^{13}}{dt_1} \frac{d\sigma^{24}}{dt_2} dt_1 dt_2.
\]

D’s are double generalized parton distributions.
\[ \frac{1}{\sigma_{eff}} = \int \frac{d^2 \Delta}{(2\pi)^2} \frac{D_a(x_1, x_2, -\Delta)D_b(x_3, x_4, \Delta)}{D_a(x_1)D_a(x_2)D_b(x_3)D_b(x_4)}, \]

Independent particle approximation which could be reasonable for small \( x_1, x_2 \)

\[ D(x_1, x_2, p_1^2, p_2^2, \Delta) = G(x_1, p_1^2, \Delta)G(x_2, p_2^2, \Delta) \]

one line calculation \[ F_{2g}(x \sim 0.03, t) = (1 - t/m_g^2)^{-2}, m_g^2 \sim 1.1 \text{GeV}^2 \]

\[ \sigma_{eff} = \frac{28\pi}{m_g^2} \sim 32 \text{ mb}. \] Practically the same number with \( \exp(\text{Bt}) \) fit.

Hence our result of 03 is pretty stable since \( F_{2g}^2(\Delta) \) is measured directly.

\[ A \text{ factor of at least 2 is missing} \text{!!!!} \]
Origin of correlations? Perturbative vs non-perturbative. Delicate interplay

Qualitatively pQCD mechanism is -- parton at $Q_0$ scale can resolve into two or more partons at higher scale with all partons localized in transverse area $1/Q_0^2$

$$D_{h^1,2}^{1,2}(x_1, x_2, q_1^2, q_2^2; \Delta) = [2] D_h(x_1, x_2, q_1^2, q_2^2; \Delta) + [1] D_h(x_1, x_2, q_1^2, q_2^2; \Delta)$$

The two contributions do not enter the physical DPI cross section in arithmetic sum, driving one even farther from the familiar factorization picture based on universal (process independent) parton distributions.

A short evolution contributions to $1 \otimes 2$
Geometry and combinatorics enhance $1 \otimes 2$ by a factor of 5

Lengthy eqs & numerical calculations. Result: if pQCD evolution starts at low $Q_0 = 0.7 \div 1$ GeV scale we explain a factor of $\sim 2$ enhancement $1/\sigma_{\text{eff}}$ for large $p_T$. $\sigma_{\text{eff}}$ grows with decrease of $p_T$ while in MC's it is assumed to be $p_T$ and process independent

$$
\sigma_{\text{eff}} = \frac{28\pi}{m_g^2} \cdot \frac{1}{1 + R} \approx \frac{32 \text{ mb}}{1 + R}
$$

$$
R \equiv \frac{\sigma_{1\otimes 2}}{\sigma_{2\otimes 2}} \sim 1
$$
MPI in nuclei: probing parton correlations in nucleons and nuclei
+ implications for minijets

\[ \sigma = \sigma_1 \cdot A + \sigma_2 \]

\[ R \equiv \frac{\sigma_2}{\sigma_1 \cdot A} \approx \left( \frac{A-1}{A^2} \right) \cdot \sigma_{\text{eff}} \int T^2(b) \, d^2b \approx 0.68 \cdot \left( \frac{A}{12} \right)^{0.39} \]

\[ T(b) = \int_{-\infty}^{\infty} dz \rho_A(z, b), \int T(b) \, d^2b = A. \]

"Antishadowing effect": For A=200, and \( \sigma_{\text{eff}} = 14 \) mb \[ \frac{\sigma_{pA}}{A \sigma_{pp}} \approx 3 \]
linear in \( \sigma_{\text{eff}} \) !!

Measurement of R allows to separate longitudinal and transverse correlations of partons as it measures \( f(x_1, x_2)/f(x_1)f(x_2) \)
First observation of MPI in collisions with nuclei: \( d\text{Au} \rightarrow 2\pi^0 \) forward +X

STAR, RHIC

\begin{align*}
\text{STAR measured} & \\
pp \rightarrow & \pi^0 + \pi^0 + X \\
d-Au \rightarrow & \pi^0 + \pi^0 + X \\
\eta_{1,2} \leq 4 \ (x_F \leq 0.5), \ p_T > 1.5 \text{ GeV/c}
\end{align*}

MS & Vogelsang 2010

Trigger for two forward pions selects large \( x_q \) larger than the single pion trigger (for which \( x \) is also quite large)

In forward kinematics it is possible to observe MPI using just two particles (not 4 jets)!
\[
\frac{d^4 \sigma_{\text{double}}}{dp_{T,1} d\eta_1 dp_{T,2} d\eta_2} = \frac{1}{\pi R_{\text{int}}^2} \sum_{a \leq b \leq c} \int dx_a dx_b dz_c dx_{a'} dx_{b'} dz_{c'} f_{a a'}^p(x_a, x_{a'}) f_b^p(x_b) f_{b'}^p(x_{b'}) \\
\times \frac{d^2 \hat{\sigma}^{ab \rightarrow c X}}{dp_{T,1} d\eta_1} \frac{d^2 \hat{\sigma}^{a'b' \rightarrow c' X'}}{dp_{T,2} d\eta_2} D_{c}^{\pi^0}(z_c) D_{c'}^{\pi^0}(z_{c'}). 
\]

\[
f_{qq'}(x_q, x_{q'}) = \frac{1}{2} \left[ f_q^p(x_q) \times \phi \left( \frac{x_{q'}}{x_q} \right) + (q \leftrightarrow q') \right] 
\]

\[
\phi(\xi) = \frac{c}{\sqrt{\xi}} (1 - \xi)^n 
\]
We estimated: pedestal/away peak ~ 2; consistent with the data.
CHECK: much larger pedestal in dA

Accounting for effecting energy losses effect and LT gluon shadowing reduces \((4\rightarrow4)/(2\rightarrow2)\) ratio - still \(\Delta\varphi\) independent pedestal in dA is \(2.5 \div 4\) times larger in pp. We also find suppression of \(\Delta\varphi = 180^\circ\) peak by a factor ~ four

Black curve is the pp data peak above pedestal for \(\varphi \sim \pi\) scaled down by a factor of 4
pQCD theory of MPI in pA

MS & Treleani treatment of pA was based on partonic model. In QCD in addition to 4 ⊗ 4 there is an important 3 ⊗ 4 contribution - need to perform a new analysis, to calculate GPD of nuclei, estimate uncertainties of the nuclear effects.

Summary

Impulse approximation - two partons (or one parton in the case of 3 ⊗ 4) are taken from the same nucleon. For x > 0.03 when no shadowing effects are

\[ G_A^{1N}(x_1, x_2, \Delta) = A G_N^{1N}(x_1, x_2, \Delta) \left( 1 + O \left( \int (\alpha - 1)^2 \rho_A^N(\alpha, p_t) \frac{d\alpha}{\alpha} d^2 p_t \right) \right) \]

light-cone nucleon density matrix
Double nucleon interaction: diagonal case

\[
G_{2N}^A(x_1, x_2, \tilde{\Delta}) = A(A-1) \int \frac{1}{\alpha_1 \alpha_2} \prod_{i=1}^{i=A} \frac{d\alpha_i d^2p_{ti}}{\alpha_i} \delta \left( \sum_i \alpha_i - A \right) \delta^{(2)} \left( \sum_i p_{ti} \right) \psi_A^* (\alpha_1, \alpha_2, p_{t1}, \ldots) \psi_A (\alpha_1, \alpha_2, p_{t1} + \tilde{\Delta}, p_{t2} - \tilde{\Delta}, \ldots) G_N (x_1/\alpha_1, |\tilde{\Delta}|) G_N (x_2/\alpha_2, |\tilde{\Delta}|).
\]
\[
G_{A}^{2N}(x_1, x_2, \Delta) = A(A - 1)G_N(x_1, |\Delta|)G_N(x_2, |\Delta|) F_A^{\text{double}}(\Delta, -\Delta),
\]
\[
F_A^{\text{double}}(\Delta, -\Delta) = \int \prod_{i=1}^{i=A} \frac{d\alpha_i d^2p_i}{\alpha_i} \delta \left( \sum \alpha_i - A \right) \delta^{(2)} \left( \sum p_i \right) \psi_A^*(\alpha_1, \alpha_2, p_{t1}, p_{t2}, \ldots) \times \psi_A(\alpha_1, \alpha_2, p_{t1} + \Delta, p_{t2} - \Delta, \ldots).
\]

Here, \( F_A^{\text{double}}(\Delta,-\Delta) \) is the double nucleon nuclear form factor which is the same as in the Glauber model.

\[
F_A^{\text{NR double}}(\Delta, -\Delta) = \int \left( \prod_{i=1}^{i=A} d^3p_i \right) \psi_A^*(p_1, p_2, \ldots) \psi_A(p_1 + \Delta, p_2 - \Delta, p_3, \ldots) \delta^{(3)} \left( \sum_{i=1}^{A} p_i \right)
\]
\[
F_A^{\text{double}}(\Delta, -\Delta) \approx \left| \int d^3r \frac{1}{A} \rho_A(r) \exp \left[ i\Delta \cdot \vec{r} \right] \right|^2 = F_A(\Delta)^2
\]
\[
F_A^{\text{double}}(\Delta, -\Delta) \approx \left| \int d^3r \frac{1}{A} \rho_A(r) \exp \left[ i\Delta \cdot \vec{r} \right] \right|^2 = F_A(\Delta)^2 \approx \exp \left[ -\frac{1}{3} \Delta^2 R_A^2 \right]
\]

Derived expressions allow very compact calculations both for heavy and light nuclei.
Since \( <\Delta^2> \) \( r_N^2 \ll 1 \)

\[
G_A^{2N}(x_1, x_2, \Delta) \rightarrow f_N(x_1) f_N(x_2) F_A^{\text{double}}(\Delta, -\Delta)
\]

\[
\frac{\sigma_4(x'_1, x'_2, x_1, x_2)}{dT_1 dT_2} = \frac{f_p(x'_1, x'_2)}{f_p(x'_1) f_p(x'_2)} \frac{d\sigma_{\text{jet}}(x'_1, x_1)}{dT_1} \frac{d\sigma_{\text{jet}}(x'_2, x_2)}{dT_2} \frac{(A - 1)}{A} \int T^2(b) d^2 b \propto A^{4/3}
\]

measures longitudinal correlations

\(~ 1.2 \) in pQCD calculation of BDFS for 4 jets
Other contributions, accuracy of approximations

Schematic presentation of an interference contribution to the nucleus $^2$GPD in which the two patrons of momentum fractions $x_1$ and $x_2$ are drawn from two different nucleons in the amplitude, but in which the assignment between partons and active nucleons is swapped in the complex conjugate amplitude.

**issue emphasized by Treleani and Calucci for light nuclei**

**Our conclusions:**

(a) Longitudinal form factor kills interference for $|x_1-x_2| > 0.03$:

Suppression factor $\exp\left(- (x_1 - x_2)^2 m_N^2 \cdot R_A^2 / 3\right)$

(b) If $|x_1 - x_2| < 0.03$, $x_1 > 0.05$, finite loffe time allows interference only between nearby nucleons -- no $A^{4/3}$ term. Probably the same scale EMC effect for these $x$.

(c) For resolution scale $Q$ - all radiated partons have to be included in the exchange - otherwise very strong suppression
In summary, working in the independent nucleon approximation and neglecting the nucleon size compared to the nuclear radius, leading to the enhancement factor for the discussed cross section as compared to the impulse approximation which is expected to be small. In the case considered in (32), this correction factor will be small. In the case considered in (32), this correction factor will be small.

This terms has the same parametric integral in nuclear radius as a sum of impulse approximation, whereas this combinatorial factor 2 is obviously absent in double nucleon scattering term. This was estimated to lead to an additional contribution, if not negligible, will correct our main result (26), It thus implies that the momentum transfer $Q^2$ is significantly smaller in pA collisions considered here, the additional contribution in $1/Q^2$ integrals in (16) and (18) are almost the same and they become identical for large $Q^2$.

Interference of interactions with one and two nucleons

Subdiagram for $x_1$ parton is = first diagrams for nuclear shadowing in the theory of leading twist shadowing (Guzey, LF, MS 98 -12)
Prediction of the LT theory of nuclear shadowing based on factorization theorem for diffraction and AGK

Strong reduction of nuclear shadowing at fixed $x$ due to the DGLAP flow of partons from larger $x$

Spatial dependence of nuclear modification of gluon generalized pdfs in Ca and Pb

\[ R^g(x,Q^2) \]

\[ Q^2 = 4 \text{ GeV}^2 \]
Strong suppression of coherent $J/\psi$ production observed by ALICE (next talk) confirms our prediction of significant gluon shadowing on the $Q^2 \sim 3 \text{ GeV}^2$

$$S_{Pb} = \left[ \frac{\sigma(\gamma A \to J/\psi + A)}{\sigma_{\text{imp.approx.}}(\gamma A \to J/\psi + A)} \right]^{1/2} = \frac{g_A(x, Q^2)}{g_N(x, Q^2)}$$

Shadowing modifies impulse approx term by \( R_{\text{imp}} = R^g(x_1, Q_1^2) \ R^g(x_2, Q_2^2) \)

Double scattering term by

\[
R_2 = \frac{\int d^2 b [R^g(x_1, b) \cdot R^g(x_2, b) T(b)^2]}{\int d^2 b [R^g(x_1) \cdot T(b)] \int d^2 b [R^g(x_2) \cdot T(b)]}
\]

Suppression of “double” scattering term relative to impulse term due to shadowing effects is modest but somewhat larger when both \( x \)’s are small.
Conclusions

- p QCD leads to a strong growth of parton - parton correlation contribution to MPI

- In the 4 jet LHC kinematics MPI vs LT 2→ 4 processes are enhanced in pA by a factor of 3 as compared to pp case.
  
  \[ \text{Will allow to test MPI analyses done of pp done at LHC (D’Enterria)} \]

- Nuclear shadowing strongly reduces minijet MPI in pA (a factor of \( \sim 3 \) for central pA with \( x_1 \sim x_2 \sim 10^{-3} \) and \( p_t = 3 \text{ GeV/c} \)) --- D - mesons at LHCb?

- dA forward dipion production - strong evidence for MPI - data explained if MPI is combined with post selection effect of effective energy losses. In the future pp and pA data may allow measurement of leading quark - quark correlations in pp scattering at RHIC
Suplementary slides
FIG. 6: Rapidity dependence of the $R$ factor for two pairs of $p_{\perp} = 50$ GeV jets produced in gluon-gluon collisions.
Wavy lines stand for diffractive hard interactions “Pomerons”, H₁’s are hard probes, colored blocks are diffractive states which include the parton involved in the hard interaction.