WHAT CAN WE LEARN ABOUT THE INITIAL STAGES FROM FLOW MEASUREMENTS?

Matthew Luzum

McGill University / Lawrence Berkeley National Laboratory

International Conference on the Initial Stages in High-Energy Nuclear Collisions
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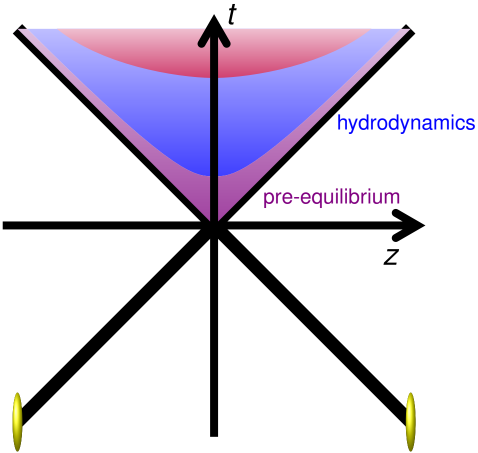
Hydrodynamic evolution



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- Depends on initial $T^{\mu\nu}(\tau = \tau_0)$

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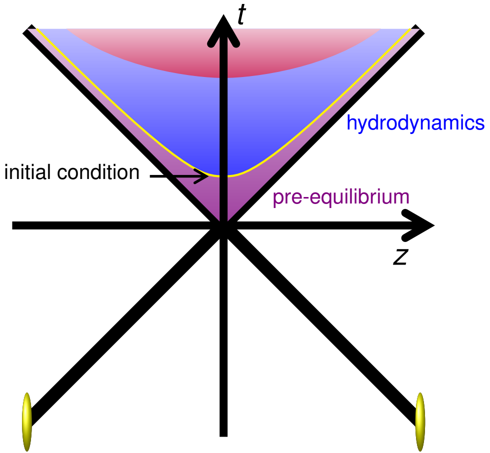

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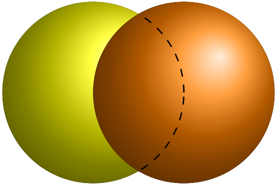
Flow

Asymmetric pressure gradients $\rightarrow$ anisotropic momentum distribution



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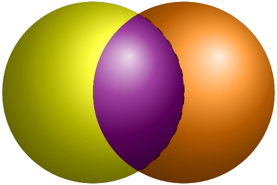

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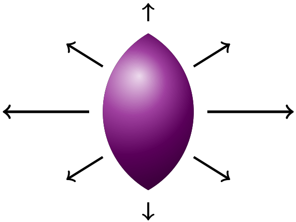

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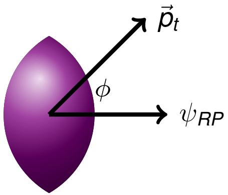
Asymmetric pressure gradients → anisotropic momentum distribution

$$\frac{2\pi}{N}\frac{dN}{d\phi} = \sum_n V_n e^{-in\phi}$$



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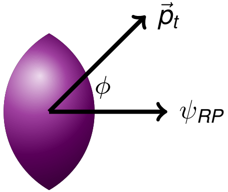

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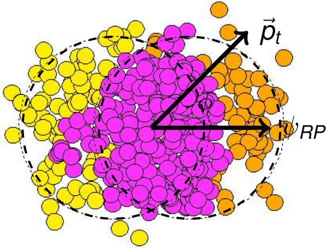
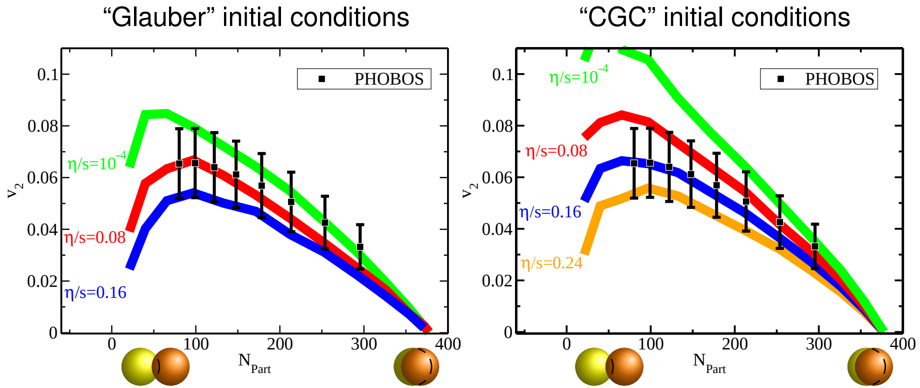
Fluctuations are important!

- Not symmetric
- Varies from event to event



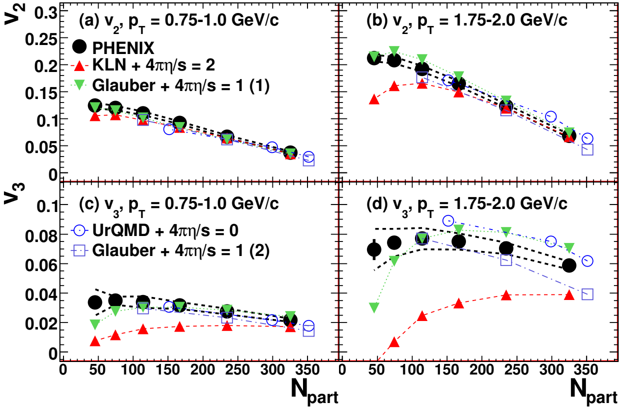
History: Elliptic Flow



(ML & Romatschke, Phys.Rev. C78 (2008) 034915)

- Depends on both medium properties (η/s) and initial conditions
- Need more information about one to constrain the other

New observables



(PHENIX, Phys.Rev.Lett. 107 (2011) 252301)

- New observables $\Longrightarrow$ more constraints
- E.g., some initial conditions incompatible with (v_2, v_3)

Constraining the initial state

Goal: Quantify direct constraints on initial state

- Usual procedure:
  1. Choose initial state model
  2. Evolve with hydro for many events
  3. Compare to data.
  4. Goto step 1
- Better: find constraints that can be checked without running hydro
- Note: in each event:

$$\frac{dN}{d\phi} = \mathcal{F}\left(T^{\mu\nu}(\mathbf{x})\right)$$

- Need to understand hydro response $\mathcal{F}$

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Cumulant expansion

Idea: *(Teaney, Yan; Phys.Rev. C83 (2011) 064904)*

- Characterize density by moments (or cumulants) of 2-D Fourier transform

$$\rho(\mathbf{k}) = \int d^2x\rho(\mathbf{x})e^{i\mathbf{k}\cdot\mathbf{x}}$$

- (Small m = small power of k in Taylor series)
- Can write complete hydro response as set of functions

$$v_n e^{in\Psi_n} = f(W_{m,n})$$

- Can use other bases (e.g., Bessel expansion of k modes)
  (Coleman-Smith, Petersen, Wolpert; J.Phys. G40 (2013) 095103
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CUMULANT EXPANSION: TAYLOR SERIES IN $\varepsilon_{m,n}$

- Must make quantities with correct symmetries out of $W_{m,n} \sim \langle r^m e^{in\phi} \rangle$
- If anisotropies are small, can arrange in Taylor series. E.g., to first order:

$$V_n \equiv v_n e^{in\Psi_n} = \sum_{p=0}^{\infty} C_{n+2p,n} W_{n+2,p}$$

- If hydro is sensitive to large scale structure, first terms are most important (smaller powers of k in Fourier transform).
- If single term sufficient:

$$v_n e^{in\Psi_n} = -C \frac{\langle r^n e^{in\phi} \rangle}{\langle r^n \rangle} \equiv C \, \varepsilon_n e^{in\Phi_n}$$

- (ε_n = lowest k mode of initial density)

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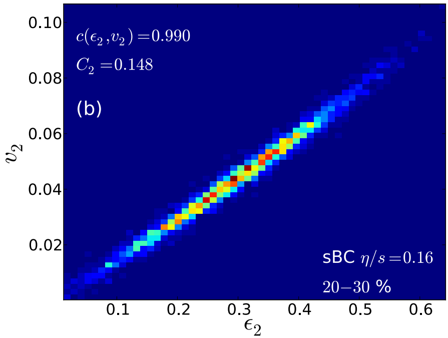
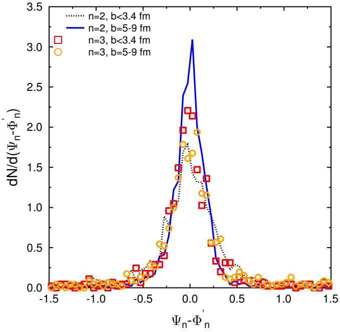
Relation works well for $n \leq 3$:

$$v_n e^{in\Psi_n} = C\, \varepsilon_n e^{in\Phi_n}$$





(Niemi, Denicol, Holopainen, Huovinen; arXiv:1212.1008)

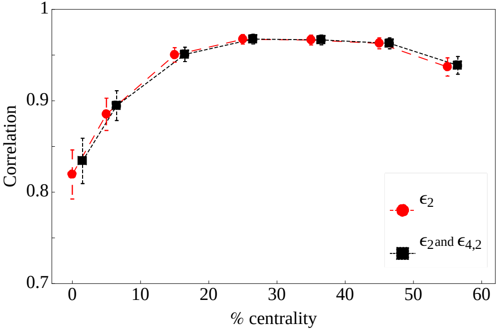
(Petersen, Qin, Bass, Muller, Phys. Rev. C 82, 041901 (2010))

Hydro response

- Hydro insensitive to higher cumulants: including several terms,

$$v_2 e^{i2\Psi_2} = C\langle r^2 e^{i2\Phi_2}\rangle + C'\langle r^4 e^{i2\Phi_2}\rangle + \dots$$

  is no better than just lowest term
- Nonlinear terms are important for $n \geq 4$



(Gardim, Grassi, ML, Ollitrault; Phys.Rev. C85 (2012) 024908)

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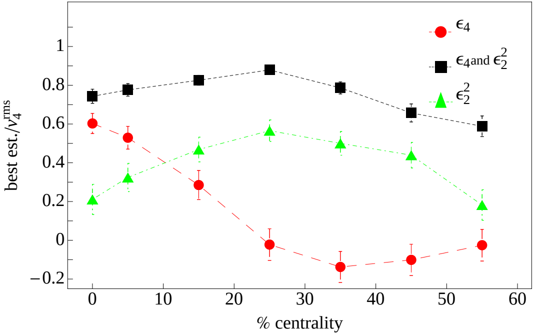

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To a good approximation:

$$\begin{aligned} v_1 e^{i\Psi_1} &= C_1\ \varepsilon_1 e^{i\Phi_1} \\ v_2 e^{i2\Psi_2} &= C_2\ \varepsilon_2 e^{i2\Phi_2} \\ v_3 e^{i3\Psi_3} &= C_3\ \varepsilon_3 e^{i3\Phi_3} \\ v_4 e^{i4\Psi_4} &= C_{4,1}\ \varepsilon_4 e^{i4\Phi_4} + C_{4,2}\varepsilon_2^2 e^{i4\Phi_2} \\ v_5 e^{i5\Psi_5} &= C_{5,1}\ \varepsilon_5 e^{i5\Phi_5} + C_{5,2}\varepsilon_2\varepsilon_3 e^{i(2\Phi_2+3\Phi_3)} \end{aligned}$$

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\end{aligned}$$

Can use these relations to derive constraints for initial state:

Hydro response

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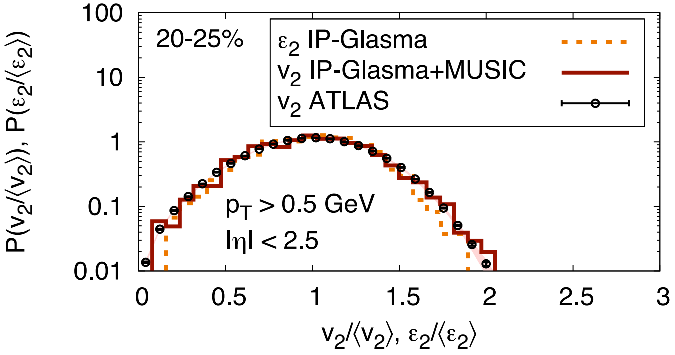


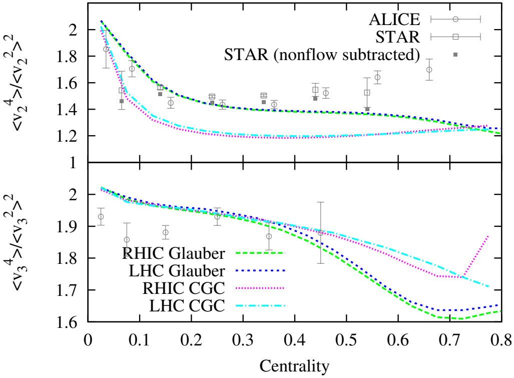
(Gale, Jeon, Schenke, Tribedy, Venugopalan, Nucl.Phys.A904-905 2013 (2013) 409c-412c)

- For $n \leq 3$, normalized v_n distribution same as ε_n distribution
- $\Longrightarrow$ can compare initial state model directly to data

Ratios of moments of v_n, Ψ_n distributions

Can take ratios to generate single numbers that can be compared to measurements. E.g.,

$$\frac{\langle v_n^4 \rangle}{\langle v_n^2 \rangle^2} \simeq \frac{\langle \varepsilon_n^4 \rangle}{\langle \varepsilon_n^2 \rangle^2}$$



(Bhalerao, ML, Ollitrault, Phys.Rev. C84 (2011) 034910

RATIOS OF MOMENTS OF v_n, Ψ_n DISTRIBUTIONS

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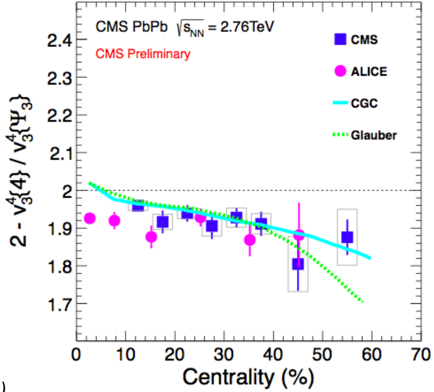
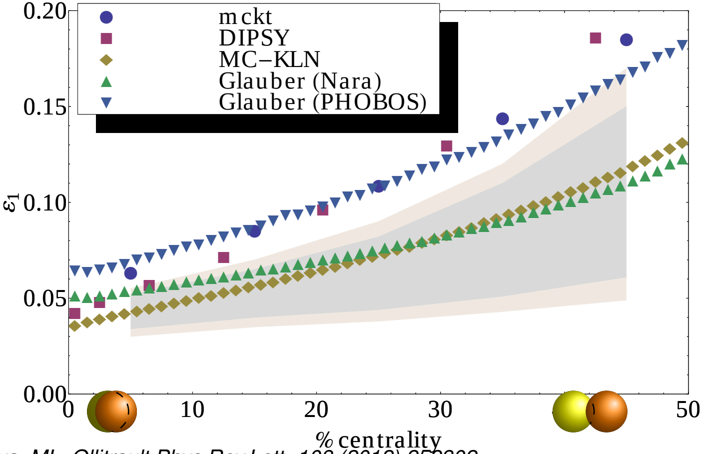


(Sanders talk on Monday)

PRECISION CONSTRAINTS: ε_1

Can also constrain magnitude of lowest moments of ε_n distribution.

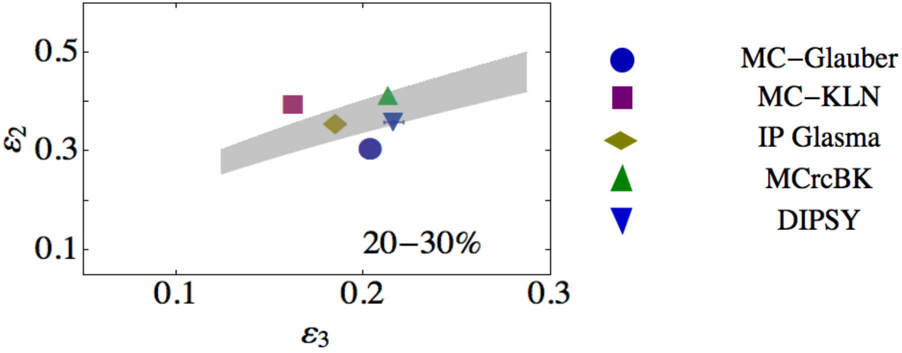
E.g., allowed values of $\sqrt{\langle \varepsilon_1^2 \rangle}$:



(Retinskaya, ML, Ollitrault Phys.Rev.Lett. 108 (2012) 252302

PRECISION CONSTRAINTS: ε_2, ε_3

Can also constrain magnitude of lowest moments of ε_n distribution.
E.g., allowed values of $\langle\varepsilon_2^2\rangle/\langle\varepsilon_3^2\rangle^{0.6}$:



(Retinskaya, ML, Ollitrault — See talk on Friday)

SUMMARY

- With the many new (and forthcoming) flow measurements, we can directly constrain theory — both medium properties and the initial state
- Eccentricities ε_n represent the lowest k mode of the Fourier transformed initial density
- The set of ε_n accurately predict flow in each event (not always linearly)
- $\Longrightarrow$ medium response effectively codified in a few coefficients
- $\Longrightarrow$ can provide simple and direct quantitative constraints that must be satisfied for any model of the initial stages

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Extra slides