A state of the art l-QCD + HRG EOS:
Full vs Partial Chemical Equilibrium

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Outline

- Motivation
- The conceptual setup
- Results

NB Work done in collaboration with M. Bluhm, P. Alba, W.M. Alberico and C. Ratti and found in arXiV:1306.6188 [hep-ph].

Motivations

The QCD Equation of State affects soft-physics observables

- *directly*, entering into the hydrodynamic equations describing the medium evolution

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{with} \quad P = P(\epsilon)$$

- *indirectly*, entering into the description of the transition from fluid to particles through the Cooper-Frey formula

$$E\frac{dN_i}{d\mathbf{p}} = \frac{g_i}{(2\pi)^3}\int_{\Sigma_{\rm fo}} p_\mu d\Sigma^\mu\, f_i(x,p),$$

$$\text{where (in the ideal case)} \quad f_i^{\rm eq}(x,p) = \frac{1}{e^{[p\cdot u(x) - \mu_i(x)]/T(x)} \pm 1}$$

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  - The formula fixes unambiguously *both the shape* of the spectrum *and* its *normalization*;
  - The particle distributions must be consistent with the ones entering into the hadronic EOS.

Hadron-Resonance Gas EOS

- A system of stable hadrons interacting through the exchange of resonances (e.g. $\pi + \pi \rightarrow \rho \rightarrow \pi + \pi$) can be described as an ideal gas of hadrons (e.g. π) and resonances (e.g. ρ)
- The partition function factorizes:

$$Z_{\mathrm{GC}} = \prod_i Z_{\mathrm{GC}}^{(i)},$$

  where i is taken to run over all hadrons/resonances up to ~ 2 GeV.
- The hadronic phase is usually described by such an EOS, where

$$P = \sum_i g_i \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{E} \frac{\mathbf{p}^2}{3} f_{\mathrm{eq}}^{(i)}(\mathbf{p}),$$

  the other quantities following from thermodynamics:

$$s = (\partial P / \partial T)_\mu \quad \text{and} \quad \epsilon = Ts - P + \mu n$$

HRG-EOS: chemical composition

At least three different scenarios can be conceived:

- Full chemical equilibrium: the multiplicity of each H/R species follows from $f_i^{\rm eq}(\mathbf{p})$. In the absence of stopping (LHC conditions!) $\langle B\rangle = \langle S\rangle = \langle I_3\rangle = 0$ and all chemical potentials vanish. A unique temperature $T_{\rm fo}$ would describe the *slope of the spectra* and the *particle abundances*;

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- Early chemical freeze-out: the multiplicity of *all* H/R species is fixed at *chemical freeze-out* ($T_{\rm chem} \approx 160$ MeV). In order to keep the chemical composition fixed until *kinetic freeze-out* for each hadron/resonance an independent chemical potential is introduced:

$$f_i(\mathbf{p}) = \frac{1}{e^{(E_p - \mu_i)/T} \pm 1}$$

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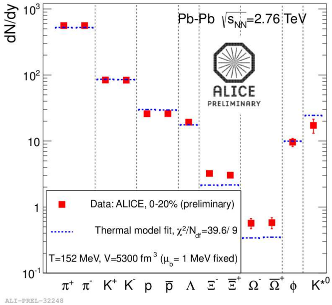
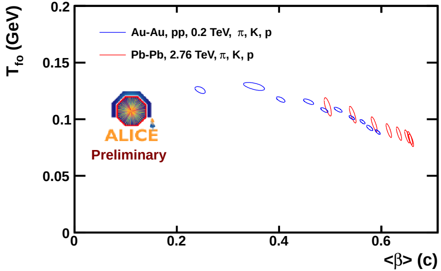
$$f_i(\mathbf{p}) = \frac{1}{e^{(E_p-\mu_i)/T}\pm 1}$$

- Partial chemical equilibrium: H/R abundances are initially fixed at $T_{\rm chem}$, but reactions mediated by short-living resonances (e.g. $\pi + p \to \Delta \to \pi + p$) are allowed.

Chemical vs kinetic freeze-out: experimental evidence





Particle yields consistent with thermal production at $T^{\rm fo}_{\rm chem} \approx 155$ MeV

Blast-wave fits of the p_T-spectra require $T^{\rm fo}_{\rm kin} \sim 100$ MeV

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$$\begin{array}{ccccc} \pi+\pi & \rightarrow & \rho & \rightarrow & \pi+\pi \\ \pi+K & \rightarrow & K^* & \rightarrow & \pi+K \\ \pi+p & \rightarrow & \Delta & \rightarrow & \pi+p \end{array}$$

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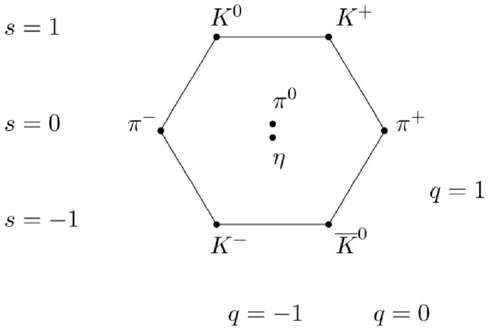
  viewed as "chemical reactions" allowed to proceed in both directions, *maintaining partial chemical equilibrium*
- The chemical potentials of all resonances can then be expressed in terms of the ones of the stable hadrons!

Stable hadrons



- Pseudoscalar meson-octet;

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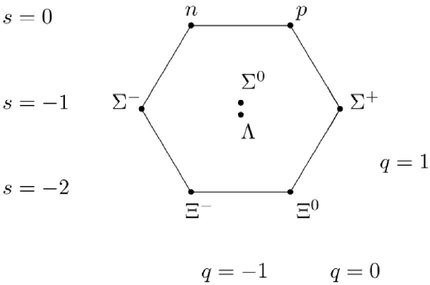

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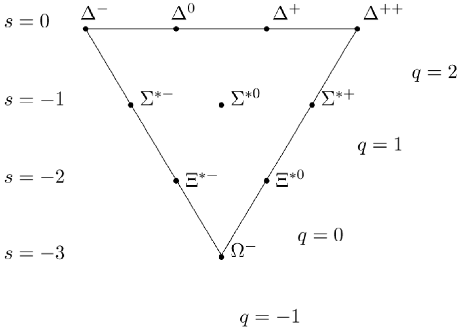

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Equilibrium relations for chemical potentials: algorithm

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- At equilibrium the Gibbs free-energy $\Phi \equiv E - TS + PV$ is stationary:

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- In the decay of P_n one has $\Delta N_n = -1 \;\rightarrow\; \Delta N_i = \sum_c \Gamma_c N_i^c$, so that

$$\mu_n = \sum_c \Gamma_c \sum_{i=1}^{n-1} N_i^c \mu_i$$

Equilibrium relations for chemical potentials: outcomes

- At the end the chemical potential of each resonance r is fixed by *8 independent chemical potentials of stable hadrons*

$$\mu_r = \sum_h \langle N_h^{(r)} \rangle \, \mu_h, \quad \text{with} \quad h = \pi, K, \eta, N, \Lambda, \Sigma, \Xi, \Omega$$

- One obtains a table like (code, mass, spin-deg, baryon-num, $\{\mu_h\}$)

2012-04-29 table_fig.dat 1

22	0.	2	0	0.	0.	0.	0.	0.	0.	0.	0.
211	0.13957	1	0	1.	0.	0.	0.	0.	0.	0.	0.
111	0.13498	1	0	1.	0.	0.	0.	0.	0.	0.	0.
-211	0.13957	1	0	1.	0.	0.	0.	0.	0.	0.	0.
321	0.49368	1	0	0.	1.	0.	0.	0.	0.	0.	0.
-321	0.49368	1	0	0.	1.	0.	0.	0.	0.	0.	0.
311	0.49765	1	0	0.	1.	0.	0.	0.	0.	0.	0.
-311	0.49765	1	0	0.	1.	0.	0.	0.	0.	0.	0.
221	0.54775	1	0	0.	0.	1.	0.	0.	0.	0.	0.
213	0.7758	3	0	2.	0.	0.	0.	0.	0.	0.	0.
113	0.7758	3	0	2.	0.	0.	0.	0.	0.	0.	0.
-213	0.7758	3	0	2.	0.	0.	0.	0.	0.	0.	0.
223	0.78259	3	0	2.803	0.	0.	0.	0.	0.	0.	0.
323	0.89166	3	0	1.	1.	0.	0.	0.	0.	0.	0.
-323	0.89166	3	0	1.	1.	0.	0.	0.	0.	0.	0.
313	0.8961	3	0	1.	1.	0.	0.	0.	0.	0.	0.
-313	0.8961	3	0	1.	1.	0.	0.	0.	0.	0.	0.
2212	0.93827	2	1	0.	0.	0.	1.	0.	0.	0.	0.
2112	0.93957	2	1	0.	0.	0.	1.	0.	0.	0.	0.
331	0.95778	1	0	1.98209	0.	0.654	0.	0.	0.	0.	0.
9010221	0.9741	1	0	1.56	0.44	0.	0.	0.	0.	0.	0.
9000211	0.9847	1	0	0.844	0.312	0.844	0.	0.	0.	0.	0.
9000111	0.9847	1	0	0.844	0.312	0.844	0.	0.	0.	0.	0.
-9000211	0.9847	1	0	0.844	0.312	0.844	0.	0.	0.	0.	0.

EOS with partial chemical freeze-out (I)

- Thermodynamic relations

$$Ts = \epsilon + P - \sum_r \mu_r n_r \qquad \text{and} \qquad dP = s\,dT + \sum_r n_r\,d\mu_r$$

r running over *all the resonances* in the cocktail.

- Entropy and particle density: $s=(\partial P/\partial T)_{\mu_r}$ and $n_r=(\partial P/\partial \mu_r)_T$
- Chemical potentials of the resonances r expressed in terms of the ones of the "stable" hadrons h:

$$\mu_r = \sum_h \langle N_h^{(r)} \rangle\, \mu_h,$$

- Density of hadrons h

$$\overline{n}_h \equiv \left(\frac{\partial P}{\partial \mu_h}\right)_T = \sum_r \left(\frac{\partial P}{\partial \mu_r}\right)_T \frac{d\mu_r}{d\mu_h} = \sum_r n_r \langle N_h^{(r)} \rangle :$$

"primary" (n_h) + "feed-down" ($\sum_{r\neq h} n_r \langle N_h^{(r)} \rangle$) contributions.

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together with the thermodynamic relations

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lead to

$$\partial_\mu[su^\mu] = -\sum_h \frac{\mu_h}{T}\partial_\mu[\overline{n}_h u^\mu]$$

In spite of partial deviations from full CE, the conservation of the "effective charges" $\overline{N}_h$ leads to the conservation of the entropy S (thanks to P. Huovinen for discussions)

EOS with PCE (III)

- ...Hence the recipe to impose partial chemical freeze-out at T_c:

$$\frac{\overline{n}_h(T,\{\mu_{h'}\})}{s(T,\{\mu_{h'}\})} = \frac{\overline{n}_h(T_c,\{\mu_{h'}=0\})}{s(T_c,\{\mu_{h'}=0\})}$$

expressing the conservation of entropy and of the "charges" $\overline{N}_h$;

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- Particle ratios are fixed at T_c:

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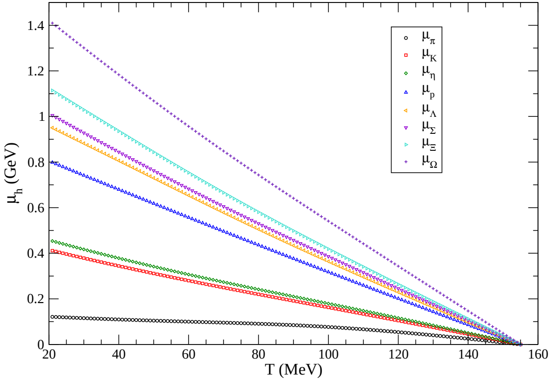
- Bibliography:

  - T. Hirano and K. Tsuda, PRC 66 (2002), 054905;
  - H. Bebie et al., NPB 378 (1992), 95;
  - R. Rapp, PRC 96 (2002), 017901;
  - P.F. Kolb and R. Rapp, PRC 67 (2003), 044903.

HRG with PCE: some results



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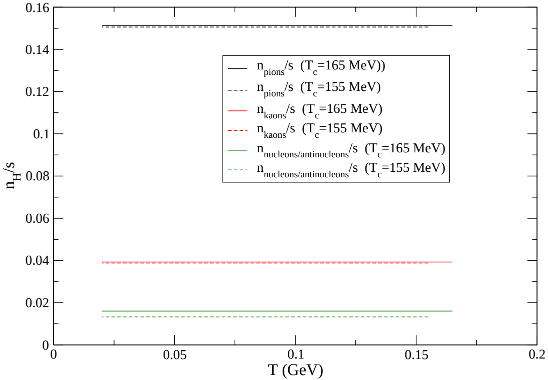

- for the chemical potentials (here $T_{\rm ch} = 155$ MeV);
- for the hadronic densities.

The full setup: l-QCD + HRG

In order to have an EOS covering the full range of temperatures of interest for the experiment and for hydro simulations...

- For high temperatures ($T > 172$ MeV) *continuum-extrapolated* lattice-QCD results with *realistic quark masses* by the *Wuppertal-Budapest* collaboration[1] were employed;

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NB in view of a generalization to the finite-density case we have treated each isospin state (e.g. π^+, π^0, π^-...) separately.

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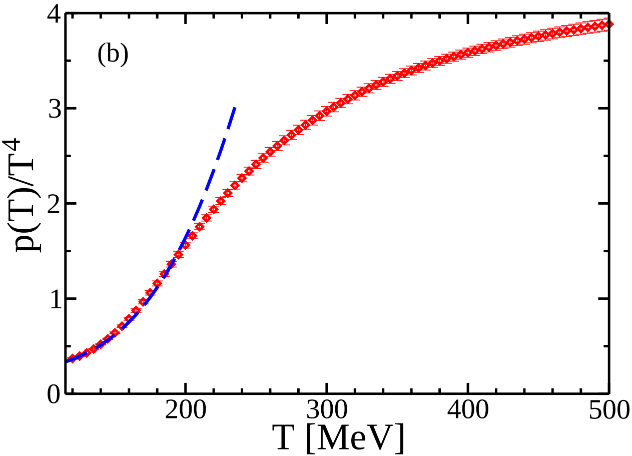

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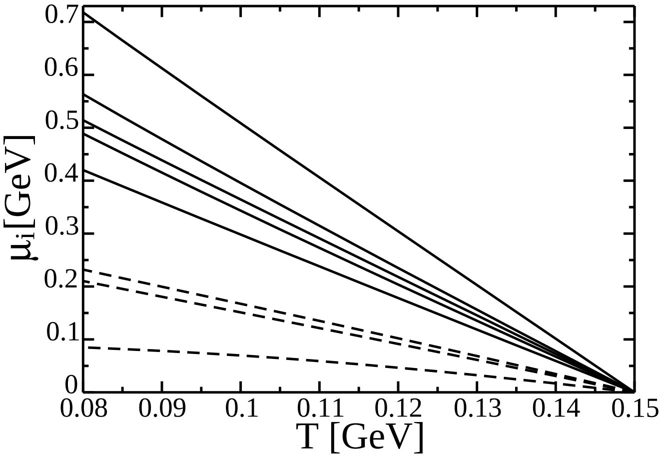

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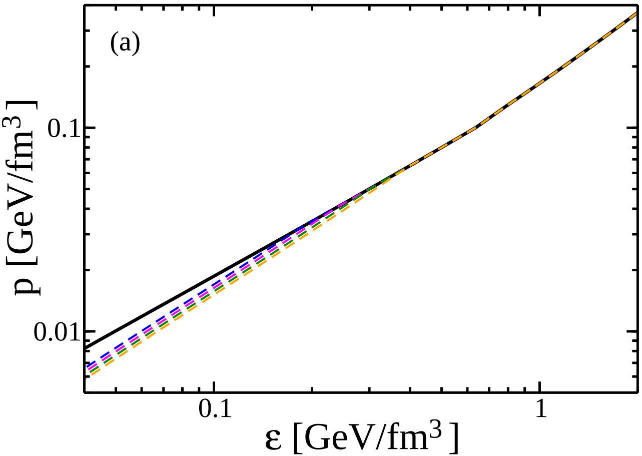

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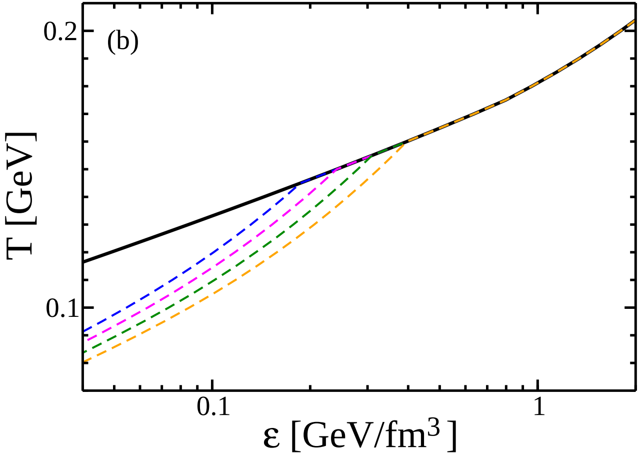

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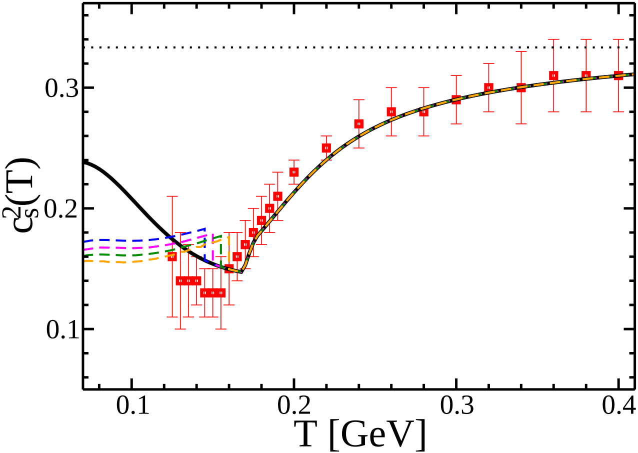

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- For the future we plan to extend our approach to the finite-density case, of interest for the low-energy scan at RHIC and for FAIR.