Bulk and Shear Viscosity Effects in Event-by-Event Relativistic Hydrrodynamics

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Outline

arXiv:1305.1981

I. WHY and HOW

WHY?

- Many works on hydro with shear viscosity and comparison with data.
  Additional difficulties: initial geometry, particle emission (δf), various formalisms, etc.
- The part played by bulk viscosity has not been so thorougly studied:
  - Monnai, Hirano, PRC80 (2009) 054906,
  - Denicol, Kodama, Koide, Mota, PRC80 (2009) 064901; JPG37 (2010) 094040,
  - Song, Heinz, PRC81 (2010) 024905,
  - Bozek, PRC81 (2010) 034909,
  - Roy, Chaudhuri, PRC85 (2012) 024909; erratum PRC85 (2012) 049902,
  - Dusling, Schafer, PRC85 (2012) 044909.
- $\longrightarrow$ Agree that $v_2(p_T)$ will be affected by bulk viscosity.
- No work on effect of bulk viscosity on higher order v_n's.
  (Above papers had smooth initial conditions.)

HOW

v-USPhydro
(viscous Ultrarelativistic Smooth Particle hydrodynamics)

Sucessor of NeXSPheRIO:

- First ($\sim$ 2000) event-by-event code for relativistic nuclear collisions (ideal fluid).
- Since 2010, various e-by-e codes have been appeared.

Description:

Modular event-by-event 2+1 hydrodynamical code that runs ideal & viscous hydro with nonzero ζ/s and η/s

- Initial conditions can easily be implemented from other sources.
- Equations of motion are solved using Smooth Particle Hydrodynamics

In progress:

- Particle decays
- 3+1

II. Effect of viscosities on the fluid expansion

Equations of Motion for bulk

Conservation of Energy and Momentum

$$D_\mu T^{\mu\nu} = 0 \tag{1}$$

The energy-moment tensor contains a bulk viscous pressure Π

$$T^{\mu\nu} = (\epsilon + p + \Pi)\, u^\mu u^\nu - (p + \Pi)\, g^{\mu\nu} \tag{2}$$

Using memory fuction method
(Denicol, Kodama, Koide, Mota, PRC75(2007)034909, PRC78(2008)034901,JPG36 (2009)035103),
Π obeys

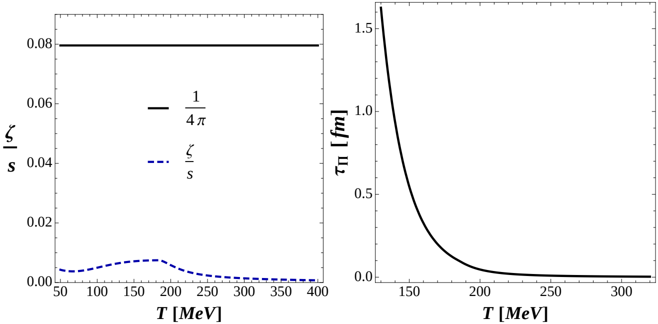
$$\tau_\Pi u^\mu D_\mu \Pi + \Pi = -(\zeta + \tau_\Pi \Pi)\, D_\mu u^\mu$$

$\Pi_{Navier-Stokes} = -\zeta D_\mu u^\mu$: it acts as a negative pressure, slowing expansion and cooling $\Rightarrow$ small effect if ζ small.

Description of Bulk Viscosity

$$\left(\frac{\zeta}{s}\right) = \frac{1}{8\pi}\left(\frac{1}{3} - c_s^2\right), \qquad \tau_\Pi = 9\,\frac{\zeta}{\epsilon - 3p}$$

Inspired by Buchel,PLB663,286(2008) and Huang, Kodama, Koide, Rischke PRC83,024906(2011)



Using alttice-based equation of state: Huovinen, Petreczky, NPA837 (2010) 26.

Conservative estimate: $\zeta/s \sim 0.1(1/4\pi)$

Equations of Motion and description of Shear Viscosity:

Energy-moment tensor

$T^{\mu\nu} = (\epsilon + p)\, u^{\mu} u^{\nu} - p\, g^{\mu\nu} + \pi^{\mu\nu}$

Equation for shear stress tensor

$\tau_\pi \Delta^{\mu\nu\lambda\rho} u^{\alpha} D_{\alpha} \pi_{\lambda\rho} + \pi^{\mu\nu} = \eta \sigma^{\mu\nu} - \tau_\pi \pi^{\mu\nu} D_{\alpha} u^{\alpha}$ (standard notations)

PRELIMINARY:

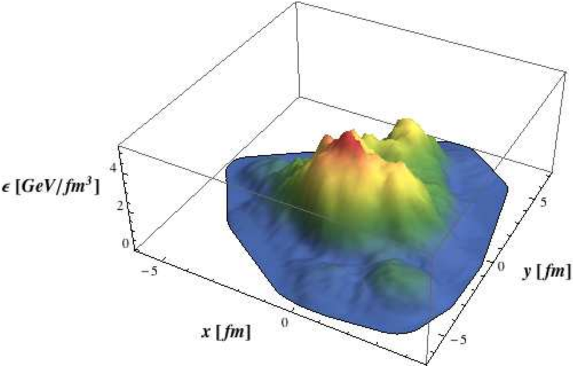
$\frac{\eta}{s} = \frac{1}{4\pi}$, $\tau_\pi = 5\, \frac{\eta}{sT}$

$\pi^{\mu\nu}_{Navier-Stokes} = \eta \sigma^{\mu\nu}$: it tends to prevent deformations of fluid cell.

Fluid expansion

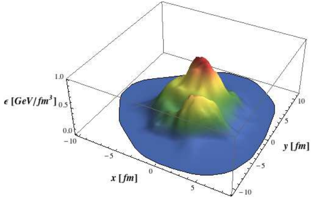
Initial Conditions:

- MC-Glauber: energy density $= cn_{coll}(\vec{r})$ (c adjusted to get midrapidity multiplicity)
- $\tau_0 = 1\,fm$ (tested)

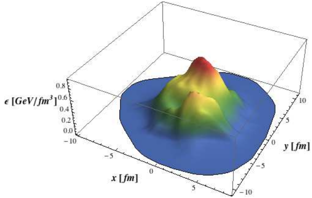


$h = 0.3, N_{SPH} \sim 3\,10^4$, nb.events/window=150.

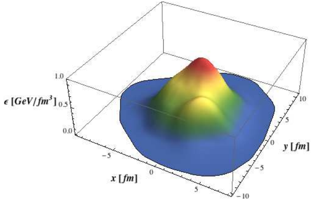
Ideal after $\tau = 5.6 fm$



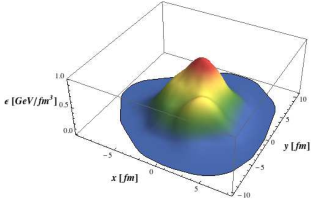
Bulk after $\tau = 5.6 fm$



Shear+Bulk after $\tau = 5.6 fm$



Shear after $\tau = 5.6 fm$



- Viscosity attenuates other forces $\rightarrow$ smearing of granularity.
- Shear dominates, bulk barely affects expansion (expected since $\zeta/s << \eta/s$).

III. Effect of viscosities at decoupling

Compute observables with Cooper-Frye formula:

Particle spectra: $E\frac{d^3N}{dp^3} = \int_{f.o.} f(x,p)p^\mu d\sigma_\mu$

$f = f_{eq} + \delta f_{shear} + \delta f_{bulk}$

Problem: compute δf.

In what follows:

Shear results: not (yet) ours

Bulk results: v-USPhydro.

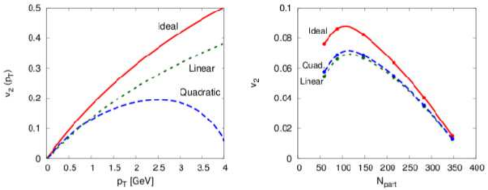
δf_{shear}

Common ansatz: $\delta f_{shear} \sim \pi_{\mu\nu} p^{\mu} p^{\nu} [(\epsilon + p) T^2]$.

Navier-Stokes limit, $\delta f_{shear} \propto (\eta/s) p^2$

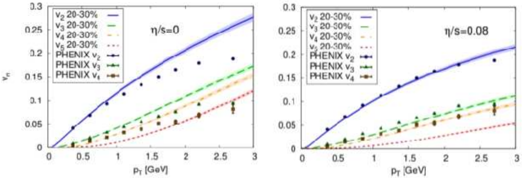
$\rightarrow$ stronger effect for larger η/s and p.

- $v_2(p_T)$: shape dominated by δf_{shear}:



Dusling, Moore, Teaney PRC81 (2010) 034907.

- $v_N(p_T)$ decreased



Schenke, Jeon, Gale PRC85 (2012) 024901

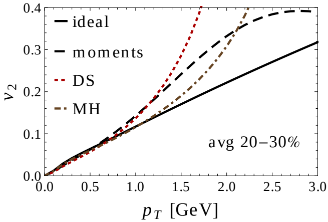
δf_{bulk}

Using method of moments as in Denicol, Niemi NPA904-905 (2013) 369c

$$\delta f_{bulk}^{(\pi)} = f_{eq} \times \Pi \times [B_0^{(\pi)} + D_0^{(\pi)} u.p + E_0^{(\pi)} (u.p)^2]$$

where

$B_0^{(\pi)} = -65.85\, fm^4$, $D_0^{(\pi)} = 171, 27\, fm^4/GeV$, $E_0^{(\pi)} = -63.05\, fm^4/GeV^2$



MH: Monnai, Hirano, PRC80 (2009) 054906

DS: Dusling, Schafer, PRC85 (2012) 044909

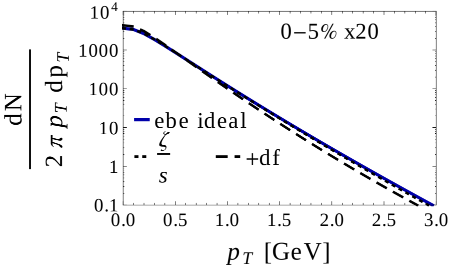
- $v_2(p_T)$: shape dominated by δf_{bulk}:
  Similar to δf_{shear}.
- $v_2(p_T)$ is enhanced relative to ideal case.
  δf_{bulk} has opposite effect to that of δf_{shear}
- Moment method leads to well-behaved $v_2(p_T)$ at high p_T.

IV. Results

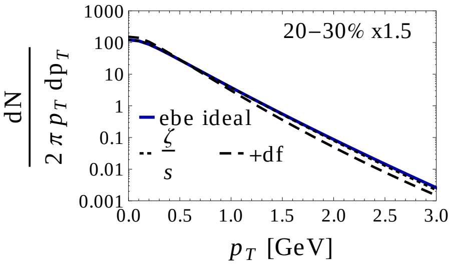
π^+ Spectrum (Direct π^+'s Only)

$T_{f.o.} = 150$ MeV

Direct $\pi^+ \approx 123$

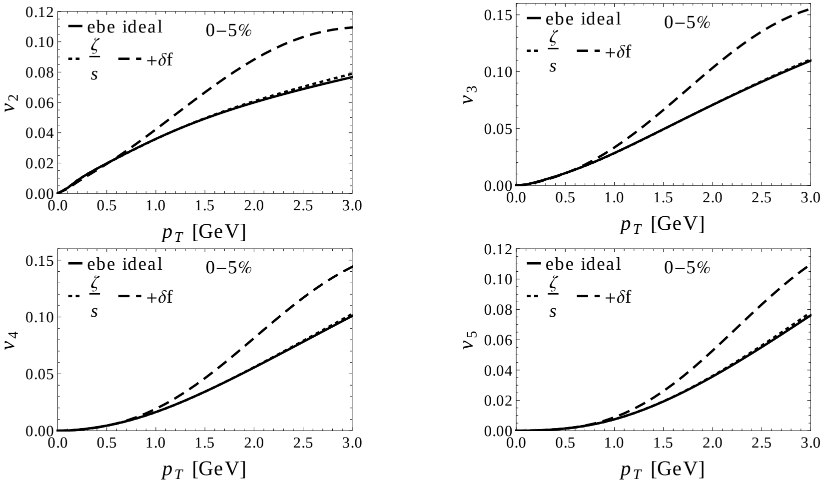


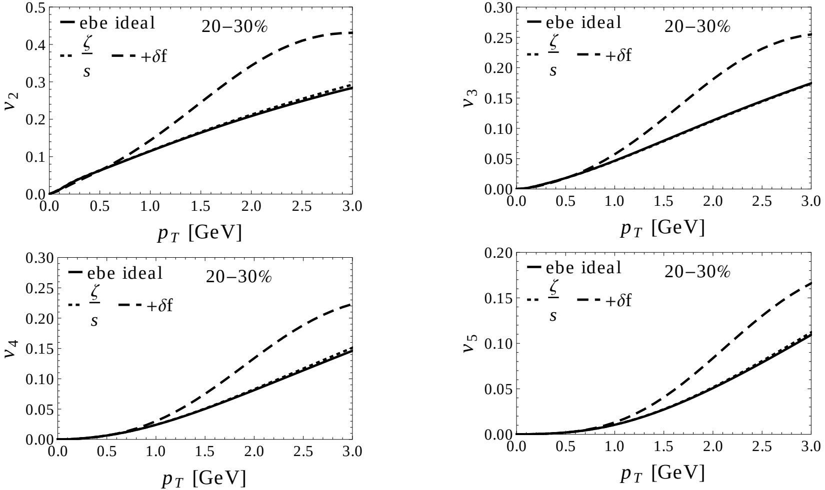
Direct $\pi^+ \approx 54$



As expected: more slow/less fast particles.

Event-by-Event higher flow harmonics





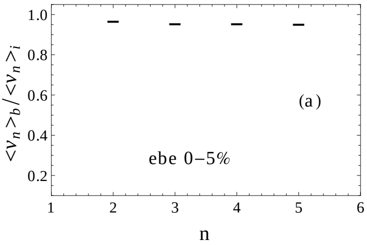
• $v_n(p_T)$ are significantly enhanced, even though ζ/s is small.

• HOW TO DISENTANGLE shear and bulk effects? (may cancel each other)

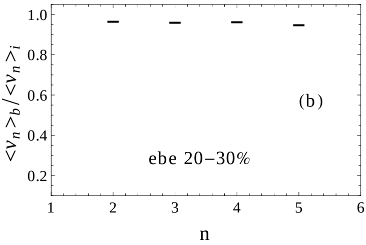
Integrated v_n's

For small ζ/s, expect $v_n^{bulk} \sim v_n^{ideal}$

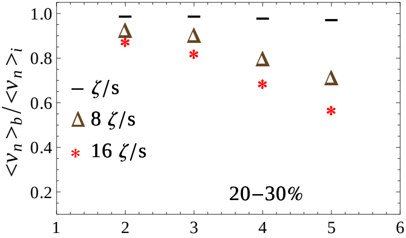
0 – 5%



20 – 30%



For large ζ/s, expect $v_n^{bulk} < v_n^{ideal}$

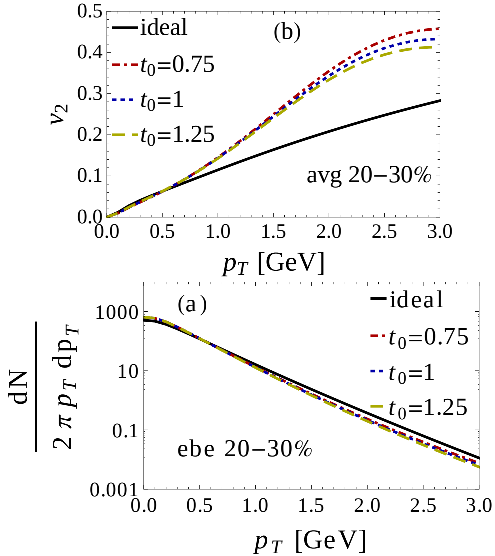


V. Conclusion

- v-USPhydro: Lagrangian 2+1 hydro code with bulk and shear viscosity, running event-by-event.
- $v_n(p_T)$ enhanced by bulk viscosity while it is decreased by shear ciscosity.
  - How to disentangle to extract η/s and ζ/s?
  - Higher ζ/s do not seem excluded.
- Integrated v_n's (or other integrated quantities) may be useful.
- δf_{bulk} plays a crucial part.
  (Here computed with moment method.)

BACK UP SLIDES

Dependence on τ_0

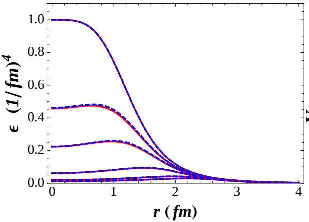


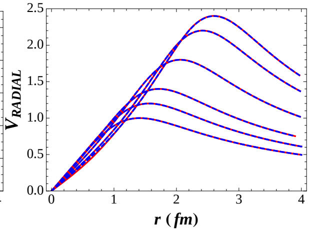
Checks

- Reproduce analytical sol. from 2+1 conformal ideal hydro

$$\epsilon = \frac{\epsilon_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1 + 2q^2\left(\tau^2 + x_\perp^2\right) + q^4\left(\tau^2 - x_\perp^2\right)\right]^{4/3}}$$

  Gubser,PRD**82**,085027(2010), Marrochio et. al. 1307.6130 [nucl-th] (first analytical solution of Israel-Stewart hydro)

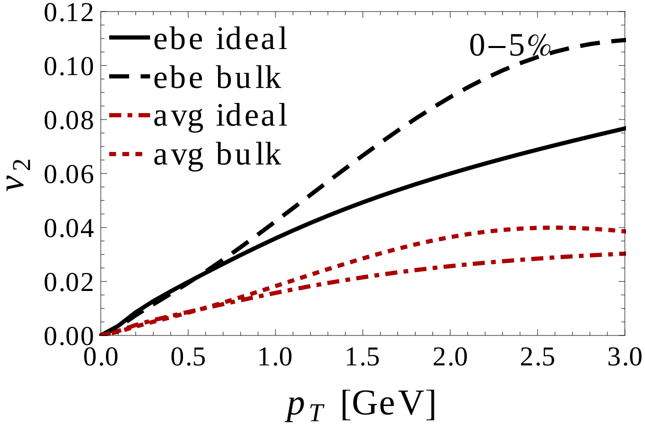




- The viscous bulk evolution converges to that computed within ideal hydrodynamics for sufficiently small ζ/s.

Averaged Initial Conditions vs. Event-by-Event

- No decays are included. We use Monte Carlo Glauber initial conditions.



- The effect of the bulk viscosity is enhanced in event-by-event studies