THERMALIZATION AND ENTROPY PRODUCTION FROM GLASMA-LIKE INITIAL CONDITION IN CYM DYNAMICS

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in collab. with

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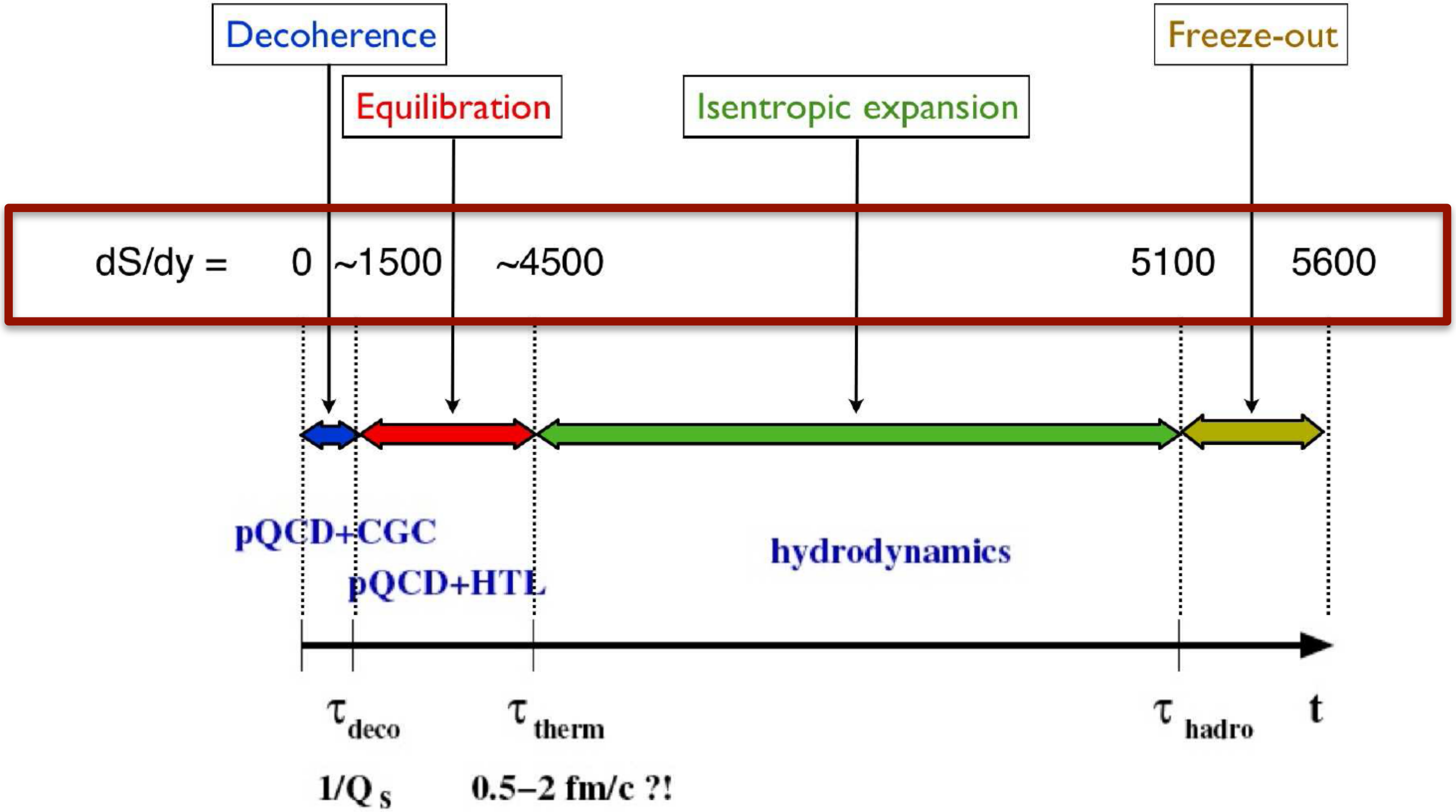
Entropy history



Entropy history of a central Au+Au collision at top RHIC energy

(from B.Mueller and A.Schaefer, Int. J. of Mod. Phys. E20, 2235-2267 (2011).)

Main part of the entropy production occurs before thermalization

… study the entropy production at the initial stage of heavy-ion collision

Kolmogorov-Sinaï entropy

We study the time evolution of entropy in classical Yang-Mills dynamics

- Kolmogorov-Sinaï (KS) entropy:

$$S_{\mathrm{KS}} = \sum_k \lambda_k \theta(\lambda_k) \quad \lambda_k : \text{Lyapunov exponent} \quad |\delta X_i(t)| \propto e^{\lambda_i t}$$

Lyapunov exponent: an index of initial-value sensitivity

- **Instability**

・Weibel instability (Mrowczynski (1993))

・Nielsen-Olesen instability (Iwazaki(2008), Fujii&Itakura (2008))

… considered to lead early thermalization

- **Chaoticity**

・Lyapunov exponent is related to mixing property of chaos (mentioned later)

KS entropy … related to instability & chaoticity

ENTROPY PRODUCTION IN CLASSICAL YANG-MILLS DYNAMICS

- Focusing on **entropy production through the instability & chaotic behavior** in Classical Yang-Mills system.

Two quantities

Distance between two trajectories in phase space

$$D_{EE} = \sqrt{\sum_x \left\{ \sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E_i'^a(x)^2 \right\}^2} \qquad D_{FF} = \sqrt{\sum_x \left\{ \sum_{a,i,j} F_{ij}^a(x)^2 - \sum_{a,i,j} F_{ij}'^a(x)^2 \right\}^2}$$

E' & F' ...slightly different from E & F at initial time, respectively

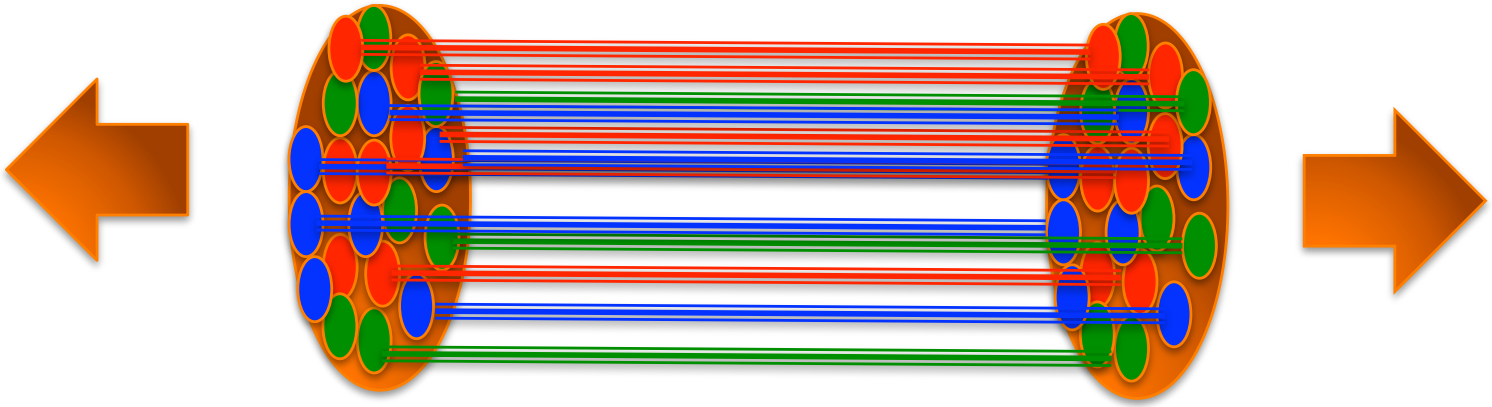
Entropy production rate (Kolmogolov-Sinaï entropy)

$$\frac{dS}{dt} = S_{\mathrm{KS}} = \sum_{\lambda_i^{\mathrm{ILE}} > 0} \lambda_i^{\mathrm{ILE}}$$

Entropy production from realistic initial condition

Entropy production from realistic initial condition
… initial condition with coherent background field

- **High energy collision… high gluon density**
  → gluons are coherent field rather than particles
  First approximation: classical treatment
- **Initial condition: color-glass condensate (CGC), Glasma**
  (L.McLerran and R.Venugopalan 1994; T.Lappi and L.McLerran 2006, …)



Classical Yang-Mills calculation with Glasma initial condition
… appropriate for the study of thermalization process

We study thermalization process from a coherent background field

Modulated initial condition

$$\begin{cases} A_i^a(\vec{r}) = \eta^a(\vec{r}) + \delta_{i2}\left(\epsilon_1 \sin\left(\frac{2x\pi}{N_x}\right) + \epsilon_2 \sin\left(\frac{2nx\pi}{N_x}\right)\sin\left(\frac{2mz\pi}{N_z}\right)\right), \\ E_i^a(\vec{r}) = 0, \end{cases}$$

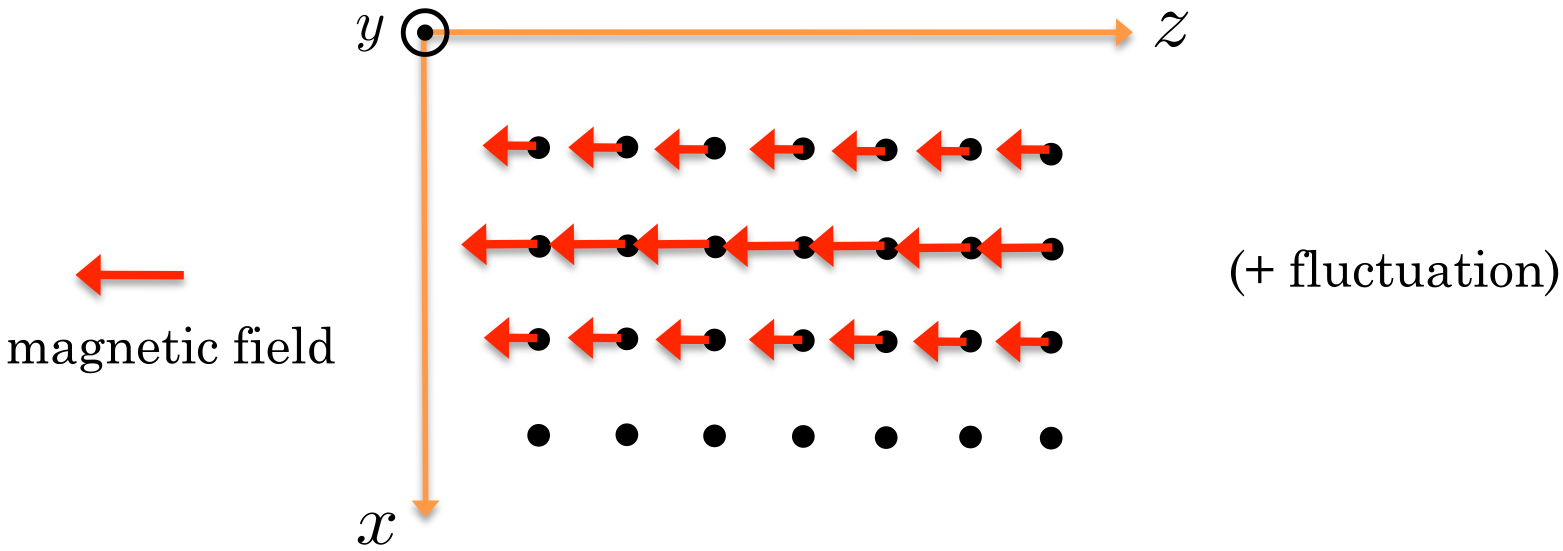
Fluctuation (noise)

$\epsilon_1 \gg \epsilon_2 \;\rightarrow$ glasma-like

Magnetic field from spatial modulation (neglecting noise)

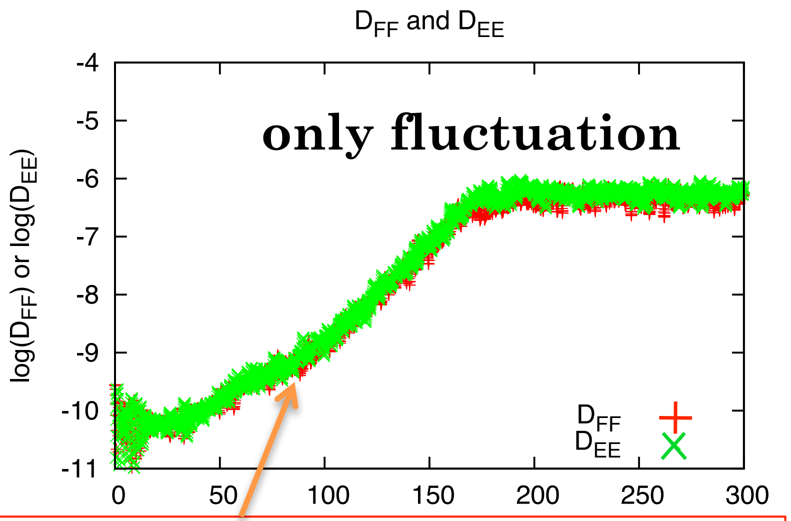
$$B_x = -2m\frac{\pi}{N_z}\epsilon_2 \sin\left(\frac{2nx\pi}{N_x}\right)\cos\left(\frac{2mz\pi}{N_z}\right) \simeq 0,\, B_y = 0$$

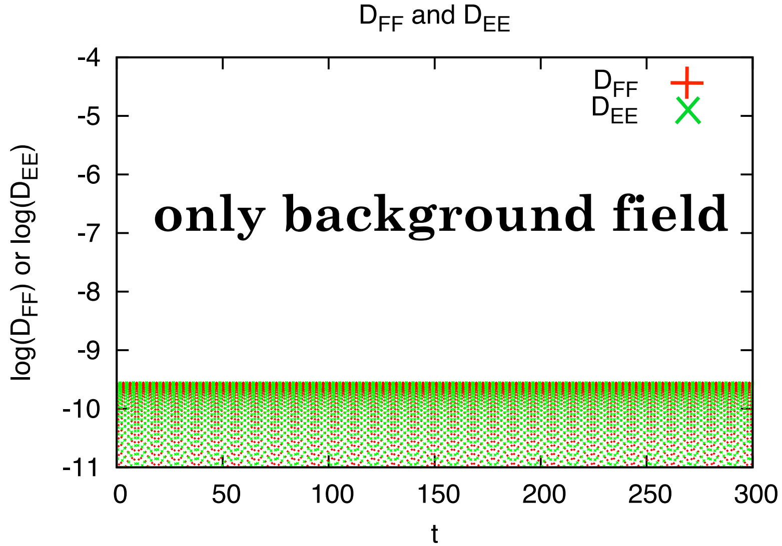
$$B_z = \frac{2\pi}{N_x}\epsilon_1 \cos\left(\frac{2x\pi}{N_x}\right) + \frac{2n\pi}{N_x}\epsilon_2 \cos\left(\frac{2nx\pi}{N_x}\right)\sin\left(\frac{2mz\pi}{N_z}\right) \simeq \frac{2\pi}{N_x}\epsilon_1 \cos\left(\frac{2x\pi}{N_x}\right)$$



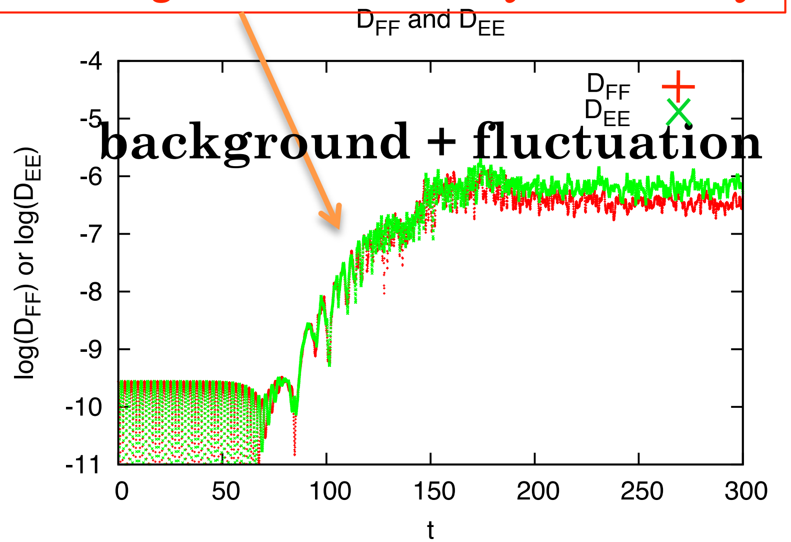
RESULTS: TIME DEP. OF DISTANCE

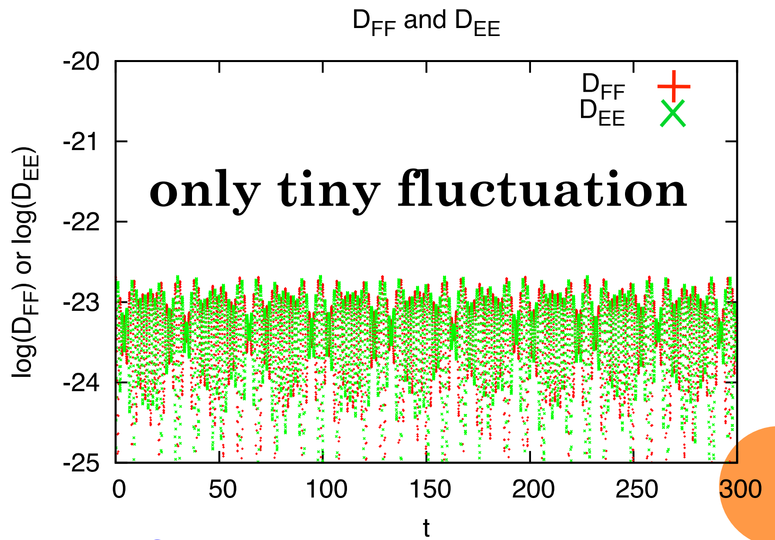
cf.) $D_{EE} = \sqrt{\sum_x \left\{ \sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E_i'^a(x)^2 \right\}^2}$





exponential growth → instability & chaoticity





- **Modulation only … No instability & chaotic behavior**
- **Modulation + (tiny) fluctuation … unstable & chaotic**

INTERMEDIATE LYAPUNOV EXPONENT & KOLMOGOROV-SINAÏ ENTROPY

… averaged Lyapunov exponent in a certain time interval

$$S_{\mathrm{KS}} = \sum_k \lambda_k^{\mathrm{int}} \theta(\lambda_k^{\mathrm{int}})$$

λ_k^{int}: Intermediate Lyapunov exponent

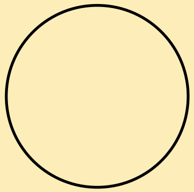
… sum of positive ILE

cf) Kunihiro, Muller, Schafer, Takahashi, Yamamoto, PRD82, 114015 (2008).

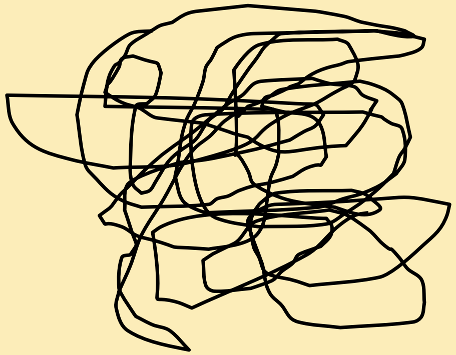
KS entropy … related to mixing property of chaos

mixing property … roughly speaking, complexity of classical orbit in phase space in a certain time interval

Orbit in phase space:



Periodic orbit → S_{KS}=0

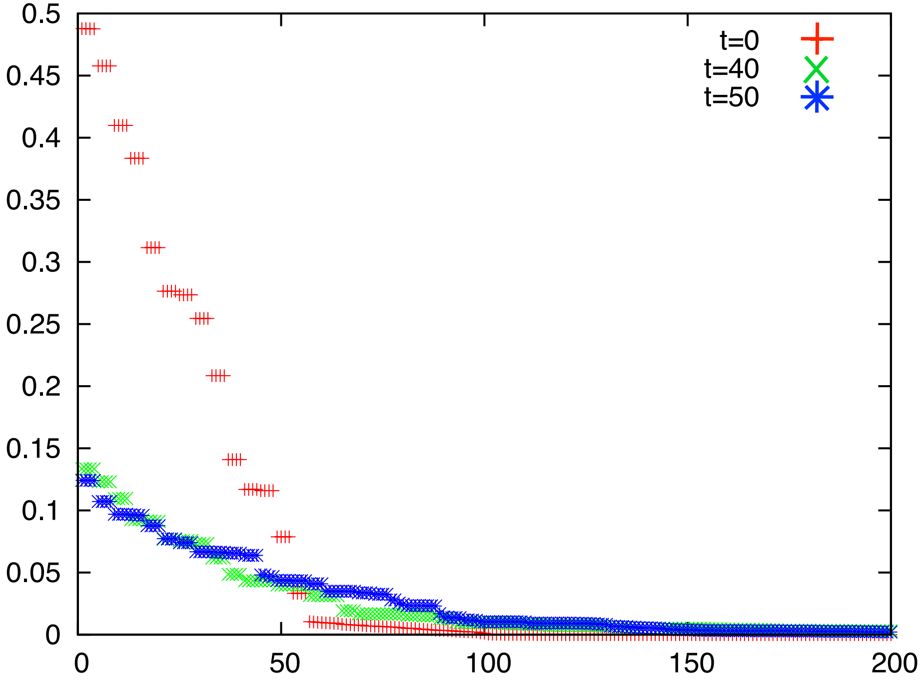


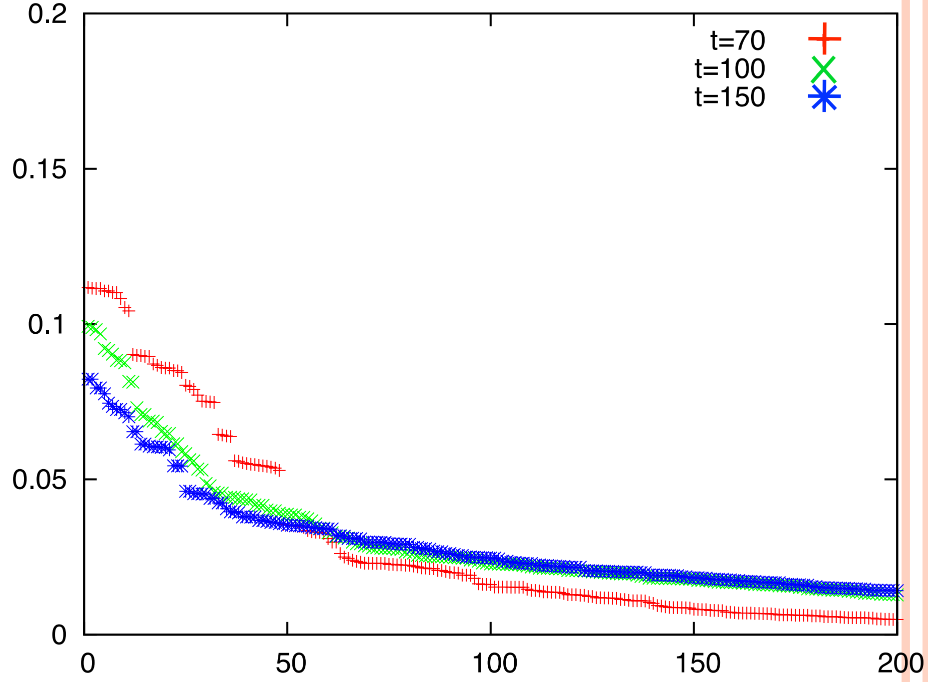
Complex orbit (dense) → $S_{KS} > 0$

※ Lyapunov exponent is invariant under the residual gauge trans. in temporal gauge

RESULTS: INTERMEDIATE LYAPUNOV EXP. ($V=4^3$)

distribution of positive intermediate Lyapunov exponent



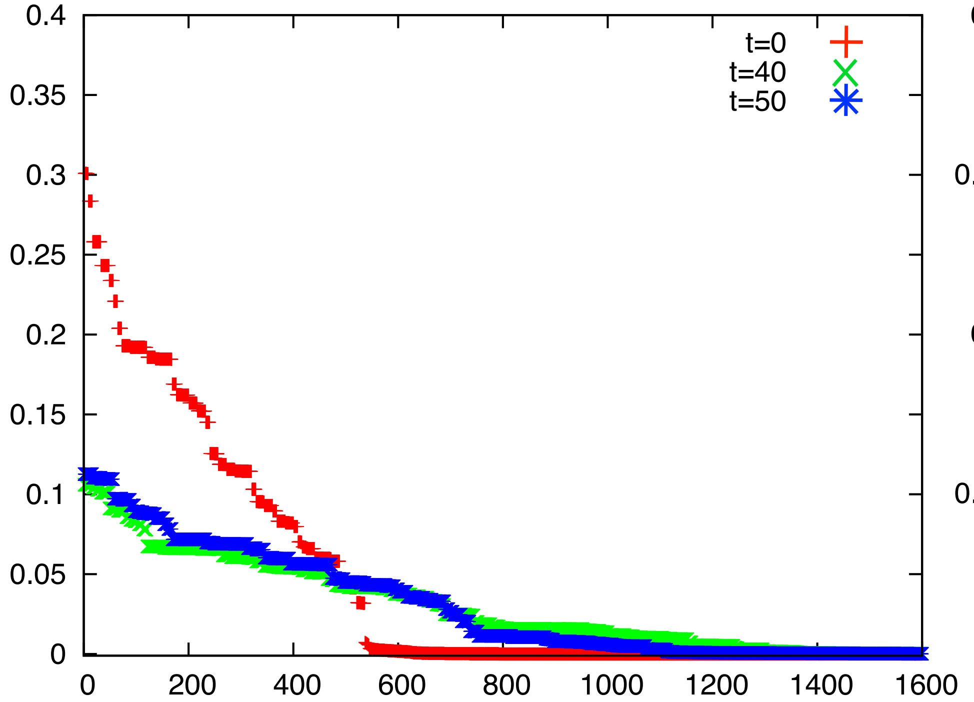


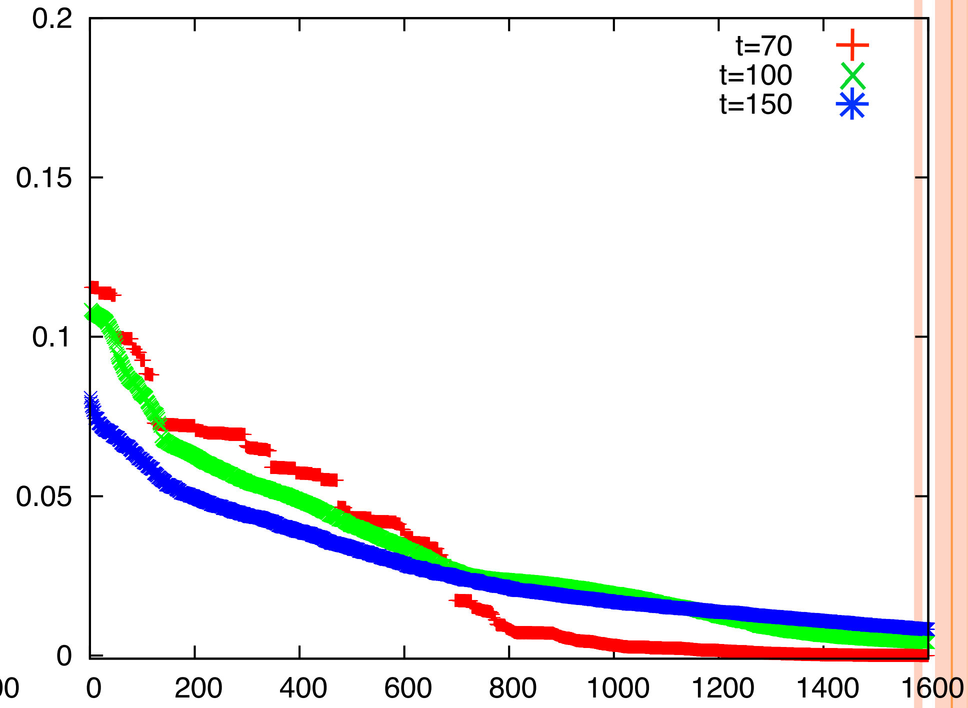
•Instability spreads to many modes already at t=40.

•Distribution of Lyapunov exp. is stable at large t, and then, significant portion of Lyapunov exponents keeps positive in a robust way

→chaotic & entropy production

INTERMEDIATE LYAPUNOV EXPONENT IN LARGER VOLUME ($V=8^3$)

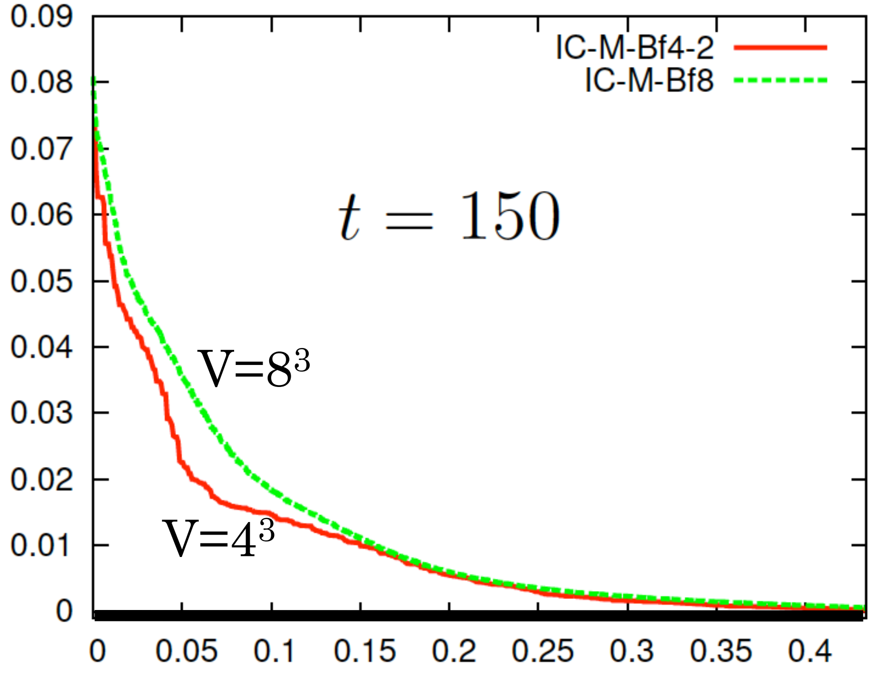




- **For large volume, qualitative behavior does not change:**

  → indicates that the entropy production occurs in infinite volume limit

Dependence of Lyapunov exponent to volume、Kolmogorov Sinaï entropy



KS entropy/volume

$s_{KS} \equiv S_{KS}/V$			
	M-Bf4-1 $\epsilon = 7.13 \times 10^{-3}$	M-Bf4-2 $\epsilon = 3.53 \times 10^{-3}$	M-Bf8 $\epsilon = 3.53 \times 10^{-3}$
t=0	0.226	0.151	0.140
40	0.088	0.063	0.095
50	0.094	0.042	0.096
70	0.098	0.052	0.092
100	0.119	0.054	0.106
150	0.122	0.079	0.099

The value of Lyapunov exp. in $V=8^3$ is always larger than that in $V=4^3$ for each mode
→ indication of entropy production in infinite volume

$\varepsilon = 3.53 \times 10^{-3}$
$V=4^3$: 0.079 @ t=150
$V=8^3$: 0.099 @ t=150

→ entropy production may occur in infinite volume limit

NON-COMPACT FORMALISM IN CYM

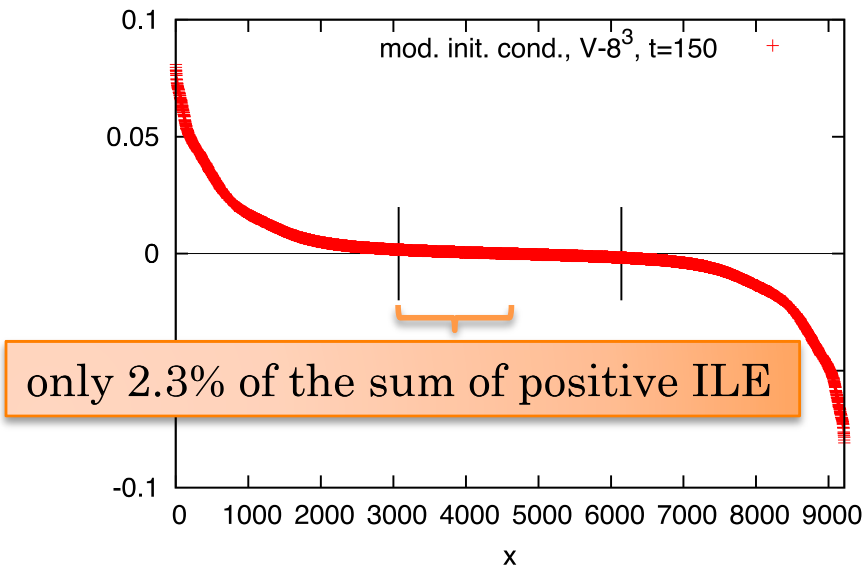
- **(A,E) formalism is used lather than (U,E) formalism**

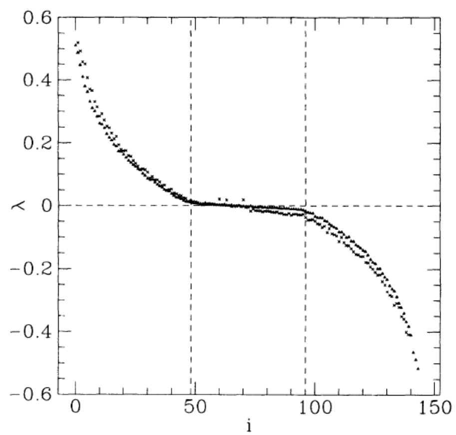
  … Gauge inv. is violated in EOM by discretization.

- **How harmful is the violation?**

The violation is reflected in the Lyapunov spectrum:

1/3 of Lyapunov exp. are zero, if gauge inv. (Gauss' law) is kept exactly.





cf) Lyapunov spectrum ($V=2^3$)
Gong, Phys.Rev.D49, 2642 (1994).

➡ **Good gauge invariance !**

Summary and conclusions

Background magnetic field + (tiny) fluctuation

(simple model of realistic initial condition)

→ **instability & chaotic behavior**

→ **entropy production**

in classical Yang-Mills dynamics

- **Entropy production proceeds already in the CYM evolution.**
- **Instability is related to entropy production.**

  … possible scenario of early thermalization
- **Fluctuations on top of the background Glasma configulation is crucial for the entropy production.**