Gluon saturation and high-energy phenomenology

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Single-inclusive production from *QCD evolution equation*: rcBK evolution + AAMQS i.c.

basic “degrees of freedom”: dipole scattering amplitude in fund. rep. **(2-point fct)**

$$\mathcal{N}_F(r, Y; b, A) \equiv \frac{1}{N_c} \mathrm{tr}\, \langle 1 - V^\dagger(y) V(z) \rangle_Y \qquad \mathbf{r} = \mathbf{y} - \mathbf{z}$$

BK equation (incl. non-linear terms → saturation of scattering amplitude!)

$$\frac{\partial \mathcal{N}(r,x)}{\partial \log 1/x} = \int d^2 r_1\, K(r, r_1, r_2) \left[\mathcal{N}(r_1, x) + \mathcal{N}(r_2, x) - \mathcal{N}(r, x) - \mathcal{N}(r_1, x)\mathcal{N}(r_2, x)\right]$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r_2}$$

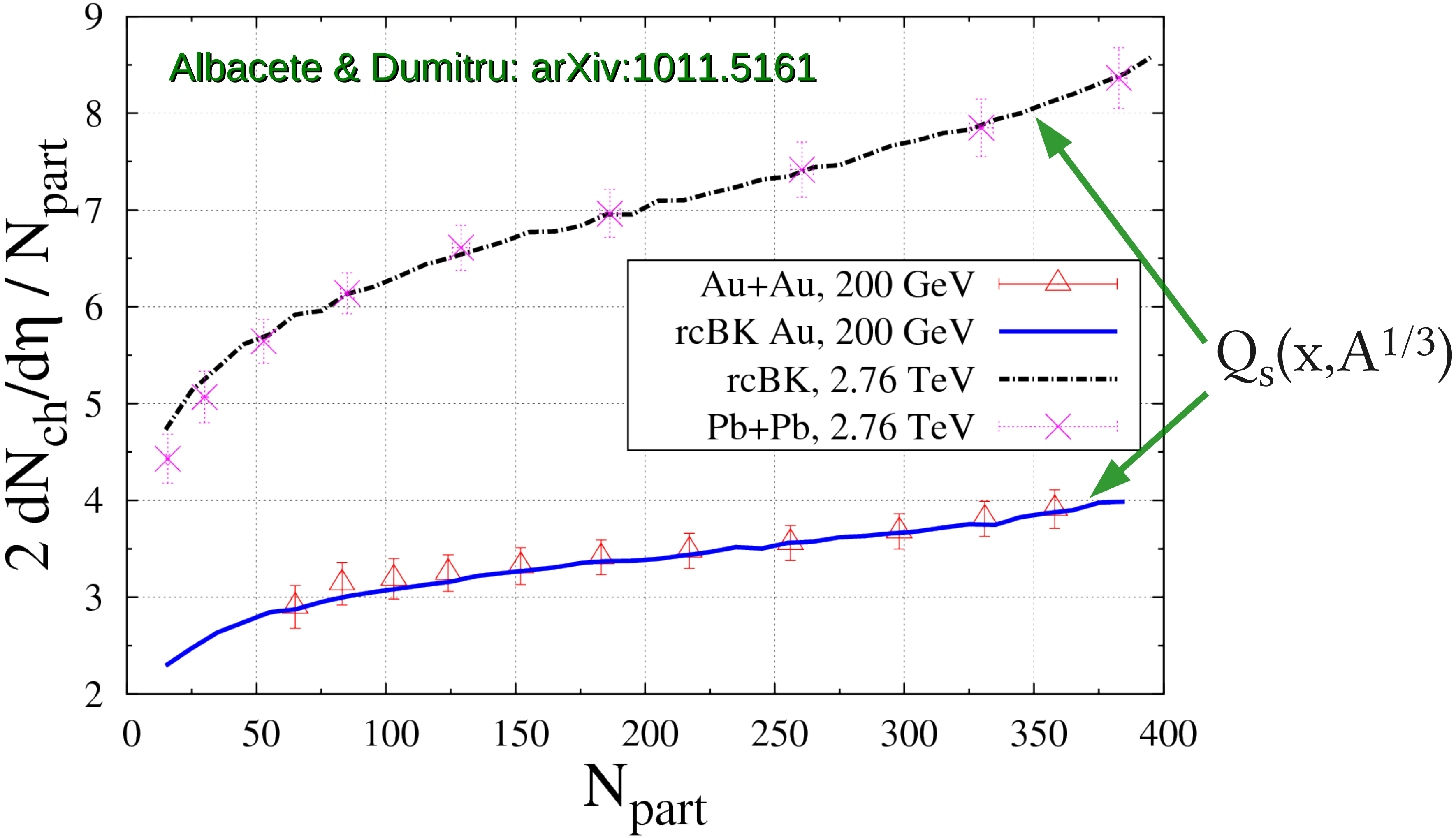
running-coupling kernel (Balitsky prescription)

$$K(\mathbf{r}, \mathbf{r_1}, \mathbf{r_2}) = \frac{N_c\, \alpha_s(r^2)}{2\pi^2} \left[\frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r^2}{r_1^2\, r_2^2} + \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

$$\alpha_s(r^2) = \frac{4\pi}{\beta \log\left(4 \frac{C^2}{r^2 \Lambda^2} + \mu\right)}$$

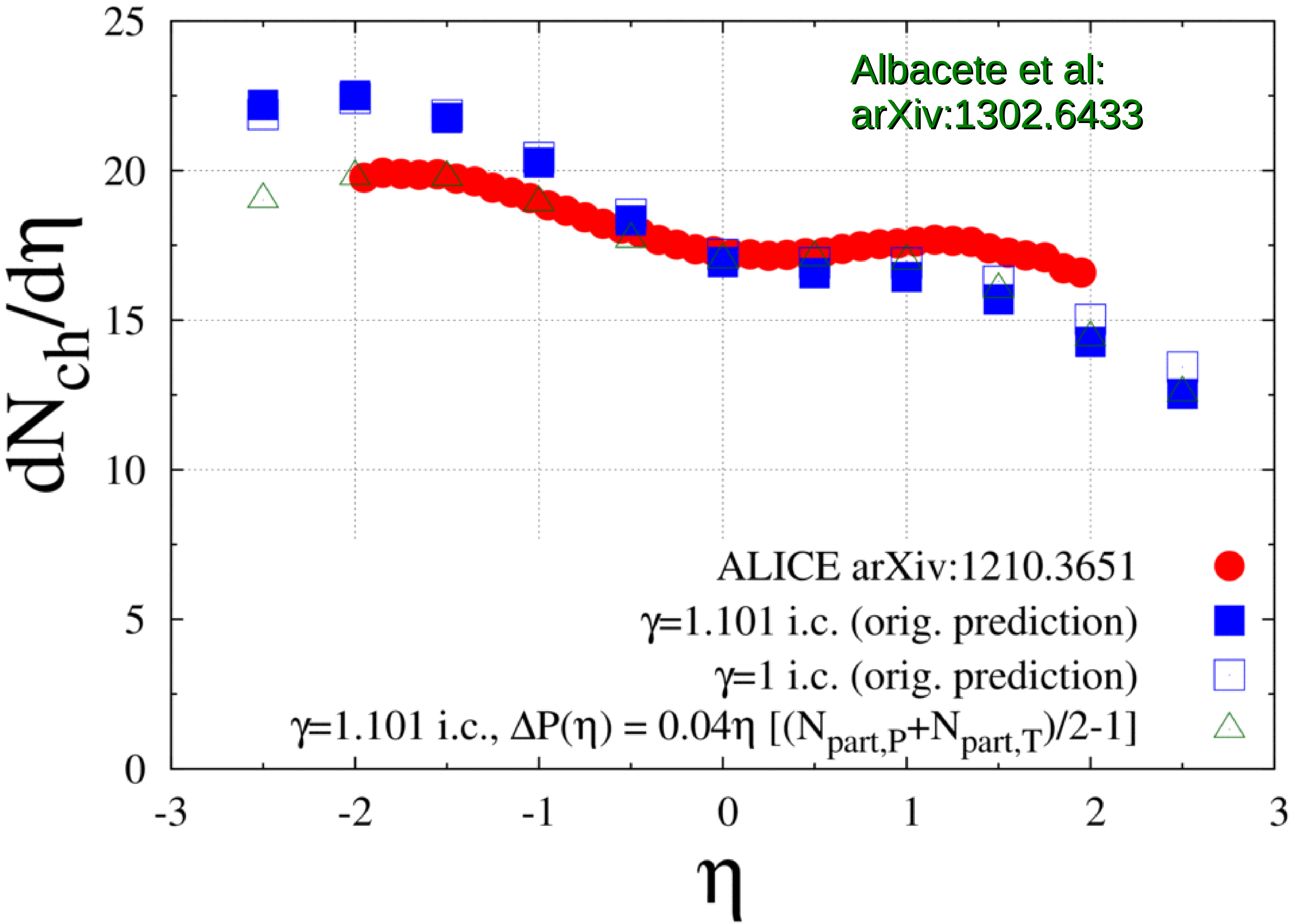
dipole scattering amplitude in adj. rep. $\mathcal{N}_A = 2\,\mathcal{N}_F - \mathcal{N}_F^2$

energy dependence *predicted by evolution eqn* !



- huge success! We have the tools to compute E-dependence of particle production in QCD *(not this accurately though)* !
- (a long way since early days of "Feynman scaling")

… and the exact same rcBK UGD for p+Pb@5000

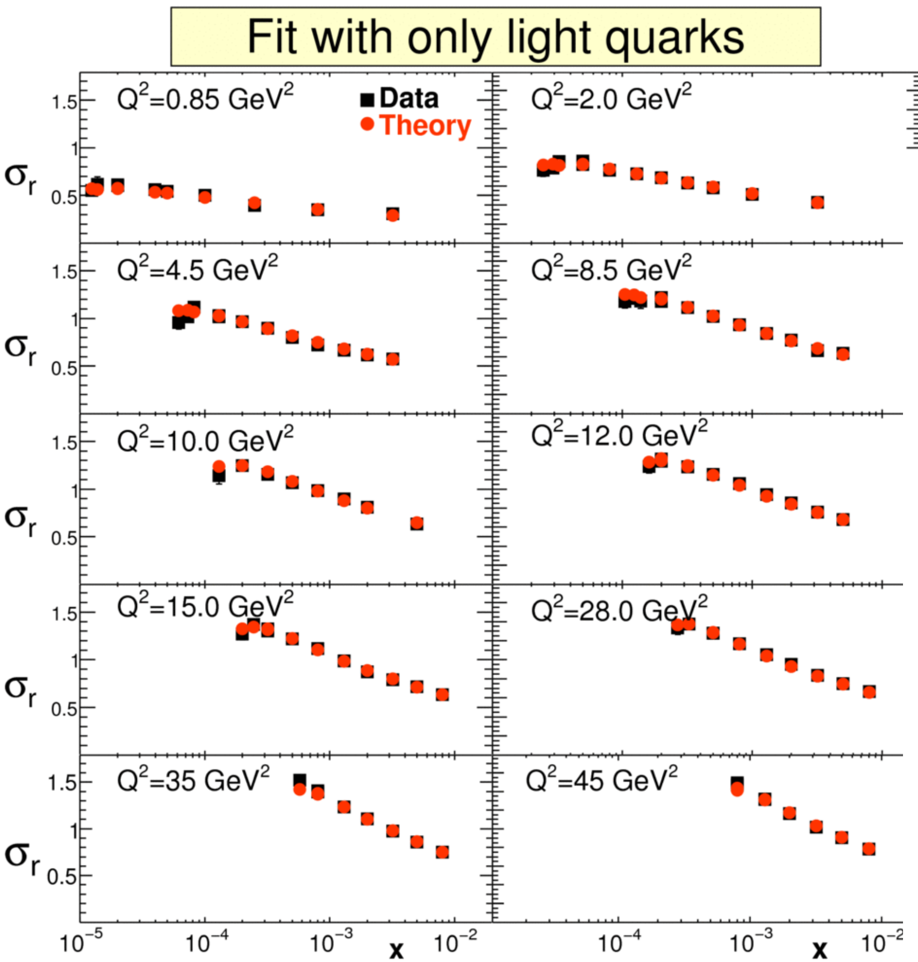


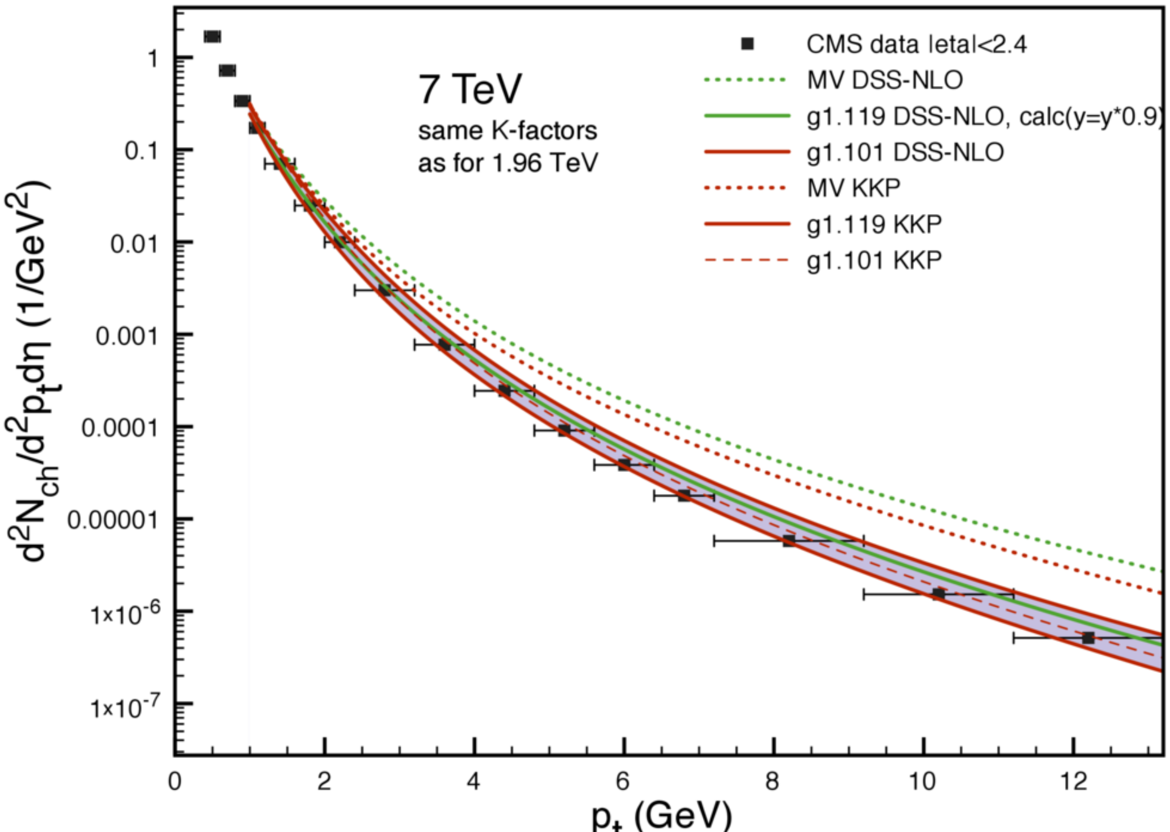
- midrapidity: bang on
- η ↔ y transformation (hadronic level !) clearly brings some uncertainty, such is life...

what is initial condition for rcBK evolution ?

- AAMQS 2011 ($\gamma > 1$!):

$$\mathcal{N}_F(r, Y=0; b) = 1 - \exp\left[-\frac{\left[r^2\, Q_{s0}^2(b)\right]^{\gamma}}{4}\, \ln\left(\frac{1}{\Lambda\, r} + e\right)\right]$$





$$\mathcal{N}_F(r, Y=0; b) = 1 - \exp\left[-\frac{\left[r^2\, Q_{s0}^2(b)\right]^\gamma}{4}\, \ln\left(\frac{1}{\Lambda\, r} + e\right)\right]$$

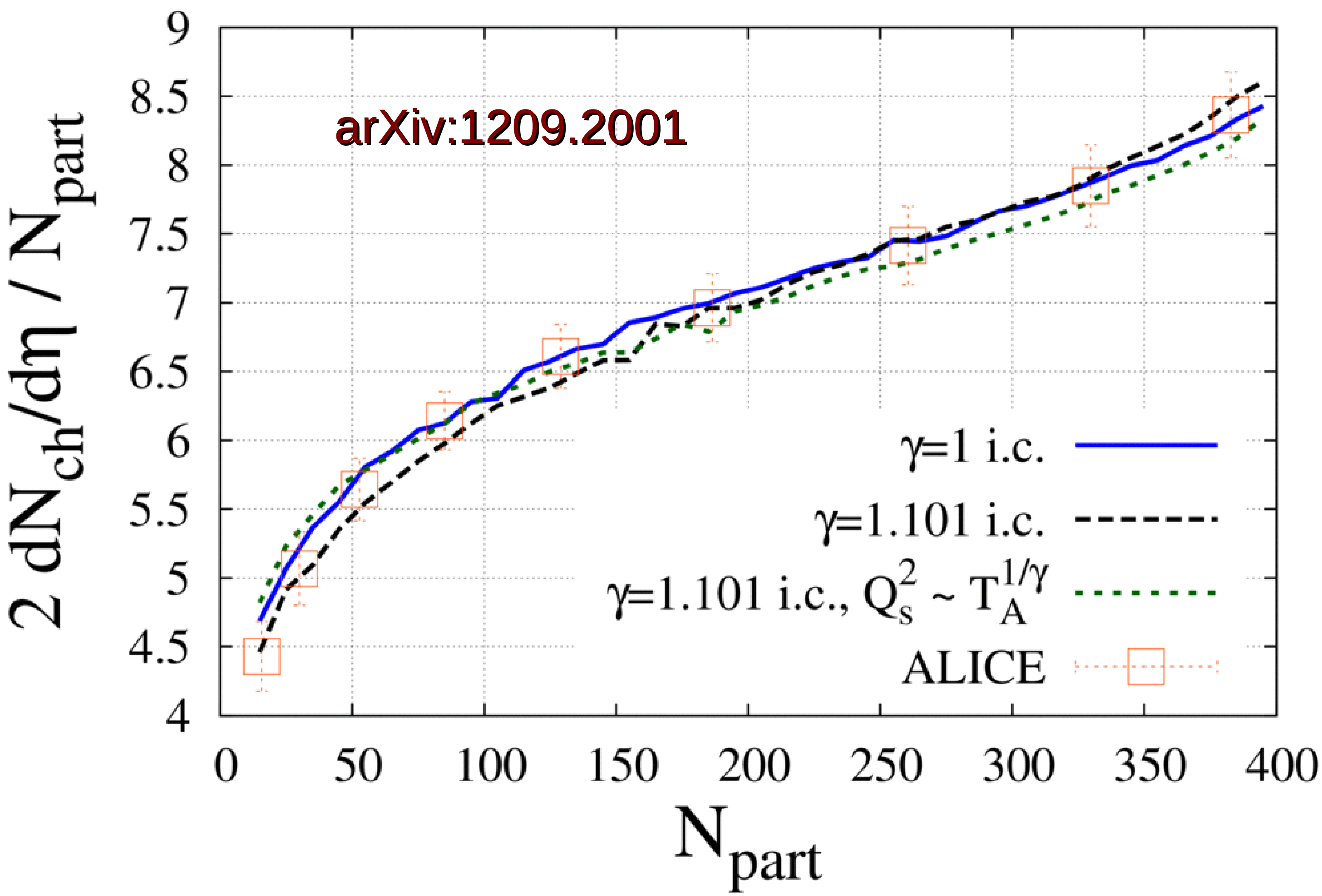
problem: A dependence...

- do we use same γ as for a proton ?
- do we assume $Q_{s0}^2 \sim T_A$ or $Q_{s0}^2 \sim T_A^{1/\gamma}$

→ w/o a better idea of where the AAMQS parameter comes from, we need to factor this into uncertainties...

(doesn't matter much for dN/dy but affects high p_T)

we already know this (the KLN legacy) :

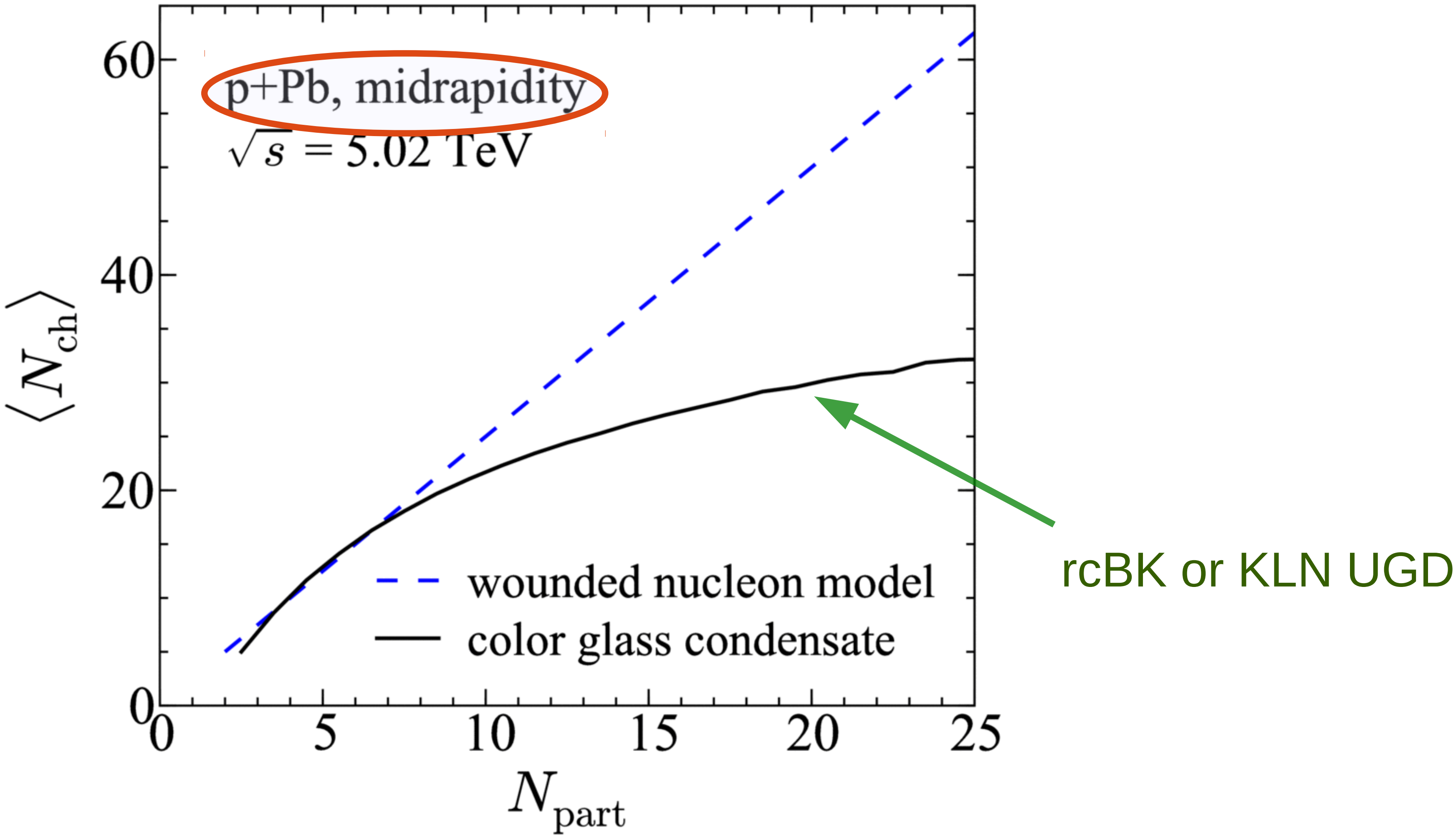


Pb+Pb @ 2.76 TeV

- “shape” very similar to RHIC: rules out simplest “minijet” models with fixed p_T ~ 1-2 GeV cutoff
- need scale $Q_s(x,N_{part})$; so that ROUGHLY $dN/dy \sim Q_s^2\, S_\perp / \alpha_s(Q_s)$

A nice illustration of *saturation*

A. Bzdak, V. Skokov: arXiv:1307.6168

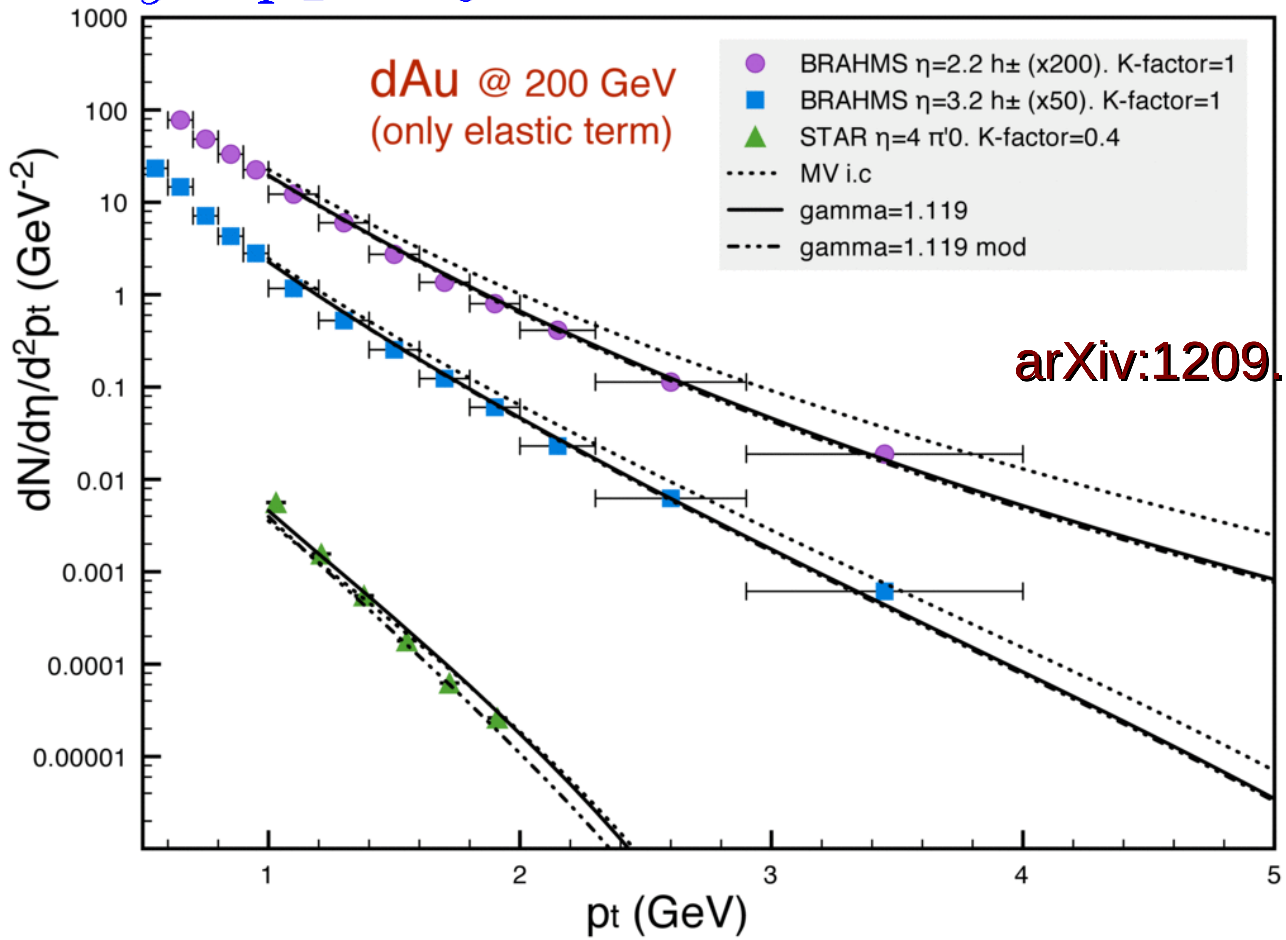


- # of particles NOT proportional to target participants …
- non-linear effect more obvious than for A+A

pA forward region: "hybrid formalism"

"elastic" term: $$\frac{dN}{dy\, d^2p_T} \sim \int f_i(x,Q^2) \otimes N(x,q_T) \otimes D_{i\to h}(Q^2,z)$$

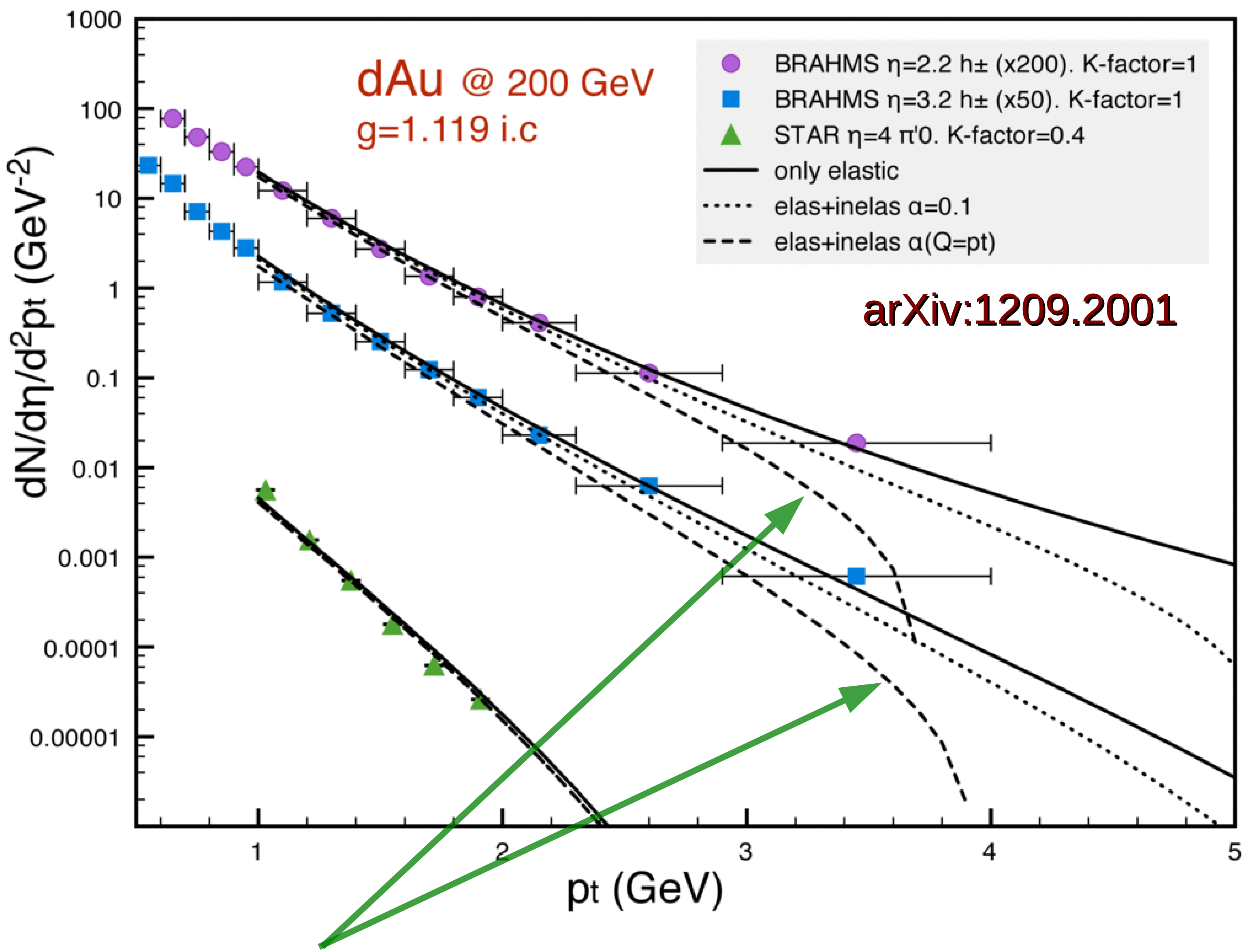
DHJ (2006)



arXiv:1209.2001

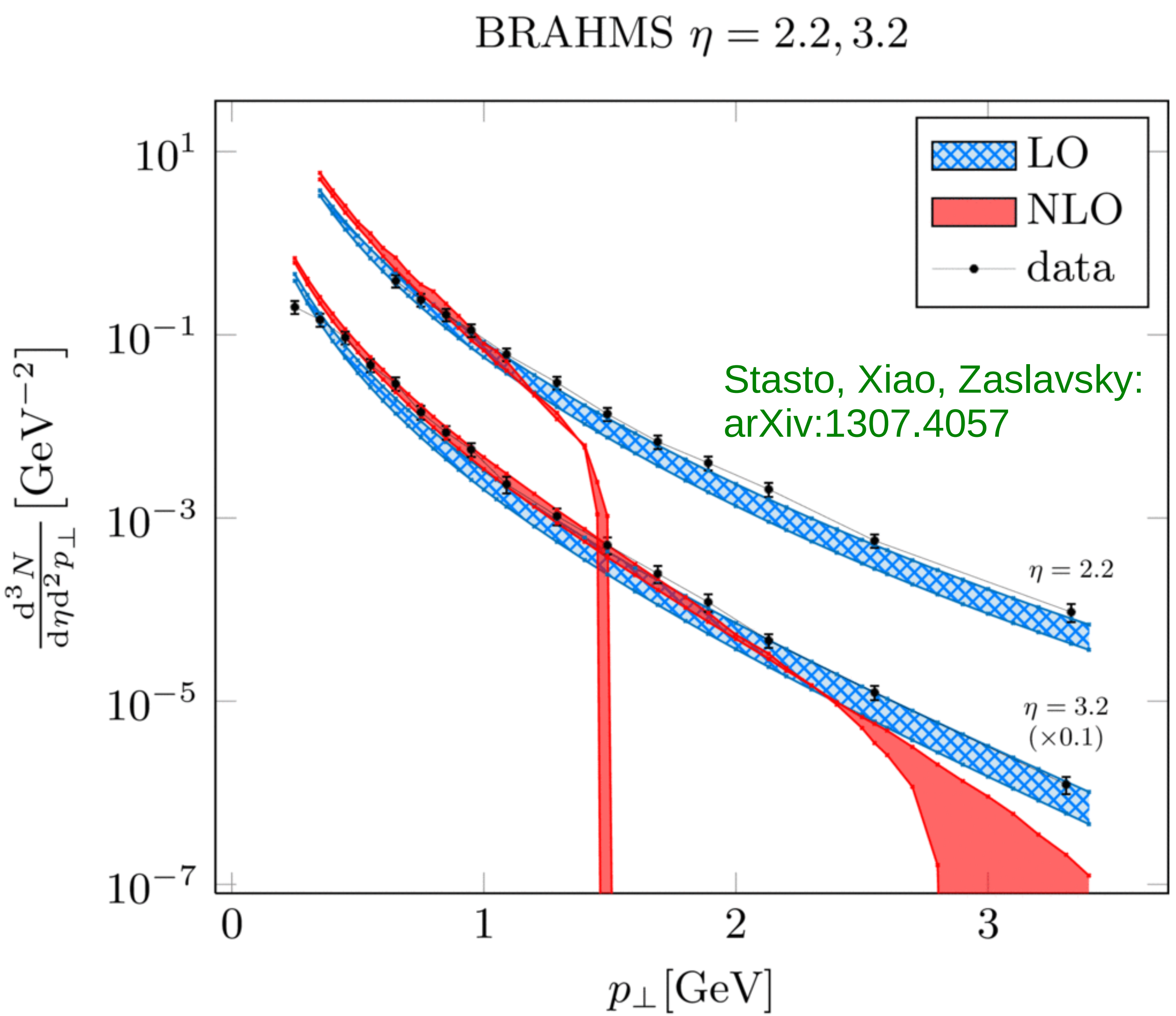
- those were the good days
  (aka we were young and we needed the money)

“inelastic” O(α_s) correction (1 extra hard gluon): Altinoluk & Kovner (2011)



- large correction at $p_T > Q_s(A)$

full NLO : Chirilli, Xiao & Yuan (2012)

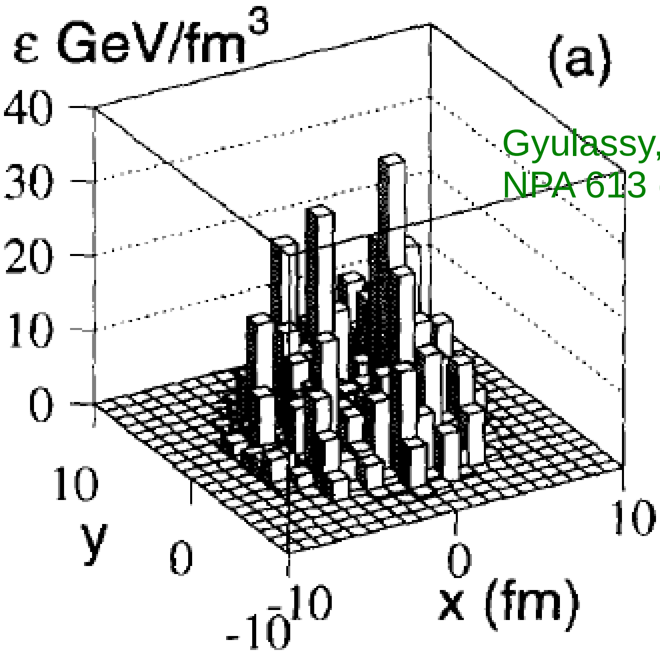


- awaits a solution ...

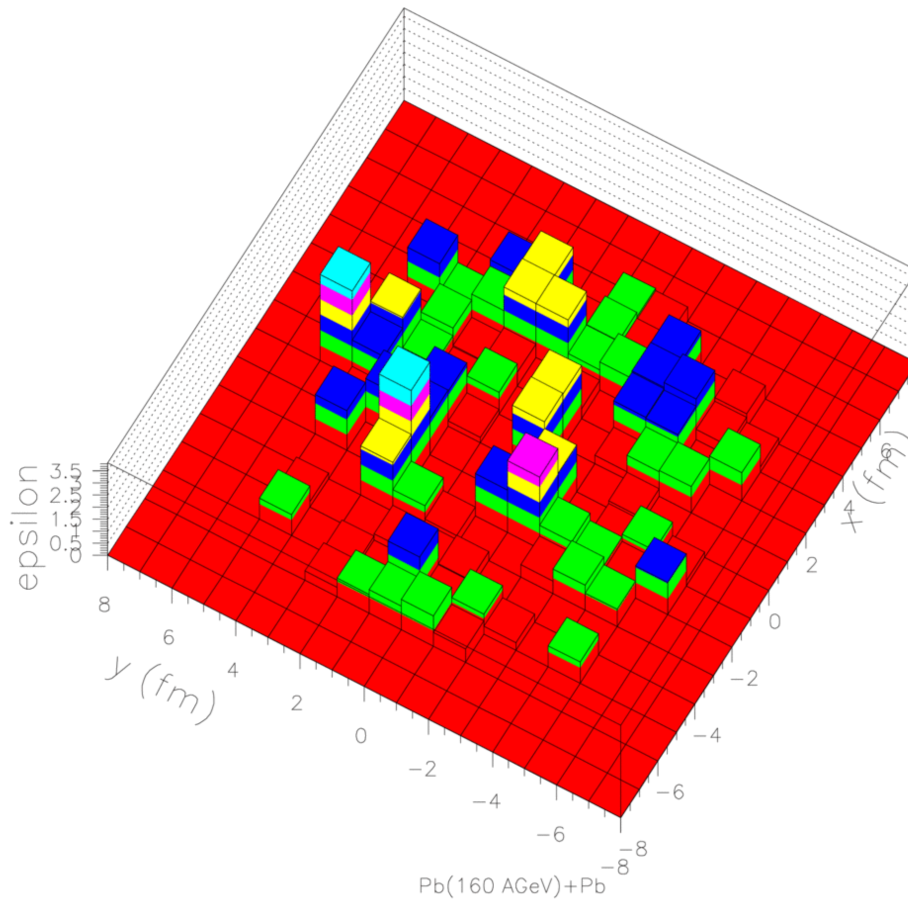
Let's move on to fluctuations

(pp, pA, AA)

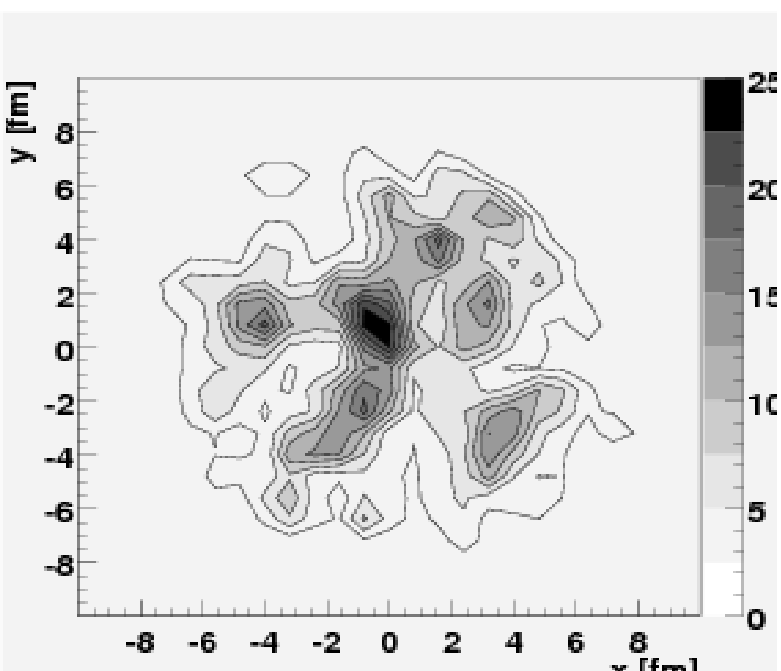
Lumpy initial conditions in AA collisions



Gyulassy, Rischke, Zhang:
NPA 613 (1997)



M. Bleicher *et al.:* QM97
NPA 638 (1998) p.391

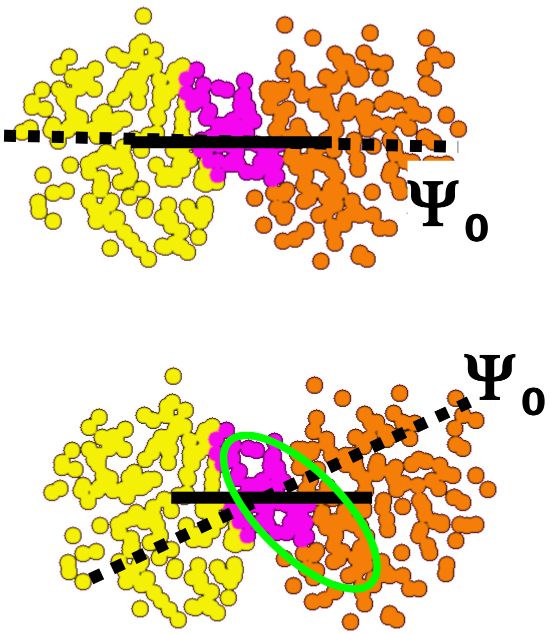


T. Kodama,
Y. Hama et al.

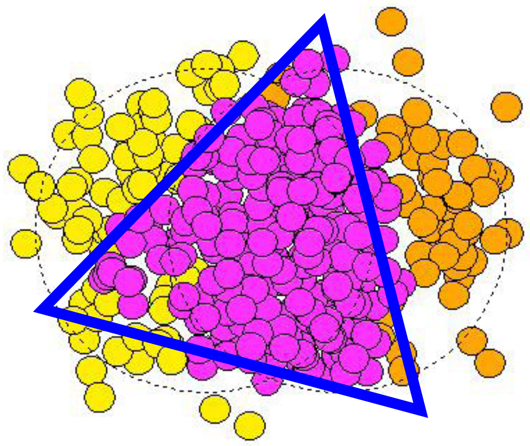
scale ~ 1 fm

Eccentricity fluctuations

Event-by-event fluctuations in the shape of the initial collision zone may be important.



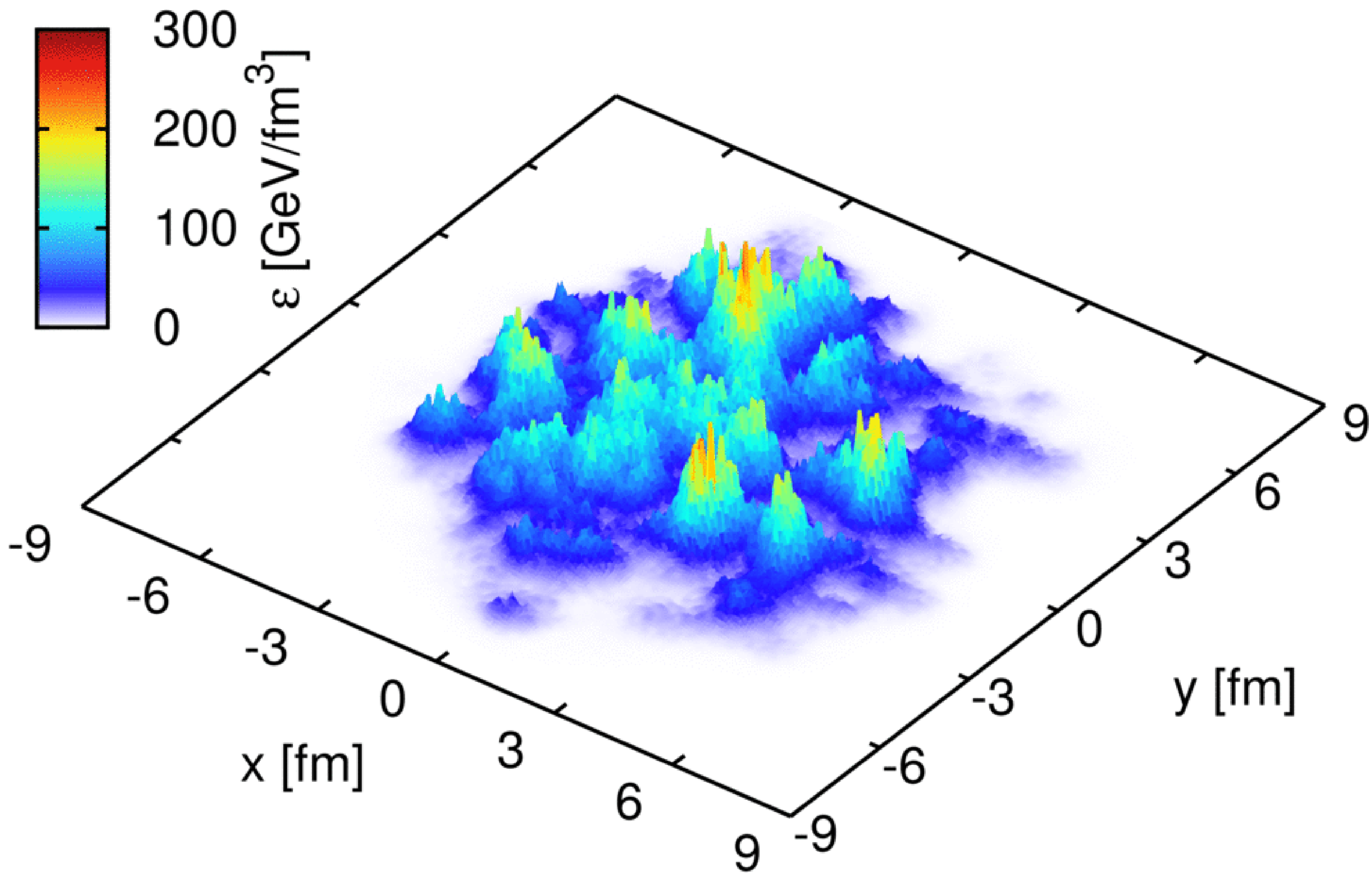
B Alver and G Roland, Phys. Rev. C 81, 054905 (2010)



→ v_3 etc

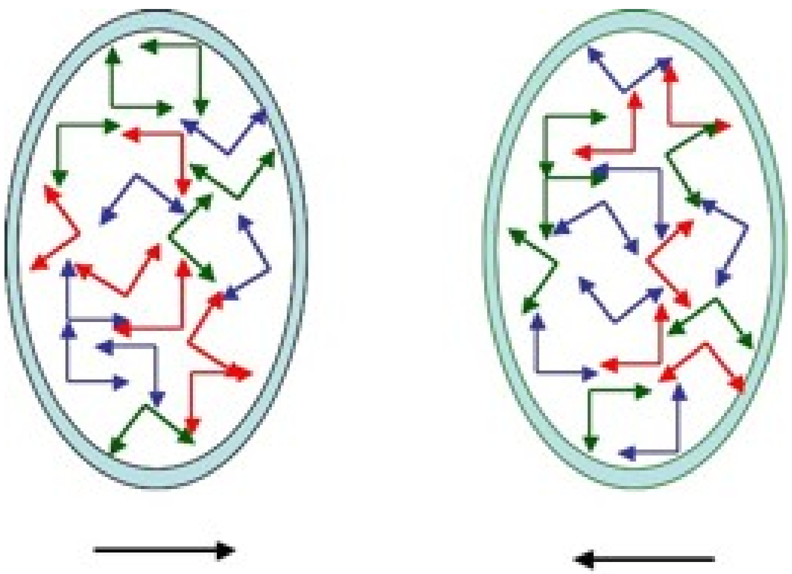
R.S.Hollis (PHOBOS) nucl-ex0707.0125

Sub-nucleon scale (~1/Q_s) fluctuations ?

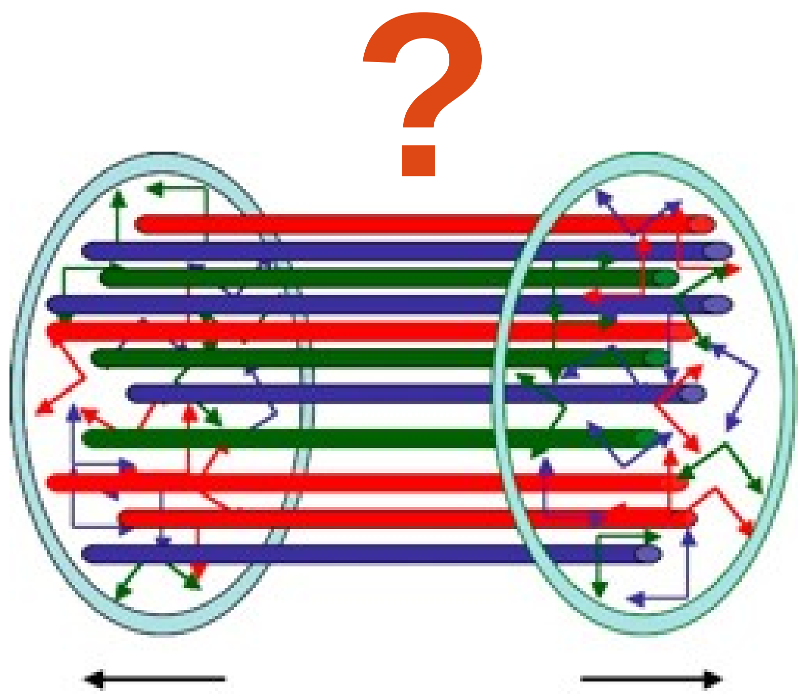


B. Schenke et al.: arXiv:1206.6805

a more detailed view: structure of B_z (E_z)



before collision



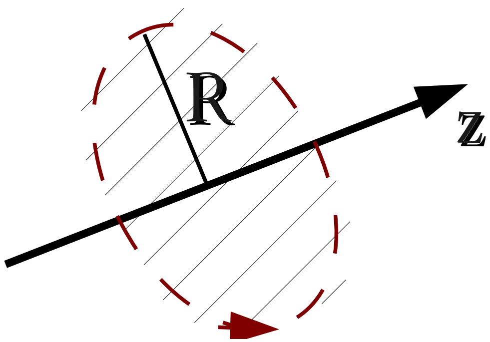
right after impact

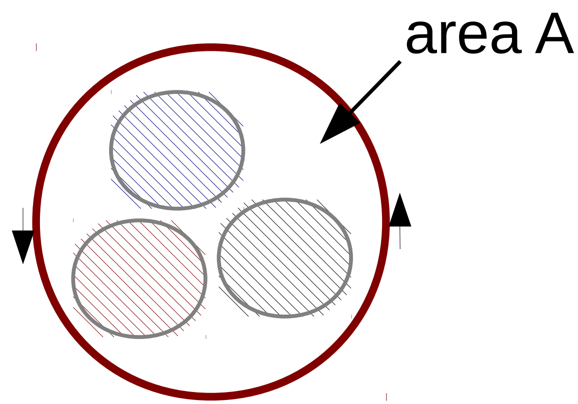
$$E^z = ig\left[A_1^i, A_2^i\right] \quad , \quad B^z = ig\epsilon^{ij}\left[A_1^i, A_2^j\right]$$

$$\nabla \cdot \mathbf{B} = ig\left[A^i, B^i\right]$$

R. Fries, J. Kapusta, Y. Li: nucl-th/0604054
Lappi + McLerran NPA 2006

Magnetic Z(N) “vortices”:

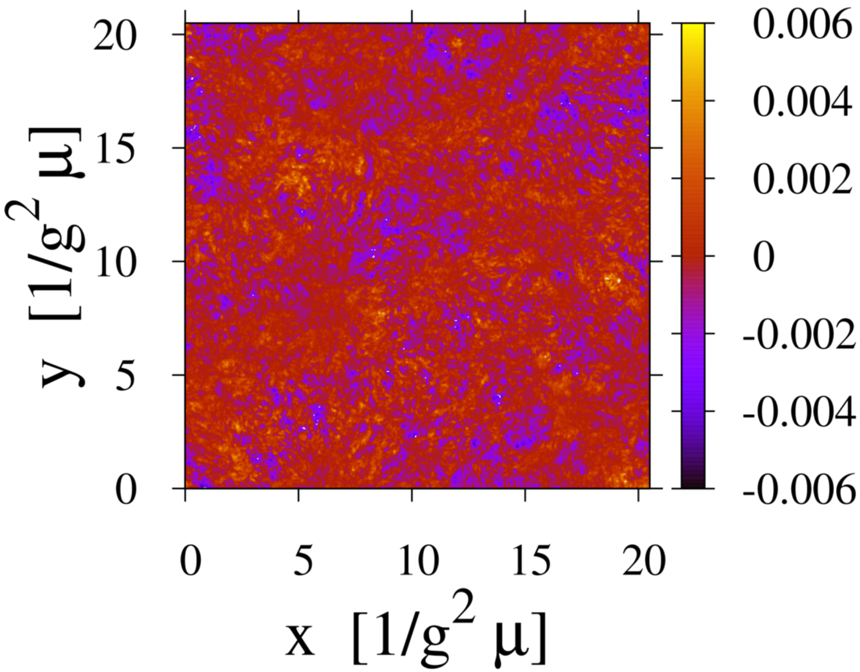




$$M(R) = \mathcal{P}\exp\left(ig\oint dx^i A^i\right)$$

$$e^{2\pi i\,(n_1+n_2+n_3)/N}$$

actual Bz field configuration



do we find :

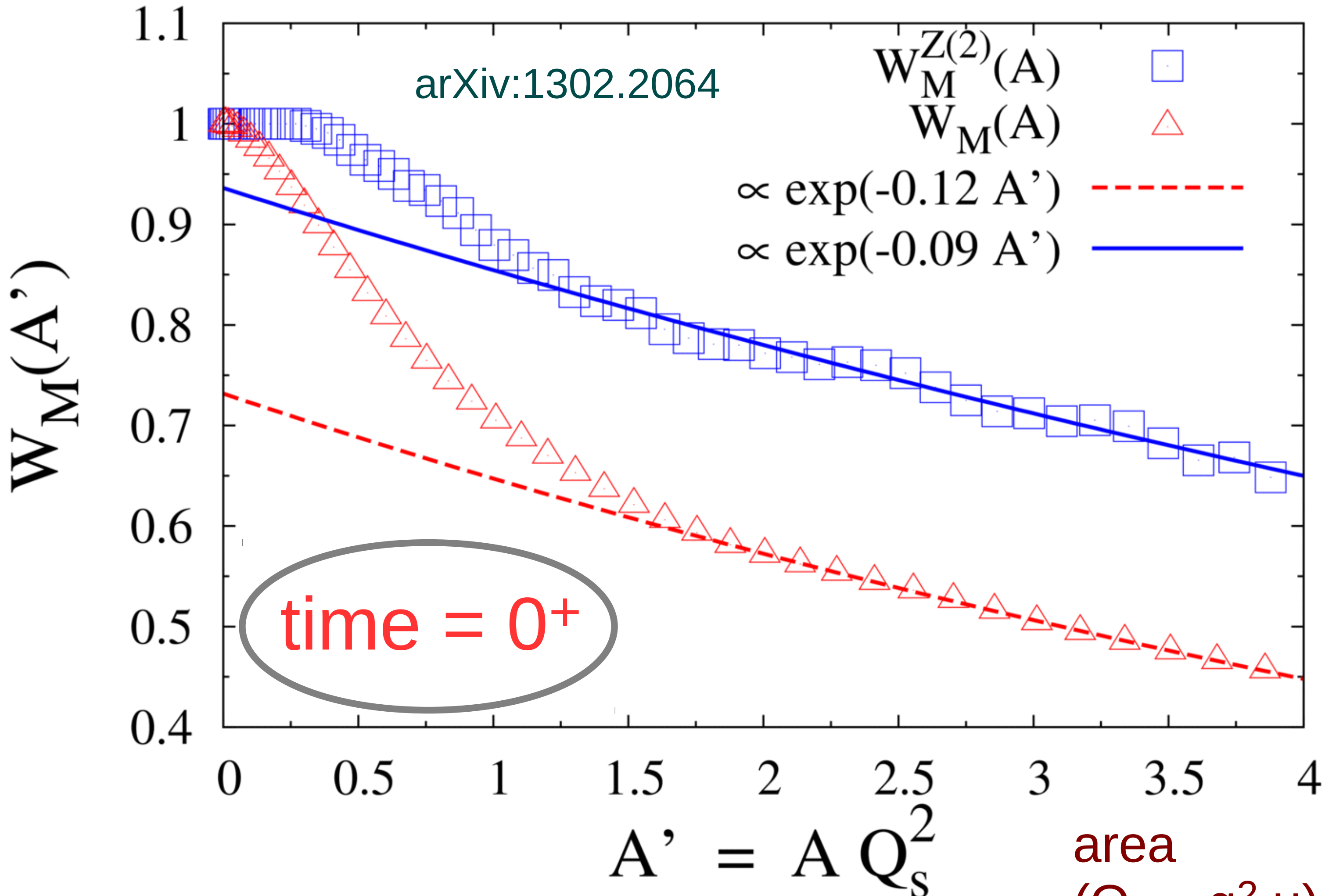
- area law ?

$$W_M(R) \sim e^{-\sigma A}$$

- loop ∈ Z(N) ?

$$\langle \mathrm{sgn\,tr}\ M\rangle \sim \frac{1}{N}\langle \mathrm{tr}\ M\rangle$$

SU(2) solution :



area ($Q_s \sim g^2\,\mu$)

- area law for loops with area $A \geq 1.5 - 2$
- $\sigma_M \sim 0.12\ Q_s^2$; thermal SU(N): $\sigma_M \sim g_{3D}^2 \sim (g^2 T)^2$
- small loops $\notin$ Z(2) but roughly ok for large ones!
- structure of B_z ~ uncorrelated vortices ?!
- $R_{vtx} \sim 1/Q_s$ from onset of area law

random uncorrelated vortex fluctuations:

(J. Preskill, "Lecture Notes on Quantum Field Theory", http://www.theory.caltech.edu/~preskill/notes.html)

$$W_M(A) \sim \exp\left(-\frac{\pi^2}{4}\, p(1-p) A\, Q_s^2\right) \;,$$

→ vortex probability in area $1/Q_s^2$

$p \approx 1/20$

magnetic screening !

$$C^{(2)}(r) = \langle \mathrm{tr}\, G(\mathbf{0})\, G(\mathbf{x}) \rangle$$

$$G(\mathbf{x}) = g\, U(\mathbf{0} \to \mathbf{x})\, F_{xy}(\mathbf{x})\, U(\mathbf{x} \to \mathbf{0})$$

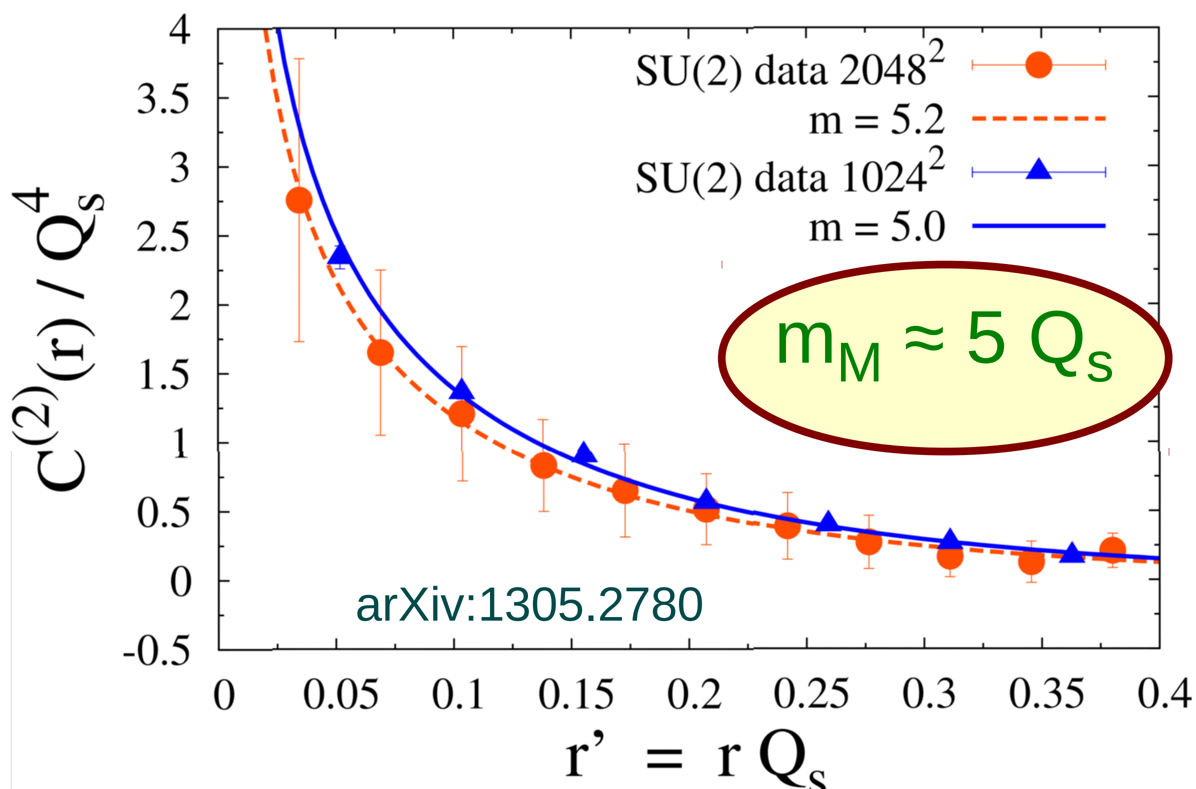
expectation:
(d=2)

$$\int d^d p \, \frac{1}{p^2 + m^2} \sim \frac{1}{r^{(d-1)/2}} \; \exp(-m\, r)$$

$$\sigma_M = \frac{1}{2} \int d^2 r \; C^{(2)}(r)$$

(Yu. Simonov et al)

satisfied to good approx



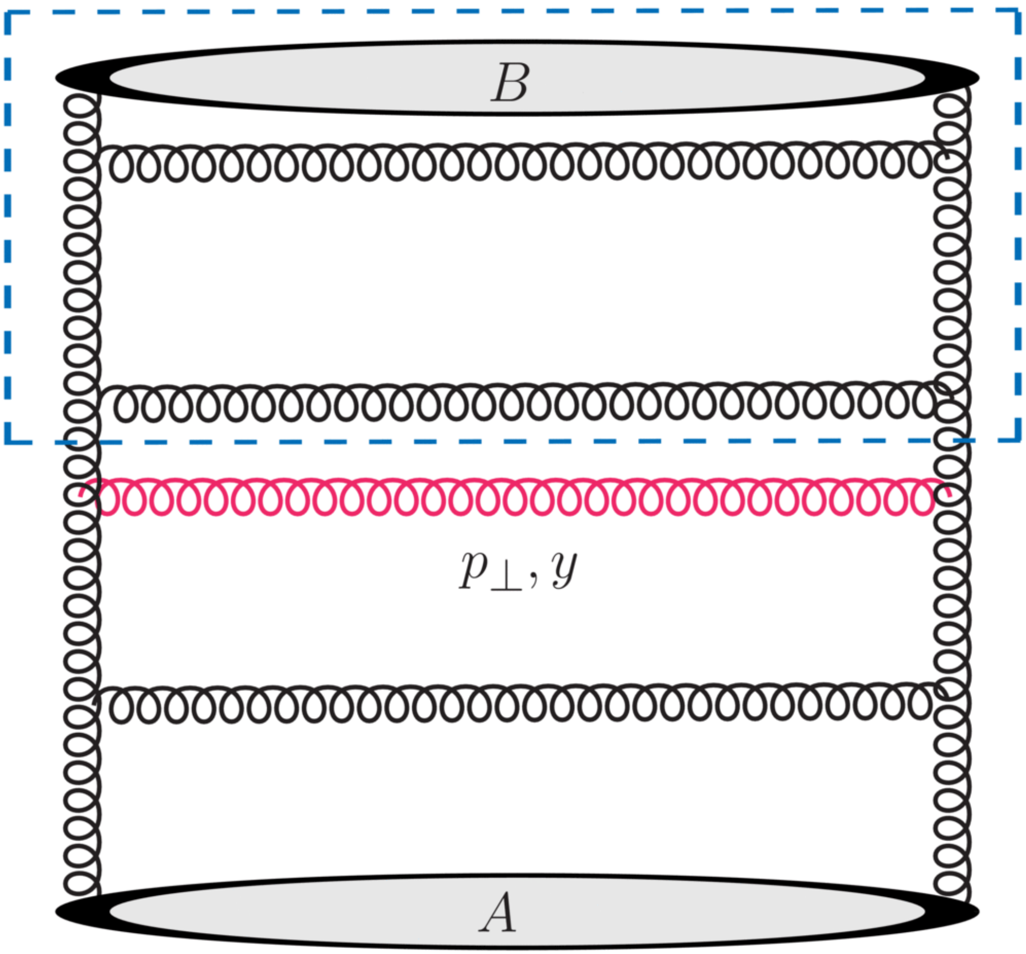
all of the above: flucs of classical field at y~0

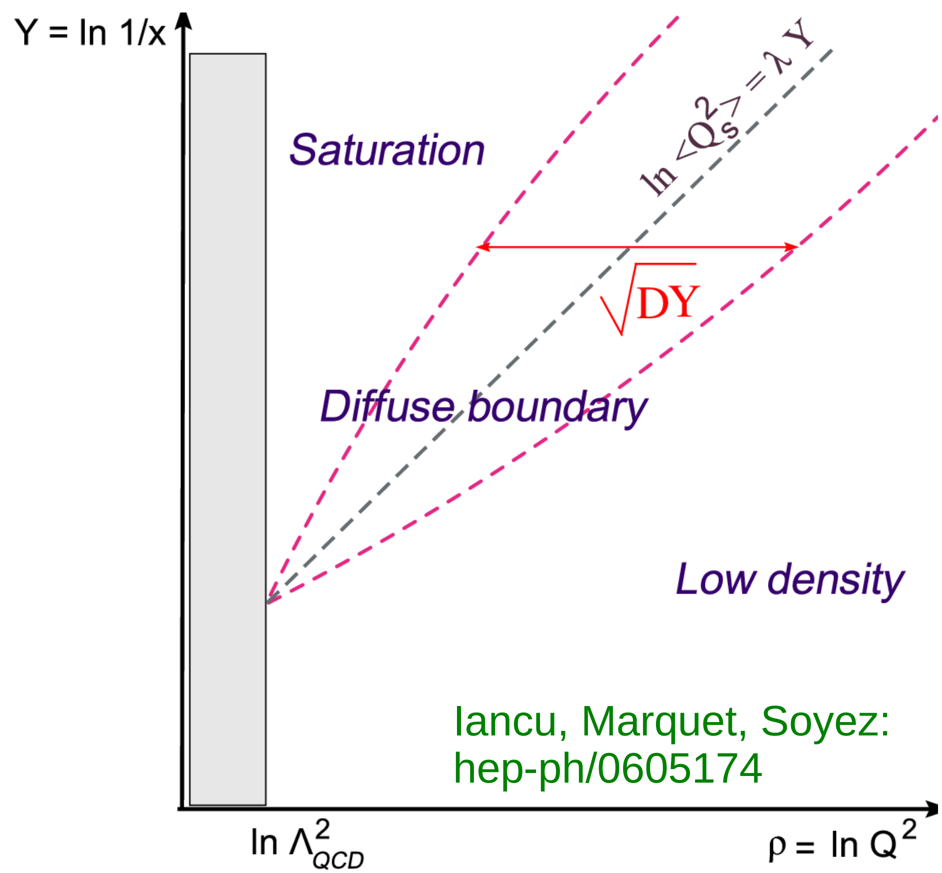
$$S_{\rm cl}[\rho] = \int d^2x_T \, \frac{\rho^a \rho^a}{Q_s^2/g^4}$$

valence charge distribution $P[\rho] = e^{-S_{\rm cl}[\rho]}$

averaged

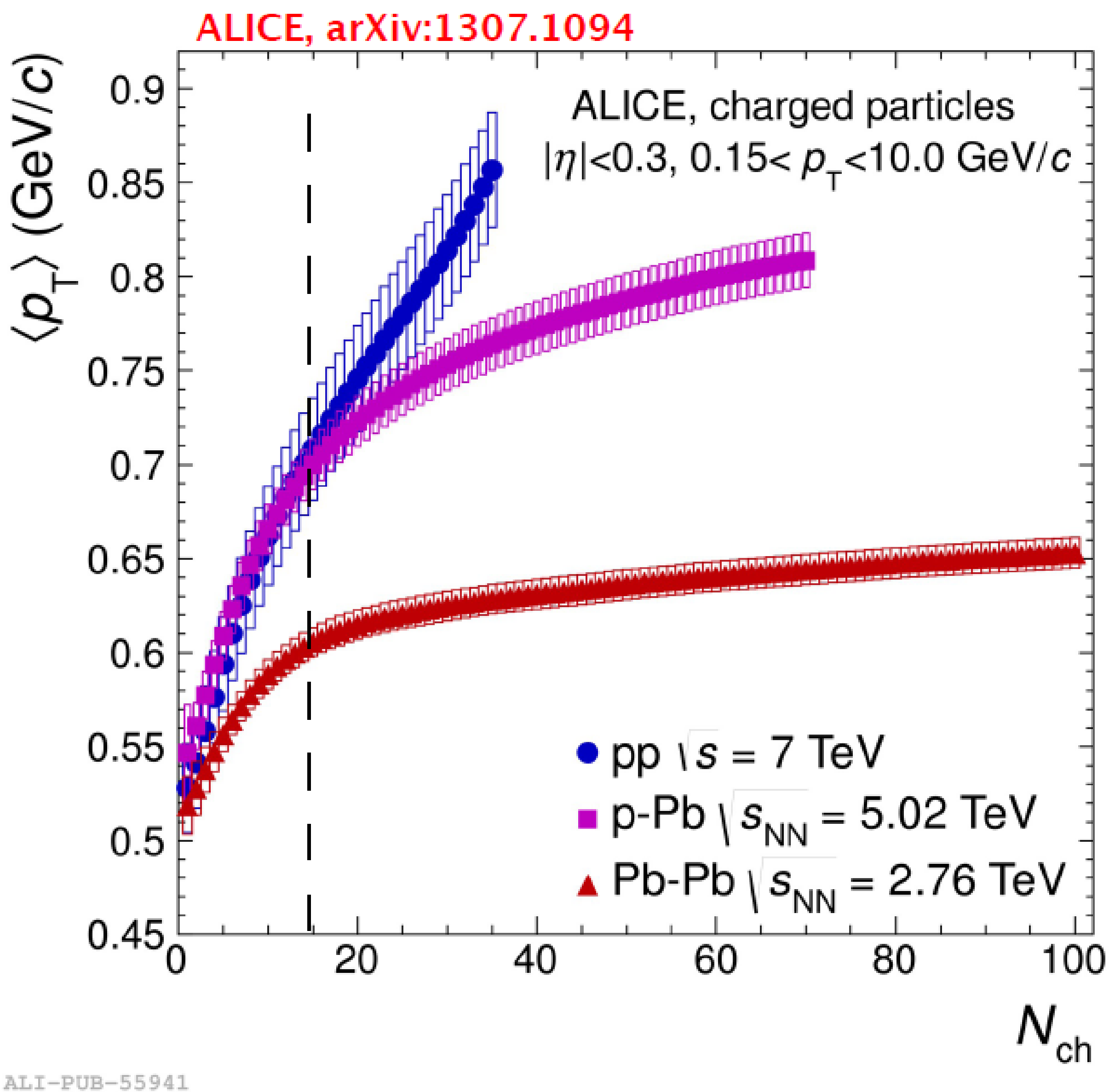
though there should also be flucs in *small-x quantum evolution*





Iancu, Marquet, Soyez: hep-ph/0605174

Mean p_T in pp, p-Pb and Pb-Pb



- Three different $\sqrt{s}$, but $\sqrt{s}$ dependence expected to be weak (known from pp)

- Much stronger increase wrt Pb-Pb

- p-Pb $<p_T>$ follows pp in region up to $N_{ch} \approx 14$

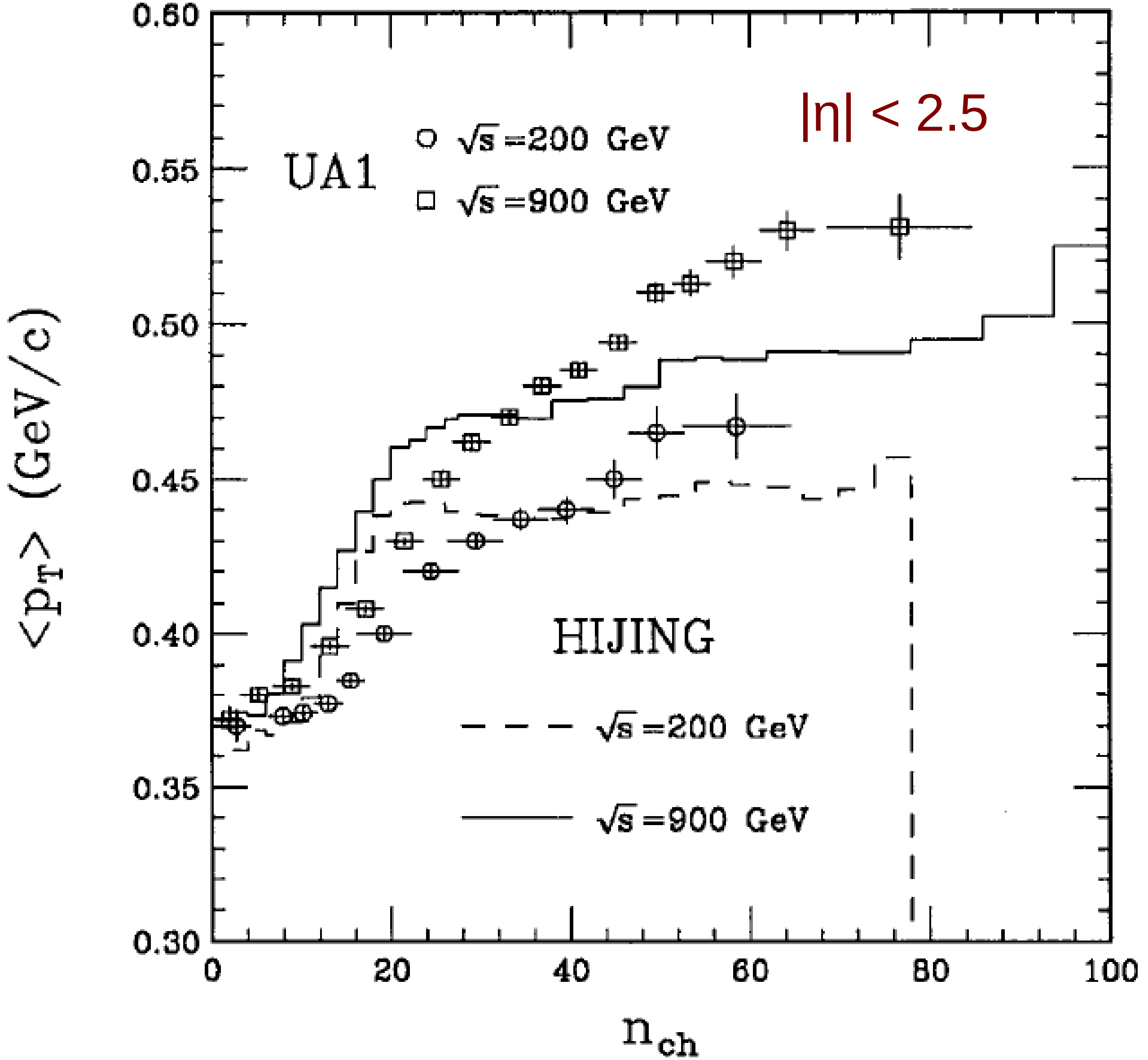
$N_{ch} > 14$ corresponds to
10% of pp x-section:
- pp already highly biased

50% of pPb x-section
- only centrality bias

Scaling with multiplicity
~ number of parton-parton interactions (number of initial strings) ?

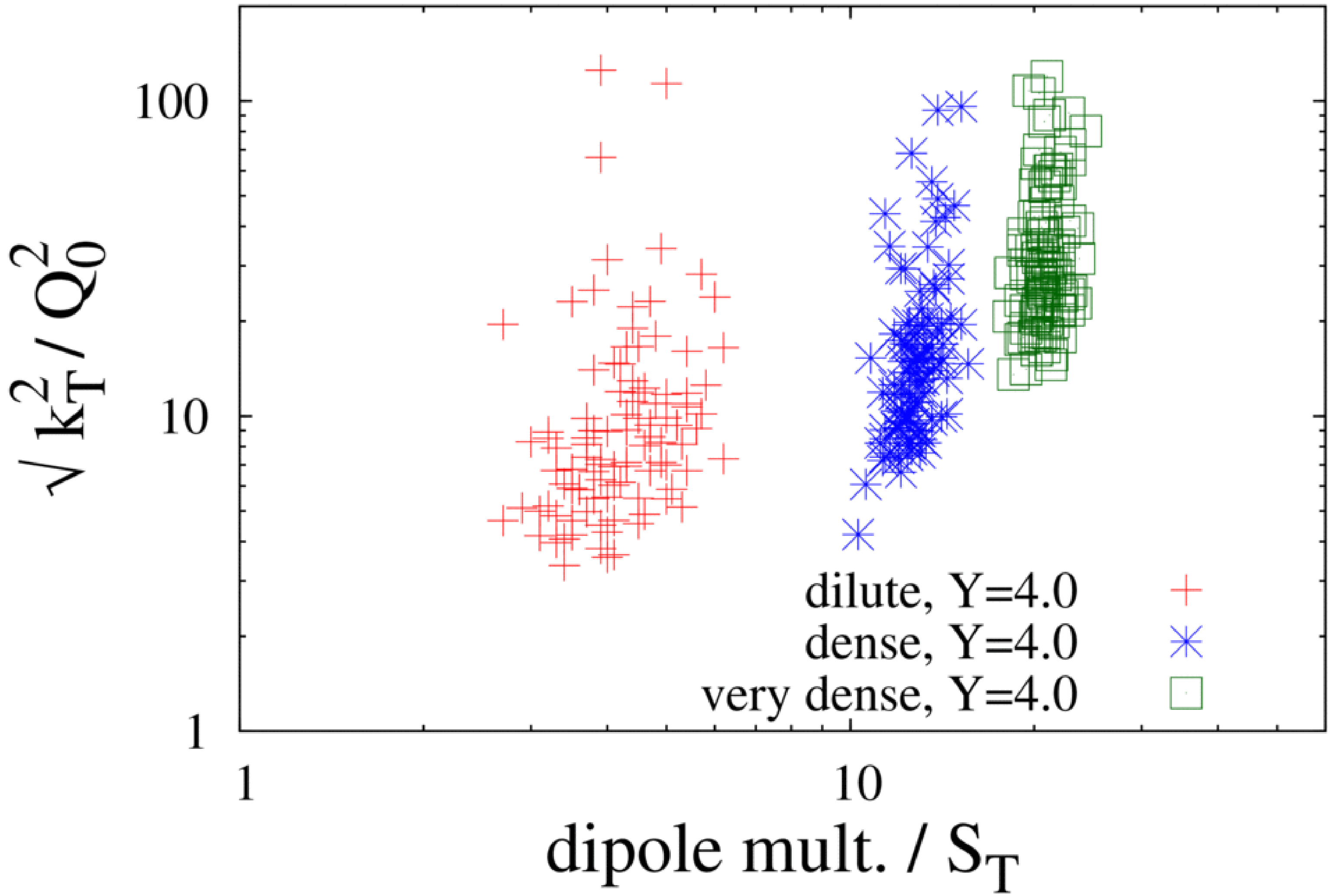
Fluctuation of # of "mini-jets" :

X.-N. Wang & M. Gyulassy: PRD (1992)



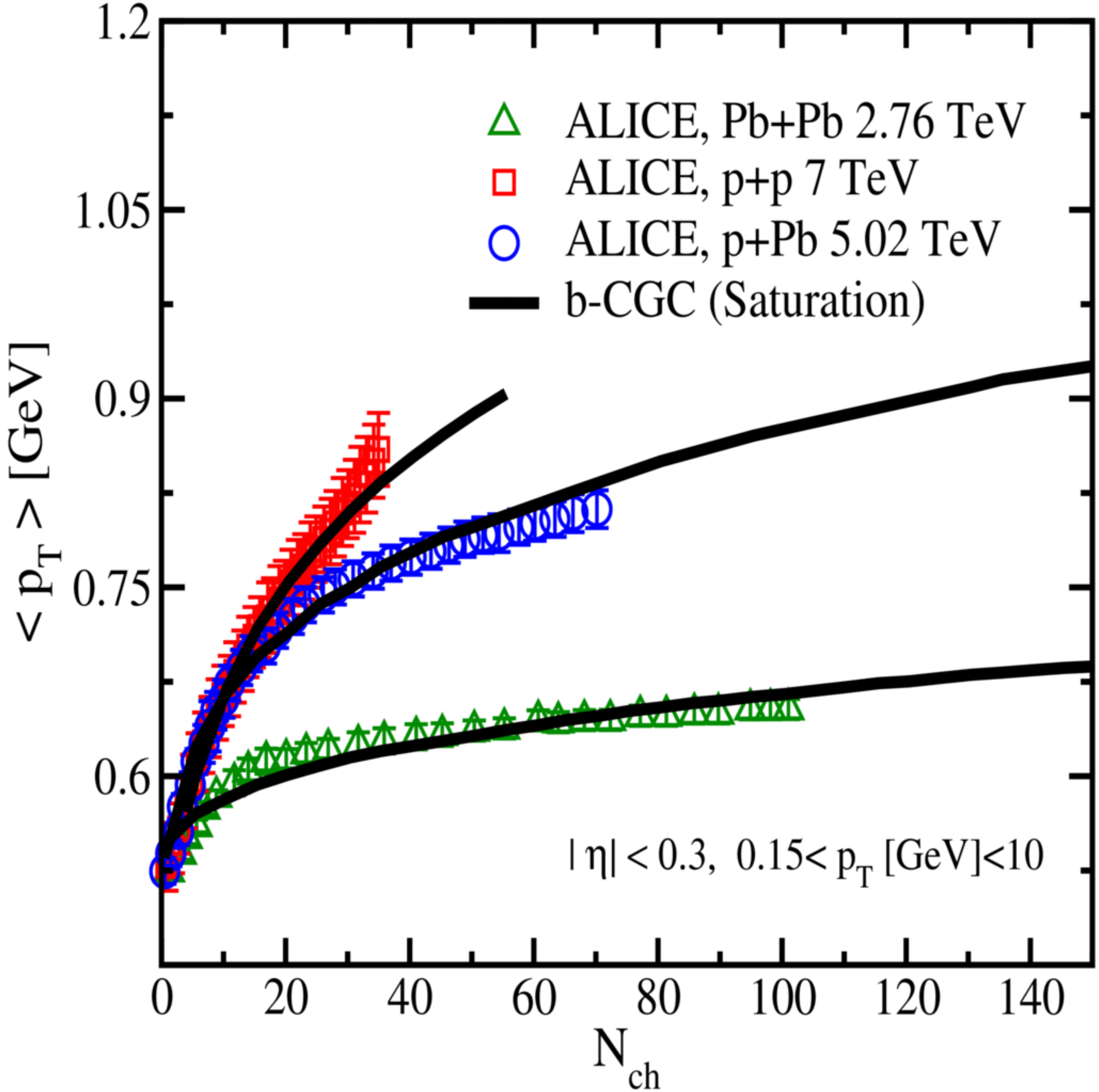
"In principle, there is no clear boundary between soft and hard processes. In HIJING, p0 is only a phenomenological scale which divides interactions into non-perturbative soft processes and PQCD hard or semi-hard processes."

r.c. evolution with fluctuations (dipole density in proton / A):



- large fluctuations, p_T increases somewhat with mult.
- is this … it ? (aka “Pomeron loops”)

Amir fits it via $Q_s^2 \to (N_{ch}/\langle N_{ch}\rangle)\, Q_s^2$



A. Rezaeian:
arXiv:1308.4736

Sources of fluctuations :

- proton profile function $\Gamma(x_T) \longrightarrow Q_s{}^2(x_0)$ (p+p)
- $N_{part} \longrightarrow Q_s{}^2(x_0)$ (p+A, A+A)
- quantum evolution $\longrightarrow S_{eff,Y}[\rho]$, $P_Y[\rho]$
- classical "valence" charge $\rho(x_T)$
- hadronization ...
- ...

Summary

- rcBK / AAMQS gluon distribution appears to have reasonably accurate x- and k_T-dependence

- A+A : fluctuations in initial state look like system of vortices, radius $R_{vtx}=1/Q_s$, 2d density $1/(20\ Q_s^2)$, string tension $\sqrt{\sigma}=0.34\ Q_s$

- p+p & p+A : better understanding of fluctuations needed :

  ✔ N_{part}

  ✗ classical fluctuations of ρ ? ($S_{cl} \sim \int \rho\rho$ for protons ?)

  ✗ fluctuations in quantum evolution ?

  ✗ hadronization of small-x gluons ?

Backup Slides

Mueller-Iancu-Triantafyllopoulos model for JIMWLK evolution (+fluctuations) in terms of dipoles

r.c. version
A.D., Iancu, Portugal, Soyez, Triantafyllopoulos: arXiv:0706.2540

dipole configuration in target wave function $n(x)$, $x \equiv \log \frac{r_0^2}{r^2}$

master equation for evolution of P[n(x)] functional :

$$\frac{\partial P[n(x),Y]}{\partial Y} = \int_z f_z[n(x) - \delta_{xz}]\, P[n(x) - \delta_{xz}, Y] - \int_z f_z[n(x)]\, P[n(x), Y]$$

gain loss

$$f_z[n(x)] = \frac{T_z[n(x)]}{\alpha(z)}$$

“deposit”/splitting rate

$$T_z[n(x)] = 1 - \exp \int_x n(x)\ \log\ 1 - \tau(z|x)$$

scattering amplitude of dipole of size *z* off target
(involves pair density etc)

$$\tau(x|y) = \alpha(x)\alpha(y)\exp(-|x-y|) \equiv \alpha(r_<^2)\,\alpha(r_>^2)\,\frac{r_<^2}{r_>^2}$$

elementary dip-dip
scattering amplitude

Balitsky hierarchy

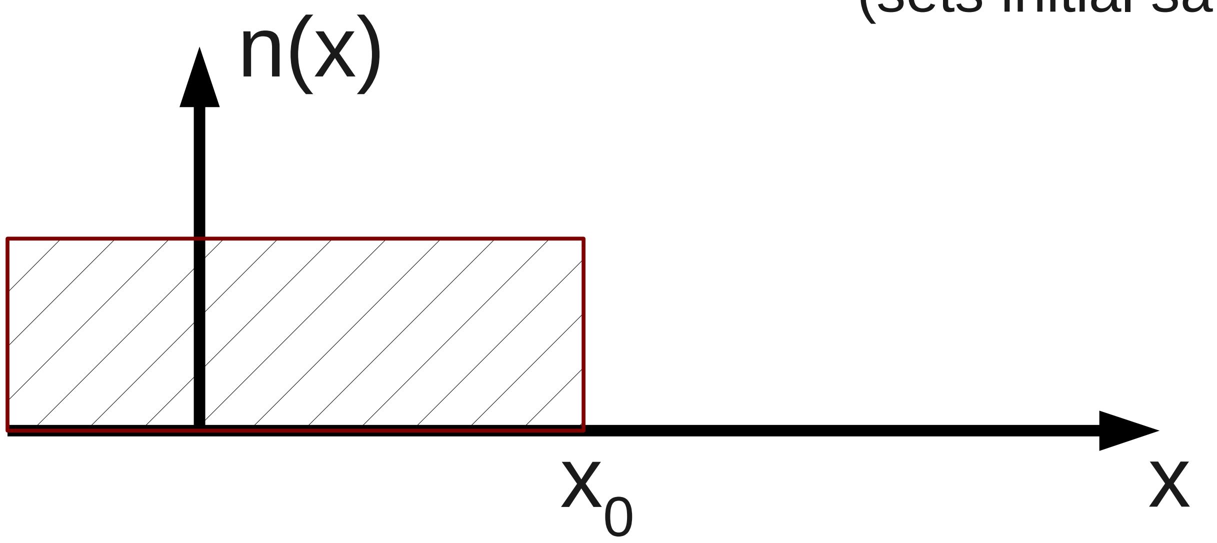
$$\frac{\partial\langle T_x\rangle}{\partial Y} = \alpha_x \int_z K_{xz}\,\langle T_z(1-T_x)\rangle$$

$$\begin{aligned}\frac{\partial\,\langle T_x T_y\rangle}{\partial Y} &= \alpha_x \int_z K_{xz}\,\langle T_z T_y(1-T_x)\rangle + \alpha_y \int_z K_{yz}\,\langle T_z T_x(1-T_y)\rangle \\ &+ \alpha_x\alpha_y \int_z \alpha_z K_{xz} K_{yz}\,\langle T_z(1-T_x)(1-T_y)\rangle\end{aligned}$$

solved with following initial condition at Y=0:

$$n(x) = 2\Theta(x - x_0) \quad ; \quad x_0 = 0.6 \text{ (proton)}, = 3.5 \text{ (nucleus)}$$

(sets initial saturation scale)



same initial condition for all configurations

but evolution introduces fluctuations !