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# Percolation Approach to Initial Stage Effects in High Energy Collisions

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***in collaboration with***

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**IS2013**
**Sept. 8-14, 2013**
**Illa de A Toxa, Galicia, Spain**

# Percolation : General

The general formulation of the percolation problem is concerned with elementary geometrical objects placed at random in a d-dimensional lattice. The objects have a well defined connectivity radius $\lambda$, and two objects are said to communicate if the distance between them is less than $\lambda$.

One is interested in how many objects can form a cluster of communication and, especially , when and how the cluster become infinite. The control parameter is the density of the objects or the dimensionless filling factor $\xi$. The percolation threshold $\xi = \xi_c$ corresponding to the minimum concentration at which an infinite cluster spans the space.

# Percolation : General

It is well known that the percolation problem on large lattices displays the features of a system undergoing a second-order phase transition.

These characteristics include critical fluctuations, quantities which diverge, and quantities which vanish as the critical percolation probability is approached. These quantities are described by a finite number of critical exponents.

*** Transition from liquid to gas**
*** Normal conductor to a superconductor**
*** Paramagnet to ferromagnet**

1. H. E. Stanley , Introduction to Phase Transitions and Critical Phenomena
2. D. Stauffer and A. Aharony, Introduction to Percolation Theory

# Percolation : General

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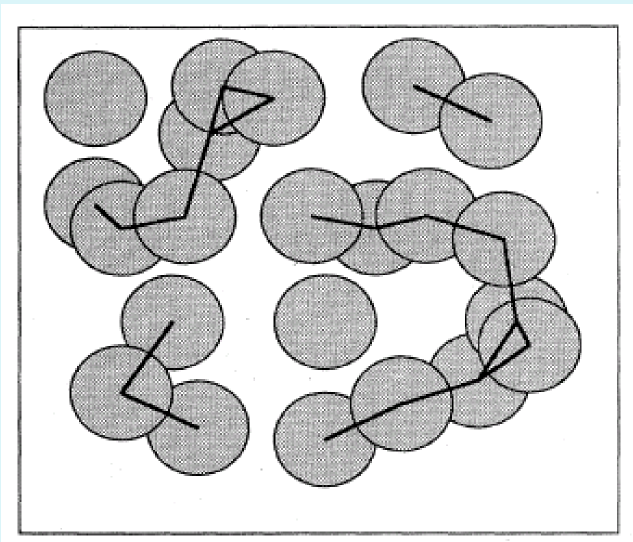
**Percolation, statistical topography, and transport in random media**

M. B. Isichenko

Reviews of Modern Physics, Vol. 64, No. 4, October 1992



$$\xi = \pi n r^2$$

**ξ is the percolation density parameter**

**n is the density and r the radius of the disc**

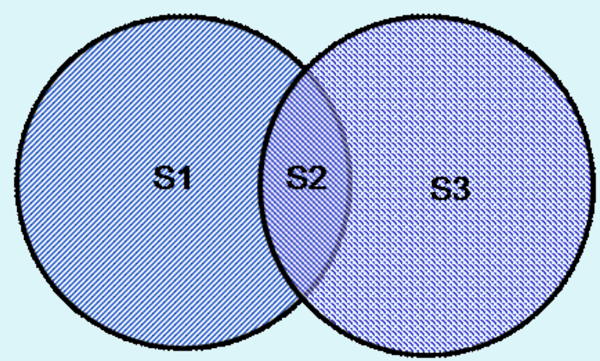
**φ is the fractional area covered by the cluster**

$$\phi = 1 - e^{-\xi}$$

**$\xi_c$ is the critical value of the percolation densit at which a communicating cluster appears**

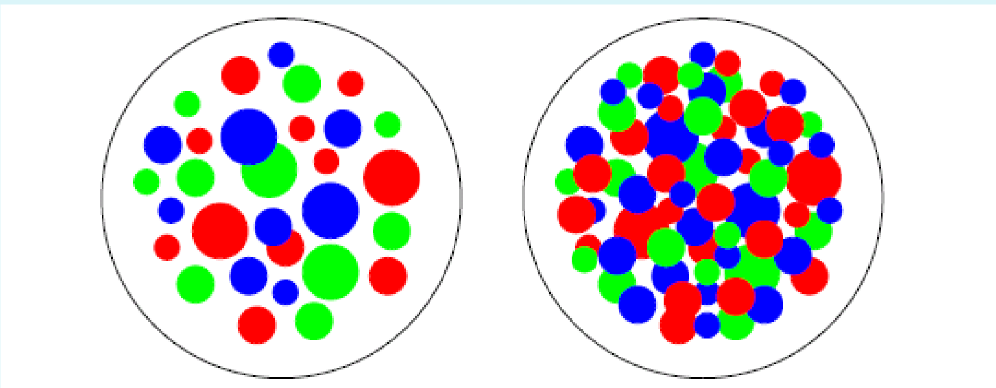
**For example at $\xi_c$ =1.2 , φ ~ 2/3**
**It means ~ 67% of the whole area is covered by the cluster**

**In the nuclear case it is the overlap area**



# Parton Percolation

Multiparticle production at high energies is currently described in terms of color strings stretched between the projectile and target. Hadronizing these strings produces the observed hadrons. The no. of strings grow with energy and the no. of participating nuclei and one expects that interaction between them becomes essential.



**Clustering of Color Sources**

De-confinement is thus related to cluster formation very much similar to cluster formation in percolation theory

Critical Percolation Density $\xi_c = 1.18$

# Color Strings + Percolation = CSPM

## Multiplicity and $<p_T^2>$ of particles produced by a cluster of *n* strings

**Multiplicity ($\mu_n$)**

$$\mu_n = F(\xi) N^s \mu_1$$

**Average Transverse Momentum**

$$< p_T^2 >_n = < p_T^2 >_1 / F(\xi)$$

$$\xi = \frac{N^s S_1}{S_N}$$

$$F(\xi) = \sqrt{\frac{1-e^{-\xi}}{\xi}}$$

= **Color suppression factor** (due to overlapping of discs).

$N^s$ = # of strings
$S_1$ = disc area
$S_N$ = total nuclear overlap area

$\xi$ **is the percolation density parameter**.

M. A. Braun and C. Pajares, Eur.Phys. J. C16,349 (2000)
M. A. Braun et al, Phys. Rev. C65, 024907 (2002)

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# Percolation and Color Glass Condensate

Both are based on parton coherence phenomena.

**Percolation : Clustering of strings**

**CGC : Gluon saturation**

- **Many of the results obtained in the framework of percolation of strings are very similar to the one obtained in the CGC.**
- **In particular , very similar scaling laws are obtained for the product and the ratio of the multiplicities and transverse momentum.**
- **Both provide explanation for multiplicity suppression and $\langle p_t \rangle$ scaling with dN/dy.**

**Momentum $Q_s$ establishes the scale in CGC with the corresponding one in percolation of strings**

$$Q_s^2 = \frac{k < p_t^2 >_1}{F(\xi)}$$

**For large value of ξ**

$$Q_s^2 \propto \sqrt{\xi}$$

**The no. of color flux tubes in CGC and the effective no. of clusters of strings in percolation have the same dependence on the energy and centrality.**

**This has consequences in the Long range rapidity correlations and the ridge structure.**

CGC : Y. V. Kovchegov, E. Levin, L McLerran, Phys. Rev. C 63, 024903 (2001).

# Results

## Bulk Properties

- ❑ Multiplicity
- ❑ pt distribution
- ❑ Particle ratios
- ❑ Elliptic flow
- ❑ Forward-Backward Multiplicity Correlations at RHIC

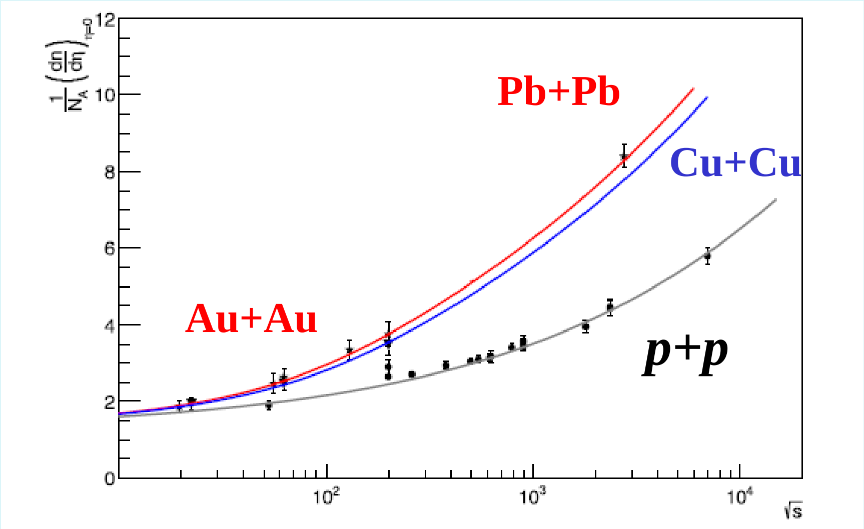
## Thermodynamics

- ❑ Temperature
- ❑ Energy Density
- ❑ Shear viscosity to Entropy density ratio
- ❑ Equation of State

**Determination of the Color Suppression Factor F(ξ) from the Data**

# Multiplicity and Ridge

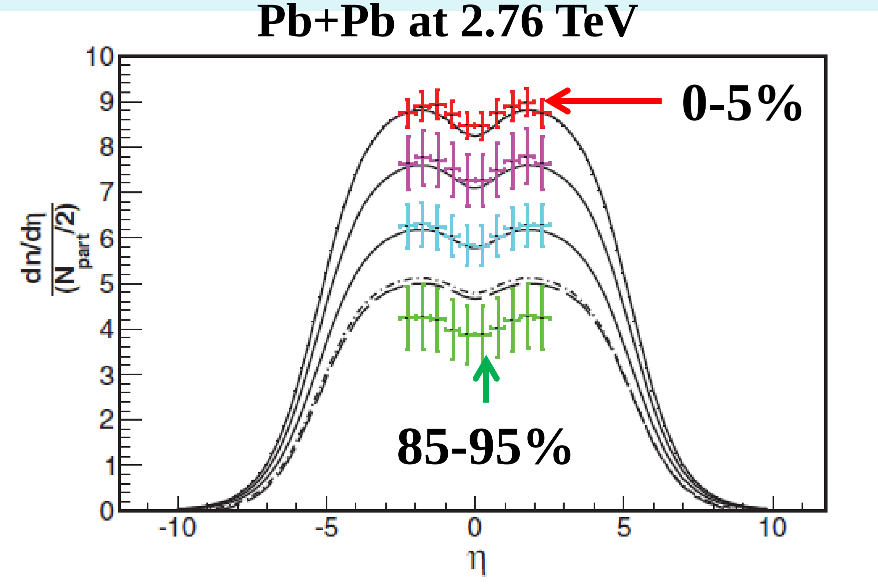
## Multiplicity dependence on √s



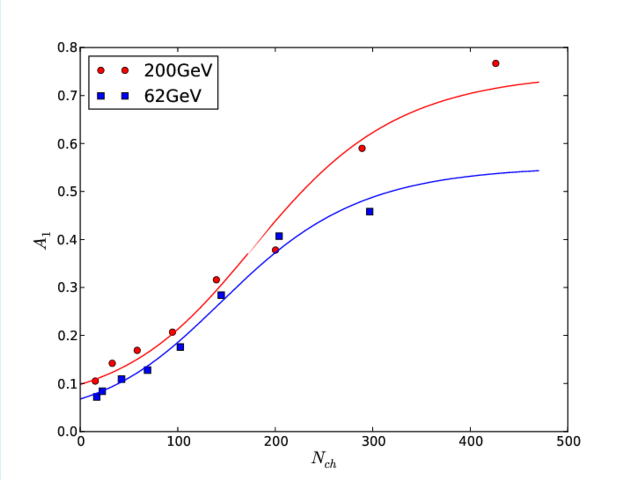

Phys. Lett., 715, 230 (2012)
ALICE Coll. Phys. Rev. Lett., 106, 032301 (2011)

## Rapidity dependence of particle densities

Pb+Pb at 2.76 TeV




Phys. Rev. C 86, 034909(2012)
CMS Coll., JHEP, 08, 141 (2011).




**Near side ridge strength in percolation and comparison with the STAR data**

**On the onset of ridge structure..**
**Alexis Moscoco**
**Parallel Session 2A , Thursday**

# Elliptic flow at RHIC and LHC

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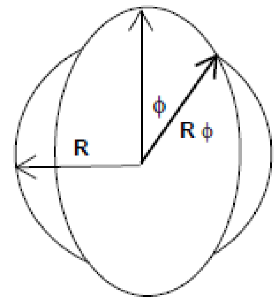



- **The cluster formed by the strings has generally asymmetric shape in the transverse plane. This azimuthal asymmetry is at the origin of the elliptic flow in percolation.**

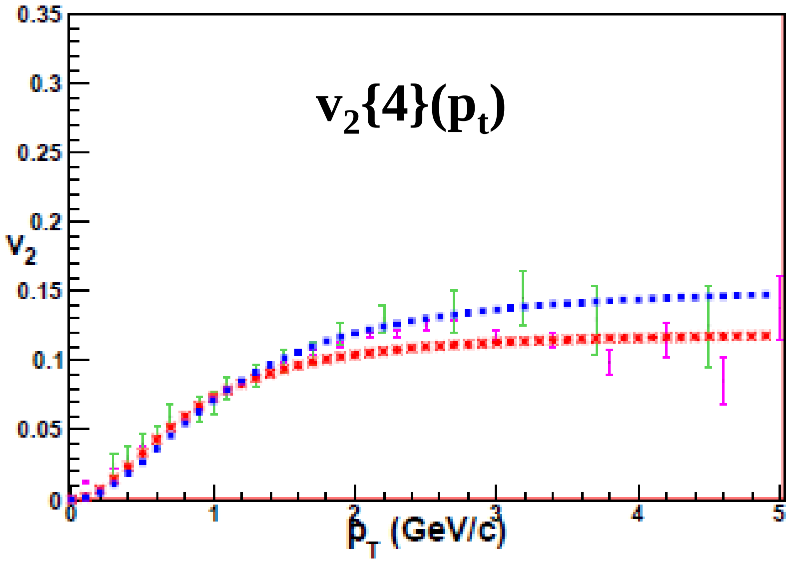
- **The partons emitted inside the cluster have to pass a certain length through the strong color field and loose energy.**

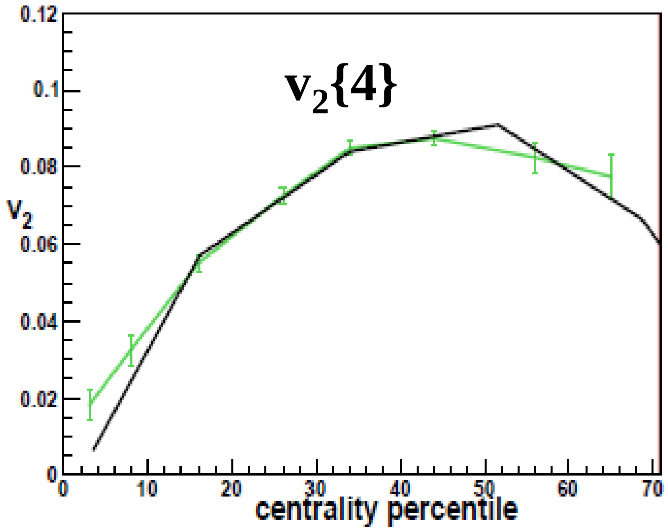
- **The energy loss by the partons is proportional to the length and therefore the $p_t$ of an particle will depend on the path length travelled and will depend on the direction of emission**

$$v_2 = \frac{2}{\pi}\int_0^{\pi} d\varphi \cos 2\phi \left(\frac{R_\phi}{R}\right)\left(\frac{e^{-\xi} - F(\xi)^2}{2F(\xi)^3}\right)\frac{R}{R-1}$$

I.Bautista, J. Dias de Deus , C. Pajares, arXiv 1102:3837
M. A. Braun , C. Pajares, Eur. J. Phys. C, 71 (2011) 1558

# Elliptic flow at RHIC and LHC




**Charged hadron elliptic flow**
ALICE: 0-10% centrality Pb+Pb@2.76 TeV
Phys. Rev. Lett, 105 (2010) 252302
STAR: 0-10% centrality Au+Au@200 GeV
Phys. Rev. C 77, (2008) 054901

**Elliptic flow as a function of centrality in Pb+Pb@2.76 TeV**
**Comparison with ALICE**

**Higher harmonics $v_3$ to $v_8$ have also been studied in percolation approach**
M. A. Braun, C. Pajares, V.V. Vechernin, arXiv:1204:5829

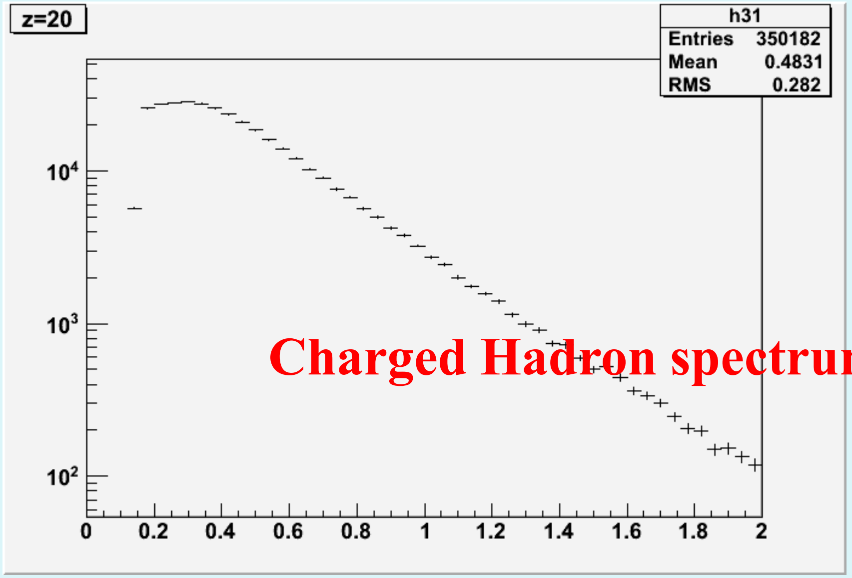
# Data Analysis

## Using the $p_T$ spectrum to calculate $\xi$

**The experimental $p_T$ distribution from pp data is used**

$$\frac{d^2N}{dpt^2} = \frac{a}{(p_0 + pt)^n}$$

$$\frac{d^2N}{dpt^2}$$




pt GeV/c

**a, $p_0$ and $n$ are parameters fit to the data.**

**This parameterization can be used for nucleus-nucleus collisions to account for the clustering :**

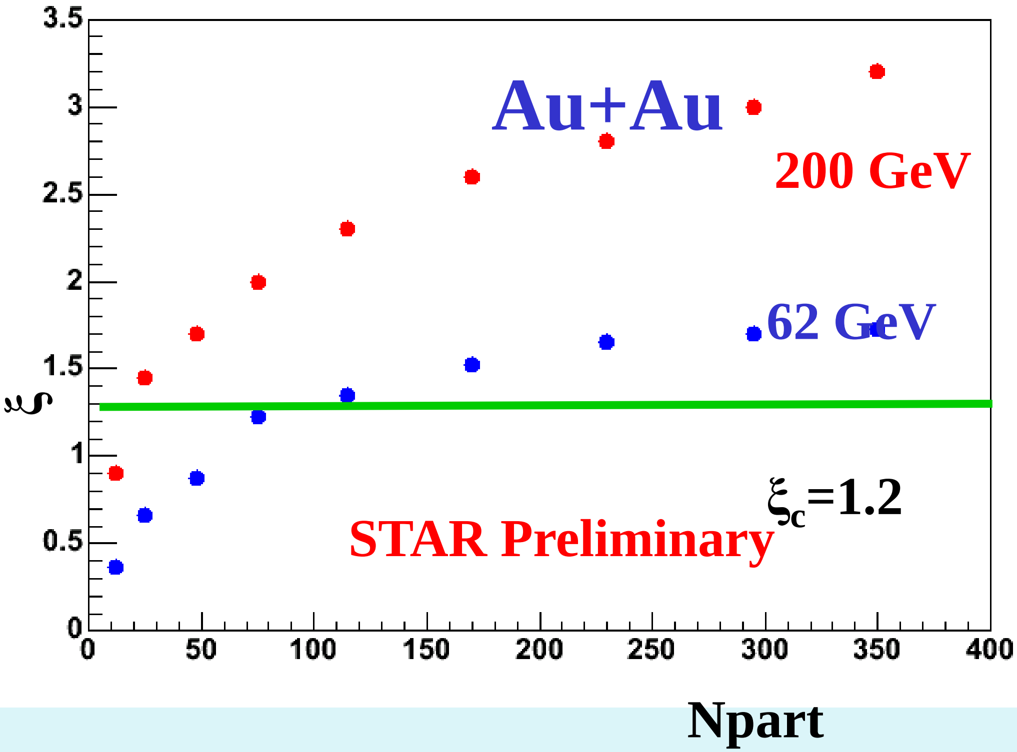
$$\frac{d^2N}{dpt^2} = \frac{b}{\left(p_0\sqrt{\frac{F(\xi_{pp})}{F(\xi_{AuAu})}} + pt\right)^n}$$

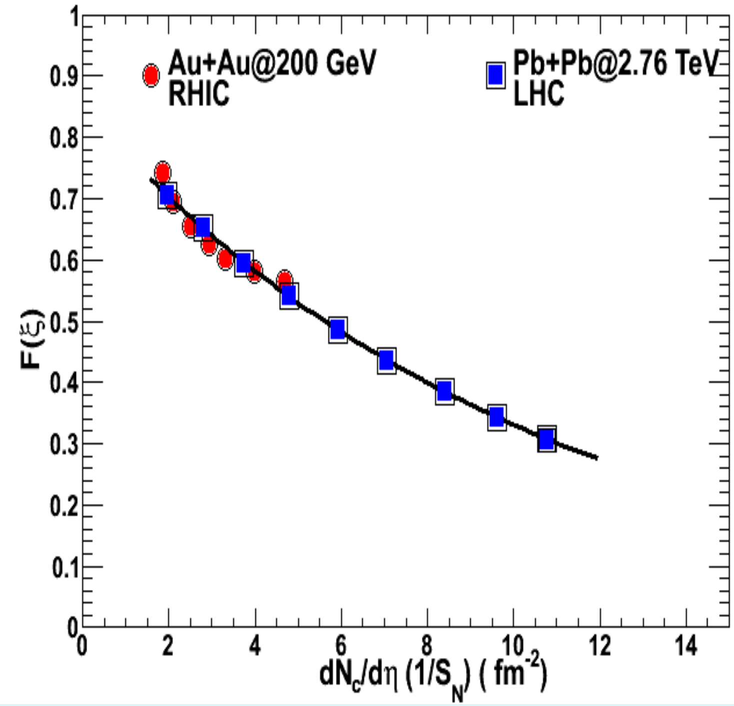
$$F(\xi)_{pp} = 1$$

$$F(\xi)_{AuAu} = 0.57$$

**For central collisions**

# Percolation Density Parameter ξ




$$F(\xi) = \sqrt{\frac{1-e^{-\xi}}{\xi}}$$

Using ALICE charged particle multiplicity
Phys. Rev. Lett. , 106, 032301 (2011).

**Now the aim is to connect F(ξ) with Temperature and Energy density**

**QM06**

# Thermalization

- The origin of the string fluctuation is related to the stochastic picture of the QCD vacuum . Since the average value of color field strength must vanish, it cannot be constant and must vanish from point to point. Such fluctuations lead to the Gaussian distribution of the string.

  **H. G. Dosch, Phys. Lett. 190 (1987) 177**

  **A. Bialas, Phys. Lett. B 466 (1999) 301**

- The fast thermalization in heavy ion collisions can occur through the existence of event horizon caused by rapid deceleration of the colliding nuclei. Hawking-Unruh effect.

  **D. Kharzeev, E. Levin , K. Tuchin, Phys. Rev. C75, 044903 (2007)**

  **H.Satz, Eur. Phys. J. 155, (2008) 167**

# Schwinger Mechanism of Particle Production

**$p_t$ distribution of the produced quarks**

$$\frac{dn}{d^2 p_\perp} \sim \exp(-\frac{\pi p_t^2}{k})$$

**k is the string tension**

**The tension of the macroscopic cluster fluctuates around its mean value because the chromoelectric field is not constant . Assuming a Gaussian form for these fluctuations one arrives at the probability distribution of transverse momentum.**

**Thermal Distribution**

$$\frac{dn}{d^2 p_\perp} \sim \exp(-\frac{\pi p_t}{T})$$

$$T = \sqrt{\frac{\langle k \rangle}{2\pi}}$$

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

**Cluster /Initial Temperature**

# Temperature

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

At the critical percolation density

$\xi_c = 1.2$ $T_c$ = 167 MeV

For Au+Au@ 200 GeV

0-10% centrality ξ = 2.88 T ~ 195 MeV

PHENIX:

Temperature from direct photon

Exponential (consistent with thermal)

Inverse slope = 220 ± 20 MeV

PRL 104, 132301 (2010)

Pb+Pb @ 2.76TeV for 0-5%

T ~265 MeV

Temperature has increased by 35% from Au+Au @ 0.2 TeV

First Results from Pb+Pb Collisions@ 2.76 TeV at the LHC

Muller, Schukraft and Wyslouch, Ann. Rev. Nucl. Sci. Oct. 2012

ALICE : Direct Photon Measurement

$T$ = 304 ± 51 MeV

QM 2012

# Energy Density

Bjorken Phys. Rev. D 27, 140 (1983)

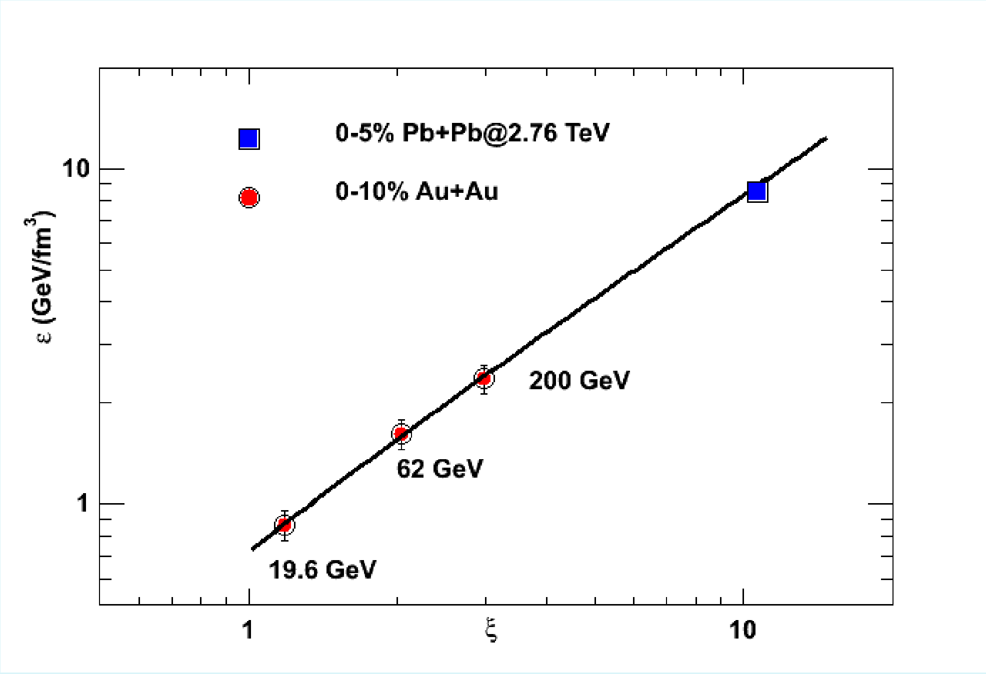
$$\varepsilon = \frac{3}{2}\frac{dN_c}{dy}\frac{<m_t>}{A}\frac{1}{\tau_{pro}} GeV/fm^3$$

Transverse overlap area

Proper Time

$\tau_{pro}$ is the QED production time for a boson which can be scaled from QED to QCD and is given by

$$\tau_{pro} = \frac{2.405\hbar}{<m_t>}$$




$$\varepsilon \propto \xi$$

Introduction to high energy heavy ion collisions
C. Y. Wong

J. Dias de Deus, A. S. Hirsch, C. Pajares, R. P. Scharenberg, B. K. Srivastava
Eur. Phys. J. C 72, 2123 ( 2012)

**Having determined the initial temperature of the system from the data one would like to obtain the following quantities to understand the properties of QCD matter**

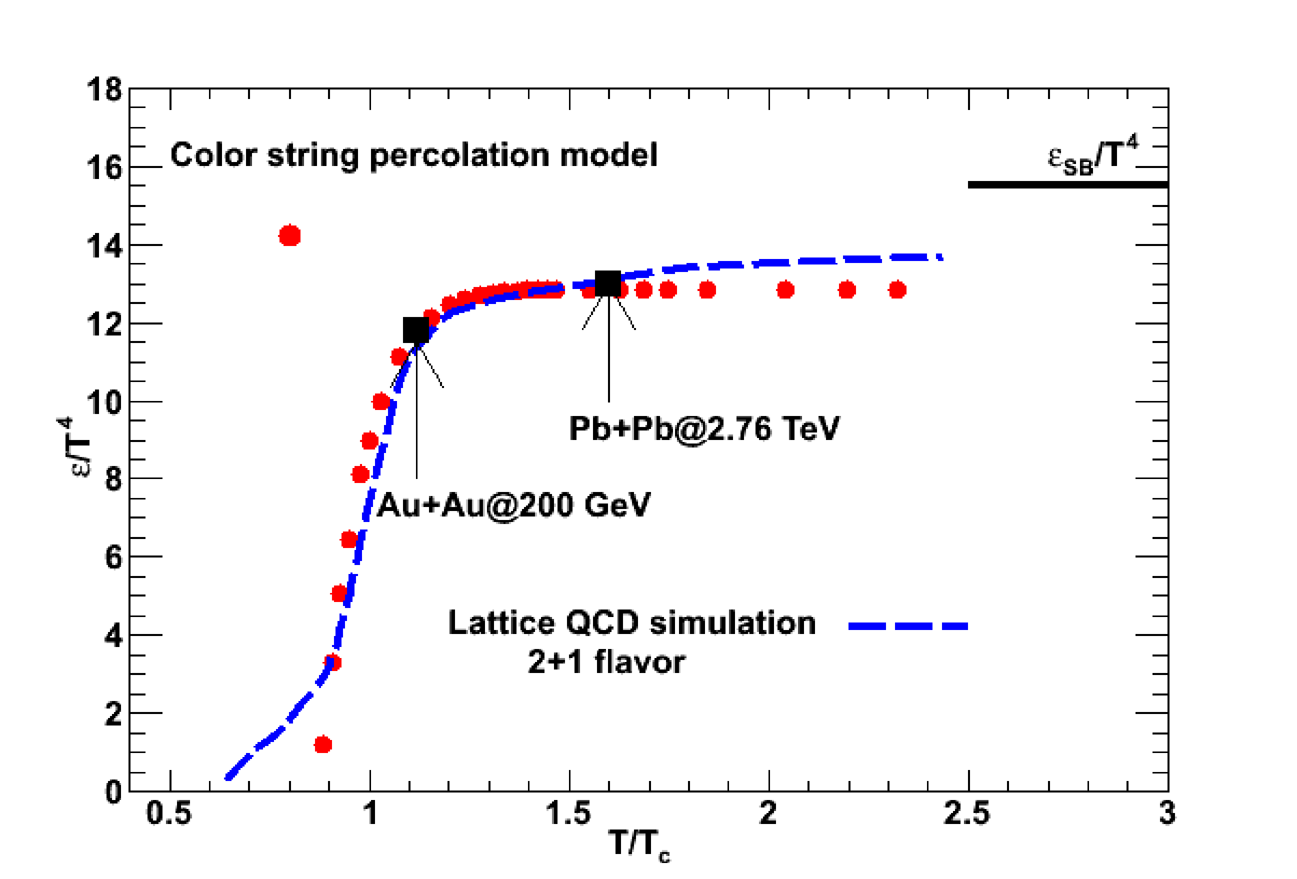
$$\varepsilon / T^4$$

**Shear Viscosity**

**Equation of State**

# Energy Density

**0-10% centrality**




**Lattice QCD : Bazavov et al, Phys. Rev. D 80, 014504(2009).**

**R. P. Scharenberg , B. K. Srivastava, A. S. Hirsch**
**Eur. Phys. J. C 71, 1510( 2011)**

PRL 97, 152303 (2006) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2006

# Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

Laszlo P. Csernai,[1,2] Joseph I. Kapusta,[3] and Larry D. McLerran[4]

[1]*Section for Theoretical Physics, Department of Physics, University of Bergen, Allegaten 55, 5007 Bergen, Norway*
[2]*MTA-KFKI, Research Institute of Particle and Nuclear Physics, 1525 Budapest 114, P. O. Box 49, Hungary*
[3]*School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA*
[4]*Nuclear Theory Group and Riken Brookhaven Center, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA*

(Received 12 April 2006; published 12 October 2006)

Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity $\eta$ to entropy density $s$ by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields $\eta/s = \hbar/4\pi k_B$, which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio $\eta/s$. We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

# The viscosity can be estimated from kinetic theory to be

$$\eta \approx \frac{4}{15}\varepsilon(T)\lambda_{mfp} \approx \frac{1}{5}\frac{T}{\sigma_{tr}}\frac{s(T)}{n(T)}$$

$$\varepsilon(T) = \frac{3}{4}Ts$$

$$\lambda_{tr} = \frac{1}{(n\sigma_{tr})}$$

$$\frac{n}{s} \approx \frac{T\lambda_{mfp}}{5}$$

**Hirano & Gyulassy, Nucl. Phys. A769, 71(2006)**

**ε Energy density**

**s Entropy density**

**n the number density**

$\lambda_{mfp}$ **Mean free path**

$\sigma_{tr}$ **Transport cross section**

$\sqrt{< pt >_1^2}$ **Average transverse momentum of the single string**

**L is Longitudinal extension of the source 1 *fm***

$$\lambda_{mfp} = \frac{L}{1-e^{-\xi}}$$

$$\frac{\eta}{s} \approx \frac{1}{5}\frac{L}{1-e^{-\xi}}T$$

# Shear viscosity to entropy density ratio

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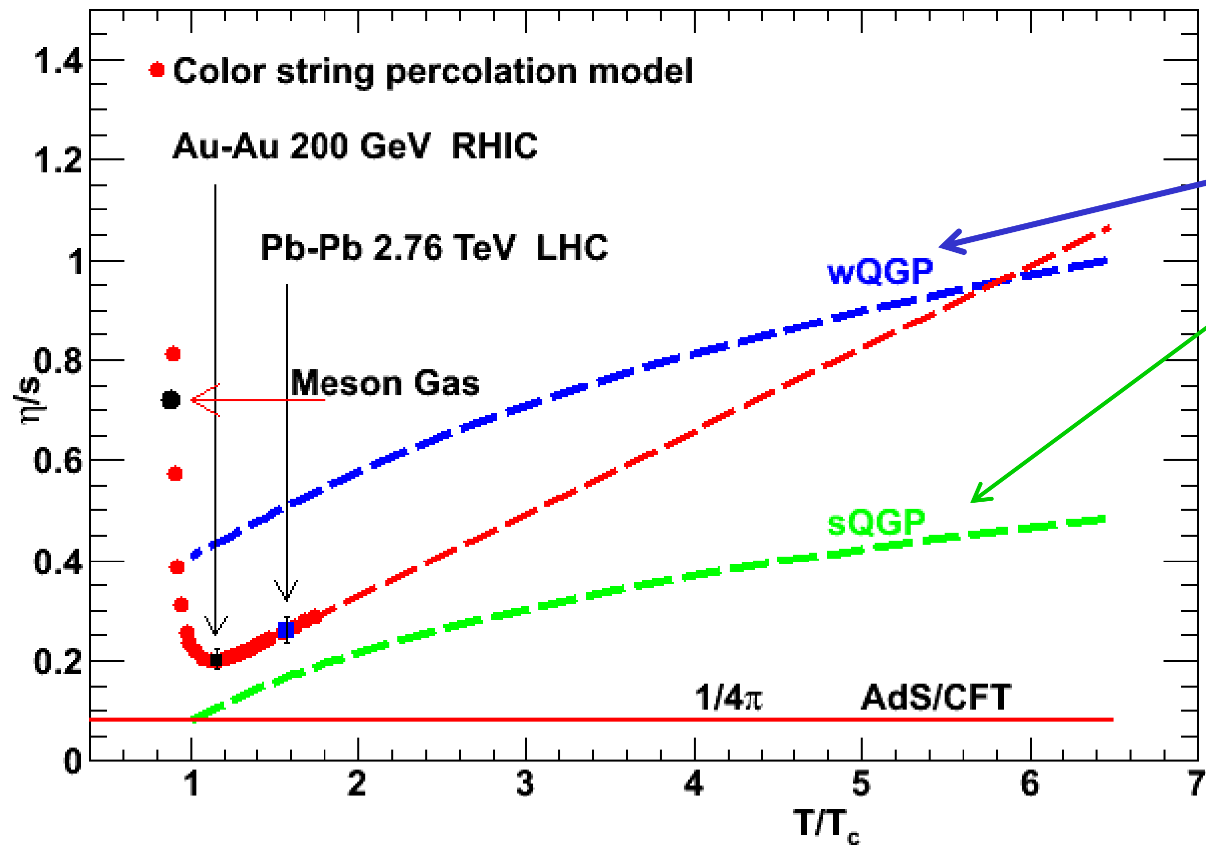
$$\frac{\eta}{s} \approx \frac{1}{5}\frac{L}{1-e^{-\xi}}T$$

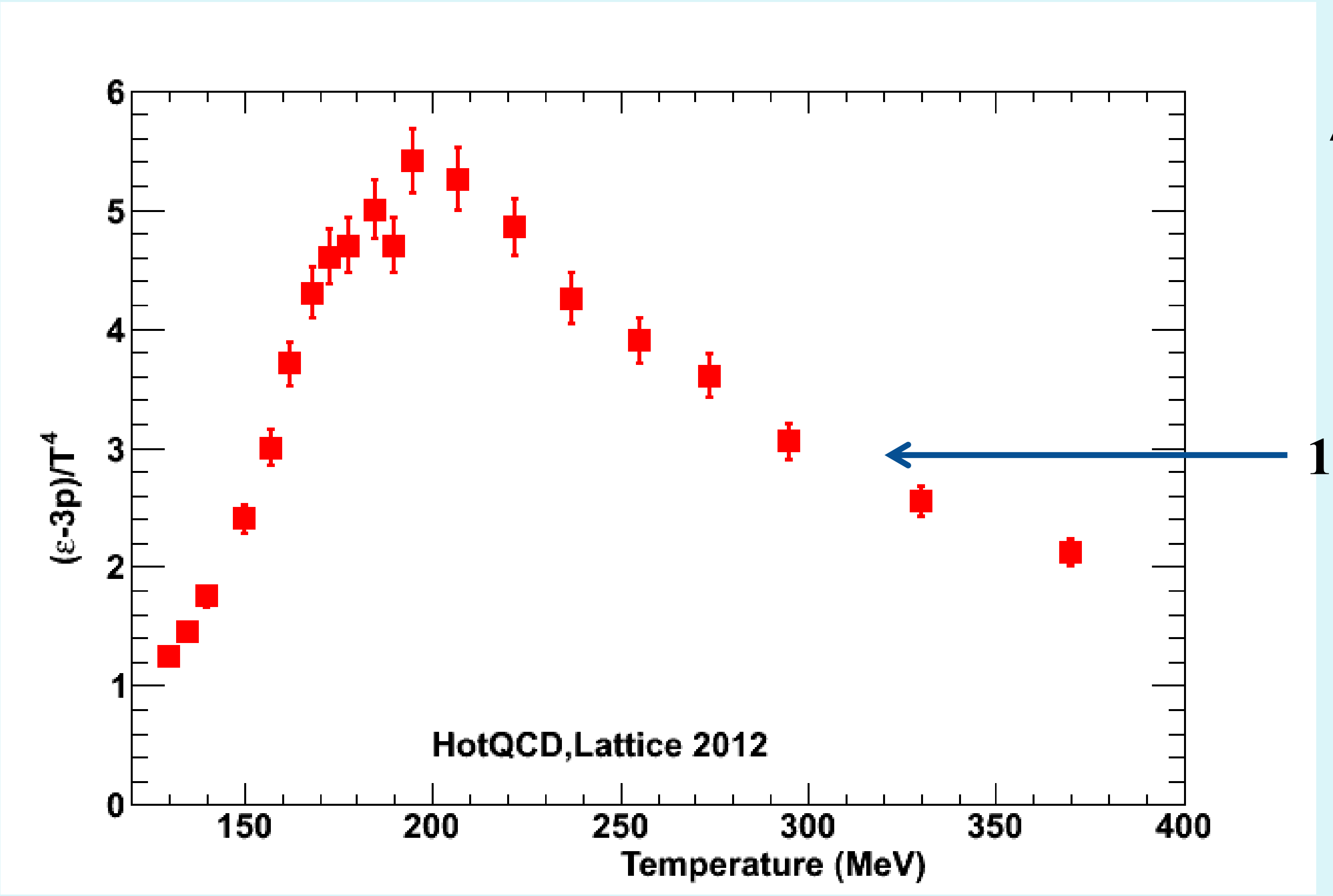



T. Hirano, M Gyulassy
Nucl. Phys.
A769, 71(2006)

J. Dias de Deus, A. S. Hirsch, C. Pajares , R. P. Scharenberg , B. K. Srivastava
Eur. Phys. J. C 72, 2123 ( 2012)

# How to obtain the pressure ?

**Interaction Measure from Lattice QCD Calculation**



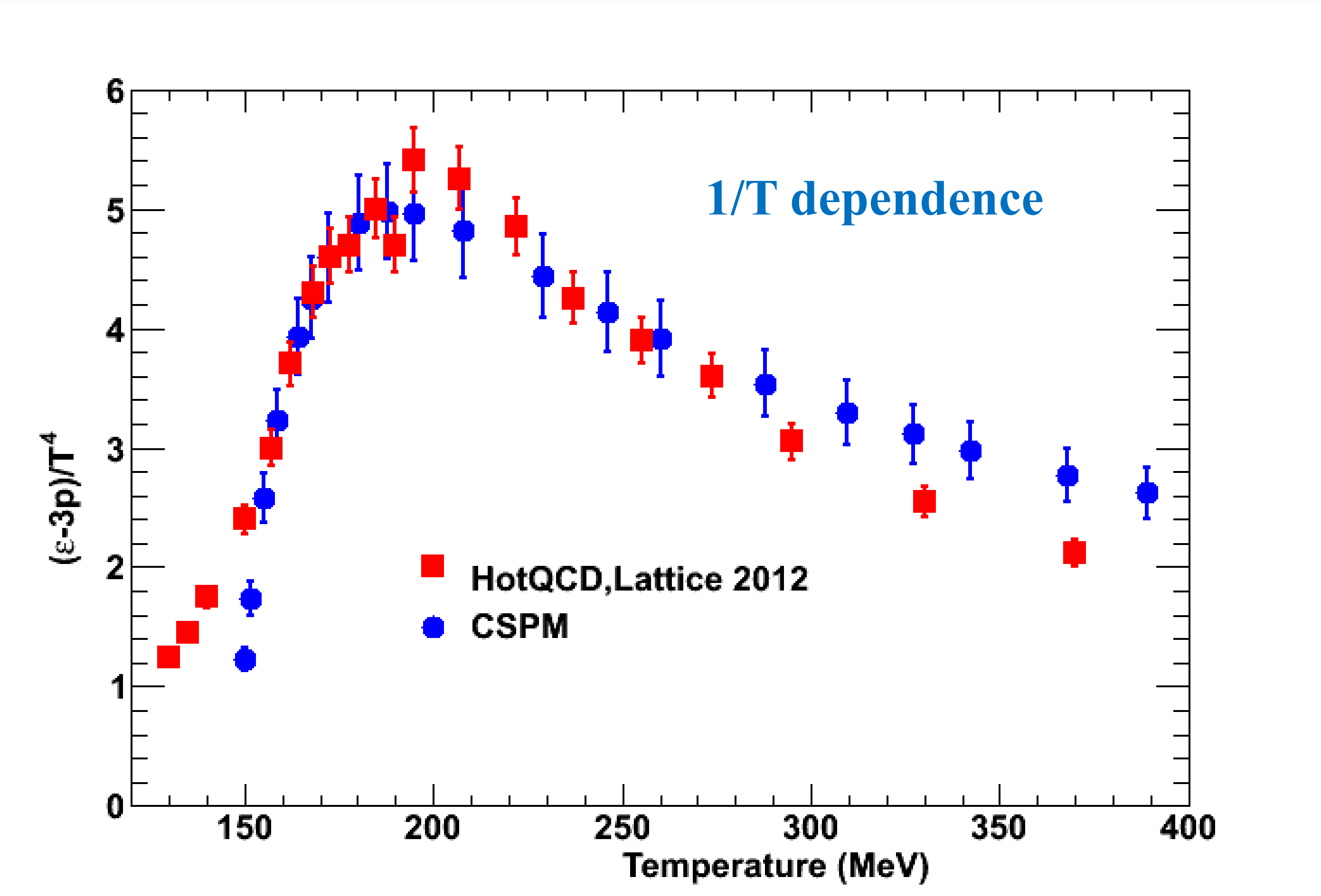

**1/T dependence**

$$\Delta = \frac{\varepsilon - 3p}{T^4}$$

**P. Petreczki ( HotQCD Collaboration), Lattice 2012**
**Cairns, Australia**

**Ansatz :** $\Delta = 1/(\eta/s)$ **Magnitude and functional dependence same as in LQCD**

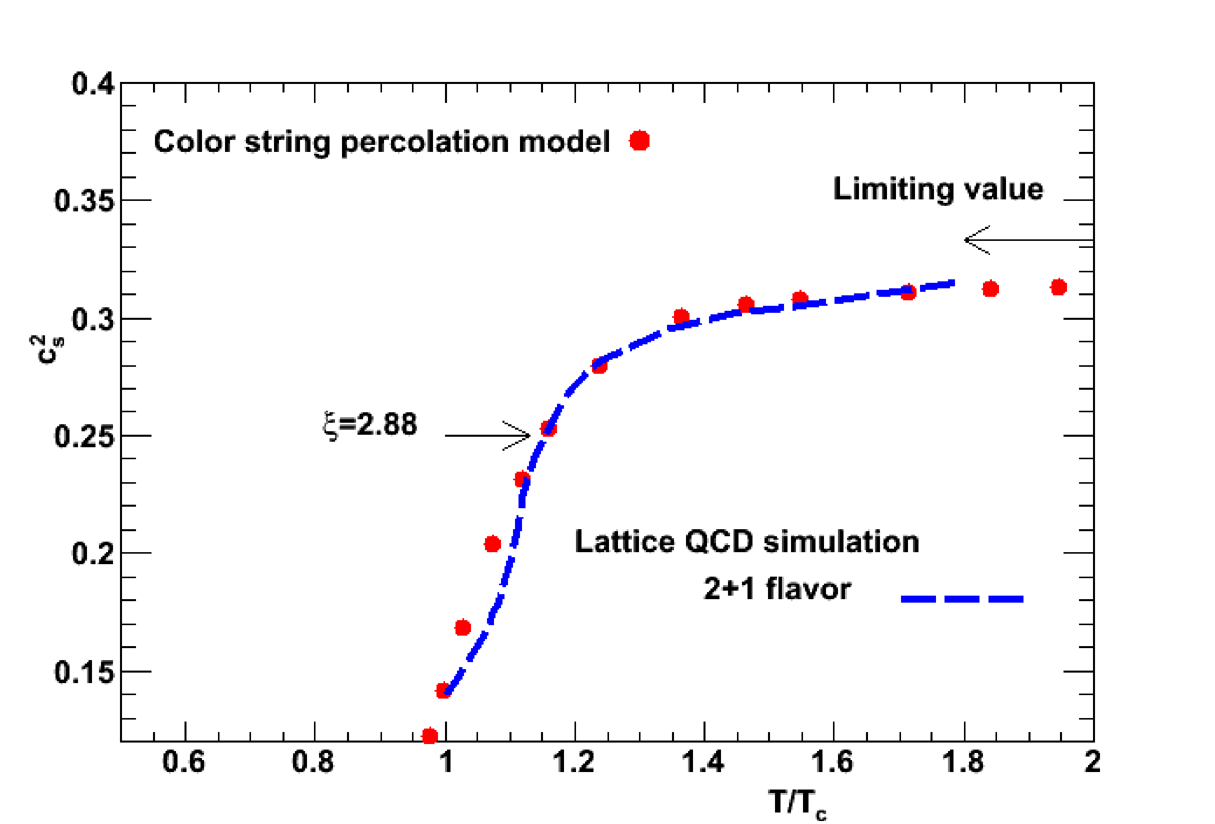
# Interaction Measure



# Sound Velocity

$$\Delta = \frac{\varepsilon - 3p}{T^4}$$

$$C_s^2 = \frac{dp}{d\varepsilon} = \varepsilon \frac{dp/\varepsilon}{d\varepsilon} + \frac{p}{\varepsilon}$$




$$C_S^2 = (-0.33)\left(\frac{\xi e^{-\xi}}{1-e^{-\xi}} - 1\right) + 0.0191(\Delta/3)\left(\frac{\xi e^{-\xi}}{(1-e^{-\xi})^2} - \frac{1}{1-e^{-\xi}}\right)$$

# Exploration of Hot QCD Matter
# The Next Decade

**Berndt Muller**
**CERN Theory Institute (HIC10)**
**Aug.16- Sept.10, 2010**

Which properties of hot QCD matter can we hope to determine from relativistic heavy ion data (RHIC and LHC, maybe FAIR) ?

Easy for LQCD

$$T_{\mu\nu} \Leftrightarrow \varepsilon, p, s$$

$$c_S^2 = \partial p / \partial \varepsilon$$

**Equation of state**: spectra, coll. flow, fluctuations

**Speed of sound**: multiparticle correlations

$$\eta = \frac{1}{T} \int d^4x \langle T_{xy}(x) T_{xy}(0) \rangle$$

**Shear viscosity**: anisotropic collective flow

Hard for LQCD

$$\hat{q} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+i}(y^-) F_i^{a+}(0) \rangle$$

$$\hat{e} = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle i\partial^- A^{a+}(y^-) A^{a+}(0) \rangle$$

$$\hat{e}_2 = \frac{4\pi^2 \alpha_s C_R}{N_c^2 - 1} \int dy^- \langle F^{a+-}(y^-) F^{a+-}(0) \rangle$$

**Momentum/energy diffusion**: parton energy loss, jet fragmentation

Easy for LQCD

$$m_D = -\lim_{|x| \to \infty} \frac{1}{|x|} \ln \langle E^a(x) E^a(0) \rangle$$

**Color screening**: Quarkonium states

# Summary

- **The Clustering of Color Sources leading to the Percolation Transition may be the way to achieve de-confinement in High Energy collisions.**

- **The Percolation framework provide us with a microscopic partonic structure which explains the early thermalization.**

- **The minimum in η/s can be studied as a function of the beam energy at RHIC that could locate the critical point/crossover in QCD phase diagram as seen in substances like He, $N_2$ and $H_2O$.**

- **A further definitive test of clustering phenomena can be made at LHC energies by comparing *h-h* and A-A collisions.**

**H. Satz : Quantum Field Theory in Extreme Environments, Paris , April 2009**

**Clustering and percolation can provide a conceptual basis for the QCD phase diagram which is more general than symmetry breaking .**

***Many Thanks to Organizers***
***for arranging this Conference in***
***such a beautiful place***

# Monte Carlo model for nuclear collisions from SPS to LHC energies

N.S. Amelin[1,a], N. Armesto[2,b], C. Pajares[3,c], D. Sousa[4,d] Eur. Phys. J. C 22, 149–163 (2001)

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**Elementary partonic collisions**
**Partonic wave functions of the colliding hadrons**
**Formation of Color String**
**Transverse momentum of both partons at the string ends**
**Given by Gaussian-> pt -broadening**
**Collectivity ->string fusion/percolation**
**SU(3) random summation of charges**
**Reduction in color charge**
**Increase in the string tension**
**String breaking leads to formation of secondaries**
**Probability rate ->Schwinger**
**Fragmentation proceeds in an iterative way**

1. **Multiplicity**
2. **pt distribution**
3. **Particle ratios**
4. **Elliptic flow**
5. **Suppression of high pt particles $R_{AA}$**
6. **J/ψ production**
7. **Forward-Backward Multiplicity Correlations at RHIC**

**Color String Percolation Model for Nuclear Collisions from SPS-RHIC-LHC**

# Monte Carlo model for nuclear collisions from SPS to LHC energies

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Eur. Phys. J. C 22, 149–163 (2001)

**Elementary partonic collisions**

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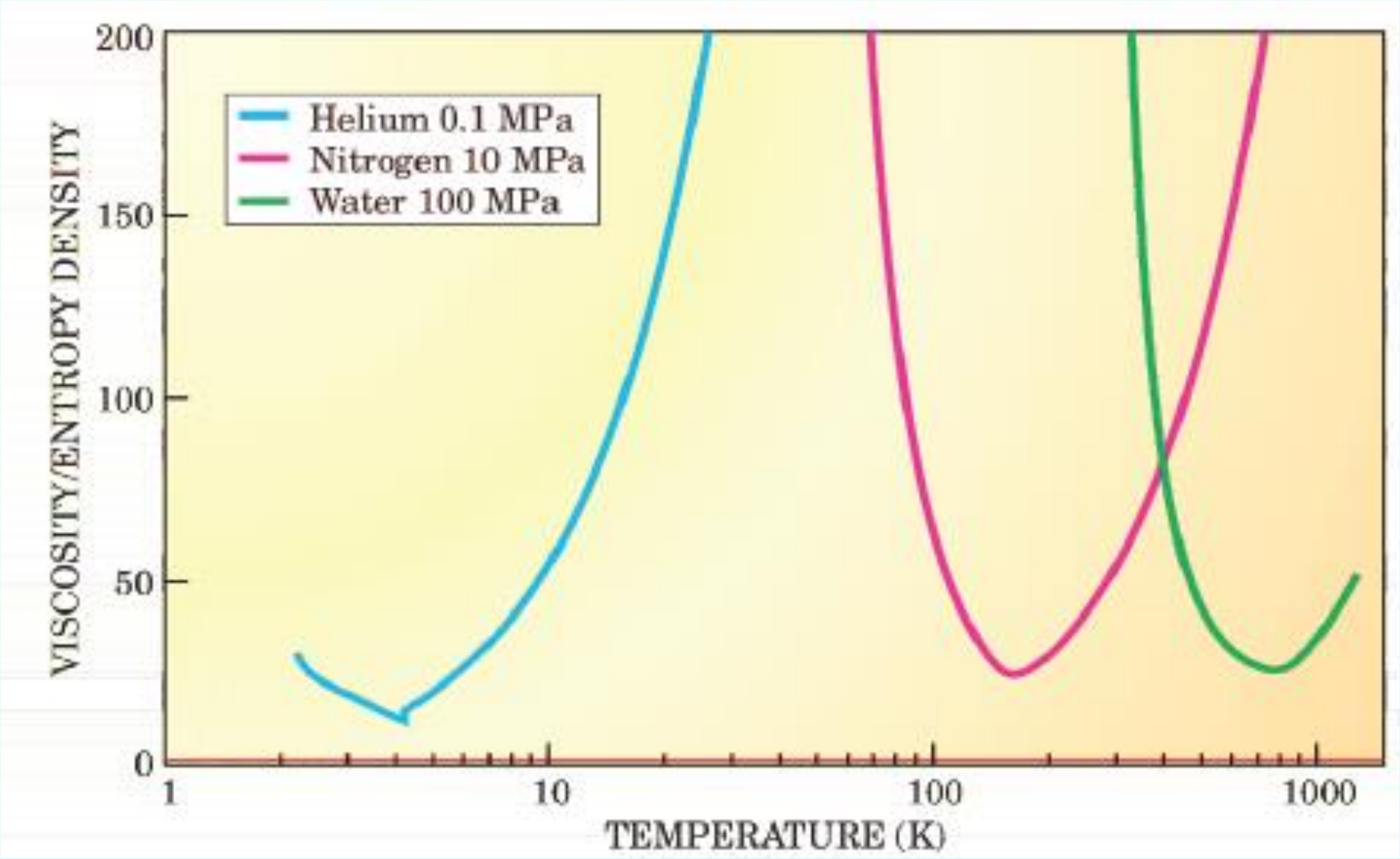
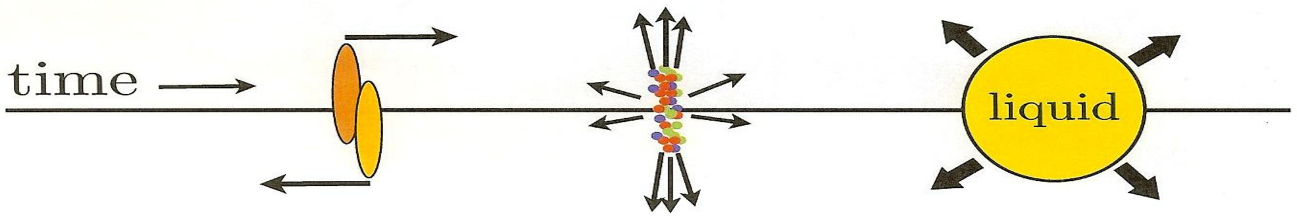
# η/s for He, $N_2$ and $H_2O$



# DIFFERENCE IN FLOW AND THERMALIZATION

Hydrodynamics & thermalization in heavy ion collisions




Basic theoretical questions:

- How long is $t_{\text{hydro}}$?
- How long is $t_{\text{therm}}$?
- How is $t_{\text{therm}}$ correlated with $t_{\text{hydro}}$?

**( Au- Au example)** **$t_{\text{hydro}} \sim 0.35$ fm/c** **$t_{\text{therm}} \sim 1.05$ fm/c**

GRAVITATIONAL COLLAPSE AND HOLOGRAPHIC THERMALIZATION

CHESLER , QM 12

# Parton Percolation

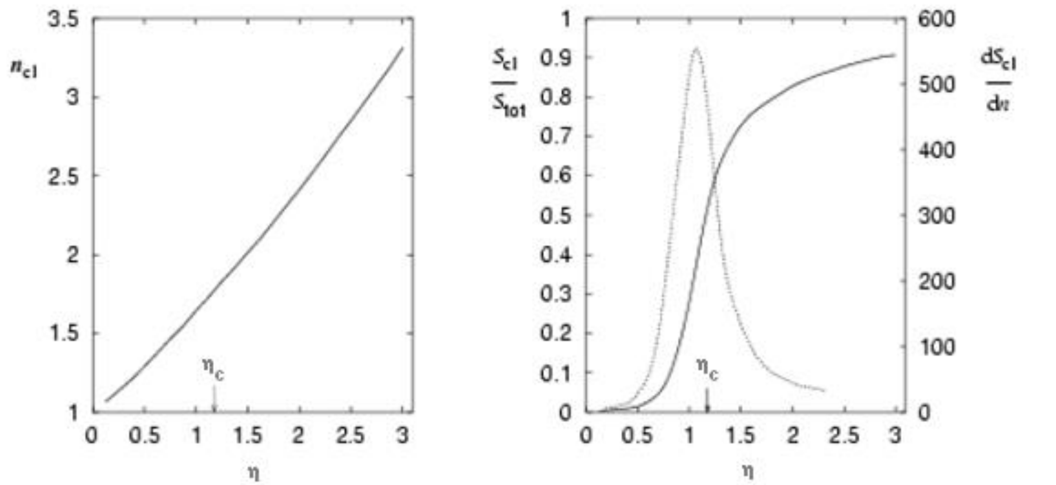



Figure 38: Average cluster density $n_{cl}(n)$ (left) and average fractional cluster size $S_{cl}(n)/S_{total}$ (right) as function of the overall density $n$ of discs, for $r/R = 1/20$; in (b), the derivative of $S_{cl}(n)/S_{total}$ with respect to $n$ is also shown (dotted line). The percolation point in the limit $r/R \to 0$ is indicated by $n_p$.

**In two dimensions, for uniform string density, the percolation threshold for overlapping discs is:**

$$\xi_c = 1.18$$ = critical percolation density

**Satz, hep-ph/0007069**

**The fractional area covered by discs at the critical threshold is:** $1 - e^{-\xi_c}$

The comparative Analysis of Statistical Hadron Production indicates that the Temperature is the same for Different Initial Collision configurations , Independent of energy (√s )

| Collision | $pp$ | Au-Au | $e^+e^-$ |
|---|---|---|---|
| Temperature [MeV] | $169.8 \pm 4.2$ | $168.5 \pm 4.0$ | $164.7 \pm 0.9$ |
| Average relative deviation data vs. fit [%] | 13.2 | 12.1 | 10.8 |
| Average relative error of data [%] | 18 | 10 | 5.7 |
| $\chi^2/\mathrm{dof}$ | $15.0/8 \simeq 1.9$ | $22.2/8 \simeq 2.8$ | $41.5/9 \simeq 4.6$ |


1) A Comparative analysis of statistical hadron production. F. Beccattini et al, Eur. Phys. J. C66 , 377 (2010).
2) Thermodynamics of Quarks and Gluons, H. Satz , arXiv: 0803. 1611v1 hep-ph 11 Mar 2008.