Non-Gaussian initial conditions for evolution of observables in high-energy collisions

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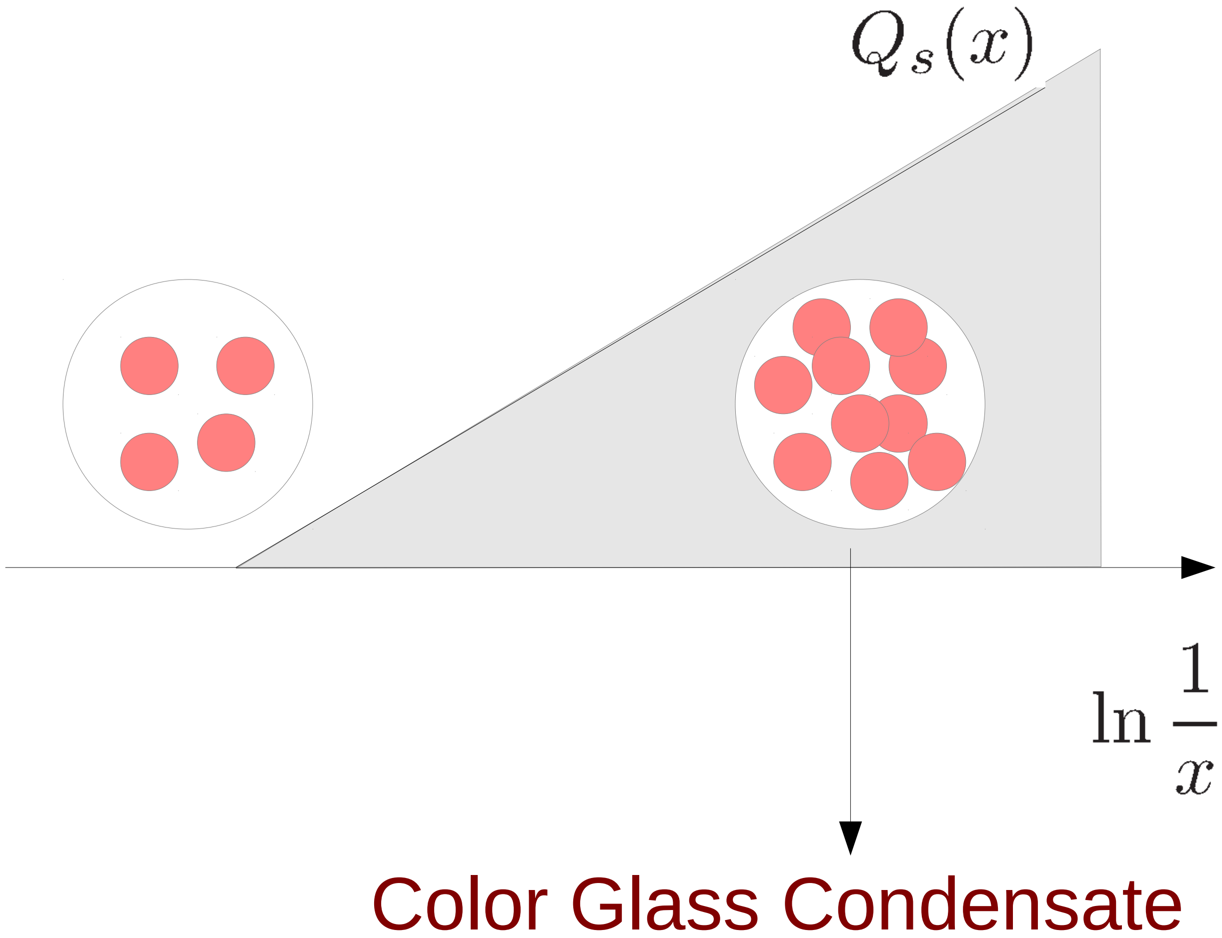
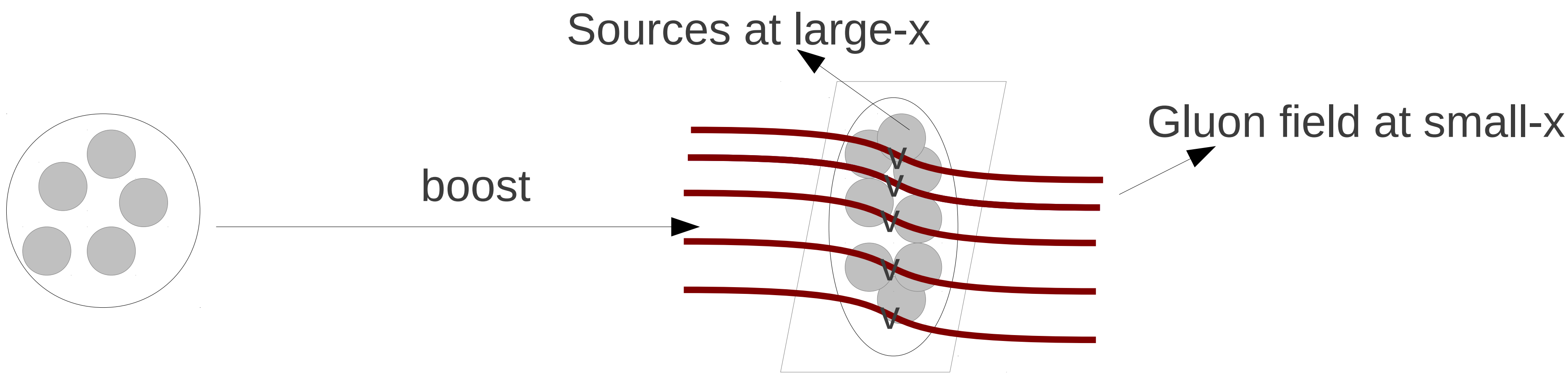
- ***Fourth order corrections to the distribution of color charges;***
- ***Dipole scattering amplitude;***
- ***Particle multiplicity distributions (KNO scaling).***



High-energy limit of nuclear collisions



Gaussian distribution of sources (McLerran-Venugopalan model):

L. D. McLerran and R. Venugopalan,

Phys. Rev. D49, 2233 (1994); 49, 3352 (1994)

$$W[\rho] = \exp\left[-\int d^2x_\perp \frac{\delta^{ab}\rho^a(x_\perp)\rho^b(x_\perp)}{2\mu^2}\right]$$

$$\mu^2 = \frac{g^2 A}{\pi R^2}$$ - Color charge squared per unit transverse area.

The model is valid for a large nucleus, $A^{1/3} \to \infty$

Higher order corrections to the MV model:

- Cubic term

  S. Jeon and R. Venugopalan, Phys. Rev. D70, 105012 (2004); 71, 125003 (2005)

- **Quartic action:**

$$S[\rho(x)] \simeq \int d^2x \Big[\frac{\delta^{ab}\,\rho^a\rho^b}{2\mu^2} - \frac{d^{abc}\,\rho^a\rho^b\rho^c}{\kappa_3} + \frac{\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}}{\kappa_4}\rho^a\rho^b\rho^c\rho^d \Big]$$

$$\mu^2 \sim O(g^2 A^{1/3})$$

$$\kappa_3 \sim O(g^3 A^{2/3})$$

$$\kappa_4 \sim O(g^4 A)$$

A. Dumitru, J. Jalilian-Marian, E.P.
Phys.Rev.D84 (2011) 014018

- ***Dipole scattering amplitude***

Global fits to e+p require initial dipole scattering amplitude with steeper fall off than MV.

J. L. Albacete, *N. Armesto, J. G. Milhano and C. A. Salgado, Phys. Rev. D80, 034031 (2009)*

J. L. Albacete, N. Armesto, J.G. Milhano, P. Quiroga-Arias and C. A. Salgado, Eur. Phys. J. C71, 1705 (2011)

Dipole scattering amplitude:

$$N(\mathbf{r}_\perp) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) \right\rangle$$

MV result:

$$N(r) = 1 - \exp\left[-\frac{r^2\, Q_s^2(x_0)}{4} \,\ln\left(\frac{1}{\Lambda\, r} + e \right) \right]$$

AAMQS (Albacete-Armesto-Milhano-Quiroga-Salgado) fits:

$$N_{\mathrm{AAMQS}} = 1 - \exp\left[-\frac{1}{4} \left(r^2 Q_s^2(x_0) \right)^\gamma \ln\left(e + \frac{1}{r\Lambda} \right) \right]$$

$$\gamma > 1$$

Dipole operator calculation

$$\langle O[\rho]\rangle \equiv \frac{\int \mathcal{D}\rho\, O[\rho]\, e^{-S_G[\rho]}\left[1-\frac{1}{\kappa_4}\int d^2u\ \rho_u^a\rho_u^a\rho_u^b\rho_u^b\right]}{\int \mathcal{D}\rho\, e^{-S_G[\rho]}\left[1-\frac{1}{\kappa_4}\int d^2u\ \rho_u^a\rho_u^a\rho_u^b\rho_u^b\right]}$$

Quartic action result

A. Dumitru, E.P.
Nucl.Phys. A879 (2012) 59-76

$$N(r)=\frac{Q_s^2r^2}{4}\log\frac{1}{r\Lambda}-\frac{C_F^2}{6\pi^3}\frac{g^8}{\kappa_4}\left[\int_{-\infty}^{\infty}dz^-\mu^4(z^-)\right]^2 r^2\log^3\frac{1}{r\Lambda}$$

Gaussian part

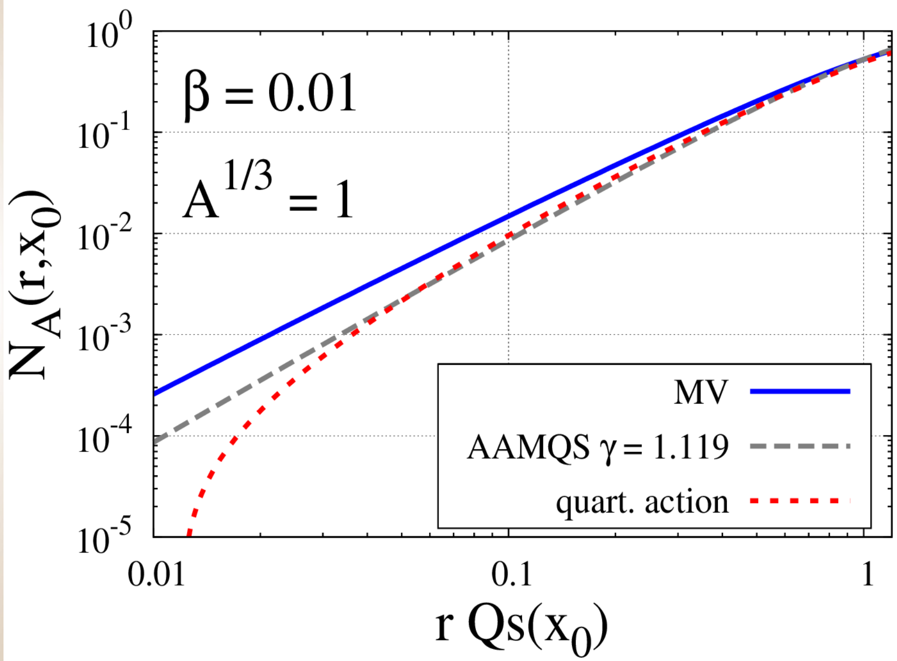
ρ^4 correction

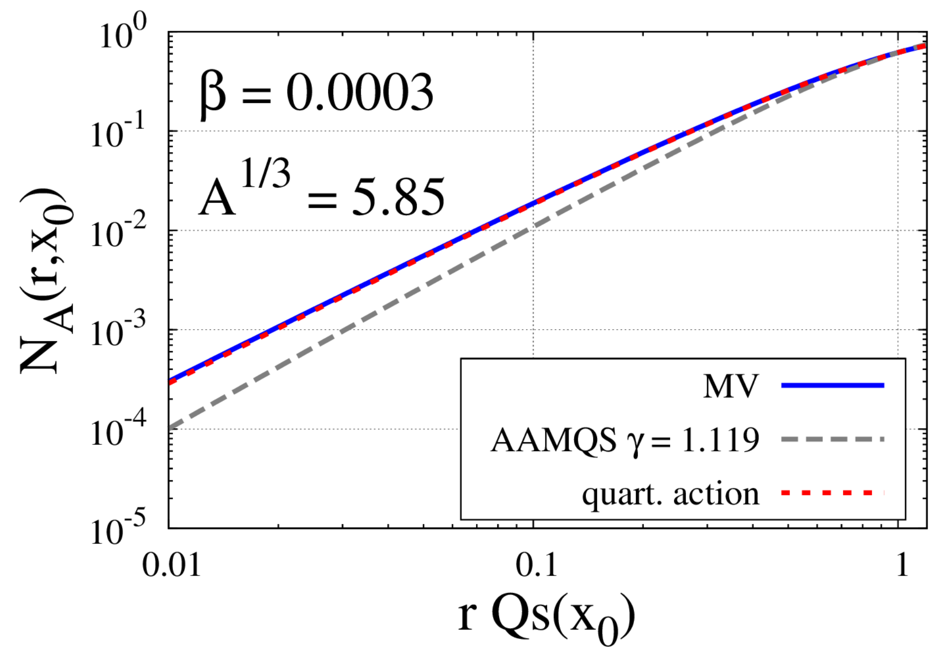
Comparison of results

$$N(r) = \frac{Q_s^2 r^2}{4} \log \frac{1}{r\Lambda} - \beta Q_s^2 r^2 \log^3 \frac{1}{r\Lambda}$$

$$\beta_A \sim A^{-2/3}$$

$$\beta \simeq \frac{1}{100} \quad , \ (A = 1)$$





ρ^4 operator may explain AAMQS model.

- ***Particle multiplicity distributions follow a negative binomial distribution (NBD)***

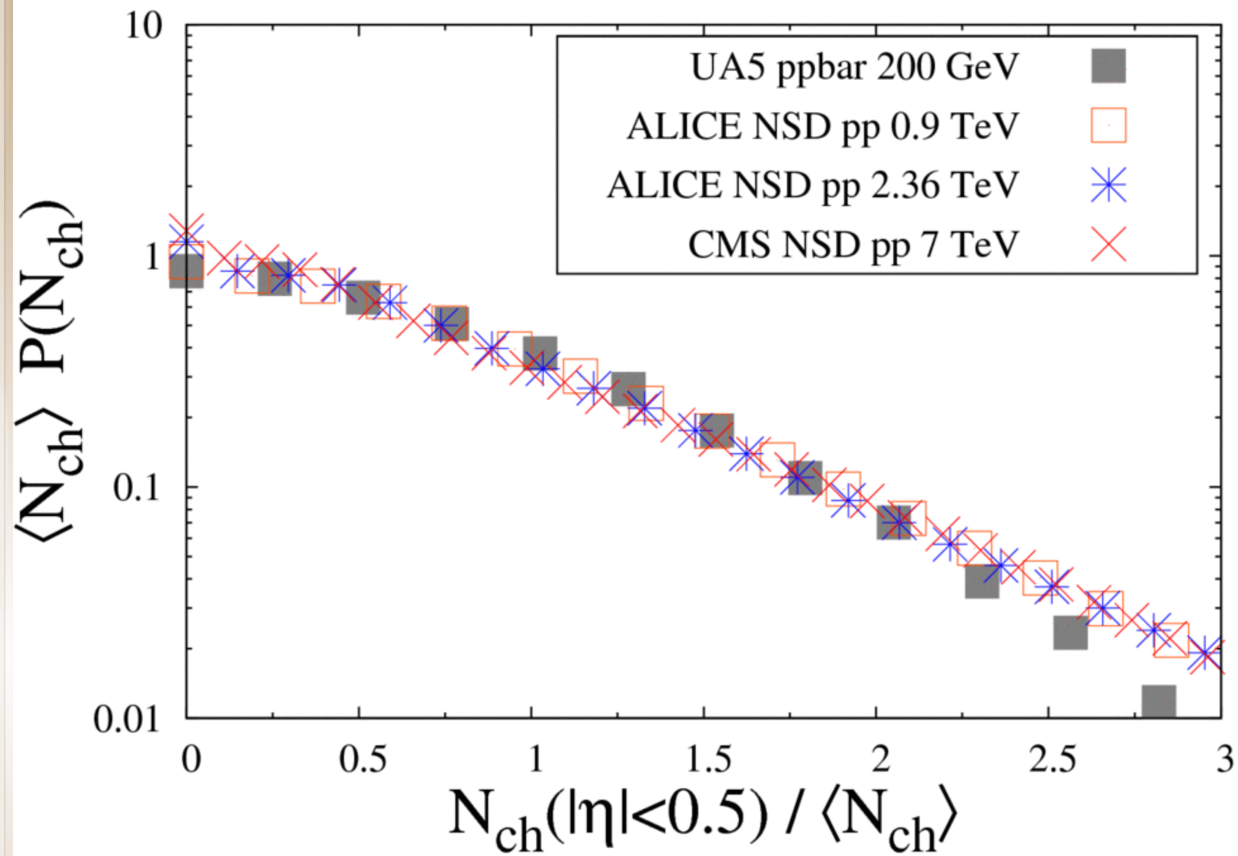
$$P(n) = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

$P(n)$ - *Probability to produce n particles;*

$\bar{n}$ - *Mean multiplicity;*

k - *Fluctuation parameter.*

Koba-Nielsen-Olesen (KNO) scaling



$$\langle N_{ch}\rangle P(N_{ch}) \equiv \Psi(z)$$

$$z \equiv \frac{N_{ch}}{\langle N_{ch}\rangle}$$

$\Psi(z)$

- Energy independent

- Requires explanation in terms of small-x gluons;

- pT-integrated multiplicities (no external hard scale) involve saturation dynamics.

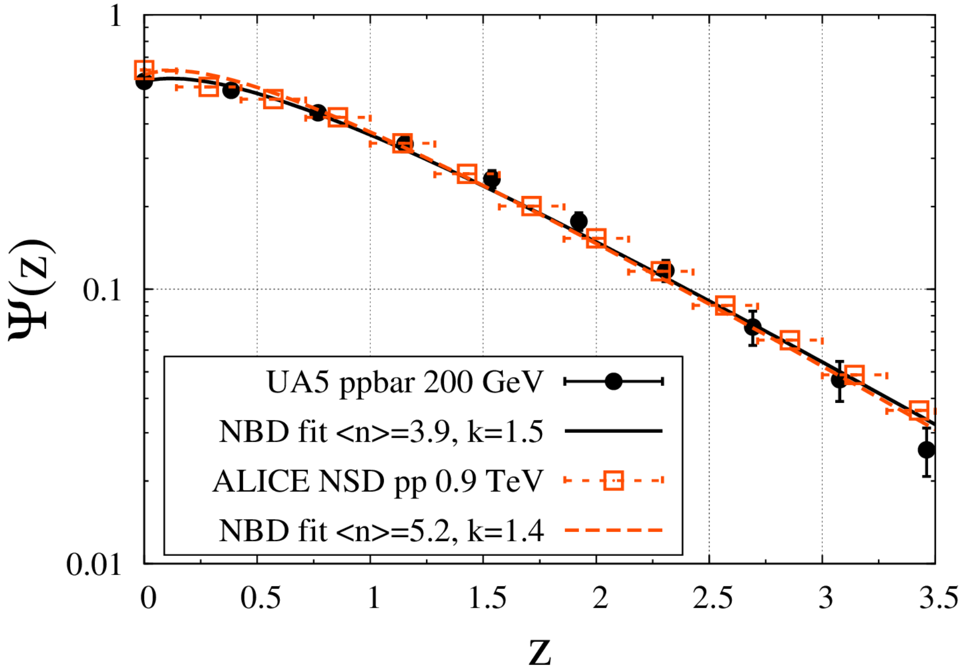
How to get *KNO* from *NBD*?

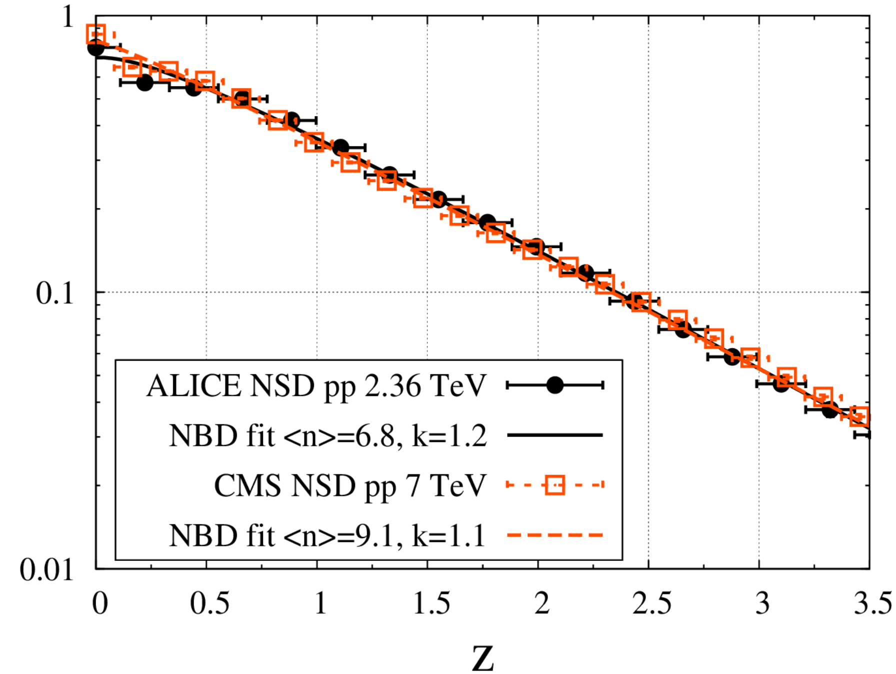
For k constant and $k \ll \bar{n}$ NBD leads to KNO scaling:

$$\bar{n}P(n)dz \sim z^{k-1}e^{-kz}dz, \qquad z \equiv \frac{n}{\bar{n}}$$

- Why is $k \ll \bar{n}$?
- k not exactly constant;

$\frac{\bar{n}}{k}$ increases with energy:





$$\langle N_{ch}\rangle\, P(N_{ch}) \equiv \Psi(z) \qquad z \equiv \frac{N_{ch}}{\langle N_{ch}\rangle}$$

NBD from MV model

F. Gelis, T. Lappi and L. McLerran, Nucl. Phys. A828, 149 (2009)

- Factorial cumulants for NBD:

$$m_n = \frac{(n-1)!}{k^{n-1}} \, \bar{n}^n$$

n - number of produced gluons

$$\bar{n} = \left\langle \frac{dN}{d^2\mathbf{p}_\perp dy_p} \right\rangle$$ - mean multiplicity

- Cross section for producing q gluons:

$$\left\langle \frac{dN}{dy_1 \cdots dy_n} \right\rangle_{\text{conn.}} = \beta_n \left\langle \frac{dN}{dy_1} \right\rangle \cdots \left\langle \frac{dN}{dy_n} \right\rangle$$

MV model result

$$\left\langle \frac{dN}{dy_1 \cdots dy_n} \right\rangle_{\text{conn.}} = \beta_n \left\langle \frac{dN}{dy_1} \right\rangle \cdots \left\langle \frac{dN}{dy_n} \right\rangle$$

$$\beta_n = (n-1)! \;\; k^{1-n}$$ → **NBD**

$$\bar{n} \sim \frac{N_c(N_c^2-1)}{\alpha_s} Q_s^2 \, \pi R^2$$

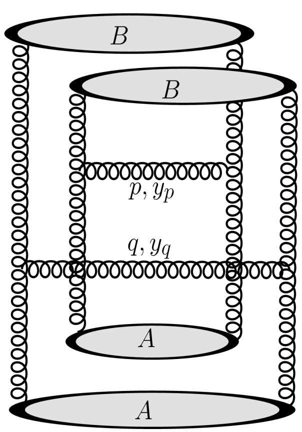
$$k \sim \frac{N_c^2-1}{2\pi} Q_s^2 \, \pi R^2$$

- **Why KNO?**

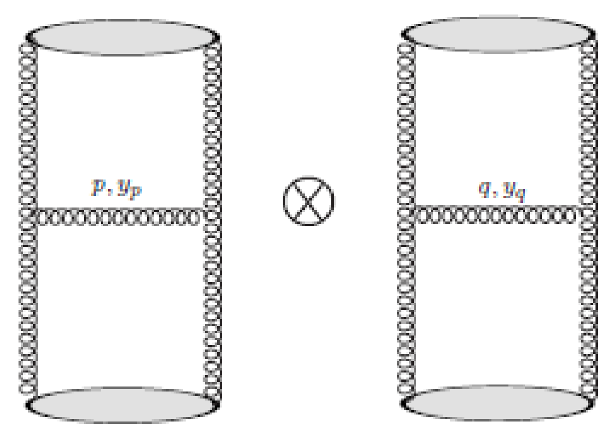
$$\frac{\bar{n}}{k} \sim \frac{N_c}{\alpha_s} \gg 1$$

- **Why is $k = \mathcal{O}(\alpha_s^0)$?**

Second cumulant: $m_2 = \frac{1}{k}\bar{n}^2$



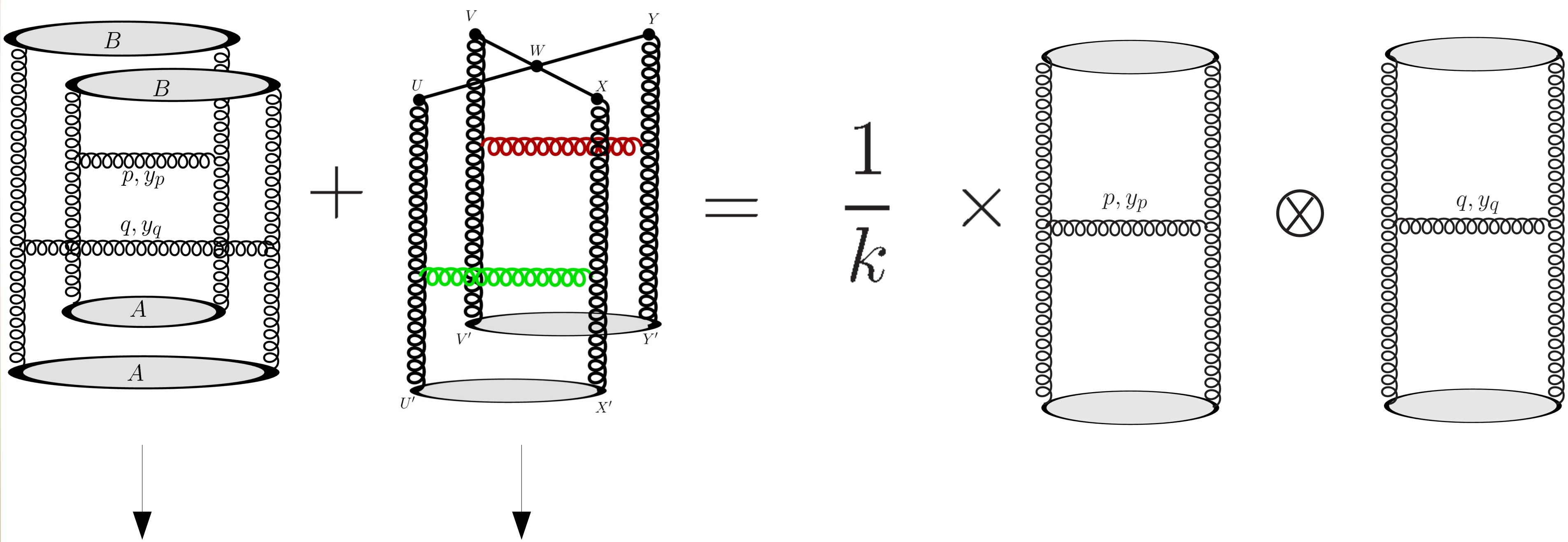
Same order in α_s



Calculation with quartic action

- Two gluon production:

$$\left\langle \frac{dN}{dp\, dy_p\, dq\, dy_q} \right\rangle = \frac{1}{k} \left\langle \frac{dN}{dp\, dy_p} \right\rangle \left\langle \frac{dN}{dq\, dy_q} \right\rangle$$



Gaussian diagrams

Correction diagrams

Quartic action result

A. Dumitru, E.P. arXiv:1209.4105

$$\frac{\bar{n}}{k} \sim \frac{N_c}{\alpha_s} \left(1 - 3\,\beta\,\left(N_c^2 + 1\right)\right)$$

↓

Correction

$$\beta \equiv \frac{C_F^2}{6\pi^3} \frac{g^8}{Q_s^2\,\kappa_4} \left[\int_{-\infty}^{\infty} dz^- \mu^4(z^-)\right]^2 \approx 0.01\,A^{-2/3}$$

$\beta > 0$ makes $\frac{\bar{n}}{k}$ smaller by a factor of 1.43 .

$\beta > 0$ **makes** $\frac{\bar{n}}{k}$ **smaller.**

- NBD fits to data show that $\frac{\bar{n}}{k}$ increases with energy

  $\Rightarrow$ might indicate flow towards a Gaussian theory;

- KNO scaling constrains the deviation of the small-x effective action from a Gaussian.

Summary

- Higher dimensional operators in the effective action may not be highly suppressed for p+p collisions;
- They may provide theoretical understanding of the AAMQS fits.
- They bring corrections to the negative binomial distribution and are constrained by the KNO scaling.

Backup slides

Non-Gaussian initial conditions for high-energy evolution

- Odderon operator $-d^{abc}\rho^a\rho^b\rho^c/\kappa_3$

S. Jeon and R. Venugopalan,
Phys. Rev. D70, 105012 (2004); 71, 125003 (2005)

- Effective action for a system of $k \gg 1$ valence quarks in SU(3);
- Random walk of SU(3) color charges in the space of representations (m,n);
- Probability $P(m,n)$ $\quad P(m,n) = e^{-S(m,n)}$

$$S(m,n;k) \simeq \frac{N_c}{k} C_2(m,n) - \frac{1}{3}\left(\frac{N_c}{k}\right)^2 C_3(m,n) + \frac{1}{6}\left(\frac{N_c}{k}\right)^3 C_4(m,n)$$

$C_2,\ C_3,\ C_4$ - Casimir operators for the representation (m,n)

- Define color charge per unit area $\rho^a \equiv g\,Q^a/\Delta^2 x$

where $|Q| = \sqrt{Q^a Q^a} \equiv \sqrt{C_2}$

Generating function: $$F(z) \equiv \sum_{n=0}^{\infty} z^n P_n$$

Factorial cumulants:

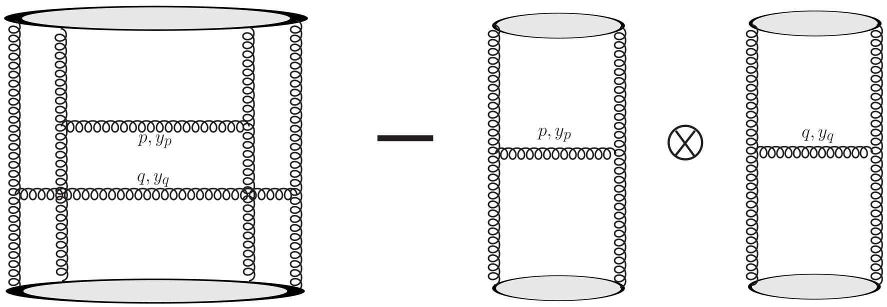
$$m_q \equiv \langle n(n-1)\cdots(n-q+1)\rangle - \text{disconnected} = \frac{d^q ln\, F(z)}{dz^q}$$

Factorial moments:

$$\langle n(n-1)\cdots(n-q+1)\rangle = \int \mathcal{D}\rho\, W[\rho]\, (n[\rho])^q$$

$$n[\rho] = \frac{dN}{d^2\mathbf{p}_\perp dy_p}$$ is multiplicity corresponding to fixed configuration of sources.

Two-gluon production:

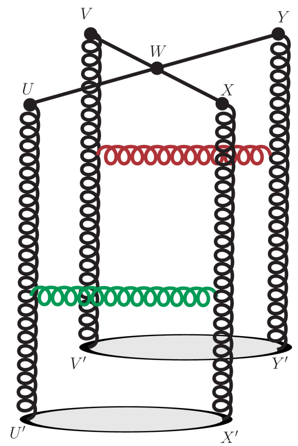


$$C(\mathbf{p},\mathbf{q}) = \left\langle \frac{dN}{d^2\mathbf{p}_{\perp 1} dy_1 p_{\perp 2} dy_2} \right\rangle - \left\langle \frac{dN}{d^2\mathbf{p}_\perp dy_p} \right\rangle \left\langle \frac{dN}{d^2\mathbf{q}_\perp dy_q} \right\rangle$$

$$C(\mathbf{p},\mathbf{q}) = \frac{g^{12}}{4(2\pi)^6} f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'} \int \prod_{i=1}^{4} \frac{d^2k_i}{(2\pi)_i^2 k_i^2} \frac{L_\mu(\mathbf{p},k_1) L^\mu(\mathbf{p},k_2) L_\nu(\mathbf{q},k_3) L^\nu(\mathbf{q},k_4)}{(\mathbf{p}-k_1)^2(\mathbf{p}-k_2)^2(\mathbf{q}-k_3)^2(\mathbf{q}-k_4)^2} \times$$

$$\langle \rho_1^{*a}(k_2)\rho_1^{*b}(k_4)\rho_1^c(k_1)\rho_1^d(k_3)\rangle \langle \rho_2^{*a'}(\mathbf{p}-k_2)\rho_2^{*b'}(\mathbf{q}-k_4)\rho_2^{c'}(\mathbf{p}-k_1)\rho_2^{d'}(\mathbf{q}-k_3)\rangle$$

Connected two-gluon diagrams from the *quartic action*:

$2\times$ 

$$= \frac{16g^{12}}{(2\pi)^8\pi^2\kappa_4}\left[\int dv^-\tilde{\mu}^2(v^-)\right]^2\left[\int dv^-\tilde{\mu}^4(v^-)\right]^2\frac{S_\perp N_c^2(N_c^2-1)(N_c^2+1)}{p^2q^2}\times$$

$$\int\frac{d^2k_1}{k_1^2}\frac{1}{(p-k_1)^2}\int\frac{d^2k_2}{k_2^2}\frac{1}{(q-k_2)^2}$$

$$= \frac{g^{12}}{4\pi^8\kappa_4}\left[\int dv^-\tilde{\mu}^2(v^-)\right]^2\left[\int dv^-\tilde{\mu}^4(v^-)\right]^2\frac{S_\perp N_c^2(N_c^2-1)(N_c^2+1)}{p^4q^4}\ln\frac{p}{Q_s}\ln\frac{q}{Q_s}$$

$$C(p,q) = \frac{2\pi - \frac{4Q_s^2(N_c^2+1)}{\kappa_4}\frac{\left[\int dv^- \tilde{\mu}^4\right]^2}{\left[\int dv^- \tilde{\mu}^2\right]^2}}{q_s^2(N_c^2-1)S_\perp}\left\langle \frac{dN}{dp\,dy_p}\right\rangle\left\langle \frac{dN}{dq\,dy_q}\right\rangle$$

Second cumulant: $m_2 = \frac{1}{k}\bar{n}^2$

$$C(p,q) = \frac{1}{k}\left\langle \frac{dN}{dp\,dy_p}\right\rangle\left\langle \frac{dN}{dq\,dy_q}\right\rangle$$