# Energy loss in Unstable Quark-Gluon Plasma
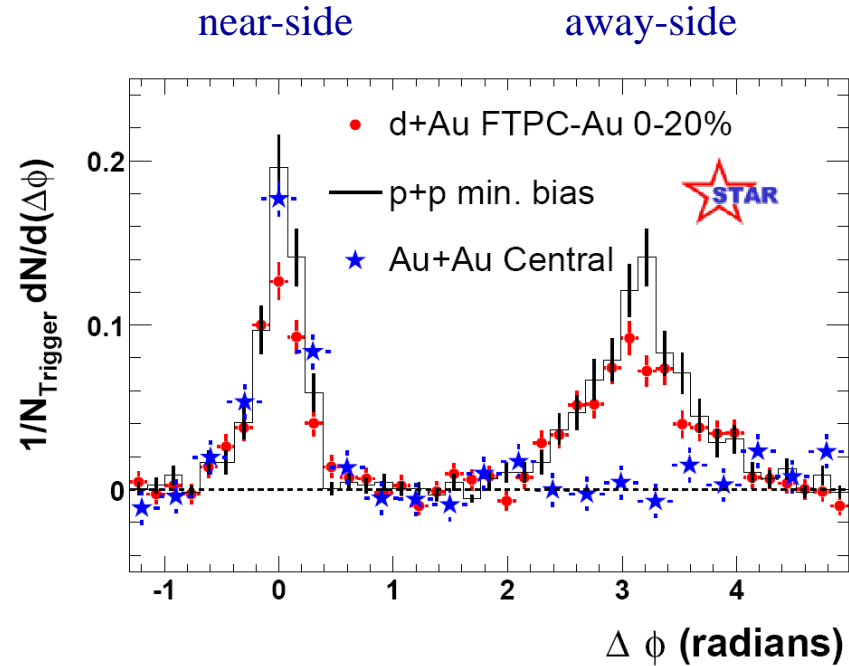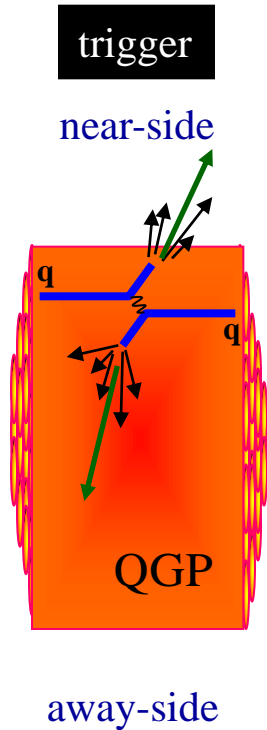
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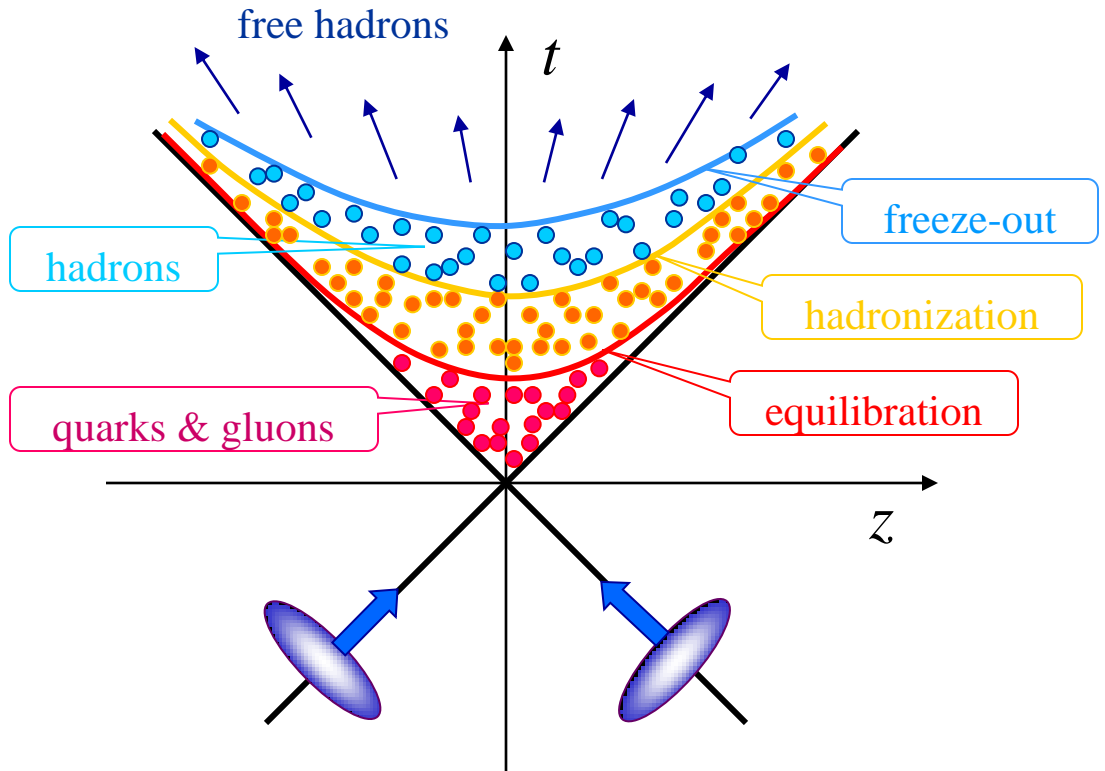
in collaboration with

**Margaret Carrington & Katarzyna Deja**

# Motivation - jet quenching



near-side          away-side

trigger

near-side

QGP

away-side

$\frac{1}{N_{Trigger}} \, dN/d(\Delta\phi)$

- d+Au FTPC-Au 0-20%
— p+p min. bias
★ Au+Au Central

STAR

$\Delta \phi$ (radians)

Away-side jet is suppressed in central collisions

J. Adams et al. [STAR Collaboration], Nucl. Phys. **A757**, 102 (2005)

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# Scenario of relativistic heavy-ion collisions

after

free hadrons

$t$

hadrons

freeze-out

hadronization

quarks & gluons

equilibration

$z$

before
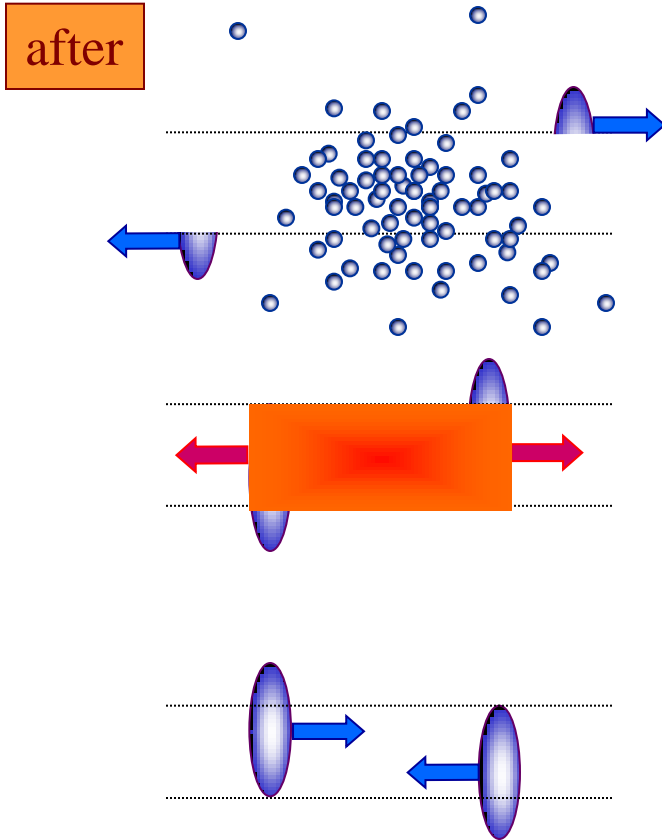
QGP is out of equilibrium at the collision early stage

# Anisotropic QGP



'prolate'

'oblate'
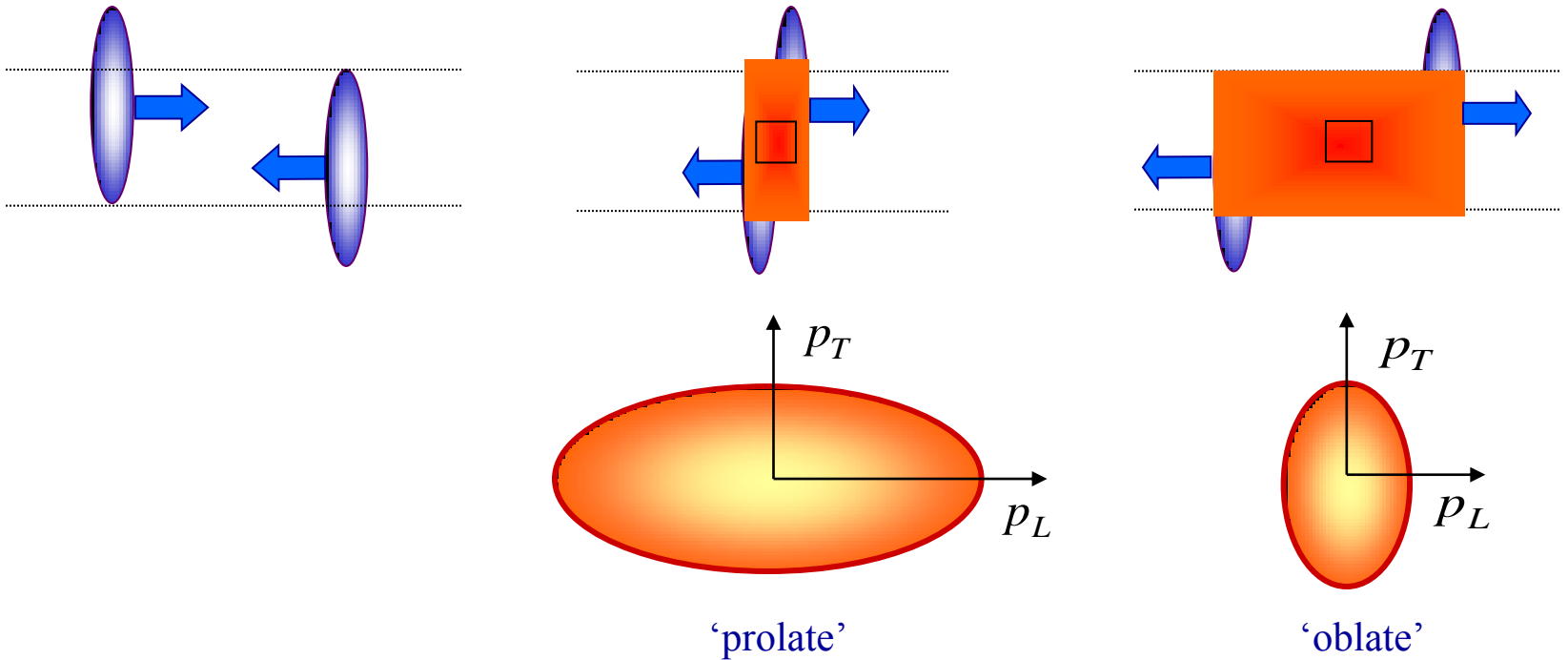
Anisotropic QGP is unstable due to magnetic plasma modes

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# Energy loss in unstable QGP

▶    Is *dE/dx* in unstable QGP sizeable?

Yes!

▶    How to compute *dE/dx* in unstable QGP?

Solve initial value problem!

# A test parton in QGP

**Wong's equation of motion** (Hard Loop Approximation)

$$
\begin{cases}
\dfrac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau) \\[2mm]
\dfrac{dp^{\mu}(\tau)}{d\tau} = g Q_a(\tau)\, F_a^{\mu\nu}(x(\tau))\, u_{\nu}(\tau) \\[2mm]
\dfrac{dQ_a(\tau)}{d\tau} = -g f^{abc}\, p_{\mu}(\tau)\, A_b^{\mu}(x(\tau))\, Q_c(\tau)
\end{cases}
$$

<u>Simplifications</u>

Gauge condition:  $p_{\mu}(\tau)\, A_b^{\mu}\big(x(\tau)\big) = 0 \;\;\Rightarrow\;\; Q_a(\tau) = \text{const}$

Parton travels with constant velocity:  $u^{\mu} = (\gamma, \gamma \mathbf{v}) = \text{const}$

# Parton's energy loss

$$\frac{dE(t)}{dt} = gQ_a \, \mathbf{E}_a(t, \mathbf{r}(t)) \cdot \mathbf{v}$$

induced & spontaneously generated chromoelectric field

parton's current: $\mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t)$
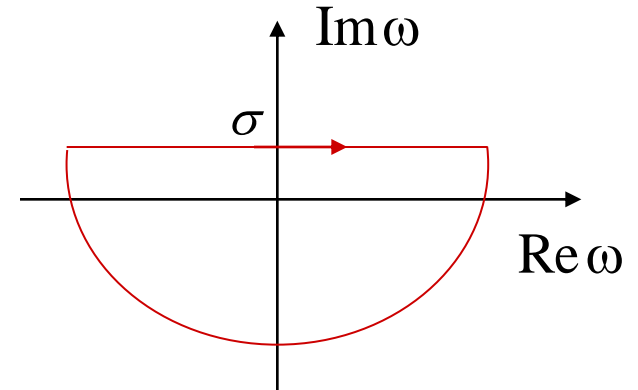
$$\frac{dE(t)}{dt} = \int d^3r \, \mathbf{E}_a(t, \mathbf{r}) \cdot \mathbf{j}_a(t, \mathbf{r})$$

# Initial value problem

One-sided Fourier transformation

$$\begin{cases} f(\omega, \mathbf{k}) = \int\limits_{0}^{\infty} dt \int d^3r \, e^{i(\omega t - \mathbf{kr})} f(t, \mathbf{r}) \\ \\ f(t, \mathbf{r}) = \int\limits_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \, e^{-i(\omega t - \mathbf{kr})} f(\omega, \mathbf{k}) \end{cases}$$

$$0 < \sigma \in R$$

$$\mathbf{j}_a(t, \mathbf{r}) = gQ_a \mathbf{v} \delta^{(3)}(\mathbf{r} - \mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega, \mathbf{k}) = \frac{igQ_a \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}}$$

$$\frac{dE(t)}{dt} = gQ_a \int\limits_{-\infty + i\sigma}^{\infty + i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} \, e^{-i(\omega - \mathbf{kv})t} \mathbf{E}_a(\omega, \mathbf{k}) \cdot \mathbf{v}$$

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# Induced Electric Field

Linearized Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$i\mathbf{k} \cdot \mathbf{D}(\omega, \mathbf{k}) = \rho(\omega, \mathbf{k}), \qquad i\mathbf{k} \cdot \mathbf{B}(\omega, \mathbf{k}) = 0,$$

$$i\mathbf{k} \times \mathbf{E}(\omega, \mathbf{k}) = i\omega\mathbf{B}(\omega, \mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$

$$i\mathbf{k} \times \mathbf{B}(\omega, \mathbf{k}) = \mathbf{j}(\omega, \mathbf{k}) - i\omega\mathbf{E}(\omega, \mathbf{k}) - \mathbf{D}_0(\mathbf{k})$$

$$D^i(\omega, \mathbf{k}) = \varepsilon^{ij}(\omega, \mathbf{k}) E^j(\omega, \mathbf{k})$$

Chromodielectric tensor

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega} \int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{k}\mathbf{v} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l} \left[ \left( 1 - \frac{\mathbf{k}\mathbf{v}}{\omega} \right) \delta^{lj} + \frac{k^l v^j}{\omega} \right] \qquad \text{dynamical information}$$

$$E^i(\omega, \mathbf{k}) = -i(\Sigma^{-1})^{ij}(\omega, \mathbf{k}) [\omega\mathbf{j}(\omega, \mathbf{k}) + \mathbf{k} \times \mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k})]^j$$

$$\Sigma^{ij}(\omega, \mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k})$$

# Energy-Loss formula

$$\frac{dE(t)}{dt} = gQ_a v^i \int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t}$$

$$\times (\Sigma^{-1})^{ij}(\omega,\mathbf{k}) \left[ \frac{igQ_a\omega\,\mathbf{v}}{\omega-\overline{\omega}} + \mathbf{k}\times\mathbf{B}_0(\mathbf{k}) - \omega\mathbf{D}_0(\mathbf{k}) \right]^j$$

$$\overline{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$$

$$\Sigma^{ij}(\omega,\mathbf{k}) \equiv -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

**Dispersion equation**

$$\det[\Sigma(\omega,\mathbf{k})] = 0$$

# Initial values of the fields

Maxwell equations & $\mathbf{j}_a(t,\mathbf{r}) = gQ_a\mathbf{v}\delta^{(3)}(\mathbf{r}-\mathbf{v}t)$

**Initial values:**

$$D_0^i(\mathbf{k}) = -igQ_a\overline{\omega}\varepsilon^{ij}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\overline{\omega},\mathbf{k})v^k$$

$$B_0^i(\mathbf{k}) = -igQ_a\varepsilon^{ijk}k^j(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})v^l$$

When the test parton enters the plasma at $t = 0$, the instabilities are initiated.

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} (\Sigma^{-1})^{ij}(\omega, \mathbf{k})$$

$$\times \left[ \frac{\omega \delta^{jl}}{\omega - \overline{\omega}} - (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega}, \mathbf{k}) + \omega \overline{\omega} \varepsilon^{jk}(\overline{\omega}, \mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega}, \mathbf{k}) \right]$$

$$\underbrace{\quad}_{\mathbf{j}(\omega, \mathbf{k})} \qquad \underbrace{\quad}_{\mathbf{B}_0(\mathbf{k})} \qquad \underbrace{\quad}_{\mathbf{D}_0(\mathbf{k})}$$

Averaging over parton's colors: $\int dQ \, Q_a \, Q_b = C_2 \delta^{ab}, \quad C_2 \equiv \begin{cases} \dfrac{1}{2} & \text{for} \quad \text{quark} \\ N_c & \text{for} \quad \text{gluon} \end{cases}$

$$C_R = \begin{cases} C_2 \dfrac{N_c^2 - 1}{N_c} = \dfrac{N_c^2 - 1}{2N_c} & \text{for} \quad \text{quark} \quad (R = F) \\ C_2 = N_c & \text{for} \quad \text{gluon} \quad (R = G) \end{cases}$$

# Stable isotropic plasma

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \varepsilon_L(\omega,\mathbf{k})\frac{k^i k^j}{\mathbf{k}^2} + \varepsilon_T(\omega,\mathbf{k})\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$(\Sigma^{-1})^{ij}(\omega,\mathbf{k}) = \frac{1}{\omega^2 \varepsilon_L(\omega,\mathbf{k})}\frac{k^i k^j}{\mathbf{k}^2} + \frac{1}{\omega^2 \varepsilon_T(\omega,\mathbf{k}) - \mathbf{k}^2}\left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right)$$

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int\limits_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i}\int \frac{d^3 k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t}(\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$

$$\times\left[\frac{\omega\delta^{jl}}{\omega-\overline{\omega}} - (k^j k^k - \mathbf{k}^2\delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\varepsilon^{jk}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k})\right]$$
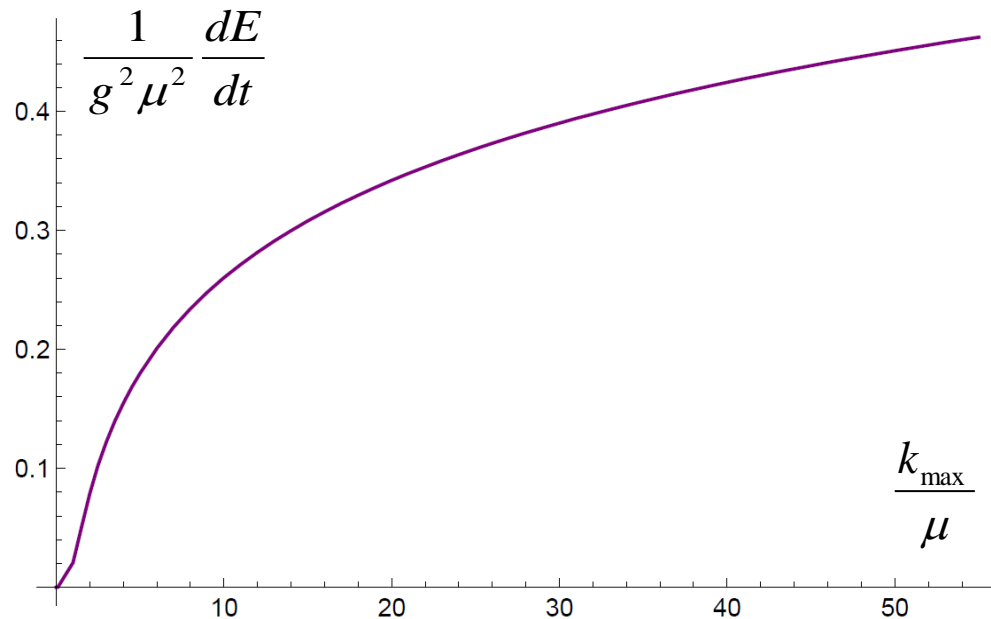
The only stationary contribution: $\omega = \overline{\omega} \equiv \mathbf{k}\cdot\mathbf{v}$

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3 k}{(2\pi)^3}\frac{\overline{\omega}}{\mathbf{k}^2}\left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2\mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2\varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2}\right]$$

equivalent to the standard result by Braaten & Thoma    13
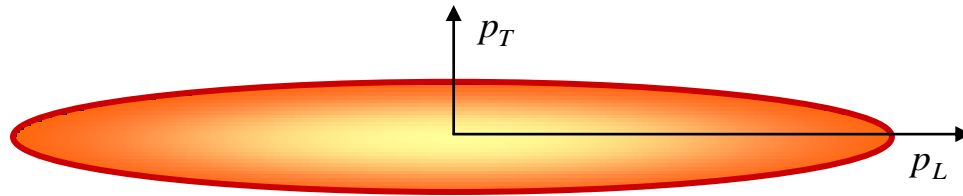
# Energy loss in equilibrium QGP

$$\frac{dE(t)}{dt} = ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[ \frac{1}{\varepsilon_L(\overline{\omega}, \mathbf{k})} + \frac{\mathbf{k}^2 \mathbf{v}^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega}, \mathbf{k}) - \mathbf{k}^2} \right]$$



Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

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$$f(\mathbf{p}) \sim \delta(p_T)$$

Collective modes

$$\det[\Sigma^{ij}(\omega, \mathbf{k})] = 0$$

$$\Sigma^{ij}(\omega, \mathbf{k}) \equiv -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega, \mathbf{k})$$

$$\varepsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega} \int \frac{d^3 p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{k}\mathbf{v} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l} \left[ (1 - \frac{\mathbf{k}\mathbf{v}}{\omega}) \delta^{lj} + \frac{k^l v^j}{\omega} \right]$$

Spectrum of collective modes

$$\omega_1(\mathbf{k}) = \mu^2 + \mathbf{k}^2$$

$$\omega_2(\mathbf{k}) = \mu^2 + (\mathbf{k} \cdot \mathbf{n})^2$$

$$\omega_{\pm}(\mathbf{k}) = \frac{1}{2} \left( \mathbf{k}^2 + (\mathbf{k} \cdot \mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k} \cdot \mathbf{n})^4 + 4\mu^2 \mathbf{k}^2 - 4\mu^2 (\mathbf{k} \cdot \mathbf{n})^2 - 2\mathbf{k}^2 (\mathbf{k} \cdot \mathbf{n})^2} \right)$$
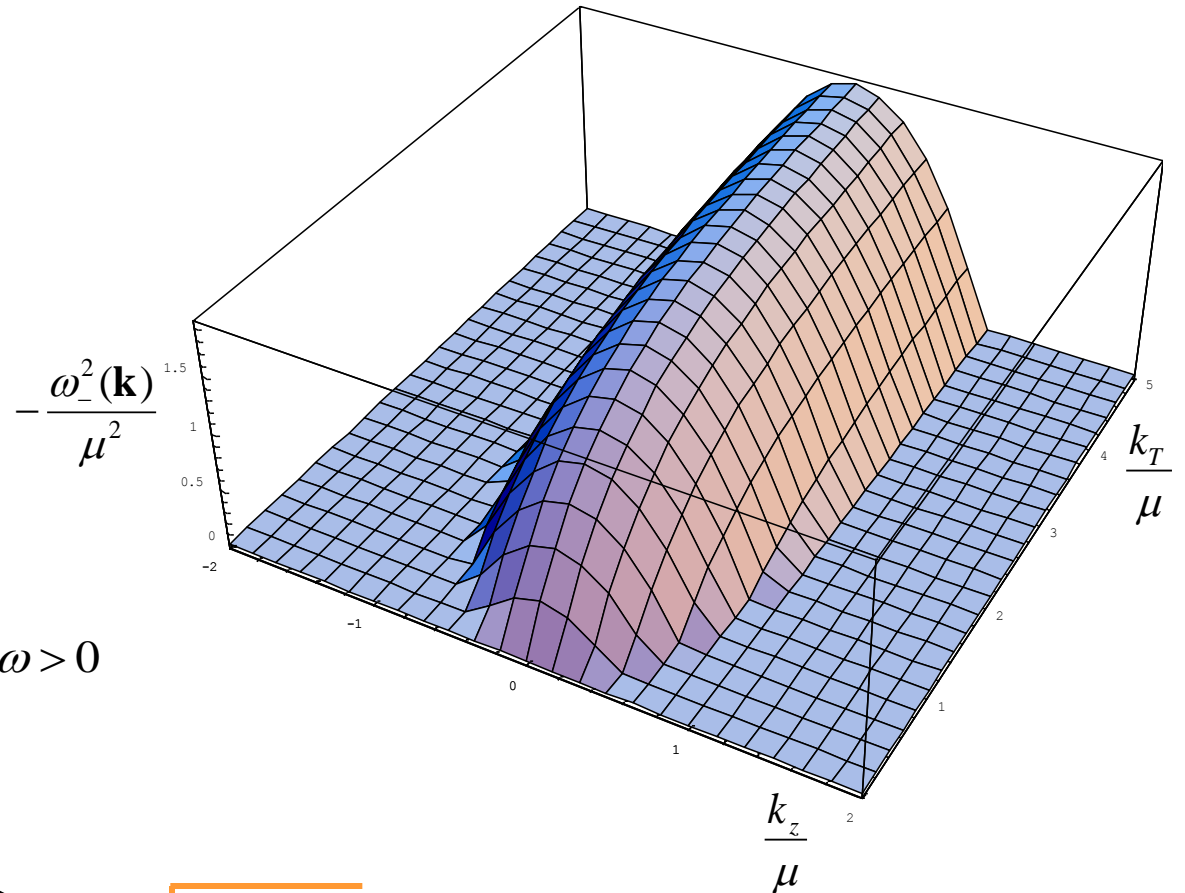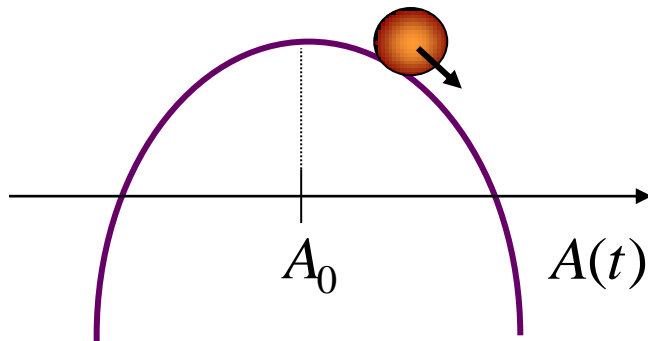
$$\mathbf{n} \equiv (0, 0, 1)$$

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stationary state

$$A(t) = A_0 + \delta A(t)$$

fluctuation

**Instability**

$$\delta A(t) \propto e^{\mathrm{Im}\,\omega t}$$

$$\mathrm{Im}\,\omega > 0$$

$$-\frac{\omega_-^2(\mathbf{k})}{\mu^2}$$

$$\frac{k_T}{\mu}$$

$$\frac{k_z}{\mu}$$

$A_0$   $A(t)$

$$\frac{dE(t)}{dt} \sim \int \frac{d^3 k}{(2\pi)^3} e^{\mathrm{Im}\,\omega t} \ldots$$

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$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} (\Sigma^{-1})^{ij}(\omega,\mathbf{k})$$

$$\times \left[ \frac{\omega \delta^{jl}}{\omega - \overline{\omega}} - (k^j k^k - \mathbf{k}^2 \delta^{jk})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\,\varepsilon^{jk}(\overline{\omega},\mathbf{k})(\Sigma^{-1})^{kl}(\overline{\omega},\mathbf{k}) \right]$$

▶ Inversion of matrix $\Sigma$ depending on $\mathbf{k}$ and $\mathbf{n}$

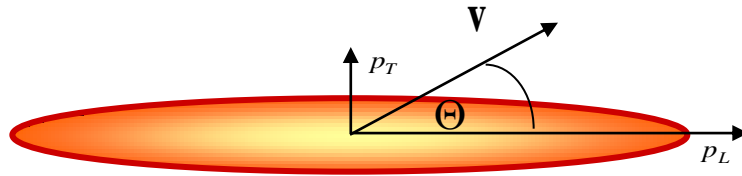$$\Sigma = aA + bB + cC + dD \qquad \Sigma^{-1} = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2},$$

$$n_T^i \equiv \left( \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) n^j$$

$$C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j$$

$$\Sigma\Sigma^{-1} = 1 \implies \alpha, \beta, \gamma, \delta$$

# Energy loss in extremely prolate QGP cont.



$$\frac{1}{g^2\mu^2}\frac{dE}{dt}$$

$k_{max} = 5\mu$

$\Theta = 0^0$
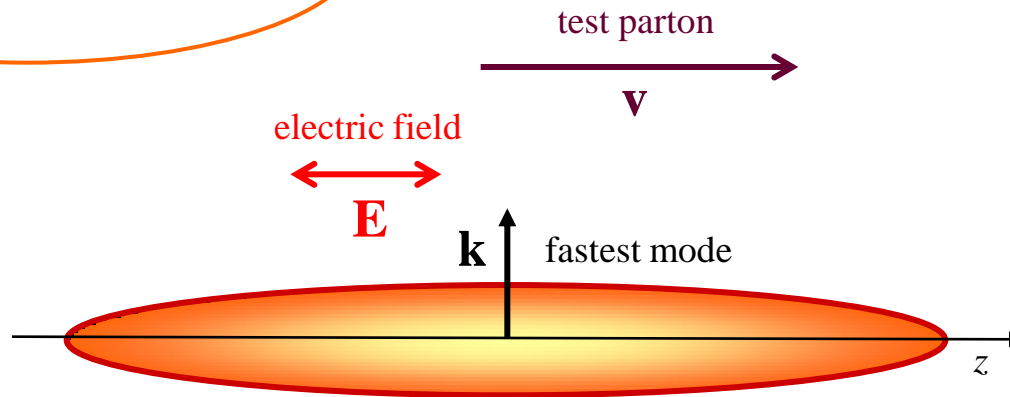$\Theta = 15^0$
$\Theta = 30^0$

$\mu t$
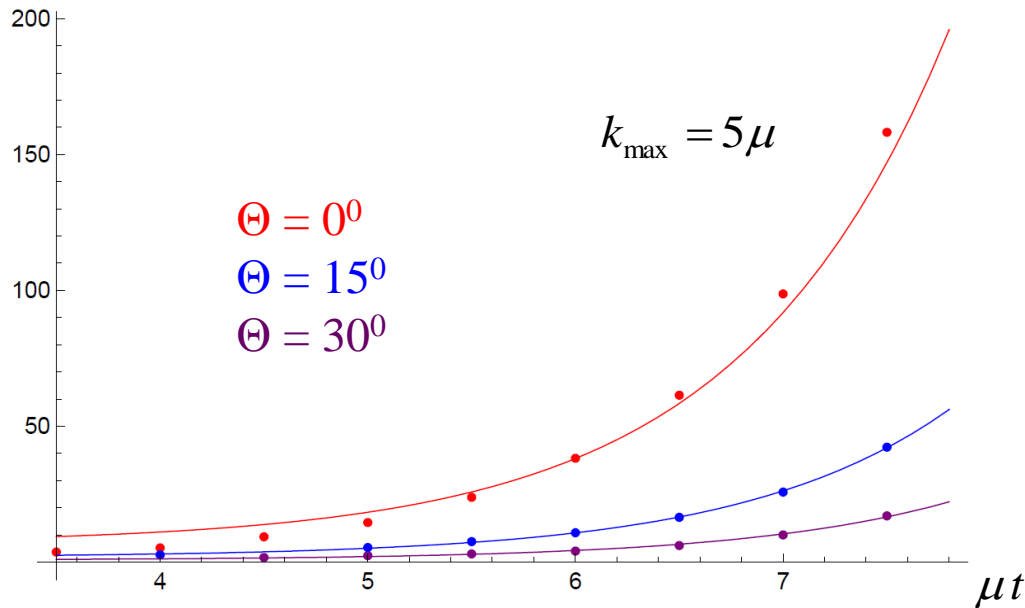
Debye mass

$$\mu^2 \equiv g^2 \int \frac{d^3 p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$$

Equilibrium value

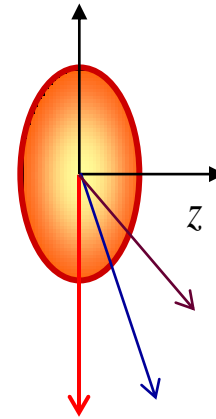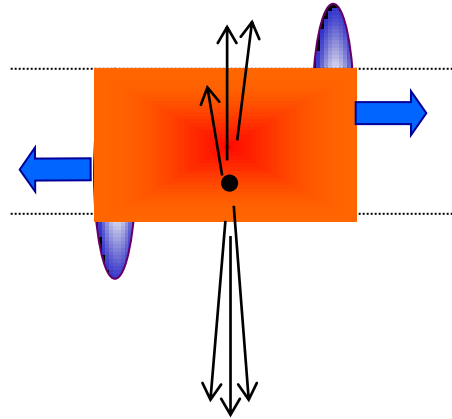$$\frac{1}{g^2\mu^2}\frac{dE}{dt} \approx 0.2$$

# Physical interpretation

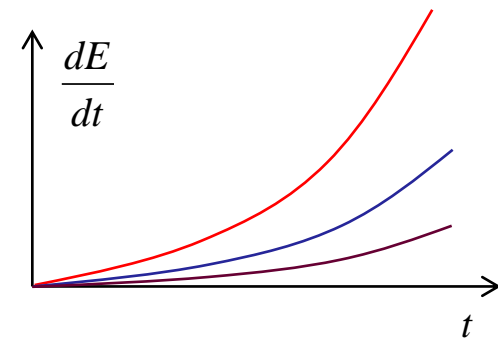$$\frac{dE}{dt} \sim \mathbf{E} \cdot \mathbf{j} \sim \mathbf{E} \cdot \mathbf{v}$$

test parton

**v**

electric field
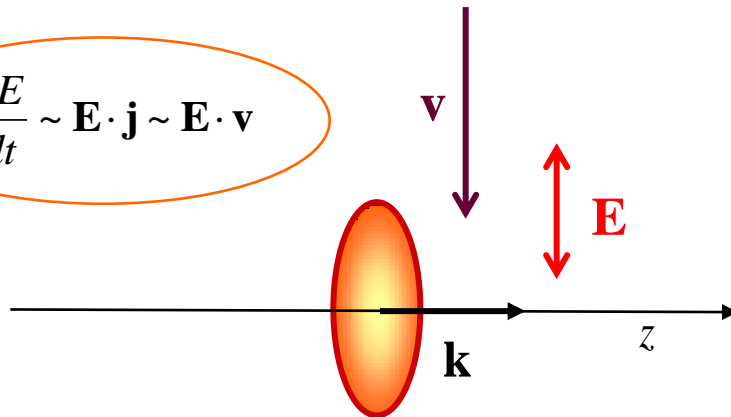
**E**

**k** fastest mode

*z*

The largest *dE/dt* for **v** along axis *z* !

# Energy loss in oblate QGP

Experimentally relevant configuration



$$\frac{dE}{dt} \sim \mathbf{E} \cdot \mathbf{j} \sim \mathbf{E} \cdot \mathbf{v}$$

The largest *dE/dt* for **v** perpendicular to the beam axis !

# Conclusions

▶ Energy loss is found as a solution of initial value problem

▶ Extremely prolate QGP is discussed as an example

▶ Strong time and directional dependence of *dE/dx* is demonstrated

▶ *dE/dx* in unstable QGP is much bigger than in equilibrium one

for more see:

M. Carrington, K. Deja and St. Mrówczyński,
Acta Physica Polonica B - Proceedings Supplement **5**, 047 (2012); *ibid* **5**, 343 (2012); *ibid* **6**, 545 (2013);
PoS Confinement X 175 (2012)