On the Perturbative Evaluation of Thermal Green's Functions in the Bulk and Shear Channels of Yang-Mills Theory

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- 13 September 2013 @ Illa da Toxa . Galicia . Spain -

Outline

- Motivation
- Correlators in SU(N_c) Yang-Mills theory
- Results
  - Correlators in the UV limit
  - Spectral densities
- Summary and Outlook

Linearized Viscous Hydrodynamics

- Macroscopic Form of Energy Momentum Tensor:

$$T^{\mu\nu} = -Pg^{\mu\nu} + (e+P)u^\mu u^\nu + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + (\frac{2}{3}\eta - \zeta)H^{\mu\nu}\partial_\rho u^\rho$$

- $\eta, \zeta =$ shear and bulk viscosity
- u^μ velocity of energy transport
- $\Delta^\mu = \partial_\mu - u_\mu u^\beta \partial_\beta, \quad H^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu}$

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Puzzles from RHIC

- Viscous hydro is compatible with experimental data only when

$$\eta/s \sim 0.2$$

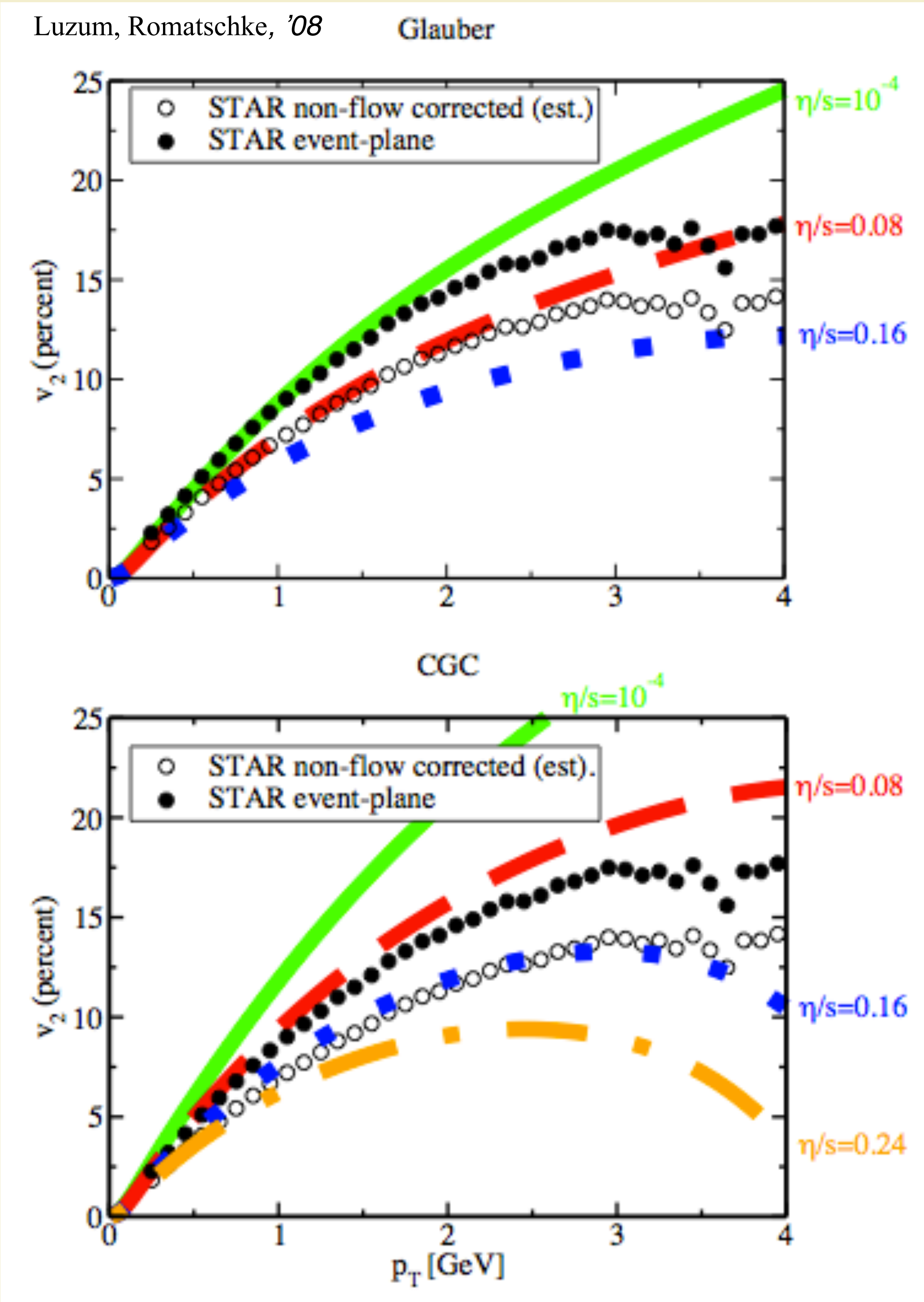
- Elliptic flow in PbPb @ 2.76TeV at LHC [ALICE: *arXiv:1011.3914 [nucl-ex]]* is identical to AuAu at RHIC

- String theory methods - AdS/CFT with gravity duals:

$$\eta/s = \frac{1}{4\pi}$$

- What are η, ζ,... in QCD? Is the plasma ‘strongly coupled’? Is N = 4 SYM really a good model for QGP?

- Ultimate answer only from non-perturbative calculations in QCD!



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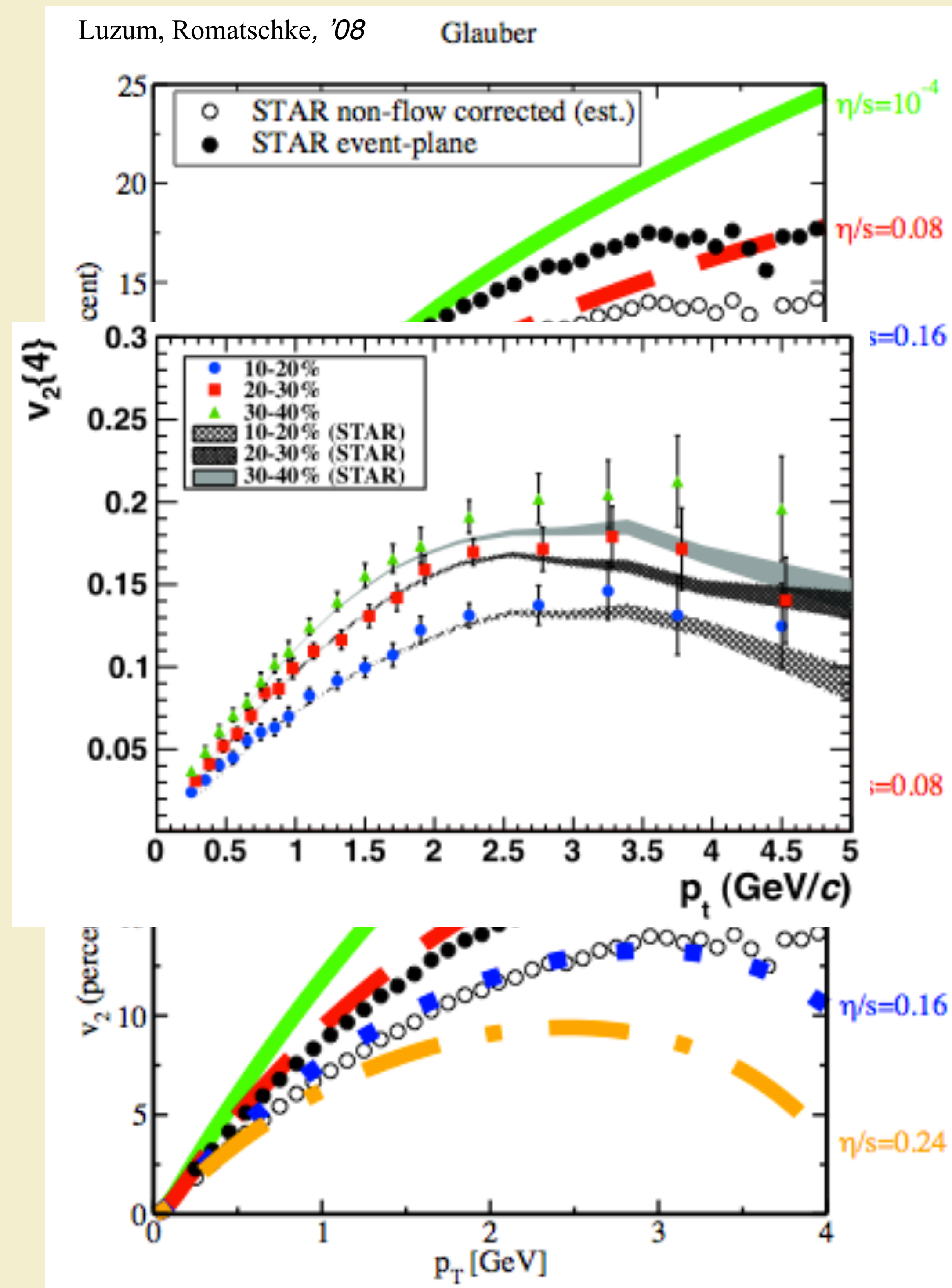


Bulk and shear viscosities: Kubo formulae

- Matching of linearized hydrodynamic and linear response description in QFT---Kubo formulae: Viscosities and other transport coeffs. are obtainable from retarded Minkowskian correlators of energy momentum tensor

$$\eta = \pi \lim_{\omega\to 0} \frac{\rho_{12,12}(\omega, \mathbf{k}=\mathbf{0})}{\omega}$$

$$\zeta = \frac{\pi}{9} \sum_{i,j=1}^{3} \lim_{\omega\to 0} \frac{\rho_{ii,jj}(\omega, \mathbf{k}=\mathbf{0})}{\omega}$$

$$\rho_{\mu\nu\rho\sigma} = \mathrm{Im} G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0})$$

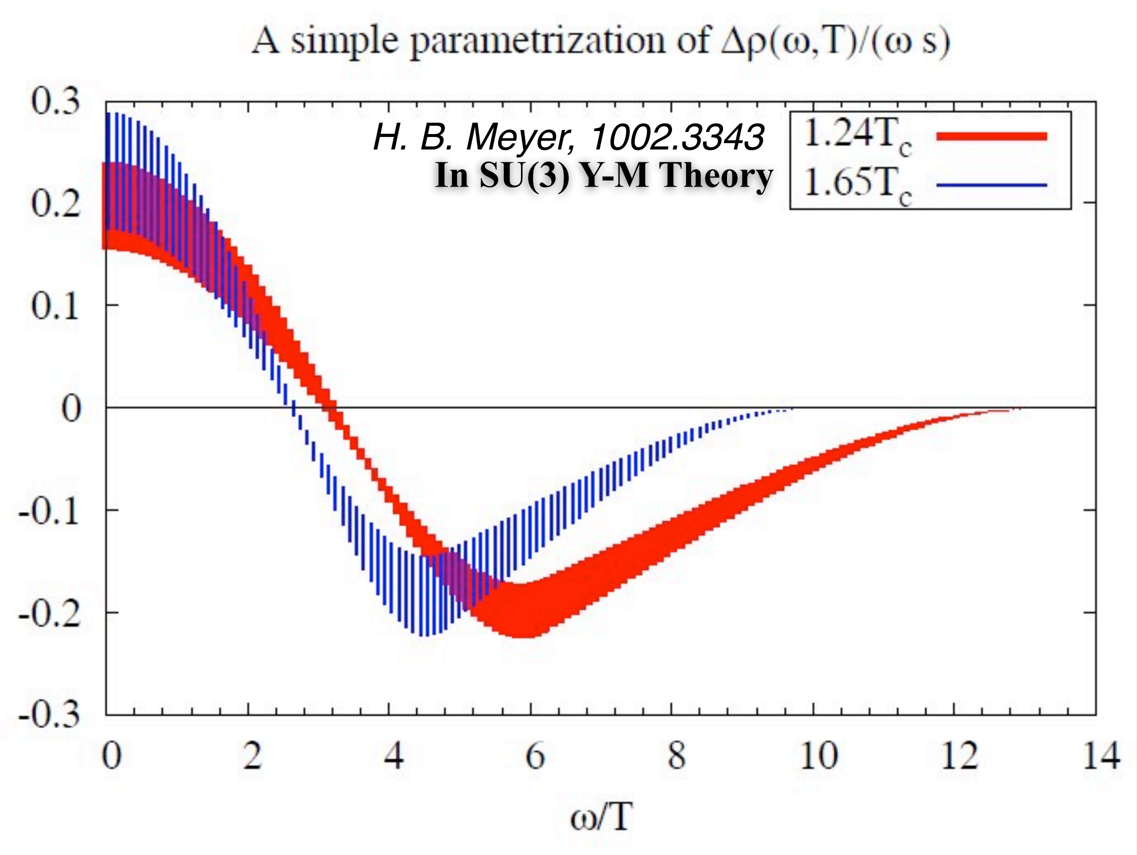
$$G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0}) \equiv i \int_0^\infty dt e^{i\omega t} \int d^3x \, \langle [T_{\mu\nu}(t, \mathbf{x}), T_{\rho\sigma}(0, \mathbf{0})] \rangle$$

$$G_R(\omega) = \tilde{G}_E(p_n \to -i[\omega + i0^+], \mathbf{0})$$

Viscosities from the lattice

- Lattice determines spectral density ρ from Euclidean correlators: Need to invert

$$G(\hat{\tau}) = \int_0^\infty \frac{d\omega}{\pi}\rho(\omega)\frac{\cosh\left(\frac{1}{2}-\hat{\tau}\right)\beta\omega}{\sinh\frac{\beta\omega}{2}}$$



- For extracting IR limit of ρ, need to understand its behavior also at $\omega \gtrsim \pi T$ — very non-trivial challenge for lattice QCD, requiring perturbative input!

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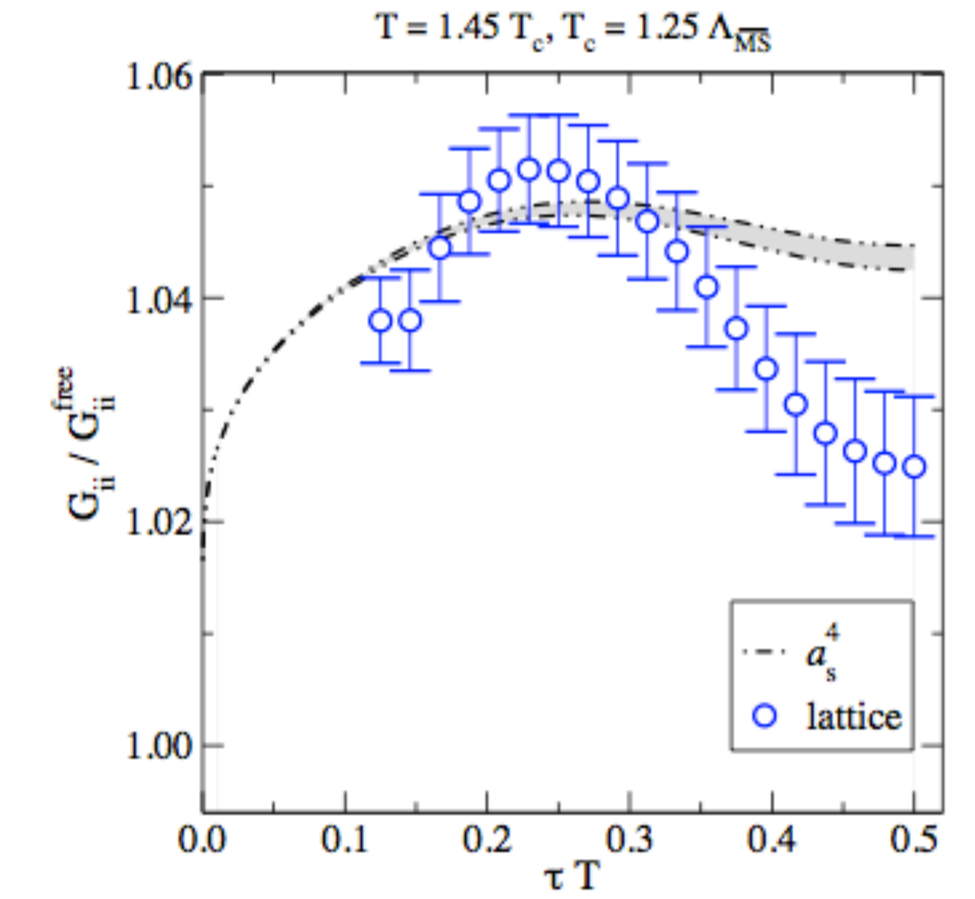
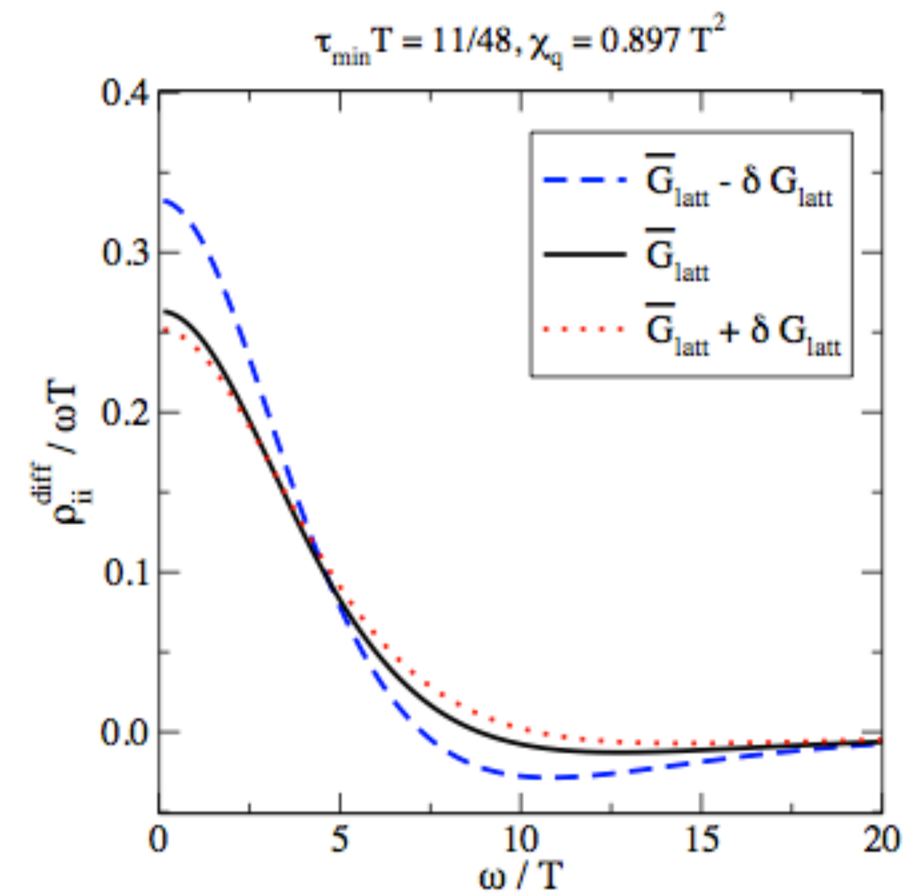
Successful application of pQCD result

- For the vector-current correlator, 5-loop vacuum limit and accurate lattice data available ⇒ Model-independent analytic continuation of Euclidean correlator à la [Burnier, Laine, Mether; *EPJC 71*] possible
- Result: Estimate for flavor current spectral density and flavour diffusion coefficient [Burnier, Laine; *EPJC 72*] $2\pi T D \gtrsim 0.8$





Setup

- SU(N$_c$) YM theory

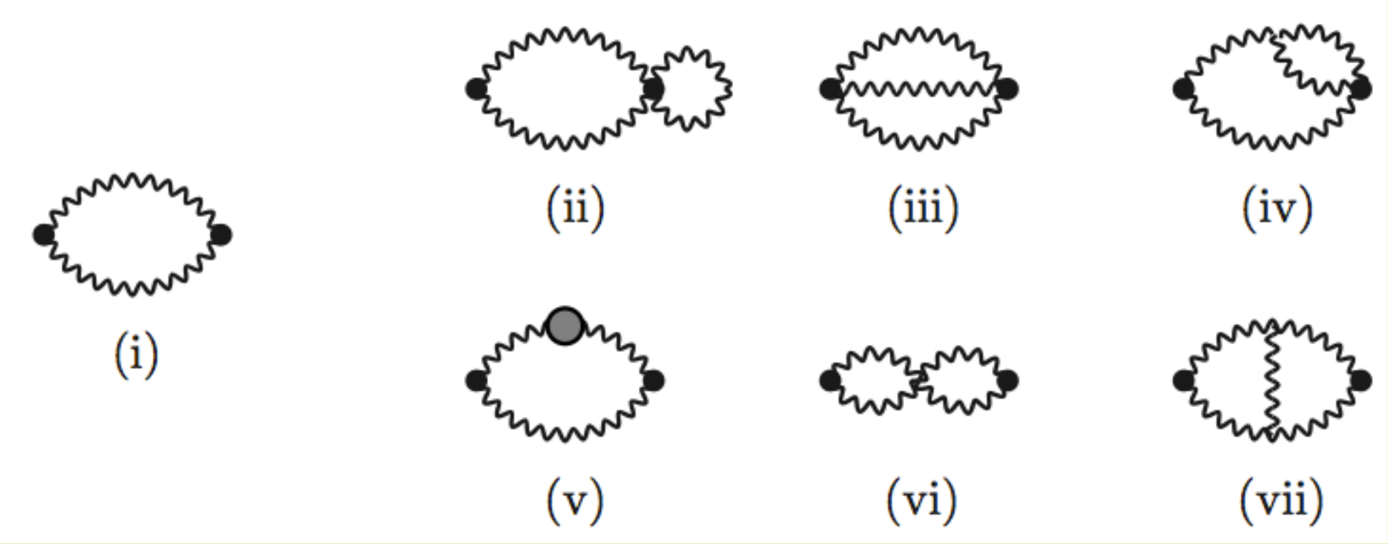
$$S_E = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^{3-2\epsilon}\mathbf{x} \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\}$$

- Define:
  - $G_\theta(x) \equiv \langle \theta(x)\theta(0) \rangle_c$, $\theta \equiv c_\theta\, g_B^2 F^a_{\mu\nu} F^a_{\mu\nu}$
  - $G_\chi(x) \equiv \langle \chi(x)\chi(0) \rangle$, $\chi \equiv c_\chi\, \epsilon_{\mu\nu\rho\sigma} g_B^2 F^a_{\mu\nu} F^a_{\rho\sigma}$
  - $G_\eta(x) = -16 c_\eta^2 \, \langle T_{12}(x)\, T_{12}(0) \rangle_c \, .$

  where $T_{\mu\nu} = \frac{1}{4}\delta_{\mu\nu} F^a_{\alpha\beta} F^a_{\alpha\beta} - F^a_{\mu\alpha} F^a_{\nu\alpha} \, ,$

Correlators to NLO

The LO and NLO Feynman graphs contributing to the correlators



- Write down diagrammatic expansions for Euclidean correlators in momentum space $\tilde{G}_\alpha(P) \equiv \int_x e^{-iP\cdot x}\tilde{G}_\alpha(x)$
- Carry out Matsubara sums by 'cutting' thermal lines and evaluate remaining 3d integrals at high P to get the OPE
- Extract the spectral densities with $\rho_\alpha(\omega) = \mathrm{Im}\tilde{G}_\alpha(p_0 = -i\omega + 0^+, \mathbf{p} = \mathbf{0})$

Spectral functions

$$\rho(\omega) = \mathrm{Im}\Big[\tilde{G}(P)\Big]_{P\to(-i[\omega+i0^+],\mathbf{0})} \, .$$

- After Matsubara sums, the imaginary part can be extracted with

$$\frac{1}{\omega \pm i0^+} = \mathbb{P}\left(\frac{1}{\omega}\right) \mp i\pi\delta(\omega)$$

- Example:

$$\mathcal{I}_{\mathrm{j}}^{0}(P) \equiv \sum\!\!\!\!\!\!\!\int_{Q,R} \frac{P^6}{Q^2R^2[(Q-R)^2+\lambda^2](Q-P)^2(R-P)^2}$$

Denoting $E_q \equiv q \, , \quad E_r \equiv r \, , \quad E_{qr} \equiv \sqrt{(\mathbf{q}-\mathbf{r})^2+\lambda^2} \, ,$

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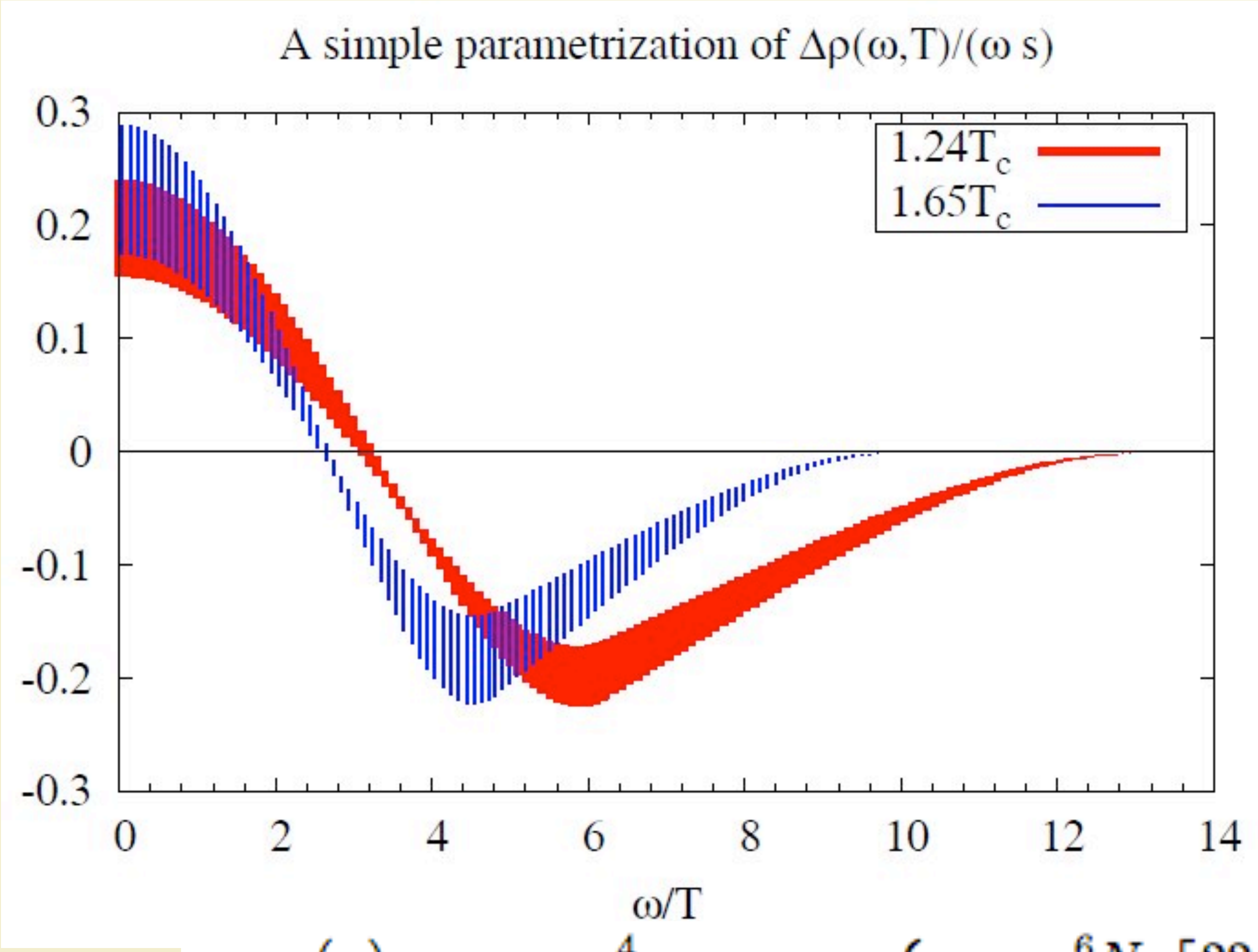
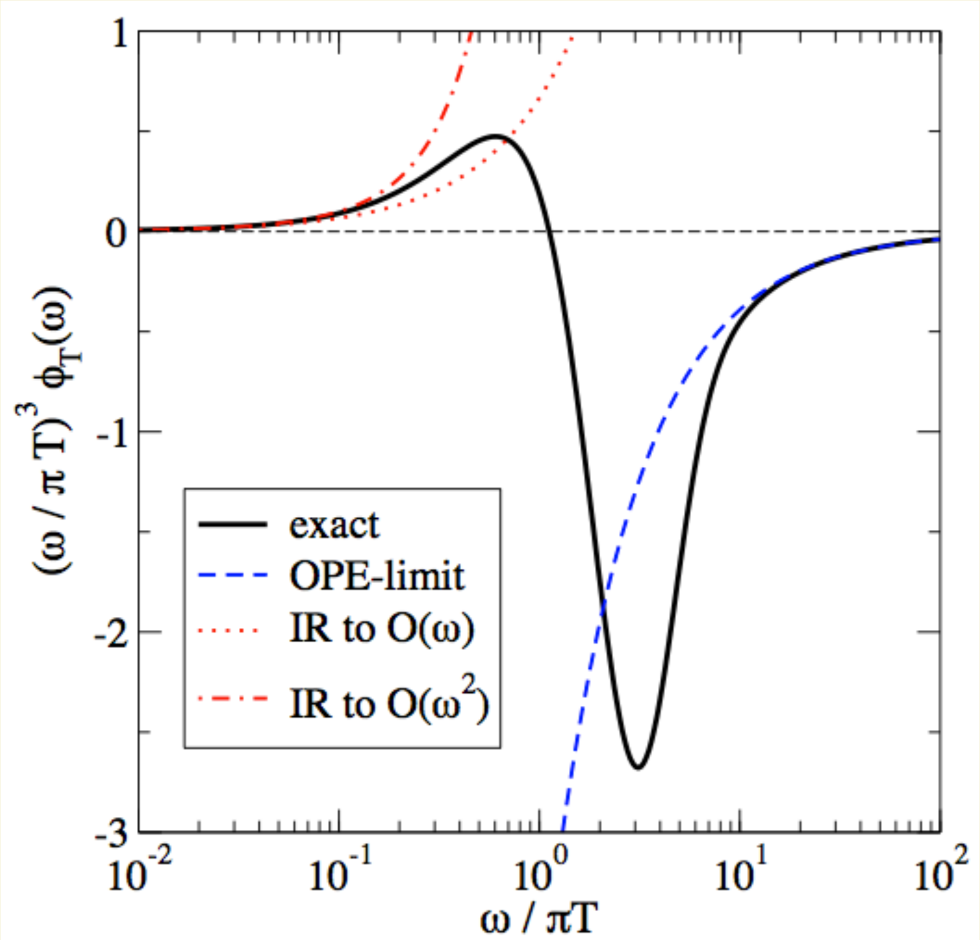
Spectral functions: Bulk channel

H. B. Meyer, 1002.3343

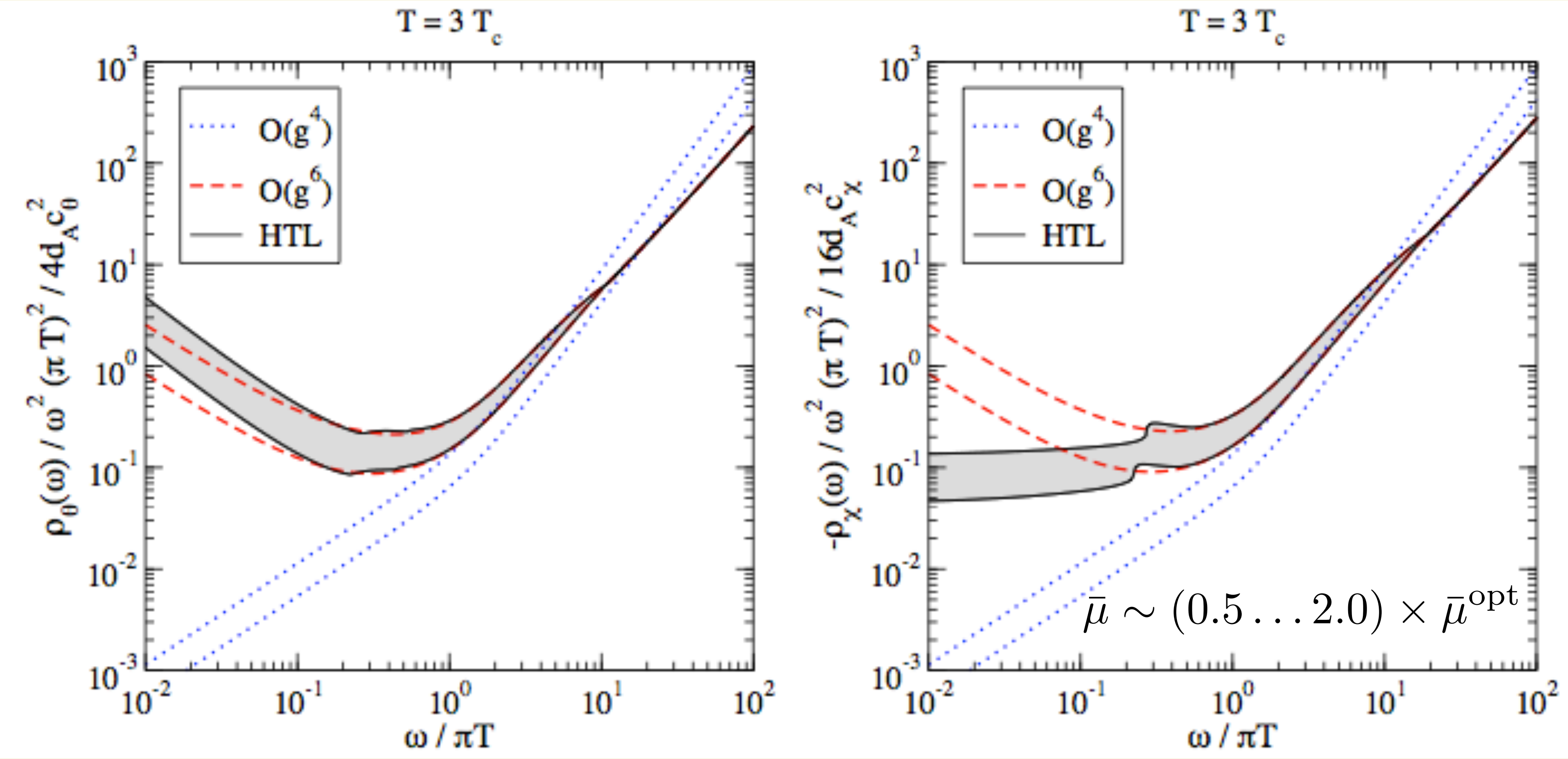




$$\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} = \frac{\pi\omega^4}{(4\pi)^2}(1+2n_{\frac{\omega}{2}})\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\frac{-\rho_\chi(\omega)}{16d_A c_\chi^2} = \frac{\pi\omega^4}{(4\pi)^2}(1+2n_{\frac{\omega}{2}})\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

Spectral functions: Bulk channel



$$\rho^{\rm QCD}_{\rm resummed} = \rho^{\rm QCD}_{\rm resummed} - \rho^{\rm HTL}_{\rm resummed} + \rho^{\rm HTL}_{\rm resummed} \approx \rho^{\rm QCD}_{\rm naive} - \rho^{\rm HTL}_{\rm naive} + \rho^{\rm HTL}_{\rm resummed} \,.$$

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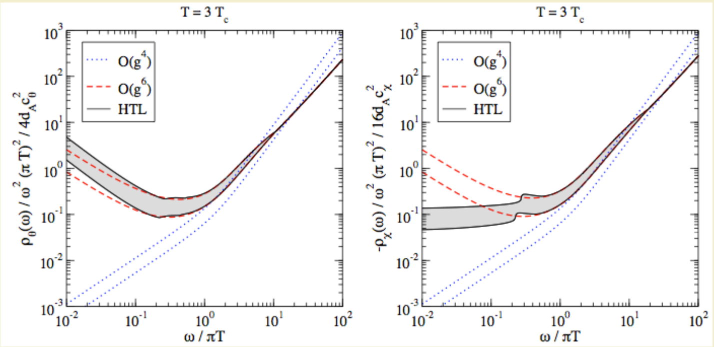
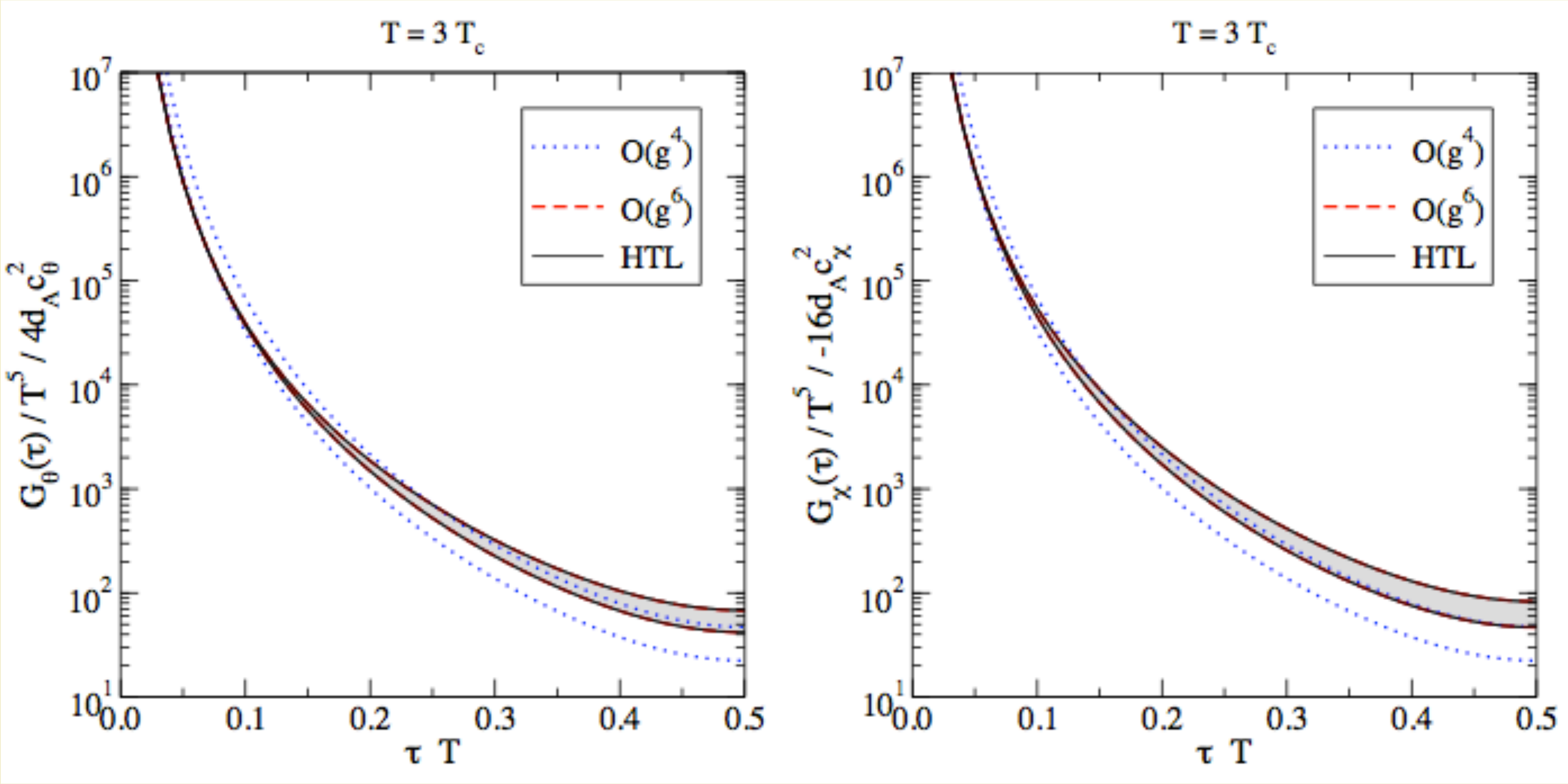
Imaginary-time correlators: Bulk channel

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega, \mathbf{0}) \frac{\cosh\left(\frac{\beta}{2} - \tau\right)\omega}{\sinh\frac{\beta\omega}{2}} \, .$$





- Considerable difference between LO and NLO in spectral function leads to a small correction to the imaginary-time correlators.

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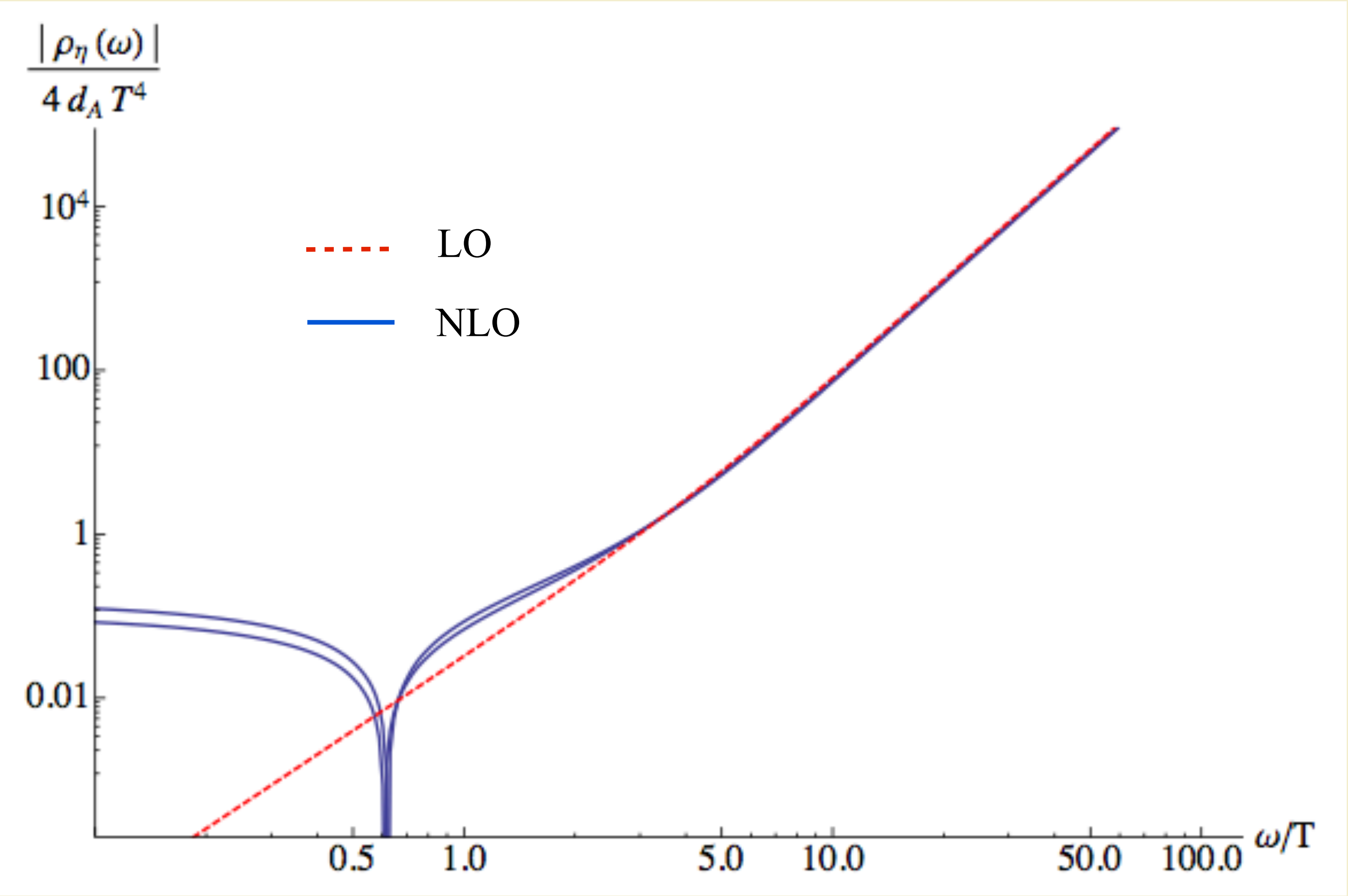
Spectral functions: Shear channel



$$\frac{\rho_\eta(\omega)}{4d_A} = \frac{\omega^4}{4\pi}\left(1+2n_{\frac{\omega}{2}}\right)\left\{-\frac{1}{10}+\frac{g^2 N_c}{(4\pi)^2}\left(\frac{2}{9}+\phi_T^\eta(\omega/T)\right)\right\}$$

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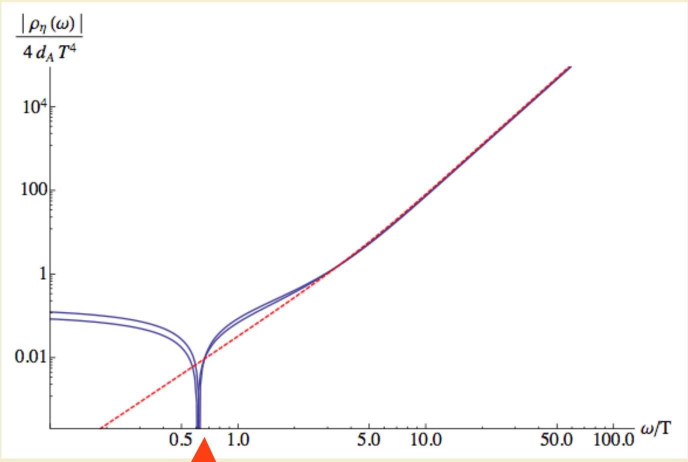
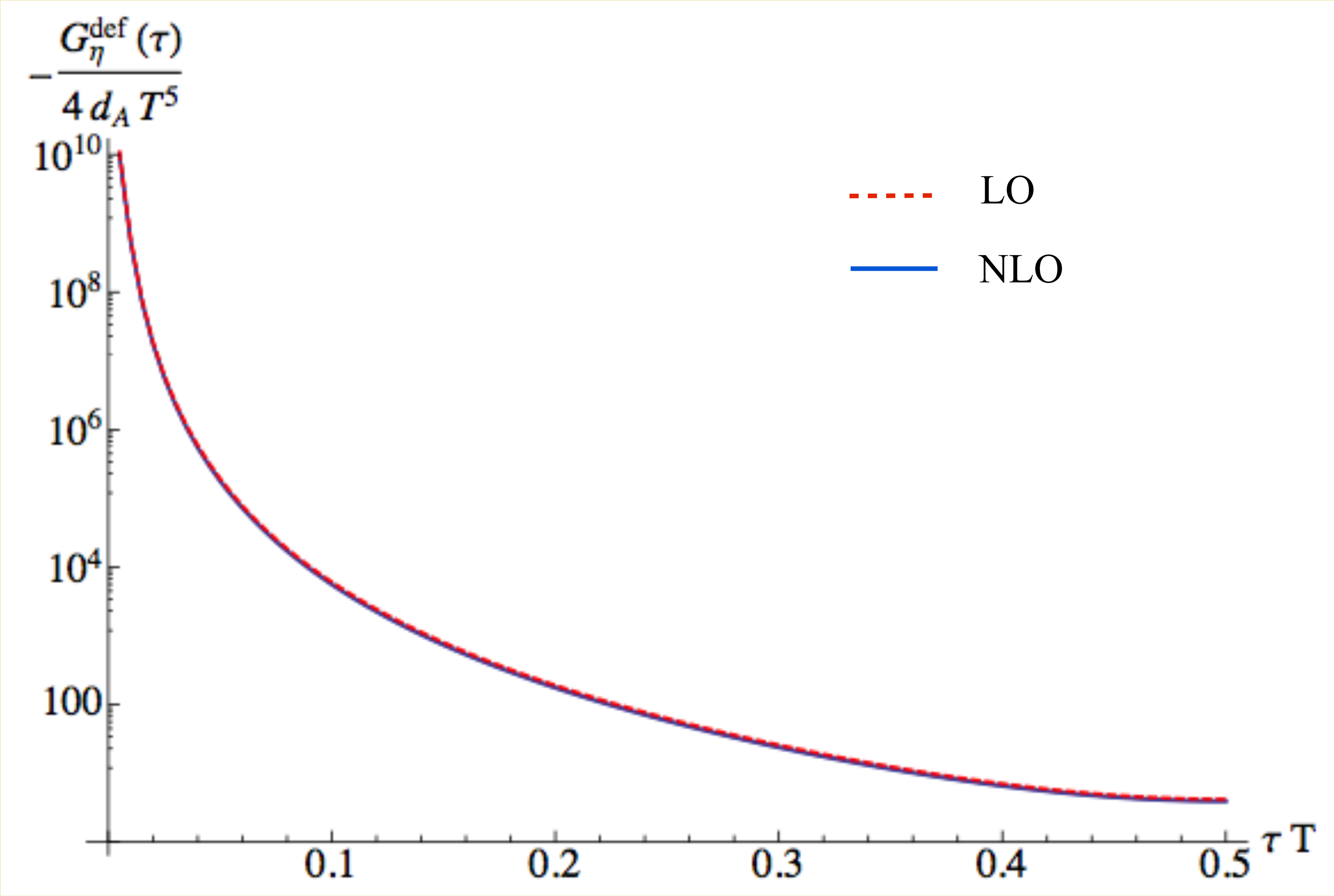
Imaginary-time correlators: Shear channel



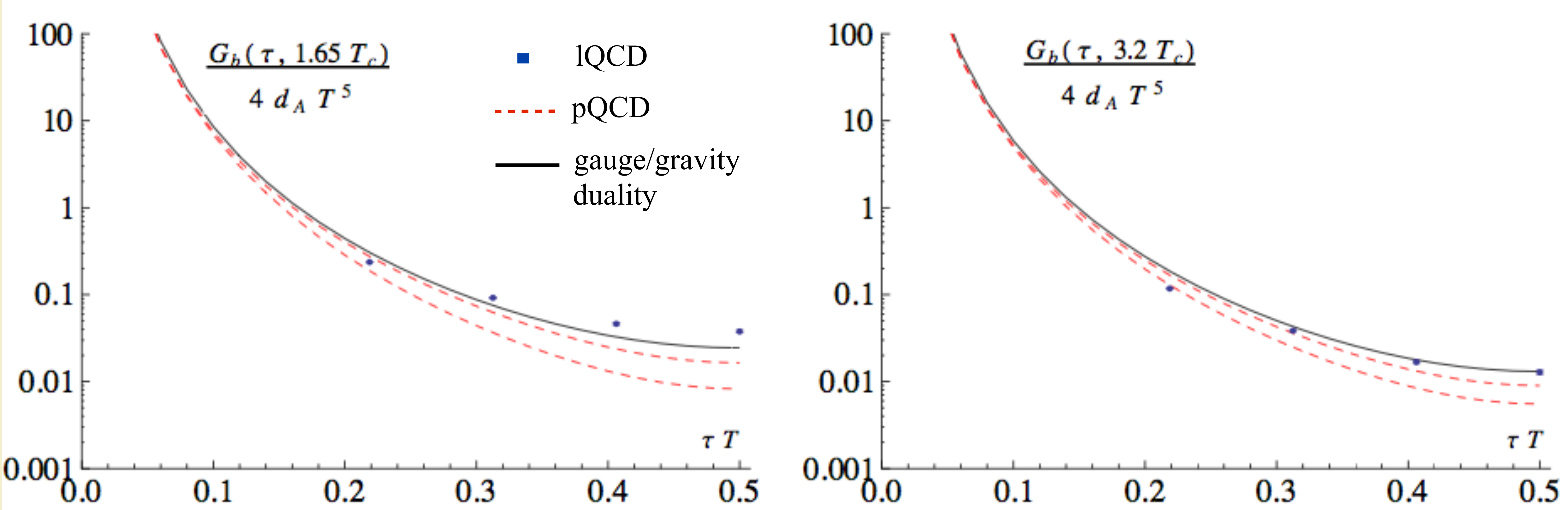
$\omega_0 \approx 0.6T$

$$G_\eta^{\rm def}(\tau) = \int_{\omega_0}^{\infty} \frac{{\rm d}\omega}{\pi} \rho_\eta(\omega) \frac{\cosh\left[\left(\frac{\beta}{2}-\tau\right)\omega\right]}{\sinh\frac{\beta\omega}{2}}$$



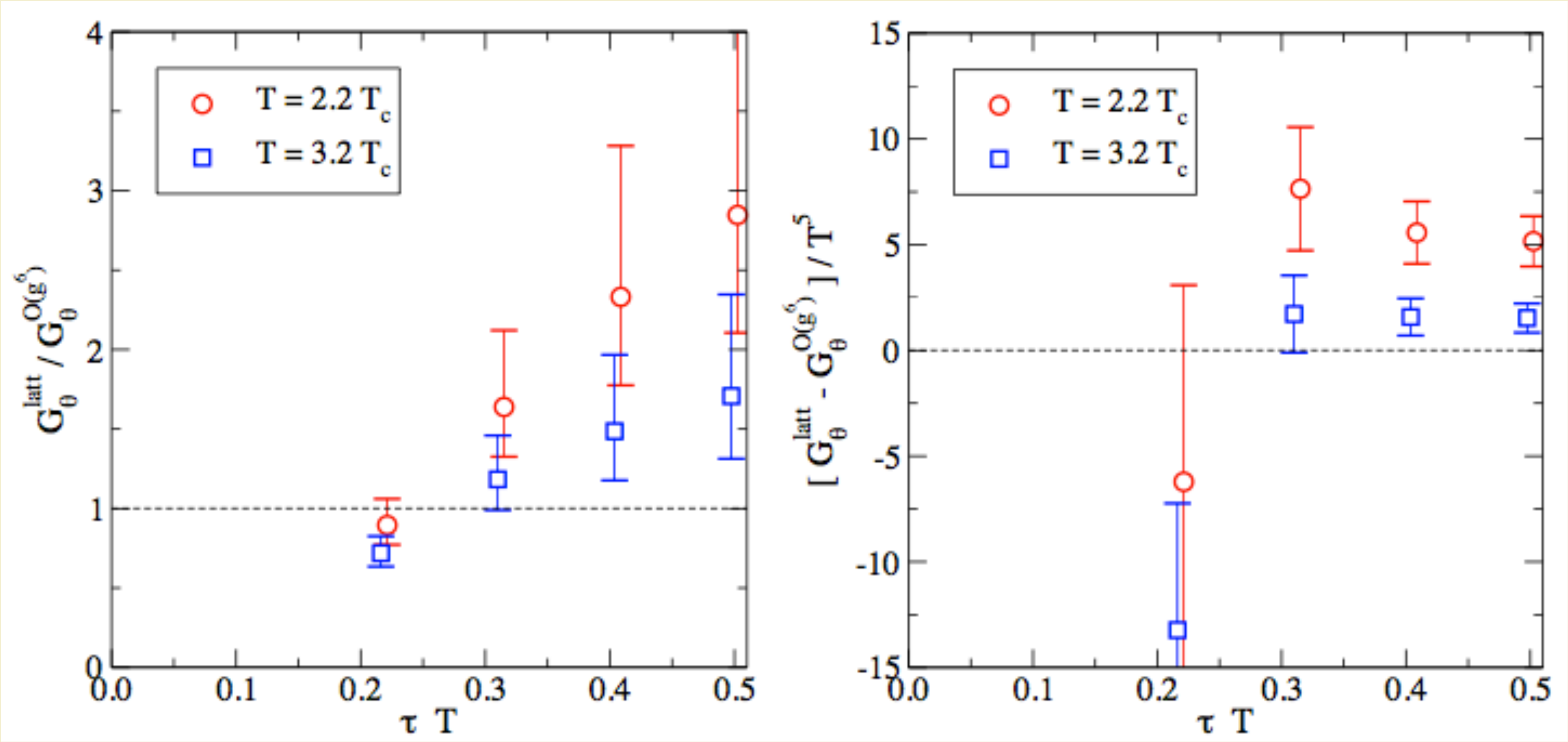
Lattice vs. pQCD vs. gauge/gravity duality: Bulk channel

K. Kajantie, M. Krssak and A. Vuorinen, arXiv:1302.1432 [hep-ph].



Lattice vs. pQCD: Bulk channel

Lattice data from H.B. Meyer, JHEP 04(2010), 099 [10023344]



- The ratio shows good agreement at short distances.
- The difference no longer shows the short distance divergence.
  A model independent analytic continuation could be attempted.

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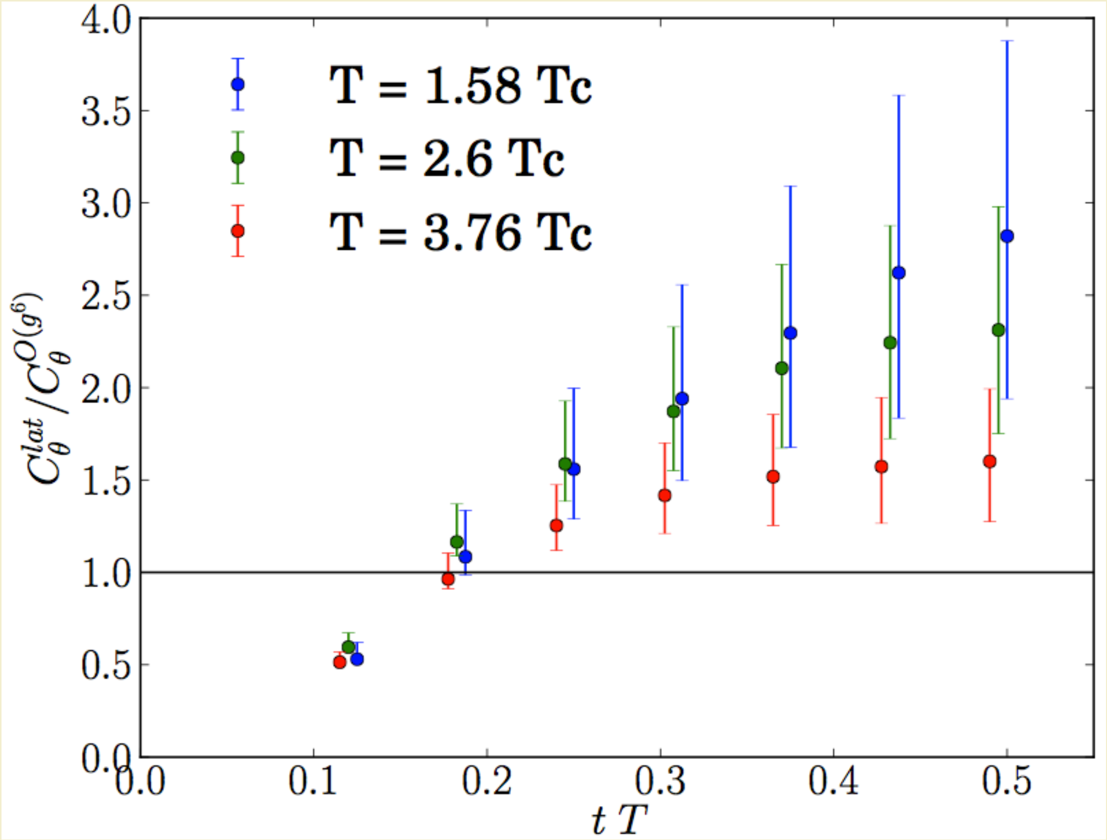
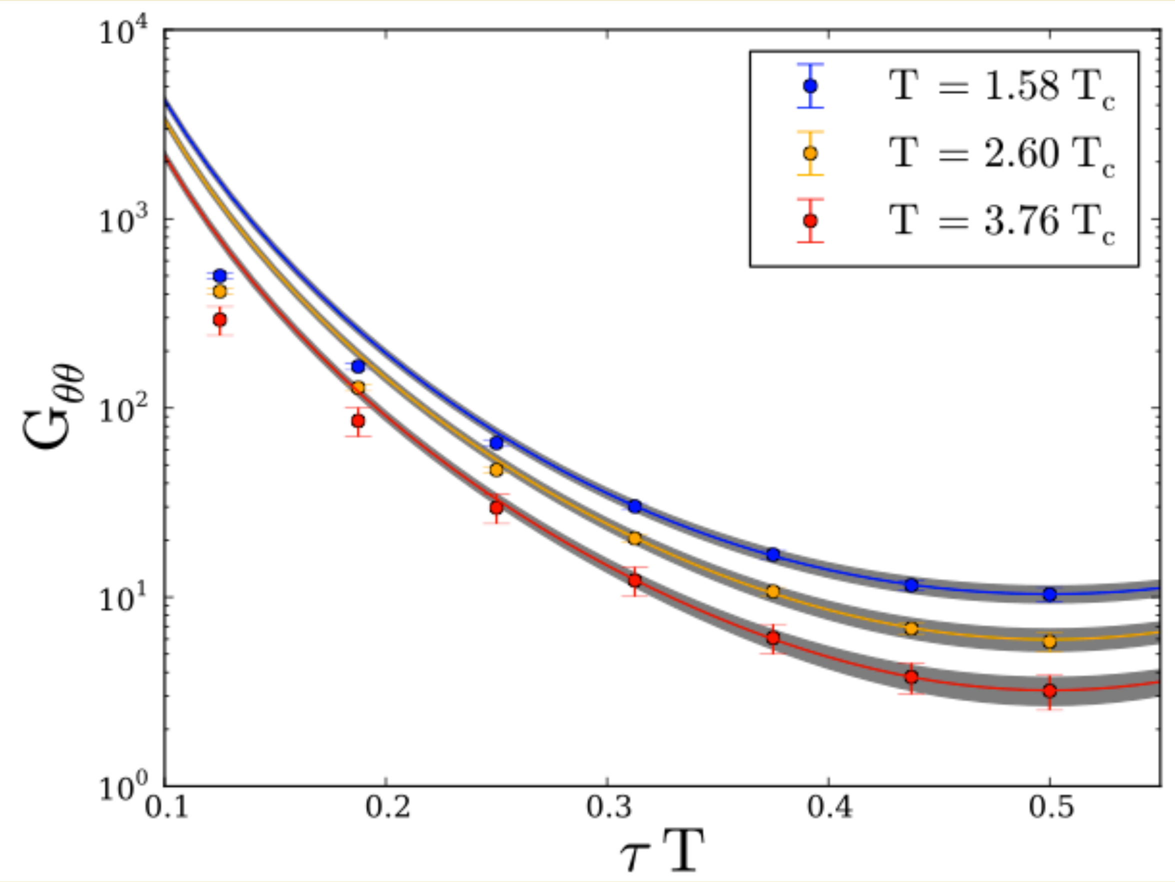
Lattice vs. pQCD: Bulk channel

Chuan Miao(CPOD2011), H. B. Meyer



Chuan Miao, H.B. Meyer (Preliminary)



- Right panel: Fit correlators with Breit Wigner formula (low frequency) + NLO results (high frequency), the width of the B-W is fixed to 0.5πT.
- NLO perturbative input is very helpful.

Summary and Outlook

- Information on correlation functions of the energy momentum tensor crucial for disentangling the properties of the QGP
  - Wilson coefficients refined and determined in the OPE
  - Spectral densities needed in extracting transport coefficients from lattice QCD data
- NLO results in the bulk and shear channels completed, HTL for the shear channel underway
  - Results promising, but quantitative comparisons await
- If pure YM results useful, inclusion of fermions straightforward