On the Perturbative Evaluation of Thermal Green’s Functions in the Bulk and Shear Channels of Yang-Mills Theory

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• Motivation
• Correlators in SU(Nc) Yang-Mills theory
• Results
  • Correlators in the UV limit
  • Spectral densities
• Summary and Outlook
Macroscopic Form of Energy Momentum Tensor:

\[ T^{\mu\nu} = -Pg^{\mu\nu} + (e + P)u^\mu u^\nu + \Delta T^{\mu\nu} \]
\[ \Delta T^{\mu\nu} = \eta(\Delta^\mu u^\nu + \Delta^\nu u^\mu) + \left(\frac{2}{3}\eta - \zeta\right)H^{\mu\nu} \partial_\rho u^\rho \]

- \( \eta, \zeta = \) shear and bulk viscosity
- \( u^\mu = \) velocity of energy transport
- \( \Delta^\mu = \partial_\mu - u_\mu u^\beta \partial_\beta \), \( H^{\mu\nu} = u^\mu u^\nu - g^{\mu\nu} \)
Puzzles from RHIC

- Viscous hydro is compatible with experimental data only when 
  \[ \frac{\eta}{s} \sim 0.2 \]

- Elliptic flow in PbPb @ 2.76TeV at LHC [ALICE: arXiv:1011.3914 [nucl-ex]] is identical to AuAu at RHIC

- String theory methods - AdS/CFT with gravity duals:
  \[ \frac{\eta}{s} = \frac{1}{4\pi} \]

- What are \( \eta, \zeta, \ldots \) in QCD? Is the plasma ‘strongly coupled’? Is \( N = 4 \) SYM really a good model for QGP?

- Ultimate answer only from non-perturbative calculations in QCD!

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Matching of linearized hydrodynamic and linear response description in QFT—**Kubo formulae**: Viscosities and other transport coeffs. are obtainable from retarded Minkowskian correlators of energy momentum tensor.

\[
\eta = \pi \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, \mathbf{k} = \mathbf{0})}{\omega} \\
\zeta = \frac{\pi}{9} \sum_{i,j=1}^{3} \lim_{\omega \to 0} \frac{\rho_{ii,jj}(\omega, \mathbf{k} = \mathbf{0})}{\omega}
\]

\[
\rho_{\mu\nu\rho\sigma} = \text{Im} G_{\mu\nu\rho\sigma}^{R}(\omega, \mathbf{0})
\]

\[
G_{\mu\nu\rho\sigma}^{R}(\omega, \mathbf{0}) \equiv i \int_{0}^{\infty} dt e^{i\omega t} \int d^{3}x \langle [T_{\mu\nu}(t, \mathbf{x}), T_{\rho\sigma}(0, \mathbf{0})] \rangle
\]

\[
G_{R}(\omega) = \tilde{G}_{E} (p_n \to -i[\omega + i0^{+}], \mathbf{0})
\]
Lattice determines spectral density $\rho$ from Euclidean correlators: Need to invert

$$G(\hat{\tau}) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho(\omega) \cosh \left( \frac{1}{2} - \hat{\tau} \right) \beta \omega}{\sinh \frac{\beta \omega}{2}}$$

For extracting IR limit of $\rho$, need to understand its behavior also at $\omega \gtrsim \pi T$ — very non-trivial challenge for lattice QCD, requiring perturbative input!
Successful application of pQCD result

For the vector-current correlator, 5-loop vacuum limit and accurate lattice data available ⇒ Model-independent analytic continuation of Euclidean correlator à la [Burnier, Laine, Mether; *EPJC* 71] possible

Result: Estimate for flavor current spectral density and flavour diffusion coefficient [Burnier, Laine; *EPJC* 72] $2\pi T D \gtrsim 0.8$
• **SU(N_c) YM theory**

\[ S_E = \int_0^\beta \int d^3-2_\mathcal{X} \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right\} \]

• **Define:**

- \[ G_\theta(x) \equiv \langle \theta(x) \theta(0) \rangle_c \]
- \[ \theta \equiv c_\theta g_B^2 F_{\mu\nu}^a F_{\mu\nu}^a \]
- \[ G_\chi(x) \equiv \langle \chi(x) \chi(0) \rangle_c \]
- \[ \chi \equiv c_\chi \epsilon_{\mu\nu\rho\sigma} g_B^2 F_{\mu\nu}^a F_{\rho\sigma}^a \]
- \[ G_\eta(x) = -16c_\eta^2 \langle T_{12}(x) T_{12}(0) \rangle_c \]

where

\[ T_{\mu\nu} = \frac{1}{4} \delta_{\mu\nu} F_{\alpha\beta}^a F_{\alpha\beta}^a - F_{\mu\alpha}^a F_{\nu\alpha}^a , \]
Write down diagrammatic expansions for Euclidean correlators in momentum space $\tilde{G}_\alpha(P) \equiv \int_x e^{-iP \cdot x} \tilde{G}_\alpha(x)$

Carry out Matsubara sums by ‘cutting’ thermal lines and evaluate remaining 3d integrals at high $P$ to get the OPE

Extract the spectral densities with $\rho_\alpha(\omega) = \text{Im}\tilde{G}_\alpha(p_0 = -i\omega + 0^+, p = 0)$
After Matsubara sums, the imaginary part can be extracted with

\[
\rho(\omega) = \text{Im} \left[ \tilde{G}(P) \right] \bigg|_{P \rightarrow (-i[\omega+i0^+],0)}.
\]

- Example:

\[
\frac{1}{\omega \pm i0^+} = \mathbb{P} \left( \frac{1}{\omega} \right) \mp i\pi\delta(\omega).
\]

\[
\mathcal{I}_0^0(P) \equiv \int_{Q,R} \frac{P^6}{Q^2R^2[(Q - R)^2 + \lambda^2](Q - P)^2(R - P)^2}
\]

Denoting \( E_q \equiv q, \quad E_r \equiv r, \quad E_{qr} \equiv \sqrt{(q - r)^2 + \lambda^2} \).
Spectral functions:

Bulk channel

A simple parametrization of $\Delta \rho(\omega,T)/(\omega \, s)$

$$\rho_{\theta}(\omega) = \frac{\pi \omega^4}{(4\pi)^2} \left(1 + 2n_{\frac{\omega}{2}}\right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[ \frac{22}{3} \ln \frac{\mu^2}{\omega^2} + \frac{73}{3} + 8 \phi_T(\omega) \right] \right\} + \mathcal{O}(g^8)$$

$$-\rho_{\chi}(\omega) = \frac{\pi \omega^4}{(4\pi)^2} \left(1 + 2n_{\frac{\omega}{2}}\right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[ \frac{22}{3} \ln \frac{\mu^2}{\omega^2} + \frac{97}{3} + 8 \phi_T(\omega) \right] \right\} + \mathcal{O}(g^8)$$
Spectral functions: Bulk channel

\[ \mu \sim (0.5 \ldots 2.0) \times \bar{\mu}_{\text{opt}} \]

\[ \rho_{\text{QCD}} \sim \rho_{\text{QCD}} - \rho_{\text{HTL}} + \rho_{\text{HTL}} \approx \rho_{\text{QCD naive}} - \rho_{\text{HTL naive}} + \rho_{\text{HTL resummed}}. \]
Considerable difference between LO and NLO in spectral function leads to a small correction to the imaginary-time correlators.
Spectral functions: Shear channel

\[
\frac{\rho_\eta(\omega)}{4d_A T^4} = \frac{\omega^4}{4\pi} \left(1 + 2n_{\frac{\omega}{2}}\right) \left\{ -\frac{1}{10} + \frac{g^2 N_c}{(4\pi)^2} \left(\frac{2}{9} + \phi^*_T(\omega/T)\right) \right\}
\]
Imaginary-time correlators: Shear channel

\[ G_\eta^{\text{def}}(\tau) = \int_{\omega_0}^{\infty} \frac{d\omega}{\pi} \rho_\eta(\omega) \frac{\cosh \left[ \left( \frac{\beta}{2} - \tau \right) \omega \right]}{\sinh \frac{\beta \omega}{2}} \]

\[ \omega_0 \approx 0.6T \]
\textbf{Lattice vs. pQCD vs. gauge/gravity duality: Bulk channel}

Lattice vs. pQCD:
Bulk channel

Lattice data from H.B. Meyer, JHEP 04(2010), 099 [10023344]

The ratio shows good agreement at short distances.
The difference no longer shows the short distance divergence.
A model independent analytic continuation could be attempted.
C. Miao, H. B. Meyer (Preliminary)

Right panel: Fit correlators with Breit Wigner formula (low frequency) + NLO results (high frequency), the width of the B-W is fixed to 0.5πT.

NLO perturbative input is very helpful.
Summary and Outlook

- Information on correlation functions of the energy momentum tensor crucial for disentangling the properties of the QGP
  - Wilson coefficients refined and determined in the OPE
  - Spectral densities needed in extracting transport coefficients from lattice QCD data
- NLO results in the bulk and shear channels completed, HTL for the shear channel underway
- Results promising, but quantitative comparisons await

🌟 If pure YM results useful, inclusion of fermions straightforward