QCD Reggeon Field Theory from JIMWLK / KLWMIJ Evolution

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IS2013, Illa da Toxa - Galicia (Spain)
September 13, 2013

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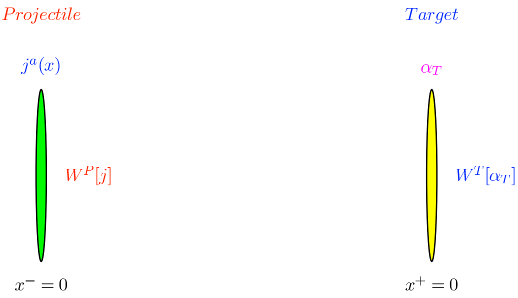
arXiv: 1306.2794 [hep-ph]

Motivation and goals

- Main question we would like to address is : What is the high energy limit of QCD?
  - Many candidates : BFKL Pomeron Calculus, Lipatov's effective action, elements of Field Theory of Bartels, JIMWLK/KLWMIJ Hamiltonians for Wilson line Operators,...
- Is it possible to derive an effective theory of QCD in terms of color singlet exchange amplitudes, the Reggeon Field Theory of QCD?
- Will this RFT reduce to BFKL Pomeron Calculus with a universal triple Pomeron vertex? Will there me more Reggeons and more vertices?

High Energy Scattering



The S-matrix : $$S(Y) = \langle T \langle P | \hat{S}(\rho_p, \alpha_T) | P \rangle T \rangle$$

More generally, any observable $\mathcal{O}(\rho_p, \alpha_T)$: $$\langle \mathcal{O} \rangle_Y = \langle T \langle P | \hat{\mathcal{O}}(\rho_p, \alpha_T) | P \rangle T \rangle$$

What about the evolution?

How do these averages change with the increasing energy of the system?

Rapidity Evolution

An observable averaged over the projectile : $\langle P|\hat{\mathcal{O}}(\rho_p, \alpha_T)|P\rangle = \int D\rho_p \hat{\mathcal{O}}(\rho_p, \alpha_T) W_Y^p[\rho_p]$

projectile wave function

Rapidity evolution $\rightarrow \frac{d}{dY}\langle P|\hat{\mathcal{O}}|P\rangle = -\int D\rho_p \;\; \hat{\mathcal{O}}(\rho_p, \alpha_T) \;\; H[\rho_p, \delta/\delta\rho_p] \;\; W_Y^p[\rho_p]$

the high energy effective Hamiltonian

$$\Rightarrow \underbrace{\frac{d}{dY} W^p[\rho_p] = -H[\rho_p, \delta/\delta\rho_p] W^p[\rho_p]}$$

the energy dependence of the average is defined by the spectrum of the Hamiltonian

Derivation of H is available in two limits:

JIMWLK limit : dense projectile - dilute target (nonlinear effects of gluon saturation is taken into account, but not multiple scattering corrections)

KLWMIJ limit : dilute projectile - dense target (multiple scattering corrections are taken into account but not nonlinear effects of gluon saturation)

JIMWLK and KLWMIJ Hamiltonians are related with a dense-dilute duality transformation. Thus they have identical spectra...

We choose to work KWLMIJ Hamiltonian $\rightarrow$ scattering of a dilute projectile on a dense target.

KLWMIJ Hamiltonian

A. Kovner & M. Lublinsky 2005

The KLWMIJ Hamiltonian is the limit of the effective high energy Hamiltonian for a dilute partonic system ($\rho_p \to 0$) which scatters on a dense target. It take into accounts the linear gluon emission in the projectile and multiple rescatterings off the target.

The explicit form of the KLWMIJ Hamiltonian :

$$H_{\text{KLWMIJ}} = \frac{\alpha_s}{2\pi^2} \int_{x,y,z} \frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2} \left[J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(y) R^{ab}(z) J_R^b(x) \right]$$

$R(x) = e^{T^a \frac{\delta}{\delta \rho^a}}$ $\quad \rightarrow \quad$ Scattering matrix of a projectile quark

The left and right SU(N) generators :

$$\begin{aligned} J_L^a(x) &= \operatorname{tr}\left[\frac{\delta}{\delta R_x^T} T^a R_x\right] - \operatorname{tr}\left[\frac{\delta}{\delta R_x^*} R_x^\dagger T^a\right] \\ J_R^a(x) &= \operatorname{tr}\left[\frac{\delta}{\delta R_x^T} R_x T^a\right] - \operatorname{tr}\left[\frac{\delta}{\delta R_x^*} T^a R_x^\dagger\right] \end{aligned}$$

Projecting KLWMIJ onto RFT

Reggeons are physical scattering amplitudes - color singlets. $H_{\rm KLWMIJ}$ defines a $2+1$ dimensional QFT of unitary matrix R. BUT this is not a QFT of Reggeons.

HOW CAN WE PROJECT $H_{\rm KLWMIJ}$ ONTO COLOR SINGLETS AND DERIVE QCD REGGEON FIELD THEORY?

First step is to choose effective degrees of freedom and make sure to preserve symmetries:

- $SU_L(N)\times SU_R(N)$ - effective degrees of freedom must be scalars.
- Charge conjugation : $R(x)\to R^*(x)$
- Time Reversal (Signature) : $R(x)\to R^\dagger(x)$

Elements of QCD Reggeon Field Theory

There is infinite number of independent color singlets but there is a natural hierarchy :

Dipole : $d(x,y) = \frac{1}{N_c}\mathrm{tr}[R^\dagger(x)R(y)]$

Quadrople : $Q(x,y,u,v) = \frac{1}{N_c}\mathrm{tr}[R^\dagger(x)R(y)R^\dagger(u)R(v)]$

Naturally decomposes into

Pomeron : C, T even $\quad P_{12} = \frac{1}{2}[2 - d_{12} - d_{21}]$

Odderon : C, T odd $\quad O_{12} = \frac{1}{2}[d_{12} - d_{21}]$

B - Reggeon : C, T even, perturbatively orthogonal to P

$$B_{1234} = \frac{1}{4}[4 - Q_{1234} - Q_{4123} - Q_{3214} - Q_{2143}] - [P_{12} + P_{14} + P_{23} + P_{34} - P_{13} - P_{24}]$$

C - Reggeon : C odd, T even $\quad C_{1234} = \frac{1}{4}[Q_{1234} + Q_{4123} - Q_{3214} - Q_{2143}]$

T odds $\quad D^{\pm}_{1234} = \frac{1}{4}[Q_{1234} - Q_{4123}] \pm \frac{1}{4}[Q_{3214} - Q_{2143}]$

$H_{\mathrm{KLWMIJ}} \to H_{\mathrm{RFT}}$

How do we get H_{RFT}? $\to$ dynamics of the Reggeons & "re-express" H_{KLWMIJ} in terms of Reggeon degrees of freedom

dipole evolution: act on the dipole operator by the KLWMIJ Hamiltonian

$$\frac{d}{dY} d_{xy} = \frac{\bar{\alpha}_s}{2\pi} \int_z M_{xyz} \big(d_{xz} d_{zy} - d_{xy} \big)$$

$$d_{xy} = 1 - P_{xy} + O_{xy} \qquad\qquad \frac{(x-y)^2}{(x-z)^2 (y-z)^2}$$

The evolution equation naturally decomposes into

$$\begin{aligned}
\frac{d}{dY} P_{xy} &= \frac{\bar{\alpha}_s}{2\pi} \int_z M_{xyz} \Big[P_{xz} + P_{zy} - P_{xy} - P_{xz} P_{zy} - O_{xz} O_{zy} \Big] \\
\frac{d}{dY} O_{xy} &= \frac{\bar{\alpha}_s}{2\pi} \int_z M_{xyz} \Big[O_{xz} + O_{zy} - O_{xy} - O_{xz} P_{zy} - P_{xz} O_{zy} \Big]
\end{aligned}$$

Suppose now instead of acting on P and O we act on an arbitrary function $W[P, O]$. In the leading N_c :

$$H_{\mathrm{KLWMIJ}} W[P, O] = -\frac{d}{dY} W[P, O] = -\int_{xy} \frac{d}{dY} P_{xy} \frac{\delta}{\delta P_{xy}} W + \frac{d}{dY} O_{xy} \frac{\delta}{\delta O_{xy}} W$$

$H_{\mathrm{KLWMIJ}} \to H_{\mathrm{RFT}}$

Thus, at large N_c we have

$$H_{KLWMIJ} = H_P + H_O$$

$$\frac{\delta}{\delta P_{xy}} W[P,O]$$

$$\begin{aligned}
H_P &= -\frac{\bar{\alpha}_s}{2\,\pi} \int_{x,y,z} M_{x,y;z} \left\{ [P_{x,z} + P_{z,y} \;-\; P_{x,y} - P_{x,z}P_{z,y} + O_{x,z}O_{z,y}]\, P^{\dagger}_{x,y} \right\} \\
H_O &= -\frac{\bar{\alpha}_s}{2\,\pi} \int_{x,y,z} M_{x,y;z} \left\{ [O_{x,z} + O_{z,y} \;-\; O_{x,y} - O_{x,z}P_{z,y} - P_{x,z}O_{z,y}]\, O^{\dagger}_{x,y} \right\}
\end{aligned}$$

$$\frac{\delta}{\delta O_{xy}} W[P,O]$$

Allowing B and C dependence on W, we get

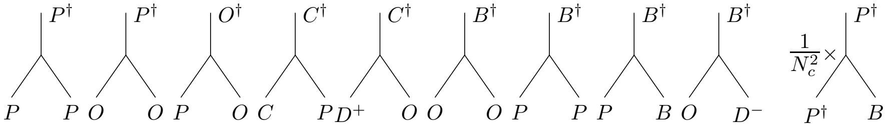
$$\begin{aligned}
H_C &= -\frac{\bar{\alpha}_s}{2\,\pi} \int \Big\{ -[M_{xyz} + M_{uvz} - L_{xuvyz}]C_{xyuv} + 4L_{xvuvz}C_{xyuz}C^{\dagger}_{xyuv} - 4L_{xvuvz}C_{xyuz}P_{zv}C^{\dagger}_{xyuv} \\
&\qquad -4L_{x,v,u,v;z}D^{-}_{xyuz}O_{zv}C^{\dagger}_{xyuv} \Big\}
\end{aligned}$$

$$\begin{aligned}
H_B &= -\frac{\bar{\alpha}_s}{2\pi} \int \Big\{ -[M_{xyz} + M_{uvz} - L_{xuvyz}]B_{xyuv}B^{\dagger}_{xyuv} + 4L_{xvuvz}B_{xyuz}B^{\dagger}_{xyuv} \\
&- 2L_{x,y,u,v;z}\Big[P_{xv}P_{uy} + O_{xv}O_{uy}\Big]B^{\dagger}_{xyuv} - 2P_{xz}P_{yz}\Big[2L_{xyuvz}B^{\dagger}_{xyuv} - \Big(L_{xuyvz} + L_{xvyuz}\Big)B^{\dagger}_{xuyv}\Big] \\
&- 4P_{xz}P_{yu}\Big[2L_{xyxvz}B^{\dagger}_{xyuv} - L_{xyxuz}B^{\dagger}_{xyvu}\Big] \Big\}
\end{aligned}$$

The splitting vertices

At leading N_c we have variety of vertices! (Anything allowed by the symmetries.)

All of them have the nature of splitting : 1 Reggeon $\rightarrow$ 2 Reggeons



WE CAN READ OF EACH VERTEX SIMPLY FROM THE HAMILTONIAN.

$$H_{\mathrm{KLWMIJ}} = H_P + H_O + H_B + H_C$$

WHAT ABOUT SUBLEADING N_c TERMS?

Beyond Leading N_c

[on going work : Altinoluk, Armesto, Kovner, Levin, Lublinksy]

When acting on W with H_{KLWMIJ}, the two color charge densities $J_{L(R)}$ can either act on the same dipole or it can act on two different dipoles.

- action on the same dipole leads to the Hamiltonian which is linear in $P^\dagger$ and $O^\dagger$.
- action on two different dipoles is $\frac{1}{N_c^2}$ suppressed and it is quadratic in the conjugate Reggeon operators.

It is explicitly given by,

$$\delta H_{P,O} = \frac{1}{N_c^2}\frac{\alpha_s}{2\pi}\int_{uvprz} L_{uvprz}[Q_{xyuv} - X_{xyzuvz}][(P^\dagger_{xy} - O^\dagger_{xy})(P^\dagger_{uv} - O^\dagger_{uv})]$$

It includes also MERGING VERTICES!

To find them we need to "project" the X-reggeon onto states perturbatively orthogonal to P, B and P^2.

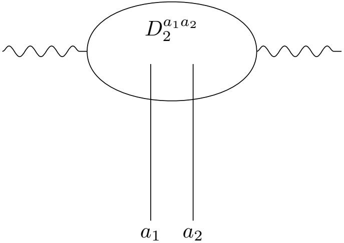
We will get merging vertices of the form $BP^\dagger P^\dagger$, $BO^\dagger O^\dagger$ and so on...

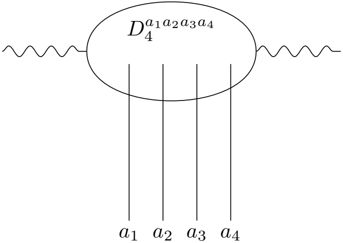
[Bartels & Whustoff (1995); Bartels & Ewerz (1999)]

Bartels' D-functions are not exactly Reggeons but they are closely related. (they are the correlators of color charge densities.)

$$D^{n}_{a_1,\ldots,a_n}(x_1,...,x_n) = (-ig)^n \langle \hat{\rho}^{a_1}(x_1),...,\hat{\rho}^{a_n}(x_n)\rangle_{photon}$$

color charge density





$$\frac{d}{dY} D^2(xy) = \bar{K}(xy, uv) \otimes D^2(uv)$$

BFKL kernel up to some kinematical factors

The relation between Reggeons and Bartels' correlators

Reggeization in D^4:

$$D^4_I = D^4 - \sum D^2$$

$$\frac{d}{dY} D^4_I = \bar{K} \otimes D^4_I + V \otimes D^2$$

conjectured to be universal

$$\frac{d}{dY} D^6_I = \bar{K} \otimes D^6_I + V \otimes D^4$$

The D-functions are related with our conjugate Reggeons:

$$D^2_{aa}(12) \approx [1 - P_{12}]P^\dagger_{12} \qquad D^4_{aabb}(1234) \approx P^\dagger_{12} P^\dagger_{34} - \delta_{12,34} P^\dagger_{12}$$

"Reggeized" terms are exactly the extra terms in the relation between D^{2n} and $(P^\dagger)^n$

The vertex V is exactly the same vertex as it appears in H_{KLWMIJ} $PPP^\dagger$ term

The advantages of our approach : once we have chosen the basic variables,

- there is no ambiguity anywhere, all reggeization corrections and vertices are determined uniquely at any order.
- Reggeons are developed at operator level. This means we can consider at the same level any dilute projectile and not to be limited to just photon.

Conclusions

- KLWMIJ is indeed the RFT of QCD. Actually, it is the half of it, we need to include also JIMWLK in order to get both merging and splitting vertices at the same order.
- We see merging vertices at KLWMIJ at subleading N_c.
- Even at leading N_c we see many Reggeons not only Pomeron and it is not clear if we can throw them away. We need to study their intercepts in more details.[on going work!]
- Are all Reggeons relevant for phenomenology? B-Reggeons might contribute to particle production![future work]