Plasma instabilities and particle production in Glasma

Shoichiro Tsutsui (Kyoto Univ.)

In collaboration with
Hideaki Iida, Teiji Kunihiro (Kyoto Univ.)
Akira Ohnishi (YITP)
Contents

- Introduction: Early thermalization problem
- Formalism: 2PI effective action
- Glasma Instabilities
  - Primary --- Nielsen-Olesen instability
  - Secondary instability
- Summary and future work
Motivation
Early thermalization problem

$\tau_{\text{thermal}} \sim 0.6 - 1.0 \text{ fm/c}$
Motivation
Early thermalization problem

thermalization = particle production in quantum field dynamics under the time dependent classical background fields
Motivation

Early thermalization problem

representative approaches

- Bottom-up thermalization scenario
  - solve the Boltzmann eq.
  - $2\rightarrow2$, $2\rightarrow3$ processes
    Baier, Mueller, Schiff, Son (2001)

- Classical statistical simulation
  - including nonlinear effects
  - appropriate to glasma instabilities
    Rommatschke, Venugopalan (2006)
Motivation

Early thermalization problem

representative approaches

- Bottom-up thermalization scenario
  - solve the Boltzmann eq.
  - 2->2, 2->3 processes
    Baier, Mueller, Schiff, Son (2001)

- Classical statistical simulation
  - including nonlinear effects
  - appropriate to glasma instabilities
    Rommatschke, Venugopalan (2006)

\[ \tau_{\text{thermal}} \sim 4.5 \text{ fm/c} \]

no background classical fields

does not reproduce the Bose-Einstein distribution

the 2PI formalism can overcome these weak points
Formalism
2PI effective action

\[ \Gamma[A, G] = S[A] + \frac{i}{2} \text{Tr} \ln G^{-1} + \frac{i}{2} \text{Tr} \ln G_0^{-1}(A)G + \Gamma_2[A, G] \]

CYM \quad 1\text{-loop} \quad 2,3,..\text{loop}

\[ \frac{\delta \Gamma[A, G]}{\delta A} \bigg|_{J=R=0} = 0 \quad \text{EOM of classical gluon fields} \]

\[ \frac{\delta \Gamma[A, G]}{\delta G} \bigg|_{J=R=0} = 0 \quad \text{KB-CJT eq (Schwinger-Dyson eq)} \]

\[ \Rightarrow G_0^{-1}G - \Pi G = 1 \]

Kadanoff, Baym (1962)
Cornwall, Jackiw, Tomboulis (1974)

a review: Berges (2004)
EOM of classical gluon fields

Loop expansion of the 2PI action

\[ \Gamma_2 = \]

EOM of classical gluon fields

\[
\partial_i^2 A_i^a - D_j F_{ji}^a = g f^{abc} (D_{xi}^b F_{jj}^c(x, y) + D_{xj}^b (F_{ji}^c(x, y) - 2F_{ij}^c(x, y)))_{y=x} + 2\text{-loop}
\]

Classical Yang-Mills eq. \hspace{1cm} quantum corrections
EOM of correlation functions

KB-CJT eq (Schwinger-Dyson eq)

\[
\left[ \partial_t^2 \delta_{ij} - (D^2 \delta_{ij} - D_i D_j - 2i g F_{ij}) \right] \mathcal{F}^{bc}_{jk}(x, y)
= g^2 \left( C_{ad,be} \mathcal{F}^{de}_{ij}(x, x) + \frac{1}{2} C_{ab,de} \mathcal{F}^{de}_{mm}(x, x) \delta_{ij} \right) \mathcal{F}^{bc}_{jk}(x, y)
- \int_{t_0}^{x_0} \Pi \rho_{ij}^{ab}(x, z) \mathcal{F}^{bc}_{jk}(z, y) + \int_{t_0}^{y_0} d^4 z \Pi \mathcal{F}^{ab}_{ij}(x, z) \rho^{bc}_{jk}(z, y) + 3\text{-loop}
\]

\( \mathcal{F} \) is the real part of \( G \)

1 to 2 process is possible

in an expanding geometry: Hatta, Nishiyama (2012)
Comparison with Classical Stat.

Lappi, McLerran (2006)

classical stat.

2PI

2PI (n-loop)

equilibrium

the range of applications
Comparison with Classical Stat.

2PI (n-loop)  

Glasma instabilities play crucial roles

Equilibrium

Linear analysis of EOM of classical gluon field

Lappi, McLerran (2006)
Glasma instabilities

perturbative expansion of $g$

$$A = \frac{1}{g} A^{(0)} + A^{(1)} + \mathcal{O}(g)$$

$$\mathcal{F} = \mathcal{F}^{(0)} + g\mathcal{F}^{(1)} + \mathcal{O}(g^2)$$

$\mathcal{F}(x, y) \equiv \frac{1}{2}\langle \hat{a}(x), \hat{a}(y) \rangle$

suppose that there is a background color magnetic field

$$A_i^a(0) = \tilde{A}(t) \left( \delta^{a2}_x \delta_{ix} + \delta^{a1}_y \delta_{iy} \right)$$

$$\begin{cases} 
B^3_z \neq 0 \\
B^a_\perp = 0
\end{cases}$$
Glasma instabilities (primary)

LO EOM
\[ \partial_t^2 \tilde{A} + \tilde{A}^3 = 0 \]

NLO EOM
\[ \partial_t^2 A^{(1)} = -\Omega[A^{(0)}]A^{(1)} \]

one of the eigenvalues of \( \Omega \) is \( \omega_{\text{NO}}^2 = p_z^2 - B \)

- lower momentum modes are unstable
- \( A_{1,2} \) are unstable

Fujii, Itakura (2008)
Berges, Scheffler, Schlichting, Sexty (2012)

Nielsen-Olesen instability
Glasma instabilities (secondary)

NNLO EOM (beyond linear analysis)

\[ \partial_t^2 A_i^{a(2)} = -\Omega A_i^{a(2)} + J_i^a [A^{(1)}, F^{(0)}] \]

induced current

LO KB-CJT

\[ \partial_t^2 F^{(0)} = -\Omega F^{(0)} \]

quantum effects become relevant to NNLO
Properties of the 1-loop current (1)

\[ J^a_i = f^{abc} \left( D^{be}_{xi} \tilde{F}^{ec}_{jj}(x, y) + D^{be}_{xj} \left( \tilde{F}^{ec}_{ji}(x, y) - 2\tilde{F}^{ec}_{ij}(x, y) \right) \right)_{y=x} \]

\[ \tilde{F}^{ab}_{ij}(x, y) = A_i^a(1)(x)A_j^b(1)(y) + F^{ab}_{ij}^{(0)}(x, y) \]

- classical part
- quantum part

\[ \text{• quantum effects affect the amplitude of the current} \]
Properties of the 1-loop current (2)

$$J_i^a = f^{abc} \left( D_x^{be} F_{ji}^e(x, y) + D_x^{be} \left( F_{ji}^e(x, y) - 2 F_{ij}^e(x, y) \right) \right)_{y=x}$$

$$J_z^a, J_\perp^3 \propto \tilde{F}_\perp \left[ A_\perp^{(1)}, F_\perp^{(0)} \right]$$

**primary**

- $A_{\perp}^{1,2}$ are unstable
- $A_{\perp}^{a}, \ A_\perp^{3}$ are stable

**secondary**

- $A_{\perp}^{1,2}$ are unstable
- $A_{\perp}^{a}, A_\perp^{3}$ can be unstable

the current triggers secondary instabilities
Properties of the 1-loop current (3)

solve the NLO EOM and LO KB-CJT

\[ A^{(1)}_\perp \propto \partial_z f(z) \sin \omega_{\text{NO}} t \]

consistent with Gauss’s law

\[ E_\perp(t = 0) = \partial_z f(z) e_\perp(x_\perp) \]
\[ E_z(t = 0) = -f(z) D_\perp e_\perp(x_\perp) \]

the size of fluctuations

no initial longitudinal fluctuations

secondary instabilities does not occur
Summarizing Picture

Nielsen-Olesen instability

sequential instabilities suggested by 2PI formalism

consistent with previous numerical results

Romatschke, Venugopalan (2006)
Berges (2012)
Iida, Kunihiro, Müller, Ohnishi, Schäfer, T. Takahashi (2013)
Summary

- The particle production in quantum field dynamics under the time dependent classical background fields plays an important role in thermalization.

**2PI formalism is best suited**

- We applied the 2PI formalism to the early stage of dynamics.
- Quantum effects affect the amplitude of the induced current.
- The induced current triggers the secondary instabilities.
- The induced current is sensitive to initial fluctuation.
Future work

- take a realistic initial condition into account
- solve EOM of classical gluon fields and KB-CJT eq. numerically to investigate the later stage of thermalization process
- check the gauge dependence (numerically)

Thank you for your attention!
Controlled gauge dependence

\[ \Gamma_2 = \mathcal{O}(g^2) \quad \mathcal{O}(g^4) \]

gauge dep. = \mathcal{O}(g^4)

the gauge dependence appears at higher order than the truncation order

Formalism

2PI effective action with CTP

2 Particle Irreducible (2PI) effective action

\[ \Gamma[A, G] = W[J, R] - \int \frac{\delta W[J, R]}{\delta J} J - \int \frac{\delta W[J, R]}{\delta R} R \]

\[ Z[J, R] = \int \mathcal{D}a \exp i \left( S[a] + \int J_a(x) a^a(x) + \frac{1}{2} \int R^{ab}(x, y) a^a(x) a^b(y) \right) \]

\[ Z[J, R] = e^{iW[J, R]} \]

Defined as Legendre transform of generating functional with respect to the sources J, R
Formalism
Statistical function

decompose the Keldysh Green function

\[ G(x, y) = \mathcal{F}(x, y) - \frac{i}{2} \rho(x, y) \left( \theta_C(x^0 - y^0) - \theta_C(y^0 - x^0) \right) \]

spectral function
\[ \rho(x, y) \equiv i \langle [a(x), a(y)] \rangle \]

statistical function
\[ \mathcal{F}(x, y) \equiv \frac{1}{2} \langle \{a(x), a(y)\} \rangle \]

statistical functions have information about particles

\[ \mathcal{F} \sim \frac{\cos(x^0 - y^0) \omega_p}{\omega_p} \left( n(p) + \frac{1}{2} \right) \]

occupation number (converge to the BE distribution)