Plasma instabilities and particle production in Glasma

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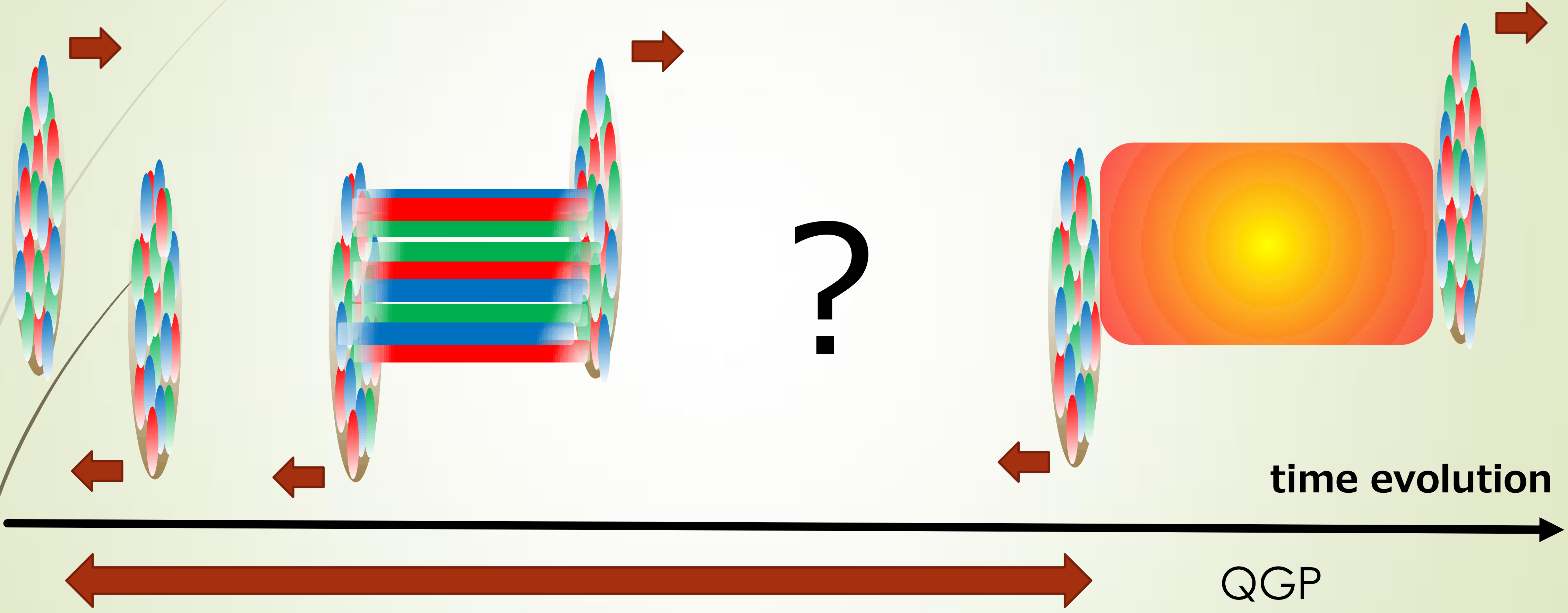
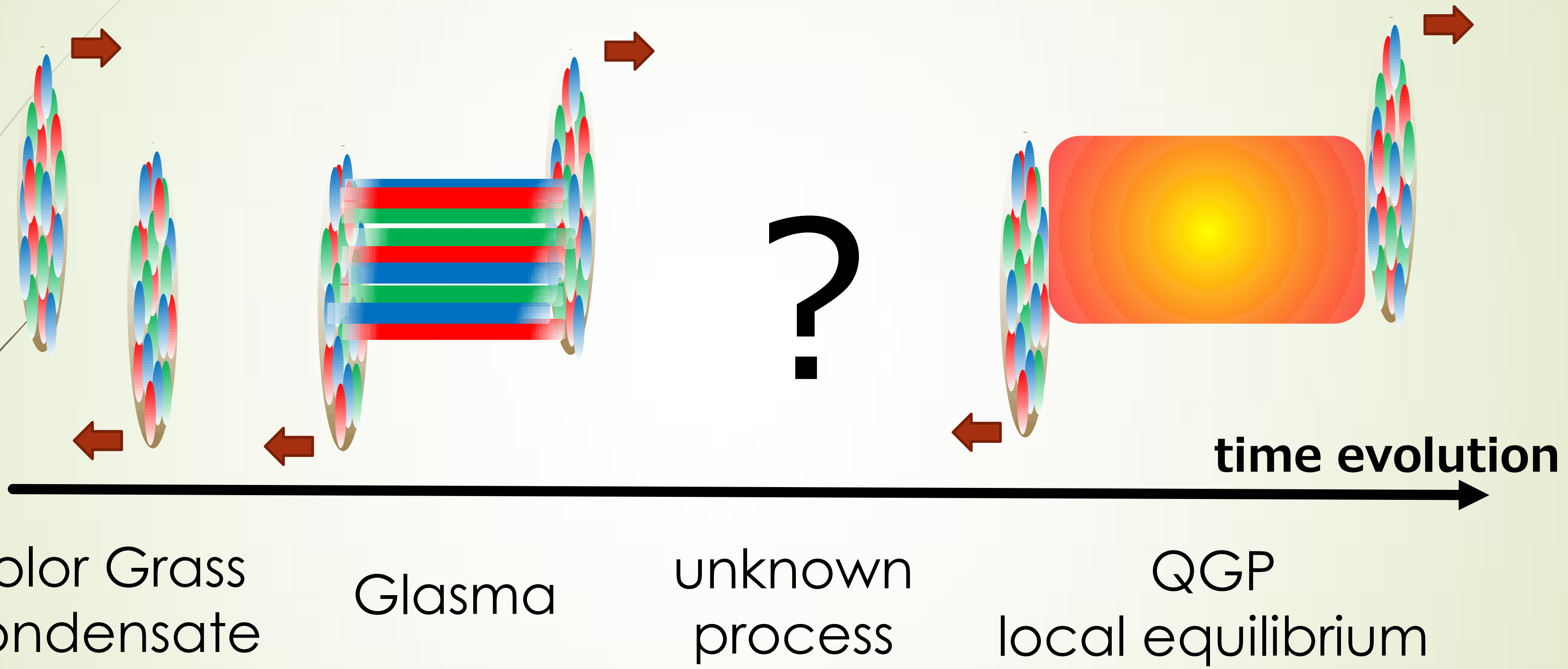
Motivation
Early thermalization problem



$$\tau_{\rm thermal} \sim 0.6 - 1.0\,{\rm fm/c}$$

Motivation
Early thermalization problem



thermalization = particle production in **quantum field dynamics**
under the **time dependent classical background fields**

Motivation
Early thermalization problem

representative approaches

- Bottom–up thermalization scenario
  - solve the Boltzmann eq.
  - 2->2 , 2->3 processes

    Baier, Mueller, Schiff, Son (2001)

- Classical statistical simulation
  - including nonlinear effects
  - appropriate to glasma instabilities

    Rommatschke, Venugopalan (2006)

Motivation
Early thermalization problem

representative approaches

- Bottom–up thermalization scenario
  - solve the Boltzmann eq.
  - 2->2 , 2->3 processes

  Baier, Mueller, Schiff, Son (2001)

$$\tau_{\mathrm{thermal}} \sim 4.5\,\mathrm{fm/c}$$

no background classical fields

- Classical statistical simulation
  - including nonlinear effects
  - appropriate to glasma instabilities

  Rommatschke, Venugopalan (2006)

does not reproduce **the Bose-Einstein distribution**

the 2PI formalism can overcome these weak points

Formalism
2PI effective action

a review: Berges (2004)

$$\Gamma[A,G] = \underbrace{S[A]}_{\text{CYM}} + \underbrace{\frac{i}{2}\,\mathrm{Tr}\ln G^{-1} + \frac{i}{2}\,\mathrm{Tr}\ln G_0^{-1}(A)G}_{\text{1-loop}} + \underbrace{\Gamma_2[A,G]}_{\text{2,3,..loop}}$$

$$\left.\frac{\delta\Gamma[A,G]}{\delta A}\right|_{J=R=0} = 0$$ EOM of classical gluon fields

$$\left.\frac{\delta\Gamma[A,G]}{\delta G}\right|_{J=R=0} = 0$$ KB-CJT eq (Schwinger-Dyson eq)

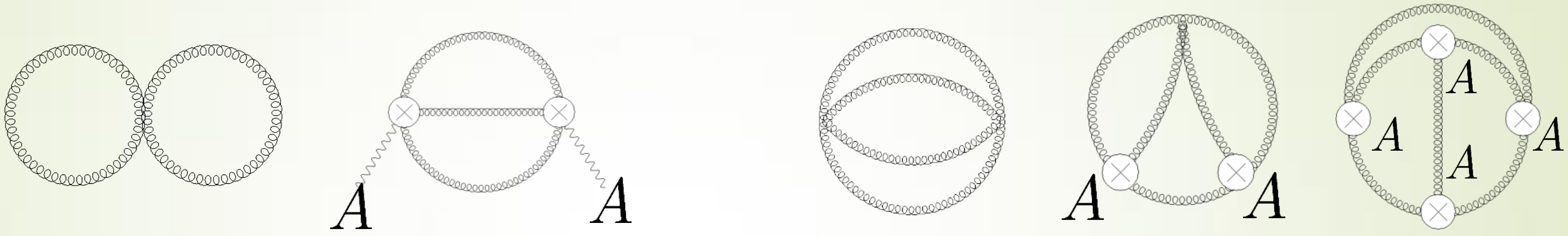
$$\Rightarrow G_0^{-1}G - \Pi G = 1$$

Kadanoff, Baym (1962)
Cornwall, Jackiw, Tomboulis (1974)

EOM of classical gluon fields

loop expansion of the 2PI action

$$\Gamma_2 =$$



EOM of classical gluon fields

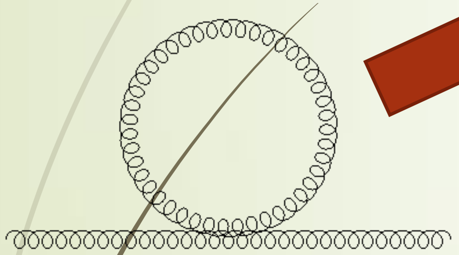
$$\underbrace{\partial_t^2 A_i^a - D_j F_{ji}^a}_{\text{Classical Yang-Mills eq.}} = \underbrace{g f^{abc}\left(D_{xi}^{be}\mathcal{F}_{jj}^{ec}(x,y) + D_{xj}^{be}\left(\mathcal{F}_{ji}^{ec}(x,y) - 2\mathcal{F}_{ij}^{ec}(x,y)\right)\right)_{y=x} + \text{2-loop}}_{\text{quantum corrections}}$$

EOM of correlation functions

KB-CJT eq (Schwinger-Dyson eq)

$\mathcal{F}$ is the real part of G

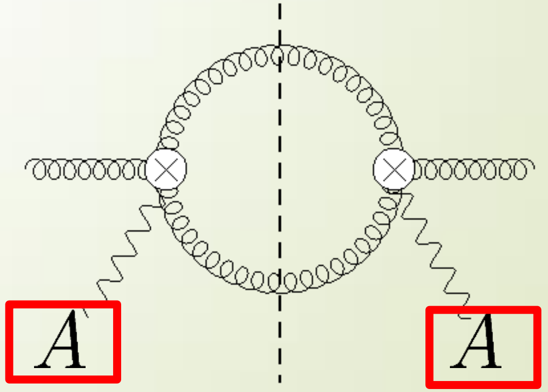
$$\left[\partial_t^2\delta_{ij} - (D^2\delta_{ij} - D_iD_j - 2igF_{ij})\right]^{ab}\mathcal{F}^{bc}_{jk}(x,y)$$



$$= g^2\left(C_{ad,be}\mathcal{F}^{de}_{ij}(x,x) + \frac{1}{2}C_{ab,de}\mathcal{F}^{de}_{mm}(x,x)\delta_{ij}\right)\mathcal{F}^{bc}_{jk}(x,y)$$

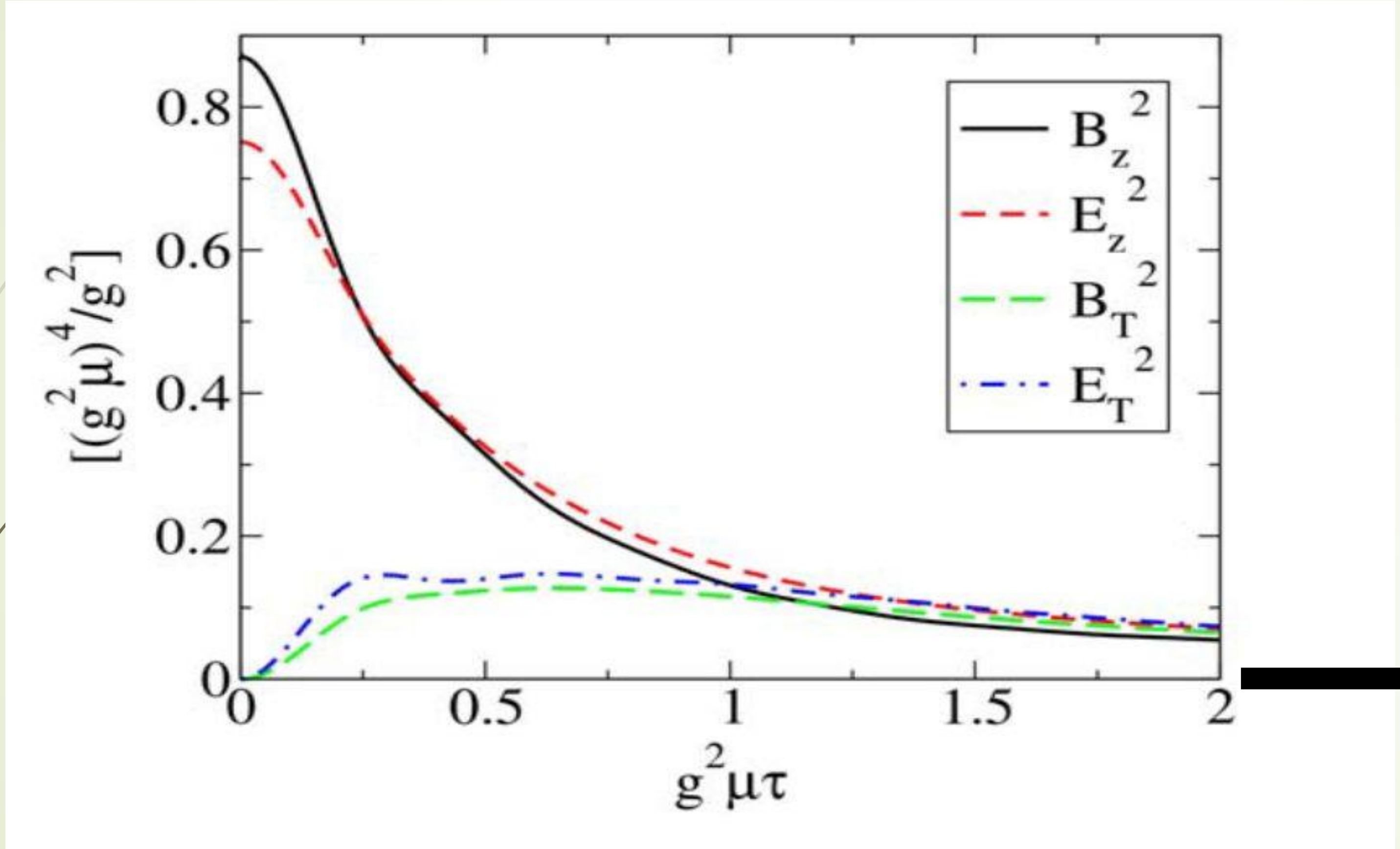
$$-\int_{t_0}^{x_0}\Pi_{\rho\,ij}^{\ ab}(x,z)\mathcal{F}^{bc}_{jk}(z,y) + \int_{t_0}^{y_0} d^4z\,\Pi_{\mathcal{F}\,ij}^{\ ab}(x,z)\rho^{bc}_{jk}(z,y) + \text{3-loop}$$

1 to 2 process is possible



in an expanding geometry: Hatta, Nishiyama(2012)

Comparison with Classical Stat.



Lappi, McLerran (2006)

equilibrium

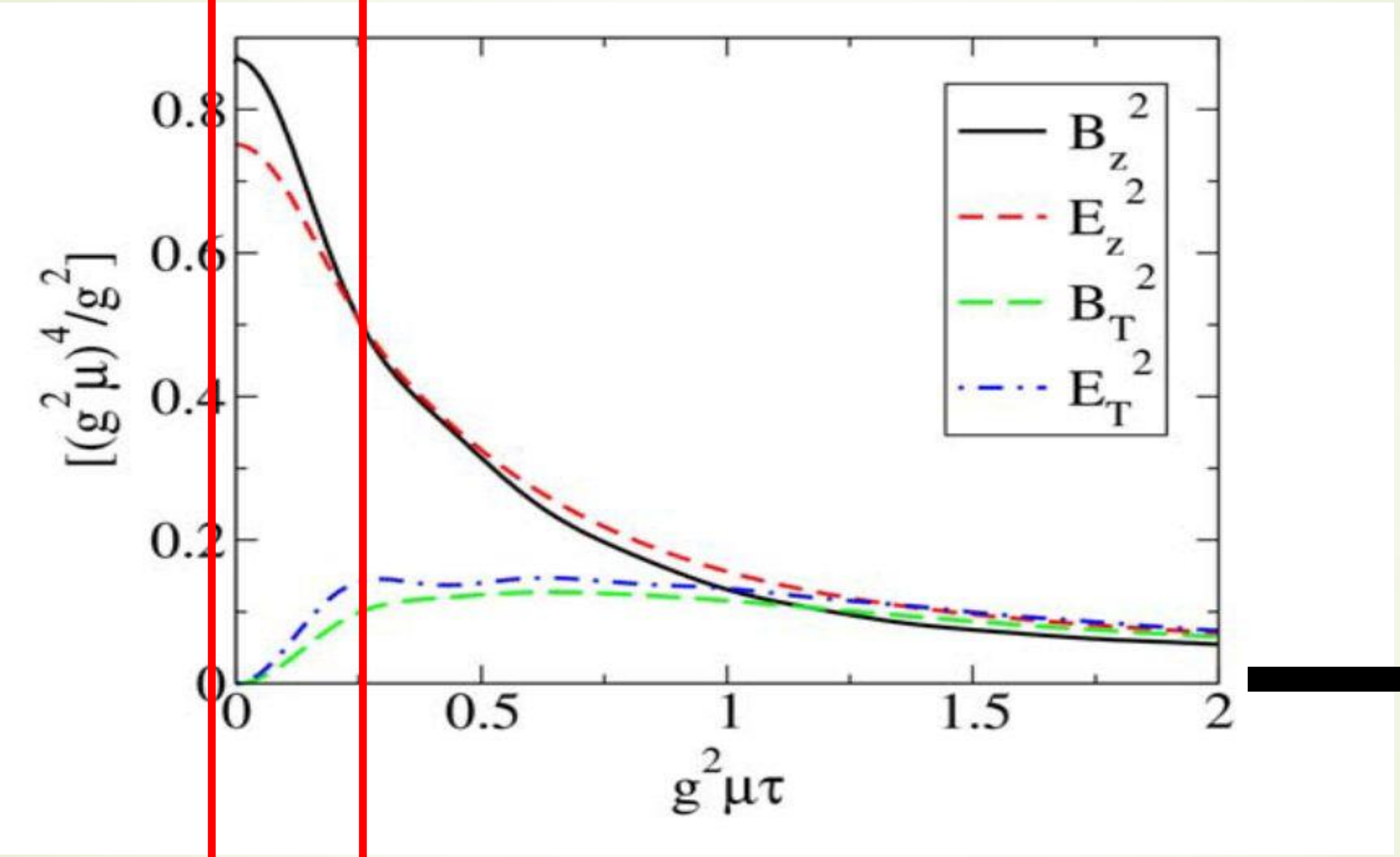
classical stat.

2PI

2PI (n-loop)

the range of applications

Comparison with Classical Stat.



Lappi, McLerran (2006)

equilibrium

glasma instabilities play crucial roles

linear analysis of EOM of classical gluon field

2PI (n-loop)

Glasma instabilities

perturbative expansion of *g*

$$A = \frac{1}{g}A^{(0)} + A^{(1)} + \mathcal{O}(g)$$

$$\mathcal{F} = \mathcal{F}^{(0)} + g\mathcal{F}^{(1)} + \mathcal{O}(g^2)$$

$$\mathcal{F}(x,y) \equiv \frac{1}{2}\langle \hat{a}(x), \hat{a}(y)\rangle$$

suppose that there is a background color magnetic field

$$A_i^{a\,(0)} = \tilde{A}(t)\left(\delta^{a2}\delta_{ix} + \delta^{a1}\delta_{iy}\right) \Rightarrow \begin{cases} B_z^3 \neq 0 \\ B_\perp^a = 0 \end{cases}$$

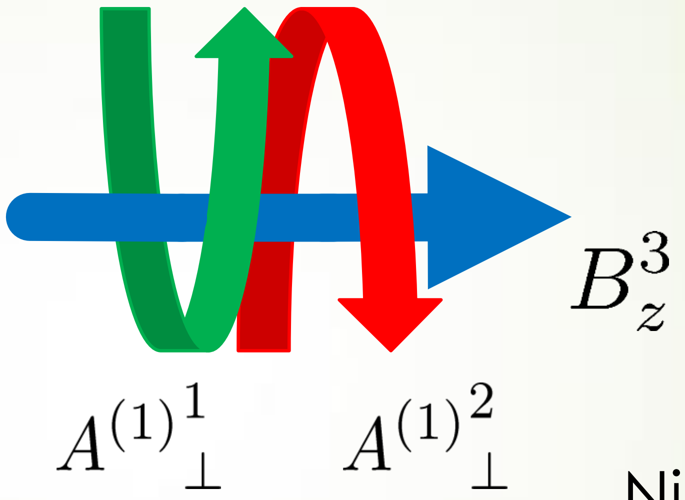
Glasma instabilities (primary)

LO EOM

$$\partial_t^2 \tilde{A} + \tilde{A}^3 = 0$$

NLO EOM

$$\partial_t^2 A^{(1)} = -\Omega[A^{(0)}]A^{(1)}$$



Fujii, Itakura(2008)
Berges,Scheffler,
Schlichting,Sexty(2012)

Nielsen-Olesen instability

one of the eigenvalues of Ω is $\omega_{\rm NO}^2 = p_z^2 - B$

- lower momentum modes are unstable
- $A_\perp^{1,2}$ are unstable

Glasma instabilities (secondary)

NNLO EOM (beyond linear analysis)

$$\partial_t^2 A_i^{a\,(2)} = -\Omega A_i^{a\,(2)} + \underline{J_i^a[A^{(1)}, \mathcal{F}^{(0)}]}$$

induced current

LO KB-CJT

$$\partial_t^2 \mathcal{F}^{(0)} = -\Omega \mathcal{F}^{(0)}$$

quantum effects become relevant to NNLO

Properties of the 1-loop current (1)

$$J_i^a = f^{abc}\left(D_{xi}^{be}\tilde{\mathcal{F}}_{jj}^{ec}(x,y) + D_{xj}^{be}\left(\tilde{\mathcal{F}}_{ji}^{ec}(x,y) - 2\tilde{\mathcal{F}}_{ij}^{ec}(x,y)\right)\right)_{y=x}$$

$$\tilde{\mathcal{F}}_{ij}^{ab}(x,y) = \underbrace{A_i^{a\,(1)}(x)A_j^{b\,(1)}(y)}_{\text{classical part}} + \underbrace{\mathcal{F}_{ij}^{ab\,(0)}(x,y)}_{\text{quantum part}}$$

- quantum effects **affect the amplitude** of the current

Properties of the 1-loop current (2)

$$J_i^a = f^{abc}\left(D_{xi}^{be}\tilde{\mathcal{F}}_{jj}^{ec}(x,y) + D_{xj}^{be}\left(\tilde{\mathcal{F}}_{ji}^{ec}(x,y) - 2\tilde{\mathcal{F}}_{ij}^{ec}(x,y)\right)\right)_{y=x}$$

$$J_z^a, J_\perp^3 \propto \tilde{\mathcal{F}}_{\perp\perp}[A_\perp^{(1)}, \mathcal{F}_{\perp\perp}^{(0)}]$$

the current triggers **secondary instabilities**

primary		***secondary***
$A_\perp^{1,2}$ are unstable	→	$A_\perp^{1,2}$ are unstable
A_z^a $A_\perp^3$ are stable		A_z^a $A_\perp^3$ can be unstable

Properties of the 1-loop current (3)

solve the NLO EOM and LO KB-CJT

$$A_\perp^{(1)} \propto \partial_z f(z) \sin \omega_{\rm NO} t$$

consistent with Gauss's law

$$E_\perp(t=0) = \partial_z f(z) e_\perp(x_\perp)$$

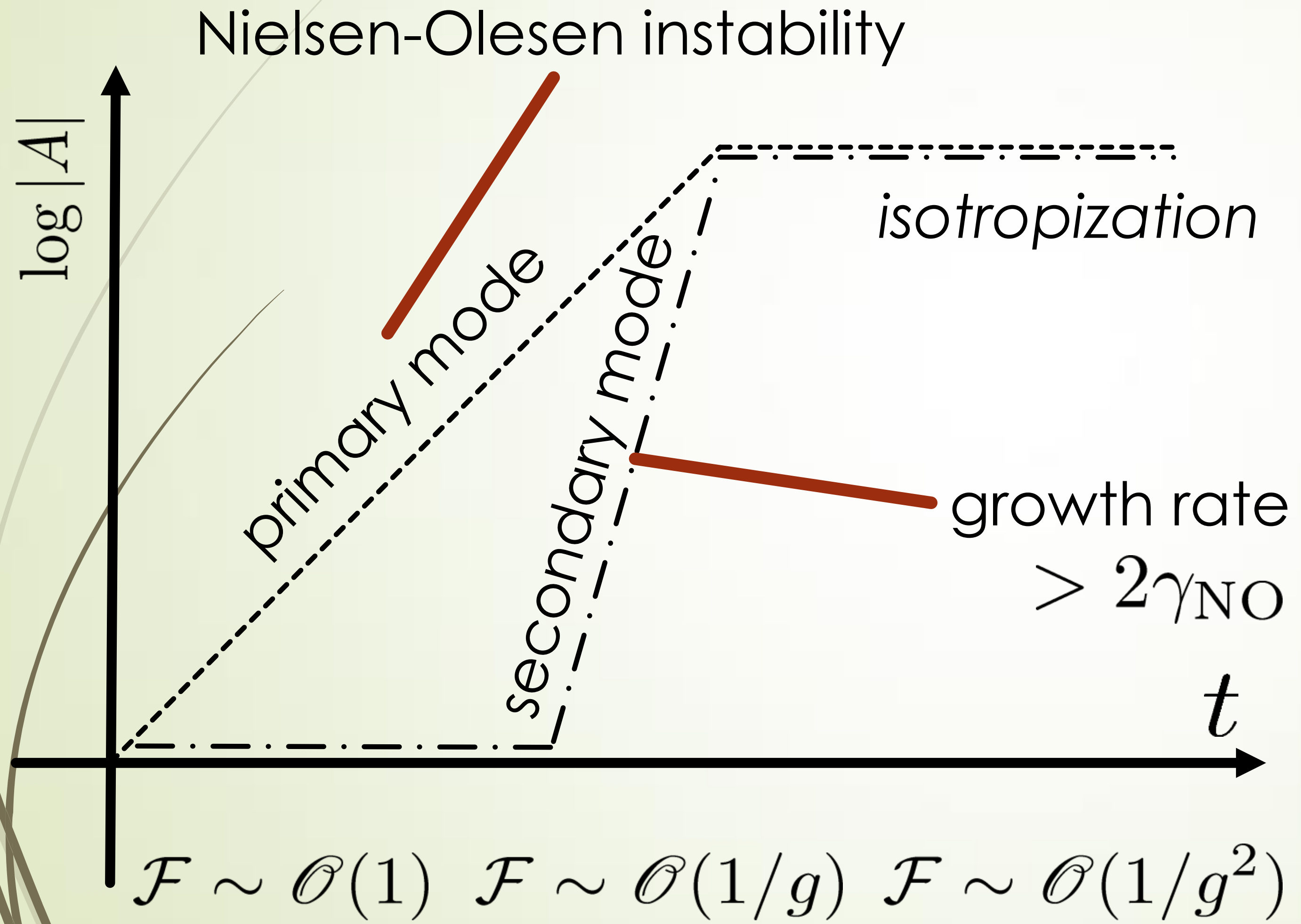
$$E_z(t=0) = -f(z) D_\perp e_\perp(x_\perp)$$

$$\mathcal{F}_{\perp\perp}^{(0)}(p_z, x_0, y_0) = \Delta \cos\left(\omega_{\rm NO} x_0 + \omega_{\rm NO} y_0\right) + \ldots$$

the size of fluctuations

no initial longitudinal fluctuations → **secondary instabilities does not occur**

Summarizing Picture



sequential instabilities suggested by 2PI formalism

consistent with previous numerical results

Romatschke,Venugopalan (2006)
Berges (2012)
Iida, Kunihiro, Müller, Ohnishi, Schäfer, T. Takahashi (2013)

Summary

- The particle production in quantum field dynamics under the time dependent classical background fields plays an important role in thermalization

  2PI formalism is best suited

- We applied the 2PI formalism to the early stage of dynamics
- Quantum effects affect the amplitude of the induced current
- The induced current triggers the secondary instabilities
- The induced current is sensitive to initial fluctuation

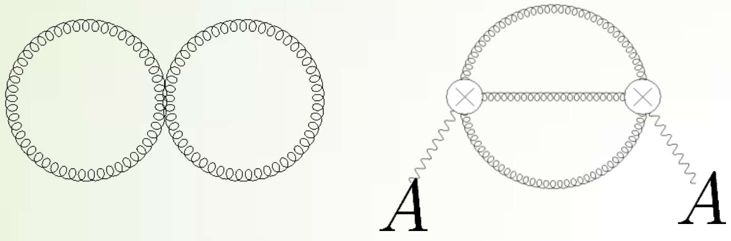
Future work

- take a realistic initial condition into account
- solve EOM of classical gluon fields and KB-CJT eq. numerically to investigate the later stage of thermalization process
- check the gauge dependence (numerically)

Thank you for your attention!

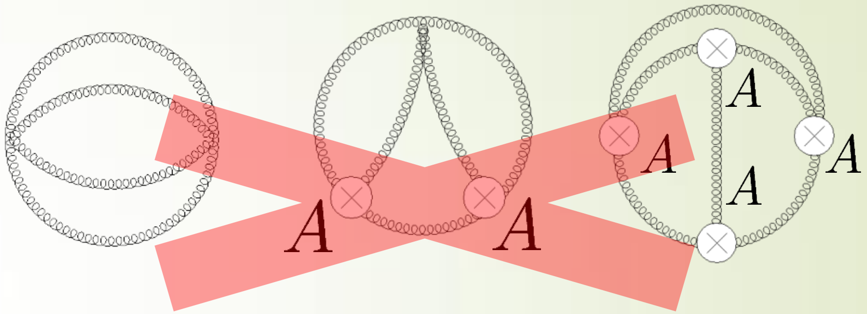
Controlled gauge dependence

$\Gamma_2 =$



$\mathcal{O}(g^2)$

truncation



$\mathcal{O}(g^4)$

$$\text{gauge dep.} = \mathcal{O}(g^4)$$

A. Arrizabalaga, J. Smit (2002)

M.E. Carrington, G. Kunstatter, H. Zaraket (2005)

the gauge dependence appears at **higher order than the truncation order**

Formalism
2PI effective action with CTP

2 Particle Irreducible (2PI) effective action

$$\Gamma[A,G] = W[J,R] - \int \frac{\delta W[J,R]}{\delta J} J - \int \frac{\delta W[J,R]}{\delta R} R$$

$$Z[J,R] = \int \mathcal{D}a \exp i \left(S[a] + \int J_a(x) a^a(x) + \frac{1}{2} \int R^{ab}(x,y) a^a(x) a^b(y) \right)$$

$$Z[J,R] = e^{iW[J,R]}$$

Defined as Legendre transform of generating functional
with respect to the sources J, R

Formalism
Statistical function

decompose the Keldysh Green function

$$G(x,y) = \mathcal{F}(x,y) - \frac{i}{2}\rho(x,y)\left(\theta_C(x^0 - y^0) - \theta_C(y^0 - x^0)\right)$$

spectral function $\rho(x,y) \equiv i\langle[a(x), a(y)]\rangle$

statistical function $\mathcal{F}(x,y) \equiv \frac{1}{2}\langle\{a(x), a(y)\}\rangle$

statistical functions have information about *particles*

$$\mathcal{F} \sim \frac{\cos(x^0 - y^0)\omega_{\boldsymbol{p}}}{\omega_{\boldsymbol{p}}}\left(n(\boldsymbol{p}) + \frac{1}{2}\right)$$

occupation number
(converge to the BE distribution)