Inclusive hadron and photon production at LHC in dipole momentum space

M. B. Gay Ducati
beatriz.gay@ufrgs.br

High Energy Phenomenology Group
Instituto de Física
Universidade Federal do Rio Grande do Sul
Porto Alegre, Brazil
http://www.if.ufrgs.br/gfpae
Outline

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⇒ Dipole formalism
⇒ Traveling wave method
⇒ Momentum space AGBS model
  ▶ Hadron and photon production
    ▶ Comparison with LHC data: $k_t$ factorization
    ▶ LHC predictions: Hybrid factorization
  ▶ Summary
Saturation and Dipole Frame

- e-P scattering at **HERA**: strong rise of the gluon dist. function for small-$x$
  - Untamed rising should violate unitarity
  - For $Q^2 \leq Q_s^2(x)$, parton recombination should happen
  - Semihard scale from pQCD $Q_s(x)$: Saturation scale

- **Dipole frame** is convenient to investigations on small-$x$

\[ \gamma^* \text{ splits into the } q\bar{q} \text{ pair and the cross section factorizes as} \]

\[ \sigma_{L,T}^{\gamma^*P}(x, Q^2) = \int_0^1 dz \int d^2 r |\Psi_{L,T}(z, r; Q^2)|^2 \sigma_{dip}(r = |r|, x) \] (1)

\[ \Psi_{L,T}(z, r; Q^2) \text{ (Describes the } \gamma^* \text{ splitting into the } q\bar{q} \text{ pair)} \]
Dipole formalism

- Considers the energy evolution ($Y = \ln(1/x)$) of the pair $q\bar{q}$

![Diagram showing energy evolution in dipole momentum space with $Y_0$, $Y_1$, $Y_2$, and $Y_3$.]

- Large $N_c$ limit: gluon $\equiv q\bar{q}$
- $\sigma_{\text{dip}}(r, Y) = \int d^2b \mathcal{N}(r, b, Y) \approx \pi R_{\text{target}}^2 \mathcal{N}(r, Y)$
- $\mathcal{N}(r, Y)$ describes the interaction through gluon cascade ($\equiv$ non abelian phases $\rightarrow$ Wilson lines)
Dipole evolution

- Balitsky-JIMWLK hierarchy

\[ \mathcal{M}(x, y, z) = \frac{(x-y)^2}{(x-z)^2(y-z)^2} \quad \bar{\alpha} = \alpha_s N_c / \pi \]

\[
\frac{\partial}{\partial Y} \langle N(x, y) \rangle_Y = \frac{\bar{\alpha}}{2\pi} \int d^2 z \mathcal{M}(x, y, z) \\
\times \left\{ -\langle N(x, y) \rangle_Y + \langle N(x, z) \rangle_Y + \langle N(z, y) \rangle_Y \right. \\
- \left. \langle N(x, z) N(z, y) \rangle_Y \right\}
\]

- Mean field: \( \langle N(x, z) N(y, z) \rangle_Y \approx \langle N(x, z) \rangle_Y \langle N(y, z) \rangle_Y \Rightarrow \)

Balitsky-Kovchegov (BK) equation

\[
\frac{\partial}{\partial Y} \langle N(x, y) \rangle_Y = \frac{\bar{\alpha}}{2\pi} \int d^2 z \mathcal{M}(x, y, z) \\
\times \left\{ -\langle N(x, y) \rangle_Y + \langle N(x, z) \rangle_Y + \langle N(z, y) \rangle_Y \right. \\
- \left. \langle N(x, z) N(z, y) \rangle_Y \right\}
\]
HEQCD amplitudes and traveling waves

- Asymptotic forms of BK solutions obtained through the mapping: QCD $\Rightarrow$ Reaction-diffusion processes
- Geometric scaling of the BK amplitudes seen as traveling waves
- In momentum space BK equation reads
  \[
  \partial_Y N_Y = \bar{\alpha} \chi(-\partial_L) N_Y - \bar{\alpha} N_Y^2,
  \]
  where
  \[
  \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)
  \]
is the BFKL kernel and $L = \log(k^2/k_0^2)$, with $k_0$ denoting a soft scale.
- Change of variables: $t \sim \bar{\alpha} Y$, $x \sim L e^{u} \sim N$, BK $\rightarrow$ FKPP [Munier e Peschanski '2004]
  \[
  \partial_t u(x, t) = \partial_x^2 u(x, t) + u(x, t) - u^2(x, t)
  \]
- Dynamics essentially determined by the linear part (BFKL)
- FKPP admits traveling wave solutions: $u(x, t) \xrightarrow{t \to \infty} u(x - v_c t)$ (“geometric scaling”)
HEQCD amplitudes and traveling waves

- TW connects \( u = 0 \) unstable and \( u = 1 \) stable fixed points.
- \( \gamma_0 > \gamma_c \)
- \( u(x,t) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2i\pi} u_0(\gamma)e^{-\gamma(x_{wf}+vt)+\omega(\gamma)t} \)

\[ \nu_c = \frac{\omega(\gamma_c)}{\gamma_c} = \partial_\gamma \omega(\gamma)|_{\gamma_c} \]

- In QCD variables: \( \omega(\gamma_c) = \chi(\gamma_c) \Rightarrow \gamma_c = 0.6275 \)
- TW translates into geometric scaling of BK amplitudes [Munier e Peschanski ’2004]

\[ N_Y(k) \approx N_s \left( \frac{k^2}{Q_s^2(Y)} \right)^{-\gamma_c} \log \left( \frac{k^2}{Q_s^2(Y)} \right) \exp \left[ -\frac{\log^2 \left( k^2/Q_s^2(Y) \right)}{2\bar{\alpha}'(\gamma_c)Y} \right] , \]

\[ Q_s^2(Y) = Q_0^2 \exp \left( \lambda Y - \frac{3}{2\gamma_c} \log Y \right) , \quad \lambda = \bar{\alpha}_c = \bar{\alpha} \chi(\gamma_c)/\gamma_c \]

- Geometric scaling window

\[ \log \left( k^2/Q_s^2(Y) \right) \lesssim \sqrt{2\bar{\alpha}'(\gamma_c)Y} \]
AGBS model for $\sigma_{dip}$

- Parametrization in momentum space for the dipole-proton scattering amplitude [Amaral, MBGD, Betemps and Soyez '2007]
  - The model uses the traveling wave BK solutions for the large $L$ (dilute) region
  - A Fourier transform of a Theta function models the saturated region

$$N(k) \overset{k \ll Q_s}{=} c - \log \left( \frac{k}{Q_s(Y)} \right)$$

- The AGBS model interpolates between the two behaviors through $(\rho \equiv \ln(k^2/k_0^2) \text{ and } \rho_s \equiv \ln(k_0^2/Q_s^2))$:

$$N_{AGBS}(\rho, Y) = L_F \left( 1 - e^{-N_{dil}} \right),$$

where

$$N_{dil} = \exp \left[ -\gamma_c (\rho - \rho_s) - \frac{\mathcal{L}^2 - \log^2(2)}{2\bar{\alpha}\chi''(\gamma_c)Y} \right],$$

$$\mathcal{L} = \ln \left[ 1 + e^{(\rho - \rho_s)} \right] \quad \text{with} \quad Q_s^2(Y) = k_0^2 e^{\lambda Y},$$

and

$$L_F = 1 + \ln \left[ e^{\frac{1}{2}(\rho - \rho_s)} + e^{-\frac{1}{2}(\rho - \rho_s)} \right]$$
AGBS model for $\sigma_{dip}$

- Model complements the coordinate space description
- Describes quite well the HERA data on $F_2^p$
- Globally fitted to HERA and RHIC data

Hybrid formalism:
large $K$-factors when compared with LHC data
Comparison with LHC data [E. Basso, MBGD and de Oliveira '2013]

▶ Central rapidity region
▶ $k_t$-factorization formalism naturally includes large-$x$ contributions from one of the colliding hadrons (not present in the hybrid formalism)
▶ Both colliding hadrons treated in the same way (emphasize $p_t$ dependence) $\eta \sim 0 \rightarrow k_t^1 \approx k_t^2$ $\eta_{forward} \rightarrow k_t^1 \ll k_t^2$
▶ Cross section reads [Braun '2000, Kovchegov e Tuchin '2003]

$$
\frac{d\sigma^{A+B\rightarrow g}}{dyd^2p_t d^2R} = K \frac{2}{C_F p_t^2} \int_{p_t}^{p_t} \frac{d^2k_t}{4} \left( \left| p_t + k_t \right|, x_1; b \right) \varphi \left( \left| p_t - k_t \right|, x_2; R - b \right),
$$

where $x_{1,2} = (p_t/\sqrt{s})e^{\pm y}$ and $C_F = (N_c^2 - 1)/2N_c$

▶ Large $N_c$ limit, UGDs are related to the dipole amplitude through [Braun '2000]

$$
\varphi (k, x; b) = \frac{N_c S_t}{2\pi^2 \alpha_s (k)} k^2 F_g (k, x; b).
$$
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$k_t$-factorization

\[ F_g(k, x; b) = \frac{1}{2\pi} (\nabla_k^2 N_G(k, x; b) + \delta(k)) \]

\[ \phi(k, x) \text{ (GeV}^{-2}) \]

\[ k^2 \text{ (GeV}^2) \]

▶ invariant cross section

\[ \frac{dN_{ch}}{d\eta d^2p_t} = \frac{h(\eta)}{\sigma_{nsd}} \int d^2R \int \frac{dz}{z^2} \frac{d\sigma^{A+B\rightarrow g}}{dy d^2p_t d^2R} D_h(z = p_t/k_t) \]

with \( D_h \) denoting the FFs and \( h \) is the Jacobian for the \( y \) to \( \eta \) transform

\[ y(\eta, p_t, m) = \frac{1}{2} \ln \left[ \frac{\sqrt{m^2 + p_t^2 \cosh^2 \eta} + p_t \sinh \eta}{\sqrt{m^2 + p_t^2 \cosh^2 \eta} - p_t \sinh \eta} \right] \]

▶ \( \sigma_{nsd} \) describes the effective area of interaction; model dependent.
$k_t$-factorization

- $\sigma_{nsp} \equiv \sigma_{nsp}(s)$, fitting the KMR model [Khoze, Martin e Ryskin '2011]

\[
\sigma_{nsp} \rightarrow \pi b_{max}^2,
\]

\[
b_{max} = b_0 + C \log(s).
\]

- If using the central produced hadrons to get $a$ and $b$

\[
b_{max} = 0.1 + 0.198 \log(s)
\]
Results: $p + p$ at LHC

- Large-$x$ effects ($\beta = 0.4$ and $\lambda_0 = 0 - 0.2$, $x_0 = 0.01$) [Gelis, Staśto and Venugopalan '2006]

$$
\phi(k, x; b) = \left( \frac{1 - x}{1 - x_0} \right)^{\beta} \left( \frac{x_0}{x} \right)^{\lambda_0} \phi(k, x_0; b)
$$

- As expected, smaller $K$ factors compared to the hybrid formalism
  - $k_t$-factorization: more suitable in the central region ($\eta \sim 0$)
Results: $p + A$ at LHC

$Q_s^A = A_{\text{eff}}^{1/3} Q_s^p$, $A_{\text{eff}} \approx 20$ for lead

$K = 4.53$

ALICE prelim. 5.02 TeV

- $p$ fragmentation region: small $x_1 \rightarrow$ nuclear target small-$x$ effects
- $Pb$ fragmentation region: large $x_1 \rightarrow$ EMC and Fermi motion effects must be included

$x_A \equiv x_1 \sim e^y$

$x_p \equiv x_2 \sim e^{-y}$
LHC predictions at forward regions [E. Basso, MBGD and de Oliveira '2013]

- Hybrid formalism more suitable in the hadron fragmentation region (small-$x$)

\[
\frac{dN_h}{dY_h d^2p_t} = K(y_h) \int_{x_F}^{1} \frac{dx_1}{x_F} \left[ f_{q/p}(x_1, \frac{p_t^2}{x_1}) N_F(q_t, x_2) D_{h/q} \left( \frac{x_F}{x_1}, \frac{p_t^2}{x_1} \right) ight] + f_{g/p}(x_1, \frac{p_t^2}{x_1}) N_A(q_t, x_2) D_{h/g} \left( \frac{x_F}{x_1}, \frac{p_t^2}{x_1} \right)
\]

- CTEQ6 LO PDF

- Nuclear modification factor for hadrons

\[
R_{pA}^h = \frac{dN_{pA \rightarrow hX}}{d^2p_t d\eta} / \frac{dN_{pp \rightarrow hX}}{d^2p_t d\eta} / N_{coll}.
\]

\[
N_{coll} = 6.9 \rightarrow \text{minimum bias } \sqrt{5.02} \text{ TeV}
\]

\[
K(y_h) \sim 1 \text{ expected}
\]
Photon production

- **LO p-A scattering** \((q + A \rightarrow \gamma + X)\)

\[
\frac{d\sigma_{\text{Direct}}}{d^2b d^2k_t d\eta_\gamma} = \frac{e_q^2 \alpha_{\text{em}}}{\pi(2\pi)^3} z^2 [1 + (1 - z)^2] \frac{1}{k_t^4} \int d^2l_t l_t^2 N_F(\bar{x}_g, b_t, l_t)
\]

- **Contributions from direct and fragmentation photons** [Rezaeian e Jalilian-Marian '2012]

\[
\frac{d\sigma_{\text{Fragmentation}}}{d^2b d^2k_t d\eta_\gamma} = \frac{1}{(2\pi)^2} \frac{1}{z} D_{\gamma/q}(z, Q^2) N_F(x_g, b_t, k_t/z),
\]

being the LO quark-photon fragmentation function given by [Owens '1986]

\[
D_{\gamma/q}(z, Q^2) = \frac{e_q^2 \alpha_{\text{em}}}{2\pi} \frac{1 + (1 - z)^2}{z} \ln \left( \frac{Q^2}{\Lambda_{QCD}^2} \right).
\]
Photon production at LHC

- Using CTEQ6 LO PDF for quarks and antiquarks densities

\[
\frac{d\sigma_{pT\rightarrow\gamma(k)X}}{d^2b d^2k_t d\eta_\gamma} = \int_{x_q^\text{min}}^1 dx_q [f_q(x_q, Q^2) + f_{\bar{q}}(X_Q, q^2)] \frac{d\sigma_{q(p)T\rightarrow\gamma(k)X}}{d^2b d^2k_t d\eta_\gamma},
\]

- Nuclear modification factor

\[
R_{pA}^\gamma = \frac{\frac{d\sigma_{pA\rightarrow\gamma X}}{d^2p_t d\eta}}{\frac{d\sigma_{pp\rightarrow hX}}{d^2p_t d\eta}} / N_{\text{coll}}
\]

- \(N_{\text{coll}} = 6.9\) for 5.02 TeV proton-lead collisions [d’Enterria ’2003]
Photon production at LHC

- Neutral pions to photon ratio

\[
\frac{\gamma^{\text{inclusive}}}{\pi^0} = \frac{d\sigma^{pT\rightarrow\gamma X}}{d^2p_t d\eta} / \frac{d\sigma^{pT\rightarrow\pi^0 X}}{d^2p_t d\eta}.
\] (2)

- \(k_t^{\pi^0} = k_t^\gamma\) e \(\eta_{\pi^0} = \eta_\gamma\)
- \(\pi^0\) produced via gluon induced processes dominant in the central region
- \(\gamma^0\) radiated by quarks enhanced contribution in the fragmentation region \(x \rightarrow 1\)
Summary

- Study of a momentum space dipole amplitude through the traveling wave method of QCD – AGBS
- Model describes quite well both DIS HERA data and RHIC inclusive charged hadron yield
- Predictions to hadron and photon production at LHC
- Two distinct factorizations:
  - $k_t$-factorization
    - Good description of LHC data at central rapidity region
  - Hybrid factorization
    - Predictions for the nuclear modification factors at forward rapidities
      - Good region to compare collinear (DGLAP) and saturation formalisms