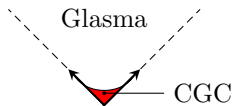


Isotropization of the Glasma

Illa da Toxa, 12th September 2013

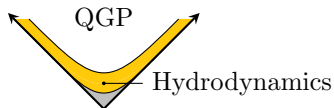
Thomas EPELBAUM
IPhT

Early transition: the problem



Huge anisotropy
(negative P_L)

**Isotropization?
Time scale?**



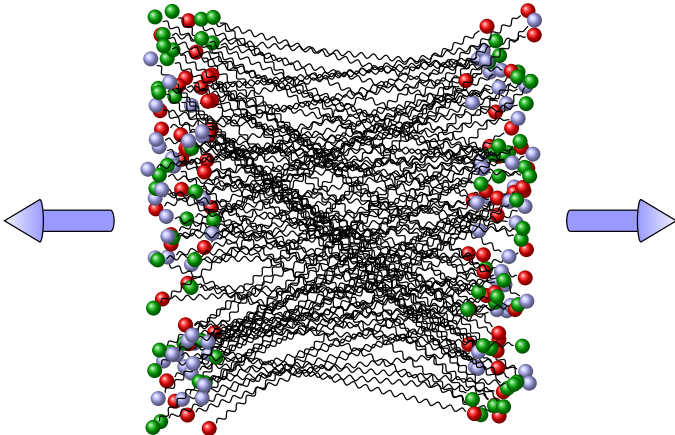
Small anisotropy

Long time puzzle: Does (fast) isotropization occur?

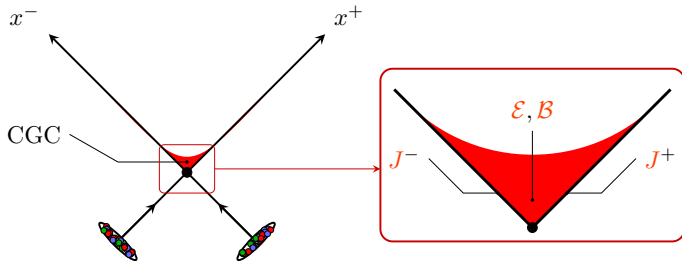
HOW TO STUDY THE TRANSITION?

Weakly coupled method at dense regime:

$$\alpha_s \ll 1 \text{ but } f_{\text{gluon}} \sim \frac{1}{\alpha_s}$$



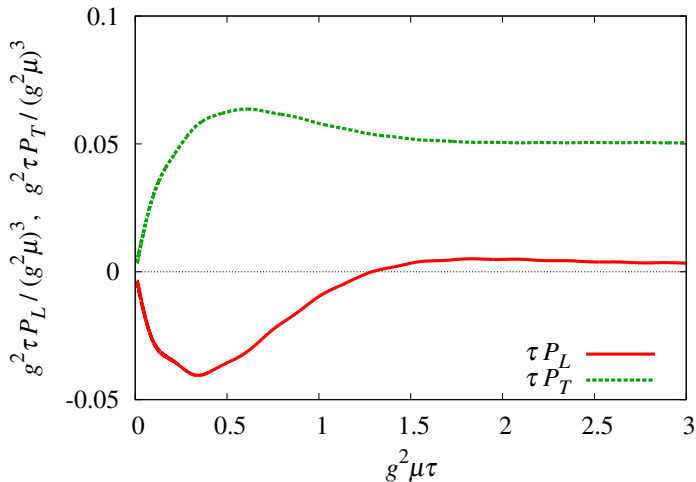
Theoretical framework (Weakly coupled but strongly interacting)



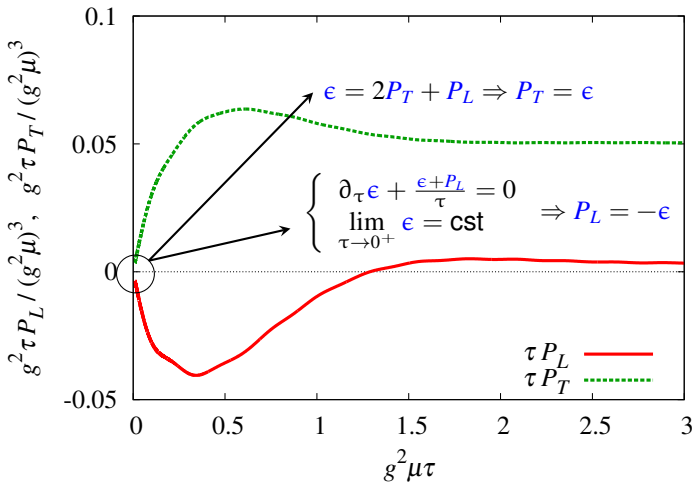
LO:
$$\epsilon = \frac{1}{2} \underbrace{(\vec{\mathcal{E}}^2 + \vec{\mathcal{B}}^2)}_{\text{Classical color fields}}$$

$$\mathcal{D}_\mu \mathcal{F}^{\mu\nu} = \underbrace{J^\nu}_{\text{Color sources on the light cone}}$$

[KRASNITZ, VENUGOPALAN (1998)]

Strong anisotropy at early time

Strong anisotropy at early time

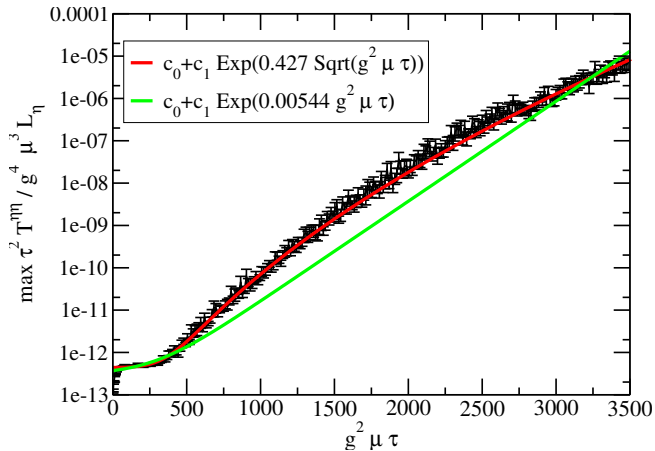


THE COLOR GLASS CONDENSATE AT NLO

$$E^2(x) = \underbrace{\mathcal{E}^2(x)}_{\text{LO}} + \frac{1}{2} \underbrace{\int_{\vec{k}} |e_{\vec{k}}(x)|^2}_{\text{NLO}} + \dots$$

$e_{\vec{k}}(x)$ perturbation to $\mathcal{E}(x)$ created by a plane wave of momentum \vec{k} in the remote past.

THE COLOR GLASS CONDENSATE AT NLO

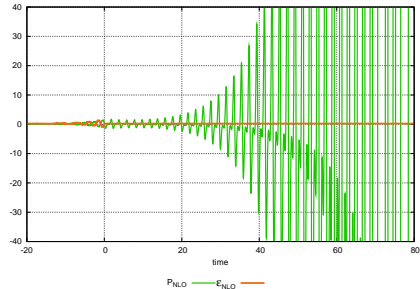
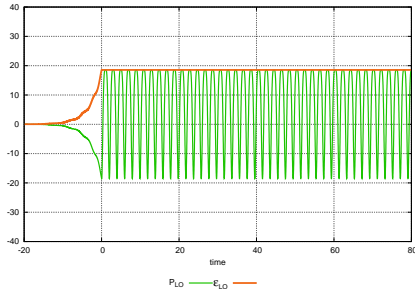


[ROMATSCHKE, VENUGOPALAN (2006)]

Small Fluctuations grow exponentially (Weibel instability)

THE COLOR GLASS CONDENSATE AT NLO

- Because of instabilities, the **NLO** correction eventually becomes as large as the **LO** \Rightarrow Important effect, should be included
- **NLO** alone will grow forever \Rightarrow unphysical effect, should be taken care of



- Such growing contributions are present at all orders of the perturbative expansion

How to deal with them?

- At the initial time $\tau = \tau_0$, take:

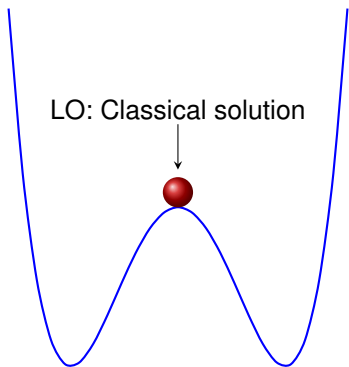
$$\vec{E}_0(\tau_0, \vec{x}) = \vec{\mathcal{E}}_0(\tau_0, \vec{x}) + \int_{\vec{k}} c_{\vec{k}} \vec{e}_{\vec{k}}(\tau_0, \vec{x})$$

where $c_{\vec{k}}$ are random coefficients: $\langle c_{\vec{k}} c_{\vec{k}'} \rangle \sim \delta_{\vec{k}\vec{k}'}$

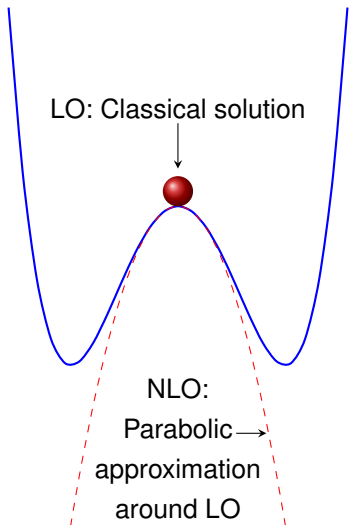
- Solve the **Classical** equation of motion $D_{\mu} F^{\mu\nu} = J^{\nu}$
- Compute $\langle \vec{E}^2(\tau, \vec{x}) \rangle$, where $\langle \rangle$ is the average on the $c_{\vec{k}}$ (Monte-Carlo)
- One can show that this resums all the fastest growing terms at each order, leading to a result that remain bounded when $\tau \rightarrow \infty$
[GELIS, LAPPI, VENUGOPALAN (2008)]

This gives: LO+NLO+Subset of higher orders

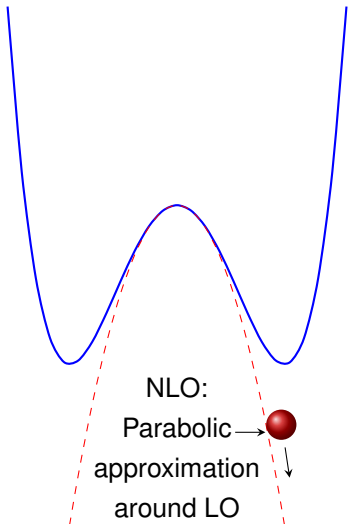
IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD



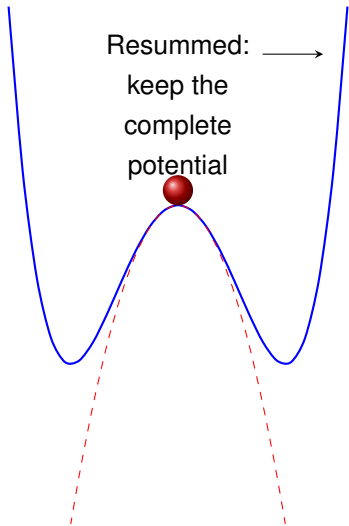
IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD



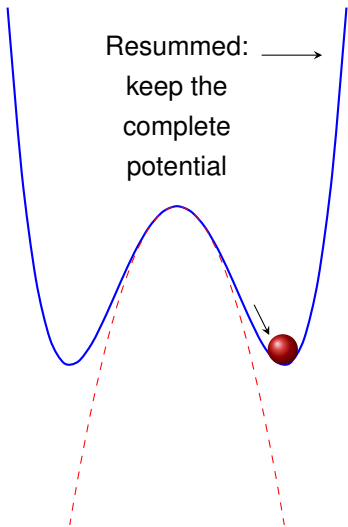
IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD



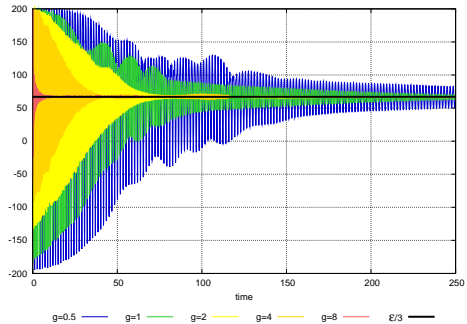
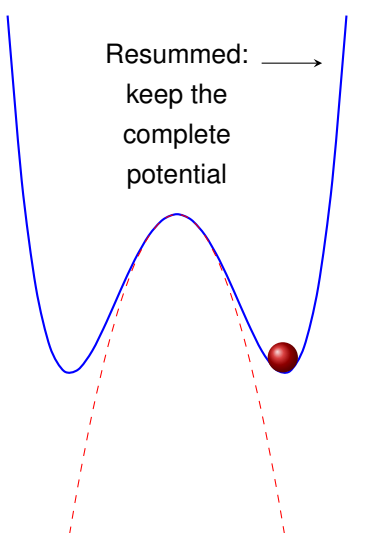
IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD



IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD

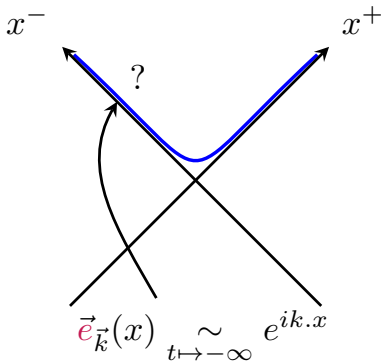


IMPLICATIONS OF THE CLASSICAL-STATISTICAL METHOD



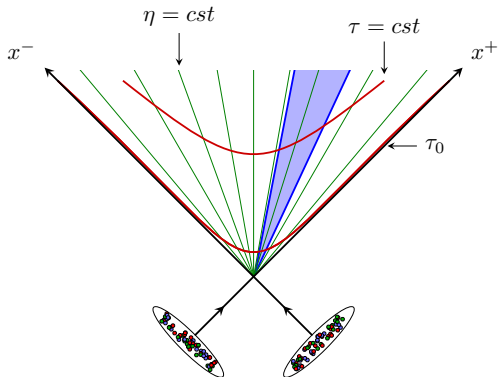
THE NLO SPECTRUM

- Need to know $\vec{e}_{\vec{k}}(\tau_0, \vec{x})$ at the time τ_0 we start the numerical simulation
- For practical reasons, we must start in the forward light cone ($\tau_0 > 0$)



This can be done analytically [TE,GELIS 1307:1765]

Proper-time/rapidity coordinate system



Result of [TE,GELIS 1307:1765]

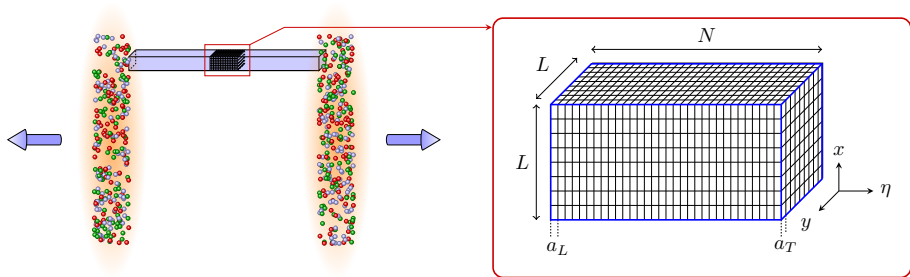
$$e_{\nu\vec{k}_\perp}^i = i\nu (F^{i,-} - F^{i,+}) \quad e_{\nu\vec{k}_\perp}^\eta(x) = \mathcal{D}^i (F^{i,-} - F^{i,+})$$

with

$$F_k^{i,+}(x) \sim e^{i\nu\eta} \mathcal{U}_1^\dagger(\vec{x}_\perp) \int_{\vec{p}_\perp} e^{i\vec{p}_\perp \cdot \vec{x}_\perp} \tilde{\mathcal{U}}_1(\vec{p}_\perp + \vec{k}_\perp) \left(\frac{p_\perp^2 \tau}{2k_\perp} \right)^{i\nu} \left[\delta^{ij} - \frac{2p_\perp^i p_\perp^j}{p_\perp^2} \right] \epsilon_{k\lambda}^j .$$

- \mathcal{U}_1^\dagger depends on the color source J^+ of the first nucleus
- Analogous formula for $F^{i,-}$.

Gauge potential $A^\mu \rightarrow$ link variables (exact gauge invariance on the lattice)

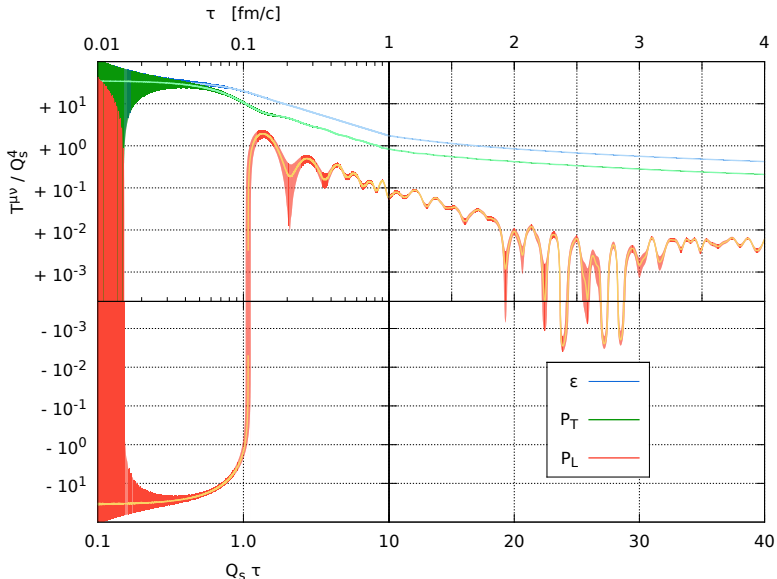


Numerical parameters

- Transverse lattice size $L = 64$, transverse lattice spacing $Q_s a_T = 1$
- Longitudinal lattice size $N = 128$, longitudinal lattice spacing $a_L = 0.016$
- Number of configurations for the Monte-Carlo $N_{\text{conf}} = 200$ to 2000
- Initial time $Q_s \tau_0 = 0.01$

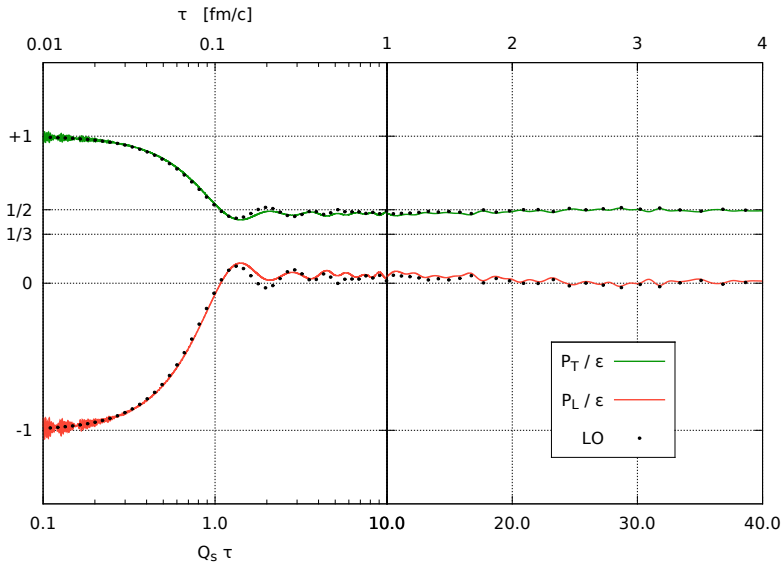
NUMERICAL RESULTS [TE,GELIS 1307:2214]

$$\alpha_s = 8 \cdot 10^{-4} \quad (g = 0.1)$$



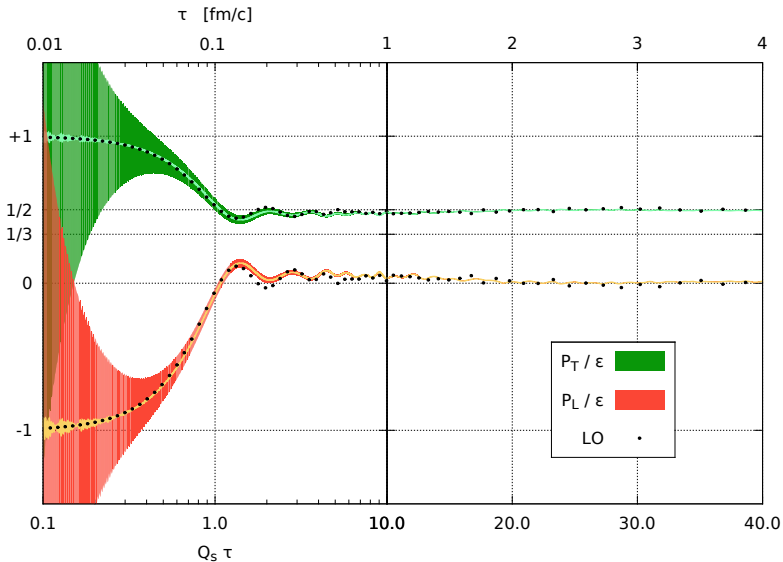
NUMERICAL RESULTS [TE,GELIS 1307:2214]

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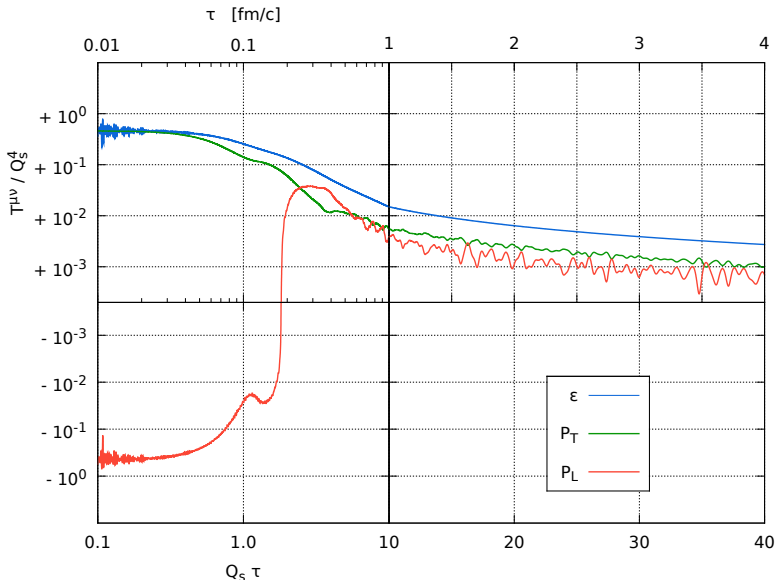
NUMERICAL RESULTS [TE,GELIS 1307:2214]

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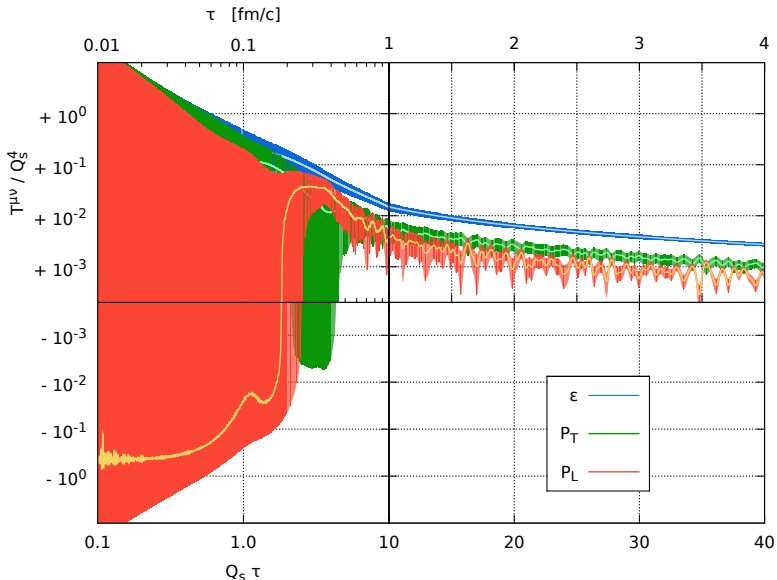
NUMERICAL RESULTS [TE,GELIS 1307:2214]

$$\alpha_s = 2 \cdot 10^{-2} \quad (g = 0.5)$$



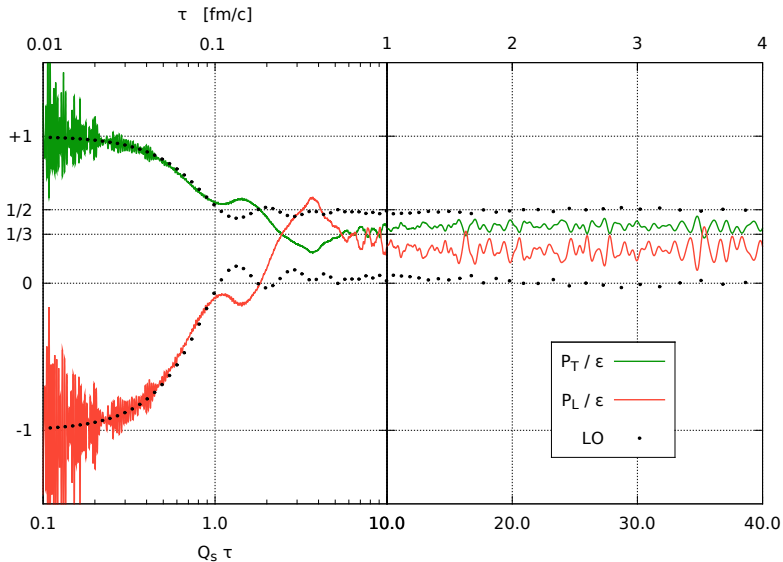
NUMERICAL RESULTS [TE,GELIS 1307:2214]

$$\alpha_s = 2 \cdot 10^{-2} \quad (g = 0.5)$$



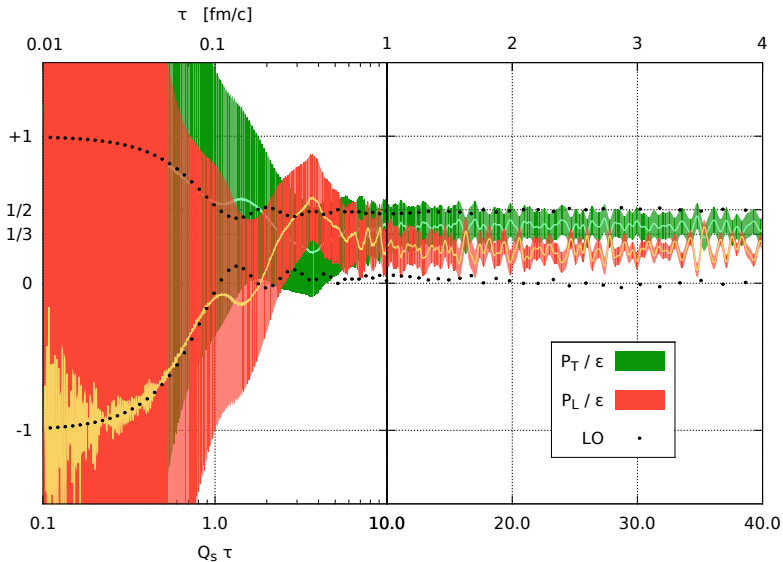
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NUMERICAL RESULTS [TE,GELIS 1307:2214]

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CONCLUSION

- Correct NLO spectrum from first principles
- Fixed anisotropy for $g = 0.5$ at $\tau \sim 1fm/c$
- No need for strong coupling to get isotropization
- Compatible with viscous hydrodynamical expansion
- Assuming simple first order viscous hydrodynamics

$$\epsilon \approx \underbrace{\epsilon_0 \tau^{-\frac{4}{3}}}_{\text{Ideal hydro}} - \underbrace{2\eta_0 \tau^{-2}}_{\text{first order correction}}$$

we can compute the dimensionless ratio ($\eta = \eta_0 \tau^{-1}$)

$$\eta \epsilon^{-\frac{3}{4}} \lesssim 1$$

In contrast, perturbation theory at LO gives $\eta \epsilon^{-\frac{3}{4}} \sim 300$

BACKUP: COMPLETELY INCOHERENT INITIAL FIELDS VS CGC

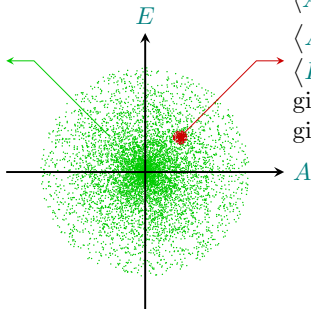
$$\langle A \rangle \sim 0, \langle E \rangle \sim 0$$

$$\langle A^2 \rangle - \langle A \rangle^2 \sim \frac{Q_s^2}{g^2}$$

$$\langle E^2 \rangle - \langle E \rangle^2 \sim \frac{Q_s^4}{g^2}$$

May give correct answer at LO

Not correct at NLO



$$\langle A \rangle \sim \frac{Q_s}{g}, \langle E \rangle \sim \frac{Q_s^2}{g}$$

$$\langle A^2 \rangle - \langle A \rangle^2 \sim Q_s^2$$

$$\langle E^2 \rangle - \langle E \rangle^2 \sim Q_s^4$$

give correct answer at LO

give correct answer at NLO