Gluon saturation beyond (naive) leading logs

Guillaume Beuf

Universidade de Santiago de Compostela

IS2013: International Conference on the Initial Stages in High-Energy Nuclear Collisions

Illa da Toxa, 8-14 September 2013

Outline

- Introduction
- Kinematical constraint for BFKL
- Leading Logs from DIS at NLO
- Kinematical constraint for the BK equation

G.B., arXiv:1310.xxxx *to appear*

See also the summarized version: G.B., arXiv:1301.0773

Gluon saturation/CGC in dense-dilute collisions

DIS observables and forward particle production in pp/pA:

- theoretically best understood observables sensitive to gluon saturation at high energy
- abundant available or incoming data from HERA, LHC, RHIC
- successful phenomenology within the Color Glass Condensate

see A. Dumitru's talk

However: saturation effects difficult to see clearly from the most inclusive observables.

Gluon saturation/CGC in dense-dilute collisions

DIS observables and forward particle production in pp/pA:

- theoretically best understood observables sensitive to gluon saturation at high energy
- abundant available or incoming data from HERA, LHC, RHIC
- successful phenomenology within the Color Glass Condensate

see A. Dumitru's talk

However: saturation effects difficult to see clearly from the most inclusive observables.

$\rightarrow$ 2 directions for improvement:

1. Study to less inclusive observables, like multi-particle correlations
   see C. Marquet's talk
2. Go to higher orders or other refinements for more precision

Gluon saturation/CGC at higher orders

Going to higher orders is necessary for precision studies.

For the simpler, inclusive observables, the calculation of higher order corrections has started:

- NLL corrections to the BK equation
  Balitsky, Chirilli (2008)
- NLO corrections to DIS structure functions
  Balitsky, Chirilli (2011)
  G.B. (2012)
- NLO corrections to forward single inclusive particle production in pA or pp
  Chirilli, Xiao, Yuan (2012)

Need for further resummations

However, besides running coupling effects, pathologically large corrections of two types plague higher order results and have to be resummed to obtain reliable results from BK at NLL.

- Kinematical corrections: due to a too naive treatment of the high-energy limit.
  $\rightarrow$ Main topic of the rest of this talk.
- Dynamical corrections: induced from DGLAP evolutions of the projectile and of the target, due to the duality between low x_{Bj} and high Q^2 evolutions.
  $\rightarrow$ Left for further studies.

The same problems appear in the linear regime for the BFKL equation, and the corresponding resummations have been fully performed.
Ciafaloni, Colferai, Salam, Staśto (1998-2007)
Altarelli, Ball, Forte (1999-2008)

Kinematical issues for BFKL in momentum space

Conventional derivations of the BFKL evolution require kinematical approximations for the t-channel gluons propagators, or for the energy denominators in the dipole model derivation.

Usual justification for those approximations: multi-Regge kinematics

- Strong ordering in rapidity y (or in k^+, or in k^-) of the emitted gluons
- and all $\mathbf{k}_\perp$'s of the same order

Sufficient but not necessary condition for the kinematical approximations to be valid.

Kinematical issues for BFKL in momentum space

Conventional derivations of the BFKL evolution require kinematical approximations for the t-channel gluons propagators, or for the energy denominators in the dipole model derivation.

Usual justification for those approximations: multi-Regge kinematics

- Strong ordering in rapidity y (or in k^+, or in k^-) of the emitted gluons
- and all $\mathbf{k}_\perp$'s of the same order

Sufficient but not necessary condition for the kinematical approximations to be valid.

Problem: unrestricted integral over $\mathbf{k}_\perp$ in the BFKL equation
$\Rightarrow$ *A posteriori* not consistent to assume all $\mathbf{k}_\perp$'s of the same order!

Kinematical issues for BFKL in momentum space

Necessary and sufficient condition for the required kinematical approximations:

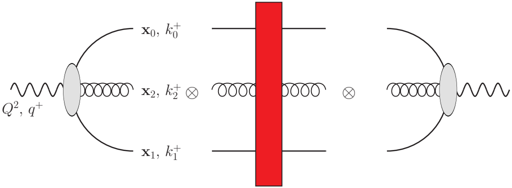
Strong ordering of the emitted gluons *both* in k^+ and in k^- simultaneously

Strong ordering is guarantied only for the evolution variable chosen for the BFKL equation: y, k^+ or k^-, depending on the factorization scheme.

$\Rightarrow$ Need to impose by hand the missing k^- and/or k^+ ordering in the equation via a kinematical constraint in the BFKL kernel.

In momentum space: Ciafaloni (1988)
Kwieciński, Martin, Sutton (1996)
Andersson, Gustafson, Kharraziha, Samuelsson (1996)
Analog in Mellin space: Salam (1998)

DIS at high energy at NLO



$$\sigma^{\gamma}_{T,L}(Q^2, x_{Bj}) = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2\int \mathrm{d}^2\mathbf{x}_0\int \mathrm{d}^2\mathbf{x}_1\int_0^1 \mathrm{d}z_1$$

$$\times\Bigg\{\mathcal{I}^{LO}_{T,L}(x_{01}, z_1, Q^2)\,\Big[1+\mathcal{O}(\bar{\alpha})\Big]\,\Big[1-\langle\mathcal{S}_{01}\rangle_0\Big]$$

$$+\bar{\alpha}\int_{k^+_{\min}/q^+}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2,Q^2)\;\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle_0\Bigg\}$$

with $z_n = k_n^+/q^+$

G.B. (2012); see also Balitsky, Chirilli (2011).

Extraction of LL from NLO

The LL contained in the NLO term seems consistent with BK

$$\bar{\alpha} \int_{z_{\min}}^{z_f} \frac{\mathrm{d}z_2}{z_2} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\, \mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2)\ \langle \mathcal{S}_{01} - \mathcal{S}_{02}\, \mathcal{S}_{21} \rangle_0$$

$$\sim\ \mathcal{I}_{T,L}^{LO}(x_{01}, z_1)\ \ \bar{\alpha} \log\left(\frac{z_f}{z_{\min}}\right) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\ \langle \mathcal{S}_{01} - \mathcal{S}_{02}\, \mathcal{S}_{21} \rangle_0$$

because $\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2{=}0) = \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\ \ \mathcal{I}_{T,L}^{LO}(x_{01},z_1)$

Extraction of LL from NLO

The LL contained in the NLO term seems consistent with BK

$$\bar{\alpha}\int_{z_{\min}}^{z_f}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2,Q^2)\;\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle_0$$

$$\sim\;\mathcal{I}_{T,L}^{LO}(x_{01},z_1)\;\;\bar{\alpha}\log\left(\frac{z_f}{z_{\min}}\right)\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\;\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle_0$$

because $\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2{=}0)=\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\;\mathcal{I}_{T,L}^{LO}(x_{01},z_1)$

However: for any small but finite z_2

$$\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2)\ll\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\;\mathcal{I}_{T,L}^{LO}(x_{01},z_1)$$

when $z_1(1-z_1)x_{01}^2\ll z_2x_{02}^2\simeq z_2x_{21}^2$.

Extraction of LL from NLO

The LL contained in the NLO term seems consistent with BK

$$\bar{\alpha}\int_{z_{\min}}^{z_f}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2,Q^2)\;\left\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\right\rangle_0$$

$$\sim\;\mathcal{I}_{T,L}^{LO}(x_{01},z_1)\;\;\bar{\alpha}\log\left(\frac{z_f}{z_{\min}}\right)\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\;\left\langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\right\rangle_0$$

because $\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2{=}0)=\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\;\;\mathcal{I}_{T,L}^{LO}(x_{01},z_1)$

However: for any small but finite z_2

$$\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2)\ll\frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\;\;\mathcal{I}_{T,L}^{LO}(x_{01},z_1)$$

when $z_1(1-z_1)x_{01}^2\ll z_2x_{02}^2\simeq z_2x_{21}^2$.

$\Rightarrow$ Splitting of a dipole into much larger dipoles do not contribute to LL's !

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}z_2}{z_2}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21}-\frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+}$$

Ordering in $k^+ = z\,q^+$ guarantied by the choice of factorization scheme/evolution in k^+.

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}z_2}{z_2}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y_2^+}$$

Ordering in $k^+ = z\,q^+$ guarantied by the choice of factorization scheme/evolution in k^+.

Modification: forbid gluon emission in the regime
$z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2\, x_{12}^2$
$\rightarrow$ Mixed-space analog of the k^- ordering (kinematical constraint).

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}z_2}{z_2}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+}$$

Ordering in $k^+ = z\,q^+$ guarantied by the choice of factorization scheme/evolution in k^+.

Modification: forbid gluon emission in the regime
$z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2\,x_{12}^2$
$\rightarrow$ Mixed-space analog of the k^- ordering (kinematical constraint).

$\Rightarrow$ Multiply the real contribution by $\theta\left(z_f\,x_{01}^2 - z_2\,\min(x_{02}^2, x_{21}^2)\right)$

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}z_2}{z_2}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+}$$

Ordering in $k^+ = z\,q^+$ guarantied by the choice of factorization scheme/evolution in k^+.

Modification: forbid gluon emission in the regime
$z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2\,x_{12}^2$
$\rightarrow$ Mixed-space analog of the k^- ordering (kinematical constraint).

$\Rightarrow$ Multiply the real contribution by $\theta\left(z_f\,x_{01}^2 - z_2\,\min(x_{02}^2, x_{21}^2)\right)$

Same general idea as in the previous study in mixed space:
Motyka, Staśto (2009)
However: inappropriate treatment of virtual corrections there.

Calculating virtual corrections from unitarity

Assume the kinematical constraint to preserve the probabilistic interpretation of the parton cascade.
Evolution of $\langle S_{01} \rangle$ over a finite range $Y_f^+ = \log(z_f/z_{\min})$:

$$\begin{aligned} \langle S_{01} \rangle_{Y_f^+} \quad = \quad & \langle S_{01} \rangle_0 \;\; D_{01}(Y_f^+) + \bar{\alpha} \int_0^{Y_f^+} \mathrm{d}Y_2^+ \; D_{01}(Y_f^+ - Y_2^+) \\ & \times \int \frac{\mathrm{d}^2\mathbf{x_2}}{2\pi} \; \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \; \theta\left(Y_f^+ - Y_2^+ - \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2} \right) \right) \\ & \times \left\langle \mathcal{S}_{02}\, \mathcal{S}_{21} - \frac{1}{N_c^2} \mathcal{S}_{01} \right\rangle_{Y_2^+} \end{aligned}$$

with the probability $D_{01}(Y^+)$ of no splitting for the dipole 01 in the range Y^+.

Calculating virtual corrections from unitarity

In the vacuum (absence of target), $S_{01} = S_{02} = S_{21} = 1$.
$\to$ equation determining $D_{01}(Y^+)$.
Solution:

$$D_{01}(Y^+) = \exp\left[-\bar{\alpha}\frac{2C_F}{N_c}\int\frac{d^2\mathbf{x}_2}{2\pi}\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\ (Y^+ - \Delta_{012})\ \ \theta(Y^+ - \Delta_{012})\right]$$

with the notation

$$\Delta_{012} = \max\left\{0,\ \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right\}$$

Typical behavior:

$$\begin{aligned}
\Delta_{012} &= 0 \quad \text{for} \quad x_{02}^2 \lesssim x_{01}^2 \quad \text{and} \quad x_{21}^2 \lesssim x_{01}^2 \\
\Delta_{012} &\sim \log\left(\frac{x_{02}^2}{x_{01}^2}\right) \sim \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \quad \text{for} \quad x_{01}^2 \ll x_{02}^2 \sim x_{21}^2
\end{aligned}$$

Kinematically constrained BK equation

Rewriting the new evolution equation as a differential equation and discarding irrelevant terms explicitly of order NLL:

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y^+} \right\}$$

G.B., arXiv:1310.xxxx *to appear*

Kinematically constrained BK equation

Rewriting the new evolution equation as a differential equation and discarding irrelevant terms explicitly of order NLL:

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y^+} \right\}$$

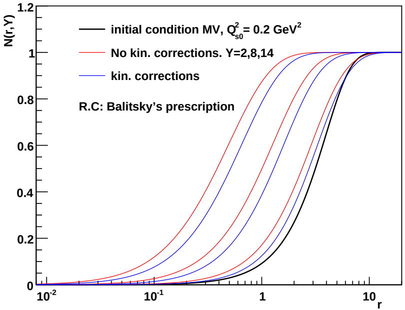
G.B., arXiv:1310.xxxx *to appear*

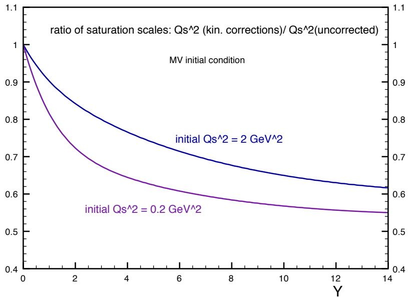
Slows down the BK evolution:

- Restriction of phase space by the theta function
- Shift of the Y^+ argument of the dipole amplitude in the real term but not in the virtual term.

Large effect especially at small Y^+.

Numerics for BK with kinematical constraint



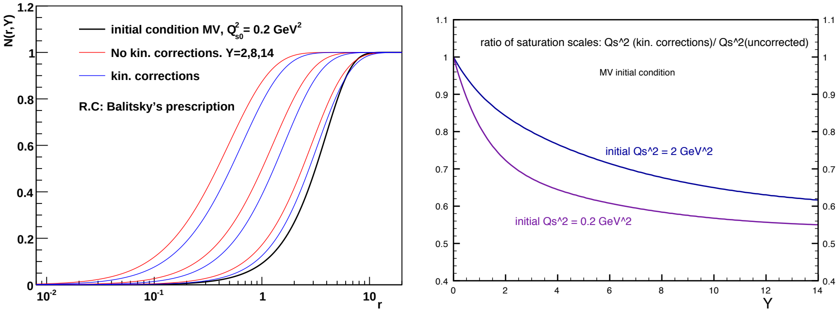


Main effects of the kinematical constraint (running coupling case):

- slows down the beginning of the Y^+ evolution, especially for softer initial Q_s
- at large Y^+: $\sim$ constant rescaling of the saturation scale

Work in progress; Albacete, Armesto, G.B., Milhano

Numerics for BK with kinematical constraint



Main effects of the kinematical constraint (running coupling case):

- slows down the beginning of the Y^+ evolution, especially for softer initial Q_s
- at large Y^+: $\sim$ constant rescaling of the saturation scale

Impact on the DIS fits of AAMQS?

Work in progress; Albacete, Armesto, G.B., Milhano

Conclusions

Kinematical constraint in BK/BFKL/... :

- Makes the derivation of the equation more consistent:
  $\rightarrow$ from naive LL to consistent LL
- Allows to subtract exactly the LL's actually present in observables at NLO
- Resum the largest corrections in the (anti-)collinear limit appearing in the evolution kernel at N^{n}LL

Conclusions

Kinematical constraint in BK/BFKL/... :

- Makes the derivation of the equation more consistent:
  $\rightarrow$ from naive LL to consistent LL
- Allows to subtract exactly the LL's actually present in observables at NLO
- Resum the largest corrections in the (anti-)collinear limit appearing in the evolution kernel at N^{n}LL

$\Rightarrow$ New standard for CGC phenomenology: BK with running coupling and kinematical constraint.

Conclusions

Kinematical constraint in BK/BFKL/... :

- Makes the derivation of the equation more consistent: $\rightarrow$ from naive LL to consistent LL
- Allows to subtract exactly the LL's actually present in observables at NLO
- Resum the largest corrections in the (anti-)collinear limit appearing in the evolution kernel at N^{n}LL

$\Rightarrow$ New standard for CGC phenomenology: BK with running coupling and kinematical constraint.

Future developments:

- Kinematical constraint for JIMWLK?
- Resummation of large *dynamical* higher order corrections for BK, to get a full collinear-resummed BK ? at NLL ?