Thermalization at weak coupling

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...or equilibration in three acts:

1. Melting of the coherent fields
2. Competition between dynamics and expansion
3. Bottom-up thermalization
What?

Hydrodynamics provides a fantastic description of HIC, but:

- assumes local thermal equilibrium
- needs thermalization time $\tau_0$ and initial geometry $T_{\mu\nu}(\tau_0)$ as input
- What happens before?
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How?
- Extremely big and at extremely high energy
  - Saturation scale $Q_s \gg \Lambda_{QCD}$
  - weak coupling: $\alpha_s(Q_s) \ll 1$
  - Scale separation allows for effective theories: HTL, kinetic theory.
  - Strong fields, not perturbative.
- Provide description consistent with CGC assumptions
- Philosophy: Do the whole calculation at weak coupling, extrapolate to finite $\alpha_s$ only at the end (Compare to McLerran’s talk)
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Units $\alpha^# Q_s^{-1}$, not $fm/c$ in this talk
1. Melting of the coherent fields
2. Competition between dynamics and expansion
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Initial condition: $t \sim Q_s^{-1}$

Blaizot Mueller '87, McLerran Venugopalan '94, Kovner McLerran, Weigert '95, Gelis Epelbaum '13

At weak coupling, well understood: Color Glass Condensate

- Strong, boost invariant fields
- Characteristic transverse coherence length $\sim Q_s^{-1}$
- Characteristic energy density $\epsilon \sim Q^4/\alpha$

- Boost invariance broken only by quantum fluctuations
- Fields decohere and isotropize in a time scale $\tau \sim Q_s^{-1} \times \log(\alpha)^{-1}$
  - For details: Thomas Epelbaum’s talk
Once the fields have decohered, description in terms of gluons $(\eta = 0, \tau = t)$:

- **Characteristic, saturation scale:** $Q_s$
- **Highly over-occupied:**
  
  \[ f(p < Q_s) \sim \alpha^{-c}, \quad c \gtrsim 1 \]
  
  \[
  \begin{align*}
  p_t &\sim Q_s \\
  p_z &\sim \alpha^d Q_s
  \end{align*}
  \]

- **Highly anisotropic:**

**Big question:** How to reach the thermal point $c = 0, d = 0$
1. Melting of the coherent fields
2. Competition between dynamics and expansion
3. Bottom-up thermalization
Competition between dynamics and expansion: $c(t)$, $d(t)$

Subsequent evolution competition between

- Longitudinal expansion (anisotropizes)
- Momentum broadening due interactions (isotropizes, dilutes)

Initial condition

occupancy: $c \sim \alpha^{-1}$

Expansion

Under-occupied

Over-occupied

anisotropy: $d$

More anisotropic

$\sim \alpha^{-5/2}$

Less anisotropic

$\sim \alpha^{-3/2}$

$\sim \alpha^{-1}$

Thermal

Initial condition

occupancy: $c \sim \alpha^{-1}$
Longitudinal expansion

Spatial expansion translates into redshift in $p_z \sim \alpha^d Q_s$

- Reduces $p_z = \alpha^d Q$, $Q_s$ stays constant
- Reduces energy: $\varepsilon(t) \sim \alpha^{-1} Q_s^4 / (Q_s t)$
Longitudinal expansion

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Lines of constant energy density and fixed time
Longitudinal expansion

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![Diagram showing lines of constant energy density and fixed time.](attachment:diagram.png)
Elastic scattering dilutes the distribution \((\delta \equiv \alpha^d)\)

- Along the attractive solution scattering and expansion compete
- Eventually system becomes underoccupied
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Scattering: Elastic

Elastic scattering dilutes the distribution \( (\delta \equiv \alpha^d) \)

- Along the attractive solution scattering and expansion compete
- Eventually system becomes underoccupied
- Suggested other solution BGLMV, that might stay \( \mathcal{O}(1) \) isotropic.

Blaizot, Gelis, Liao, McLerran, Venugopalan 2011
Plasma instabilities: Anisotropic screening

**Isotropic distributions:**
- Screening stabilizes soft E-fields
  \[ \omega_{pl}^2 \sim m^2 \sim \alpha \int d^3p \frac{f(p)}{p} \]
- B-fields induces a rotation on \( f(p) \) → No screening for static B-fields

**Anisotropic distributions:**
- B-field induces non-trivial rotation:
  - Some B-fields stabilized
  - ...others destabilized: Plasma-unstable modes
- Unstable modes grow exponentially...
Plasma instabilities: Unstable modes

- Which $B$-modes become unstable depend on the distribution of gluons Romatshcke, Strickland '03, Arnold, Moore '07

### Strong anisotropy:

$$f(\vec{p}) \sim \alpha^{-c} \Theta(Q_s - p) \Theta(\delta Q_s - p_z)$$

### Weak anisotropy:

$$f(\vec{p}) \sim f_0(|\vec{p}|)(1 + \epsilon F(\vec{p}))$$
Plasma instabilities: Growth and saturation

Saturation when fields become non-perturbatively strong:

\[ D_\mu = k_{\text{inst}} + igA_\mu \Rightarrow \begin{cases} A \sim k_{\text{inst}}/g \\ f(k_{\text{inst}}) \sim 1/\alpha \end{cases} \]

Arnold, Moore, Yaffe ’07, Bödeker, Rummukainen ’07

Hard Loop: \( Q_s \) modes as currents, unstable modes classical YM
Plasma instabilities: Momentum diffusion $\hat{q}$

Hard parton traveling through magnetic fields receives coherent kicks from patches of same-sign magnetic fields

\[ \Delta p_{\text{kick}} \sim g B l_{\text{coh}} \]

\[ \hat{q} \sim N_{\text{kick}} (\Delta p_{\text{kick}})^2 \]

\[ \hat{q} \sim \alpha \frac{l_{\text{coh}}}{t/l_{\text{coh}}} B^2 \text{ range occupancy} \]

\[ \hat{q} \sim \alpha^{-\frac{1}{2}} d + \frac{3}{2} (1-c) \]

Kurkela, Moore
Attractor with plasma instabilities

Broadening of the hard particle distribution dominated by the plasma instabilities ($\hat{q}_{el} \ll \hat{q}_{inst}$) originating from the scale $Q_s$

Numerical approaches

- **BMSS** (elastic scattering)
- **KM** (plasma instabilities)
- **BGLMV** (const. anisotropy)

**Hard Expanding Loops:** Assumes free streaming for the particles, sees the growth of plasma instabilities.

**BBSV:** Classical YM simulations in expanding background at \((c = 1, d = 0)\)-point. Favours BMSS. Not yet evidence of plasma instabilities.
1. Melting of the coherent fields
2. Competition between dynamics and expansion
3. Bottom-up thermalization
   ...or how very dilute and anisotropic systems equilibrate

Anisotropy: $d$
Occupancy: $c$

-1 0 1

Under occupied

BMSS
KM

$\sim \alpha$
Soft radiation

Physics of under occupied qualitatively different from over occupied.

Formation of soft thermal bath:

\[ Q \delta Q \]

\[ p_t \]

\[ p_z \]

- Soft modes quick to emit
  \[ n_s \sim \alpha n_{\text{col}} \]

- Low p: easy to bend
  \[ \Rightarrow \text{thermalize quickly} \]

- Can dominate dynamics!
  (i.e. scattering, screening, \ldots)

\[ \Rightarrow \text{Right way to think: Few energetic “jets” propagating in thermal bath} \]
Hard splitting

$Q_s$ modes break before they bend!

- In vacuum: on-shell particles, no splitting
- In medium: Particles receive small kicks frequently
  - For stochastic uncorrelated kicks: Brownian motion in $p$-space

\[
\Delta p^2_\perp \sim \hat{q} t, \quad t_{\text{split}}(k) \sim l_{\text{stop}}(k) \sim \alpha^{-1} \sqrt{k/\hat{q}} \quad \text{(LPM)}
\]

- Physics of Landau-Pomeranchuk-Migdal suppression important
Thermal bath eats the hard particles away:

- Scales below $k_{\text{split}}$ have cascaded down to $T$-bath

$$t_{\text{split}}(k_{\text{split}}) \sim t \Rightarrow k_{\text{split}} \sim \alpha^2 \hat{q} t^2$$

- "Falling" particles heat up the thermal bath

$$T^4 \sim k_{\text{split}} \int d^3p f(p)$$

- Thermalization when $Q_s$ gets eaten

$$k_{\text{split}} \sim Q_s$$

Thermalization time determined by how strongly hard particle motion is affected by the thermal bath, $\hat{q}$
If the $\hat{q}$ is dominated by elastic scattering with the soft thermal bath

\[
\begin{align*}
\hat{q} & \sim \alpha^2 T^3 \\
\tau_0 & \sim \alpha^{-13/5} Q_s^{-1}
\end{align*}
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Bottom up: 

If the $\hat{q}$ is dominated by elastic scattering with the soft thermal bath

$$
\begin{align*}
\hat{q} & \sim \alpha^2 T^3 \\
T & \sim \alpha^3 Q_s (Q_s t) \\
\kappa_{\text{split}} & \sim \alpha^{13} (Q_s t)^5 Q \\
\tau_0 & \sim \alpha^{-13/5} Q_s^{-1}
\end{align*}
$$

However, the thermal bath is slightly anisotropic and can give rise to plasma instabilities and increase $\hat{q}$ to faster thermalization

Kurkela, Moore ’11

$$
\begin{align*}
\hat{q} & \sim \alpha^3 Q_s^3 \\
T & \sim \alpha Q_s (Q_s t)^{1/4} \\
\kappa_{\text{split}} & \sim \alpha^5 (Q t)^2 Q_s \\
\tau_0 & \sim \alpha^{-5/2} Q_s^{-1}
\end{align*}
$$
Conclusions:

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- Role of plasma instabilities under investigation
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  - Thermalization time estimated by how long it takes for $Q_s$-‘jets’ the quench in a thermal background generated by themselves.
- Anisotropic screening leads (always) to plasma instabilities
- Role of plasma instabilities under investigation
- Thermalization time $\tau \sim \# \alpha^{-5/2} Q_s^{-1}$.
  - Is this long or short? Depends on $. Numerics needed.
Comparison to other work

BMSS: Baier, Mueller, Schiff, Son hep-ph/0009237
HEL: Rebhan, Stricland arXiv:1207.5795
CGC: Gelis, Iancu, Jalilian-Marian, Venugopalan arXiv:1002.0333
BS: Berges, Schlichting: arXiv:1209.0817