Thermalization at weak coupling

Aleksi Kurkela
CERN (PH-TH)

...or equilibration in three acts:

- 1. Melting of the coherent fields
- 2. Competition between dynamics and expansion
- 3. Bottom-up thermalization

What?

Hydrodynamics provides a fantastic description of HIC, but:

- assumes local thermal equilibrium
  - needs thermalization time τ_0 and initial geometry $T_{\mu\nu}(\tau_0)$ as input
- What happens before?

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How?

- Extremely big and at extremely high energy
  - Saturation scale $Q_s \gg \Lambda_{QCD}$
  - weak coupling: $\alpha_s(Q_s) \ll 1$
  - Scale separation allows for effective theories: HTL, kinetic theory.
  - Strong fields, not perturbative.
- Provide description consistent with CGC assumptions
- Philosophy: Do the whole calculation at weak coupling, extrapolate to finite α_s only at the end (Compare to McLerran's talk)

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Units $\alpha^{\#} Q_s^{-1}$, not fm/c in this talk

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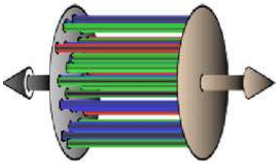
Initial condition: $t \sim Q_s^{-1}$

Blaizot Mueller '87, McLerran Venugopalan '94, Kovner McLerran, Weigert '95, Gelis Epelbaum '13

At weak coupling, well understood: Color Glass Condensate



- Strong, boost invariant fields
- Characteristic transverse coherence length $\sim Q_s^{-1}$
- Characteristic energy density $\epsilon \sim Q^4/\alpha$

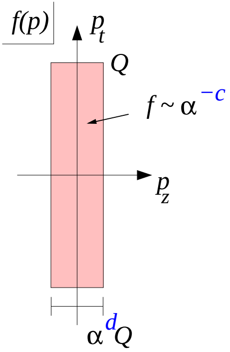
- Boost invariance broken only by quantum fluctuations
- Fields decohere and isotropize in a time scale $\tau \sim Q_s^{-1} \times \log(\alpha)^{-1}$
    - For details: Thomas Epelbaum's talk

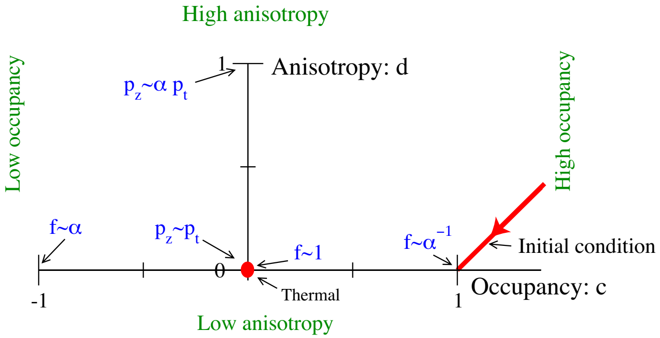
The Map:

Kurkela, Moore '11

Once the fields have decohered, description in terms of gluons $(\eta = 0, \tau = t)$:

- Characteristic, *saturation* scale: Q_s
- Highly over-occupied: $f(p < Q_s) \sim \alpha^{-c}, \quad c \gtrsim 1$
- Highly anisotropic: $\begin{cases} p_t \sim Q_s \\ p_z \sim \alpha^{d} Q_s \end{cases}$





Big question: How to reach the thermal point $c = 0, d = 0$

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- 2. Competition between dynamics and expansion
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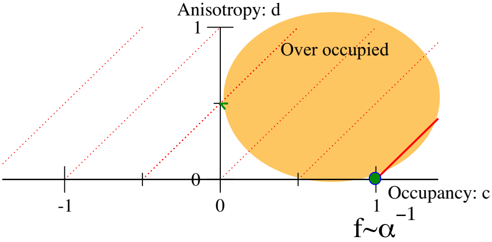
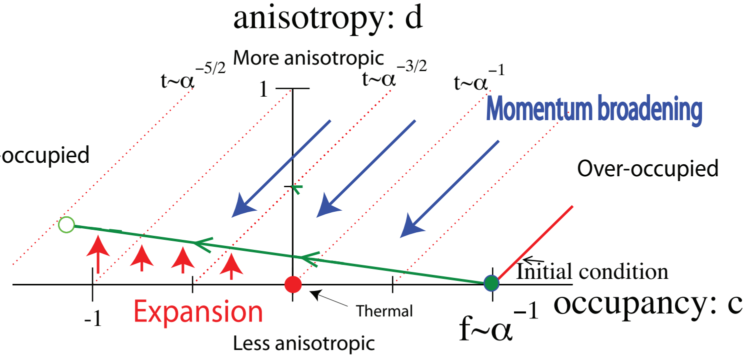


Competition between dynamics and expansion: $c(t)$, $d(t)$

Subsequent evolution competition between

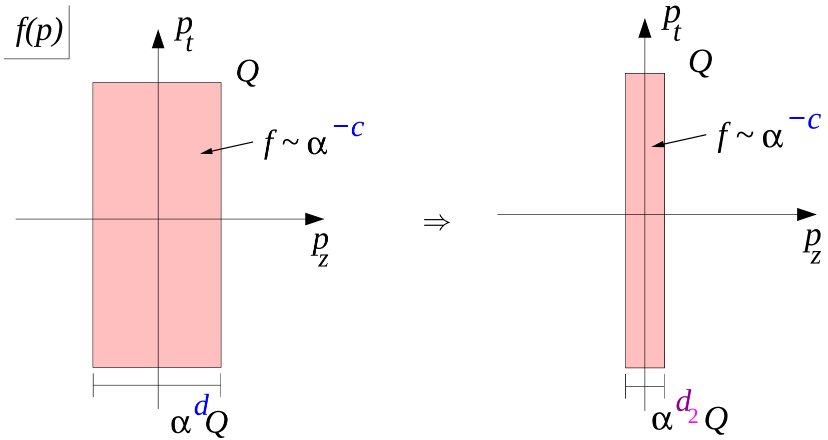
- Longitudinal expansion (anisotropizes)
- Momentum broadening due interactions (isotropizes,dilutes)



Longitudinal expansion

Spatial expansion translates into redshift in $p_z \sim \alpha^d Q_s$

- Reduces $p_z = \alpha^d Q$, Q_s stays constant
- Reduces energy: $\varepsilon(t) \sim \alpha^{-1} Q_s^4/(Q_s t)$



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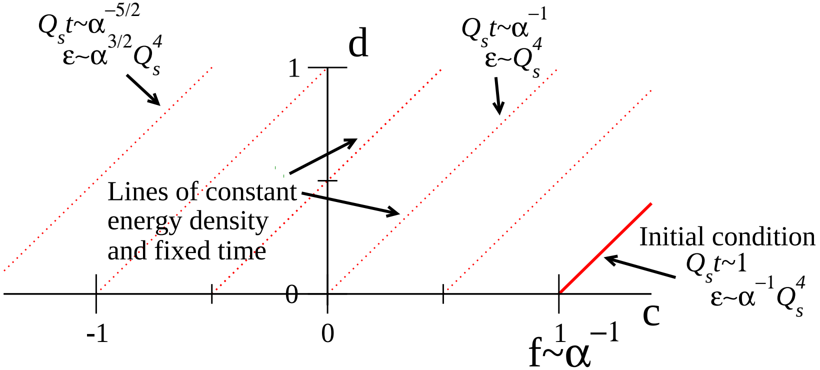

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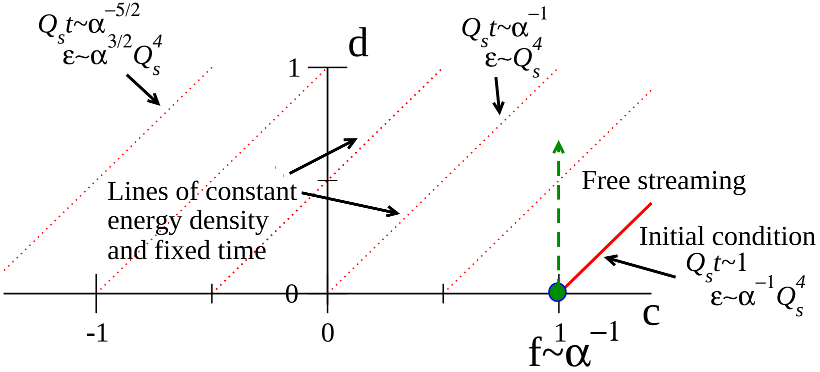
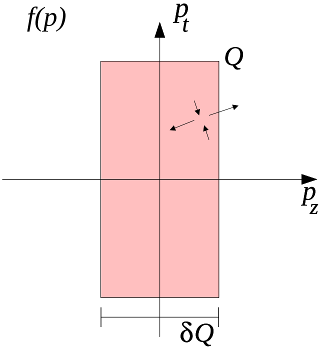
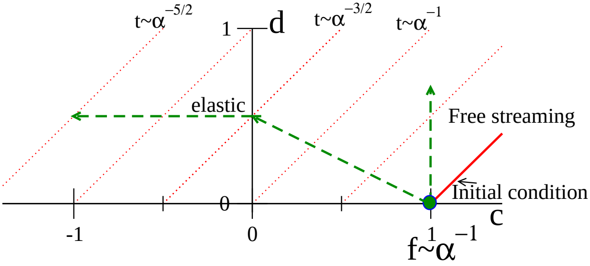


Scattering: Elastic

Baier, Mueller, Schiff, Son '00

Elastic scattering dilutes the distribution $(\delta \equiv \alpha^{d})$





- Along the attractive solution scattering and expansion compete
- Eventually system becomes underoccupied

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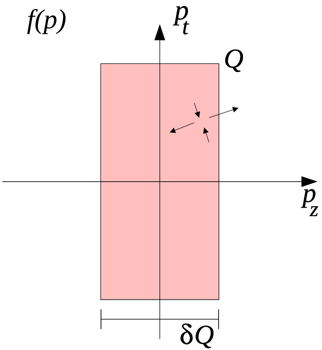

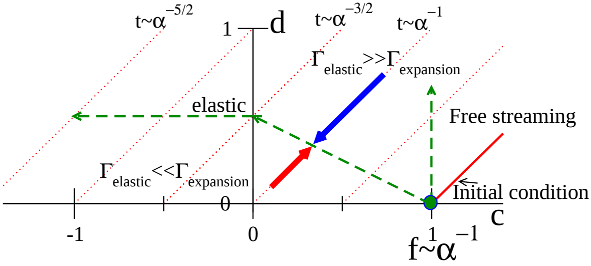

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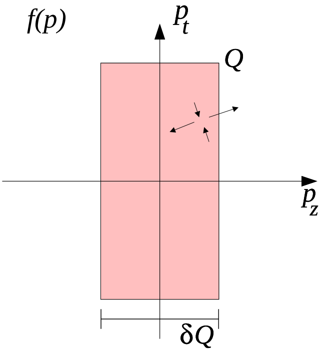

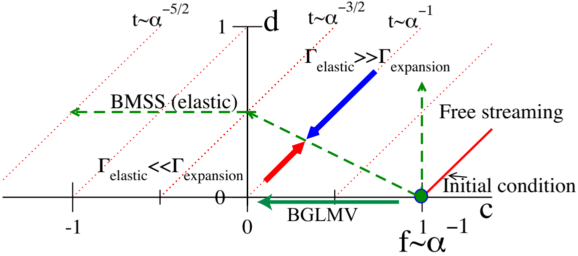

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- Suggested other solution BGLMV, that might stay $\mathcal{O}(1)$ isotropic.

Blaizot, Gelis, Liao, McLerran, Venugopalan 2011

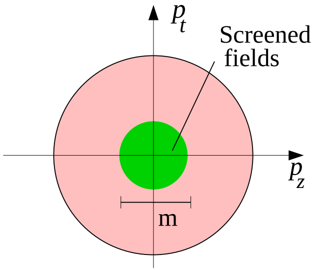
Plasma instabilities: Anisotropic screening

Mrowczynski '93, '00

Isotropic distributions:

- Screening stabilizes soft E-fields

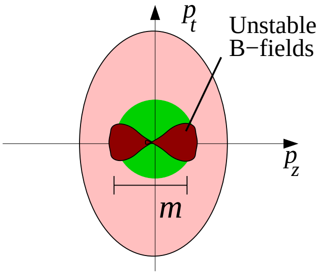
$$\omega_{pl}^2 \sim m^2 \sim \alpha \int d^3p \frac{f(p)}{p}$$



- B-fields induces a rotation on $f(p) \rightarrow$ No screening for static B-fields

Anisotropic distributions:

- B-field induces non-trivial rotation:
  - Some B-fields stabilized
  - ...others destabilized: Plasma-unstable modes



- Unstable modes grow exponentially...

Plasma instabilites: Unstable modes

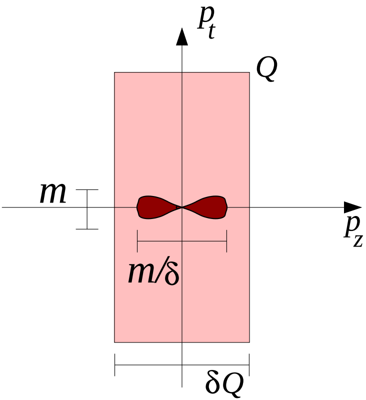
- Which B-modes become unstable depend on the distribution of gluons Romatshcke, Strickland '03, Arnold, Moore '07

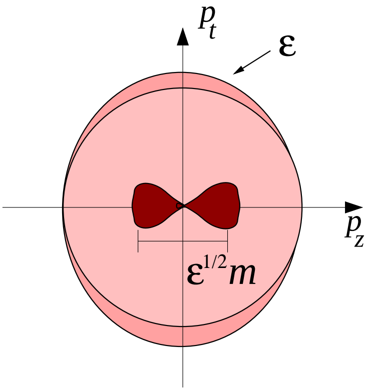
Strong anisotropy:

$$f(\vec{p}) \sim \alpha^{-c}\Theta(Q_s - p)\Theta(\delta Q_s - p_z)$$

Weak anisotropy:

$$f(\vec{p}) \sim f_0(|\vec{p}|)(1 + \epsilon F(\vec{p}))$$

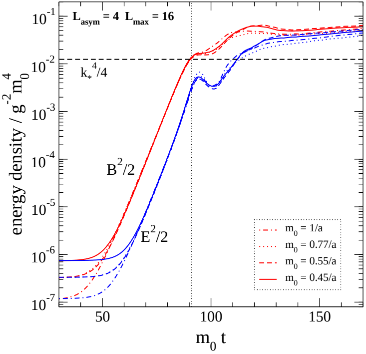




Plasma instabilities: Growth and saturation

- Saturation when fields become non-perturbatively strong:

$$D_\mu = k_{\rm inst} + igA_\mu \Rightarrow \begin{cases} A \sim k_{\rm inst}/g \\ f(k_{\rm inst}) \sim 1/\alpha \end{cases}$$

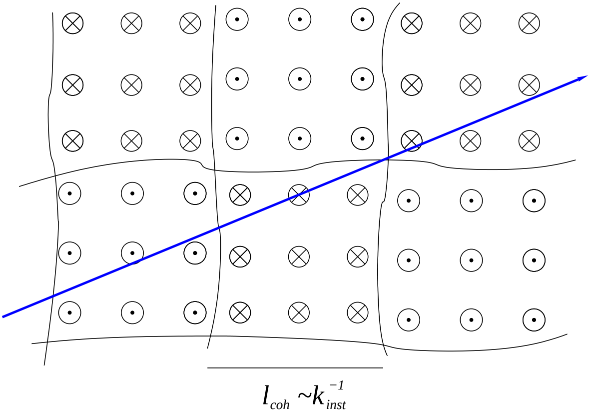


Arnold, Moore, Yaffe '07, Bödeker, Rummukainen '07

Hard Loop: Q_s modes as currents, unstable modes classical YM

Plasma instabilites: Momentum diffusion $\hat{q}$

Hard parton traveling through magnetic fields receives coherent kicks from patches of same-sign magnetic fields



- $\Delta p_{\rm kick} \sim g\, B\, l_{\rm coh}$
- $\hat{q}\, t \sim \underbrace{N_{\rm kick}}_{t/l_{\rm coh}} (\Delta p_{\rm kick})^2$

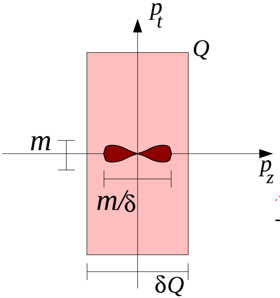
$$\hat{q} \sim \alpha \underbrace{l_{\rm coh}}_{range} \underbrace{B^2}_{occupancy}$$

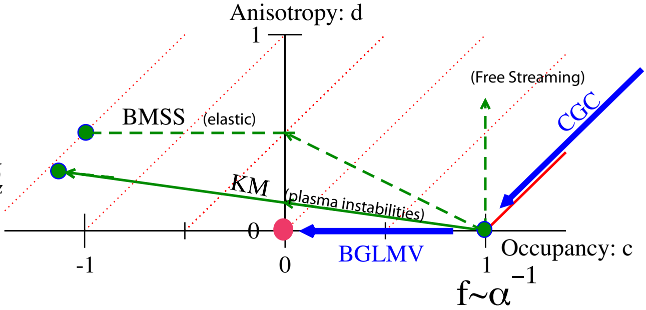
$$\hat{q} \sim \alpha^{-\frac{1}{2}d + \frac{3}{2}(1-c)}$$

Kurkela, Moore

Attractor with plasma instabilities

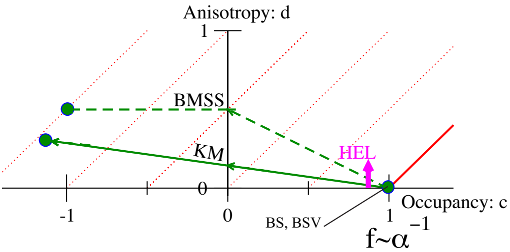
Broadening of the hard particle distribution dominated by the plasma instabilities ($\hat{q}_{el} \ll \hat{q}_{inst}$) originating from the scale Q_s

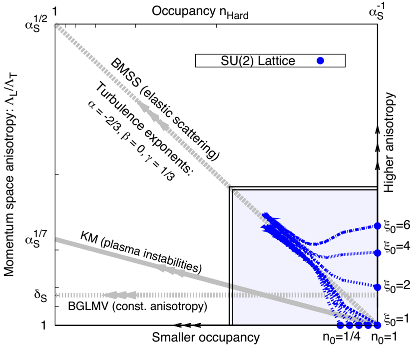




BMSS: Baier, Mueller, Schiff, Son 2009, KM: Kurkela, Moore 2011 , BGLMV: Blaizot, Gelis, Liao, McLerran, Venugopalan 2011

Numerical approaches





HEL: Attems, Rebhan, Strickland '13

BBSV: Berges, Boguslavski, Schlichting, Venugopalan '13

- Hard Expanding Loops: Assumes free streaming for the particles, sees the growth of plasma instabilities.
- BBSV: Classical YM simulations in expanding background at $(c = 1, d = 0)$-point. Favours BMSS. Not yet evidence of plasma instabilities.

- 1. Melting of the coherent fields
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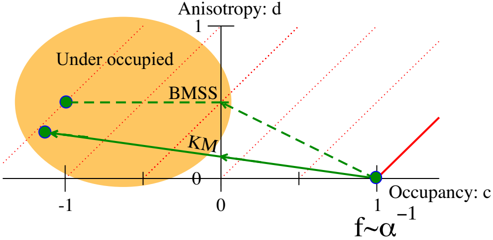
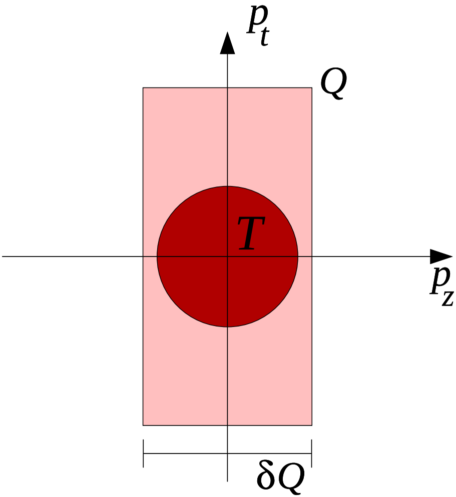
  - ...or how very dilute and anisotropic systems equilibrate



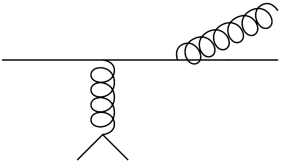
Soft radiation

Baier, Mueller, Schiff & Son '00

Physics of under occupied qualitatively different from over occupied.
Formation of soft thermal bath:



- Soft modes quick to emit
  $n_s \sim \alpha n_{\rm col}$



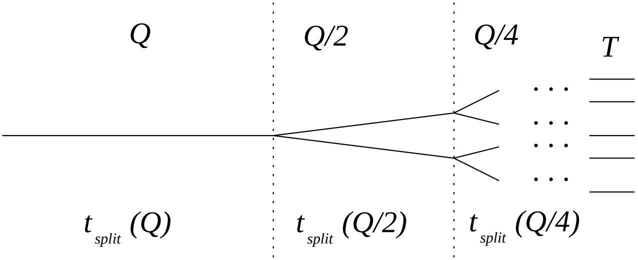
- Low p: easy to bend
  ⇒thermalize quickly
- *Can dominate dynamics!*
  (i.e. scattering, screening, ...)

⇒ Right way to think: Few energetic "jets" propagating in thermal bath

Hard splitting

Baier, Mueller, Schiff & Son '00

Q_s modes break before they bend!



- In vacuum: on-shell particles, no splitting
- In medium: Particles receive small kicks frequently
  - For stochastic uncorrelated kicks: Brownian motion in p-space

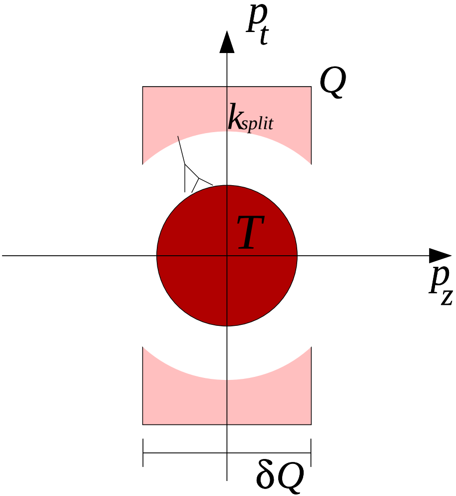
$$\Delta p_\perp^2 \sim \hat{q}\, t, \qquad t_{\rm split}(k) \sim l_{\rm stop}(k) \sim \alpha^{-1} \underbrace{\sqrt{k/\hat{q}}}_{t_{\rm form}} \qquad \text{(LPM)}$$

  - Physics of Landau-Pomeranchuk-Migdal suppression important

Bottom-Up

Baier, Mueller, Schiff, Son '00

Thermal bath eats the hard particles away:



- Scales below $k_{\rm split}$ have cascaded down to T-bath

  $$t_{\rm split}(k_{\rm split}) \sim t \Rightarrow k_{\rm split} \sim \alpha^2 \hat{q} t^2$$

- "Falling" particles heat up the thermal bath

  $$T^4 \sim k_{\rm split} \int d^3 p f(p)$$

- Thermalization when Q_s gets eaten

  $$k_{\rm split} \sim Q_s$$

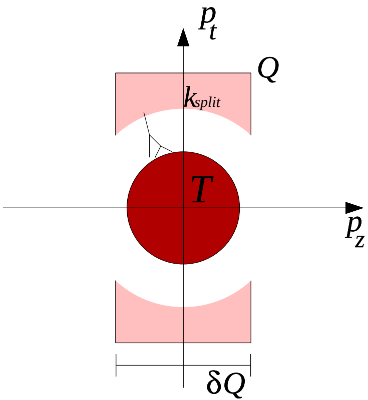
Thermalization time determined by how strongly hard particle motion is affected by the thermal bath, $\hat{q}$

Bottom up:

Baier, Mueller, Schiff, Son 2000

If the $\hat{q}$ is dominated by elastic scattering with the soft thermal bath

$$\begin{cases} \hat{q} & \sim & \alpha^2 T^3 \\ T & \sim & \alpha^3 Q_s (Q_s t) \\ {\color{blue}k_{\rm split}} & \sim & \alpha^{13} (Q_s t)^5 Q \\ {\color{red}\tau_0} & {\color{red}\sim} & {\color{red}\alpha^{-13/5} Q_s^{-1}} \end{cases}$$



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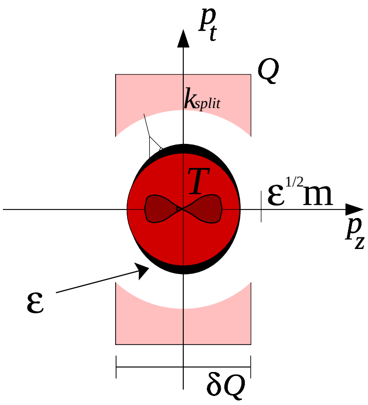
However, the thermal bath is slightly anisotropic and can give rise to plasma instabilities and increase $\hat{q}$ to faster thermalization Kurkela, Moore '11

$$\begin{cases} \hat{q} & \sim \quad \alpha^3 Q_s^3 \\ T & \sim \quad \alpha Q_s (Q_s t)^{1/4} \\ k_{\text{split}} & \sim \quad \alpha^5 (Qt)^2 Q_s \\ \tau_0 & \sim \quad \alpha^{-5/2} Q_s^{-1} \end{cases}$$



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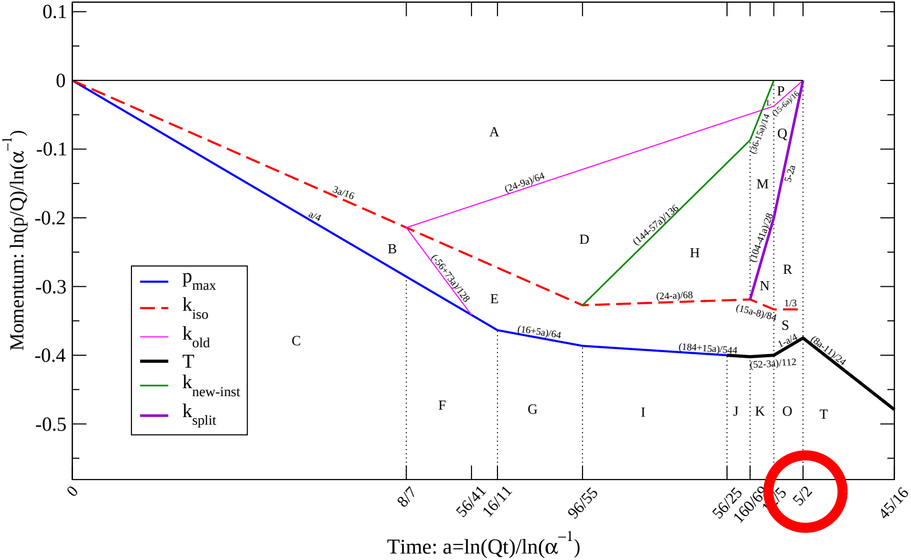
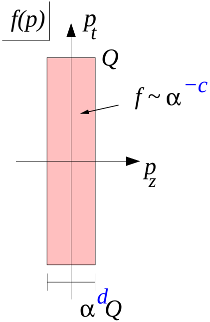
- Role of plasma instabilities under investigation
- Thermalization time $\tau \sim \#\alpha^{-5/2} Q_s^{-1}$.
  - Is this long or short? Depends on #. Numerics needed.

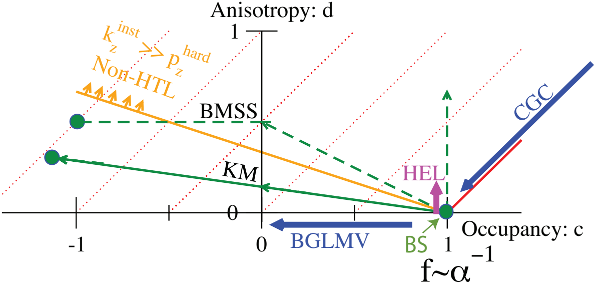
Instability driven bottom-up thermalization



Momentum scales vs. time scales

Comparison to other work





BGLMV: Blaizot, Gelis, Liao, McLerran, Venugopalan arXiv:1107.5296

BMSS: Baier, Mueller, Schiff, Son hep-ph/0009237

HEL: Rebhan, Stricland arXiv:1207.5795

CGC: Gelis, Iancu, Jalilian-Marian, Venugopalan arXiv:1002.0333

BS: Berges, Schlichting: arXiv:1209.0817