Thermalization at strong coupling

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Introduction

Why bother?

I am going to talk about recent progress in understanding far-from-equilibrium relaxation processes in strongly coupled cousins of QCD, solvable via holography.

These are large-N_c gauge theories strongly coupled at all scales in the 't Hooft sense with plasma phases resembling QGP not far above T_c as seen by soft observables.

They are clearly not QCD, most importantly they lack the asymptotic freedom.

Why bother? Because we can solve them <u>ab initio</u>, also in time-dependent settings!

Key developments near-equilibrium: $\left.\frac{\eta}{s}\right|_{\lambda>>1} = \mathcal{O}\left(\frac{1}{4\pi}\right)$ and new / anomalous transport.

~~Precision QCD physics~~, rather ballpark estimates or indications of new phenomena.

Holography

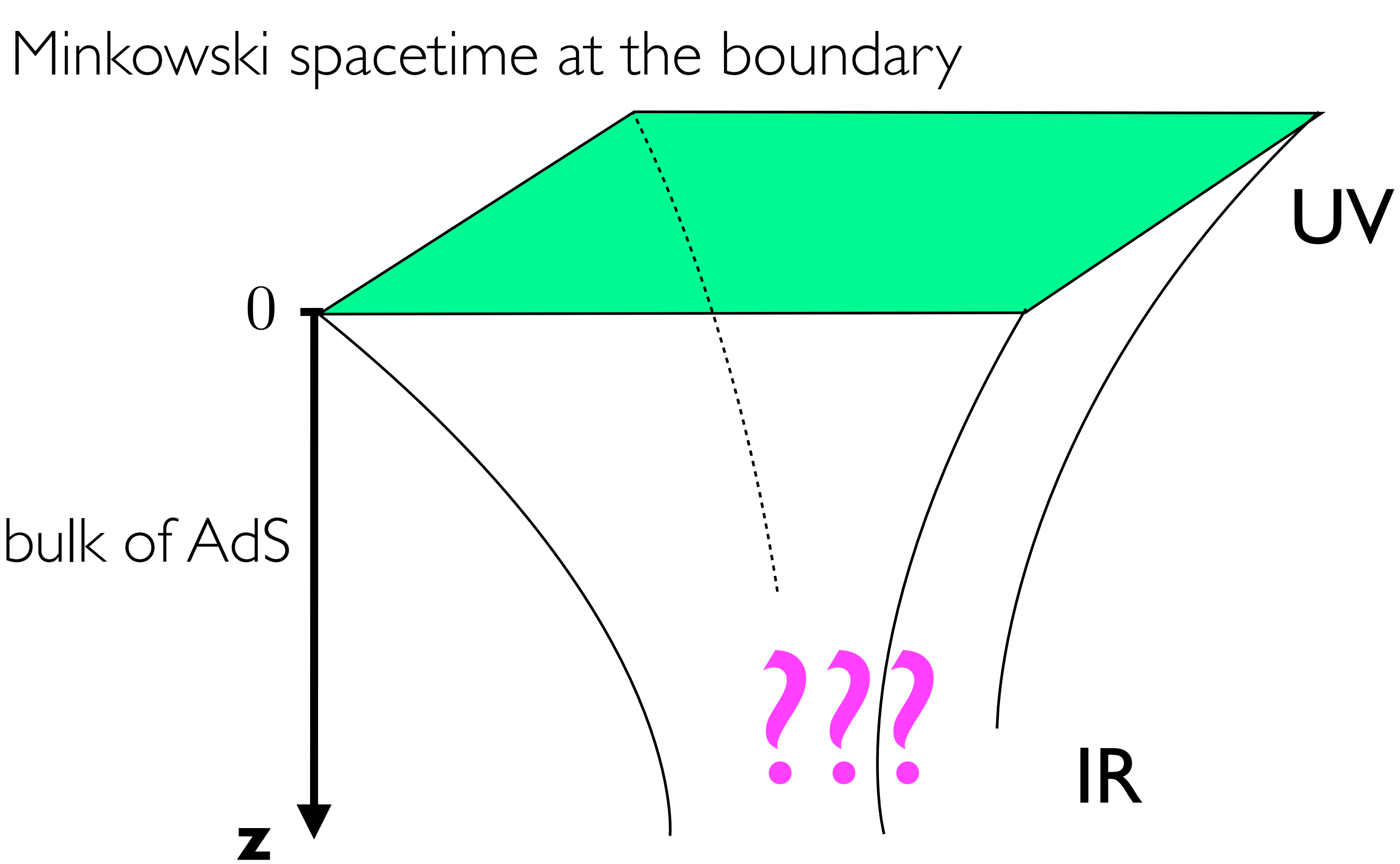
From applicational perspective holography is a tool for computing correlators in certain strongly coupled gauge theories, such as $\mathcal{N}=4$ SYM at large N_c and λ .

For simplicity I will consider AdS$_{1+4}$ / <u>CFT</u>$_{1+3}$ and focus on pure gravity sector*

$$R_{ab} - \frac{1}{2} R g_{ab} - \frac{6}{L^2} g_{ab} = 0$$

Different solutions correspond to states in a dual CFT with different $\langle T_{\mu\nu} \rangle$.

$$\frac{L^3}{G_N} \propto N_c^2 \gg 1 \text{ and } \frac{L^4}{l_s^4} \propto \underset{(=\lambda)}{g_{YM}^2 N_c} \gg 1$$



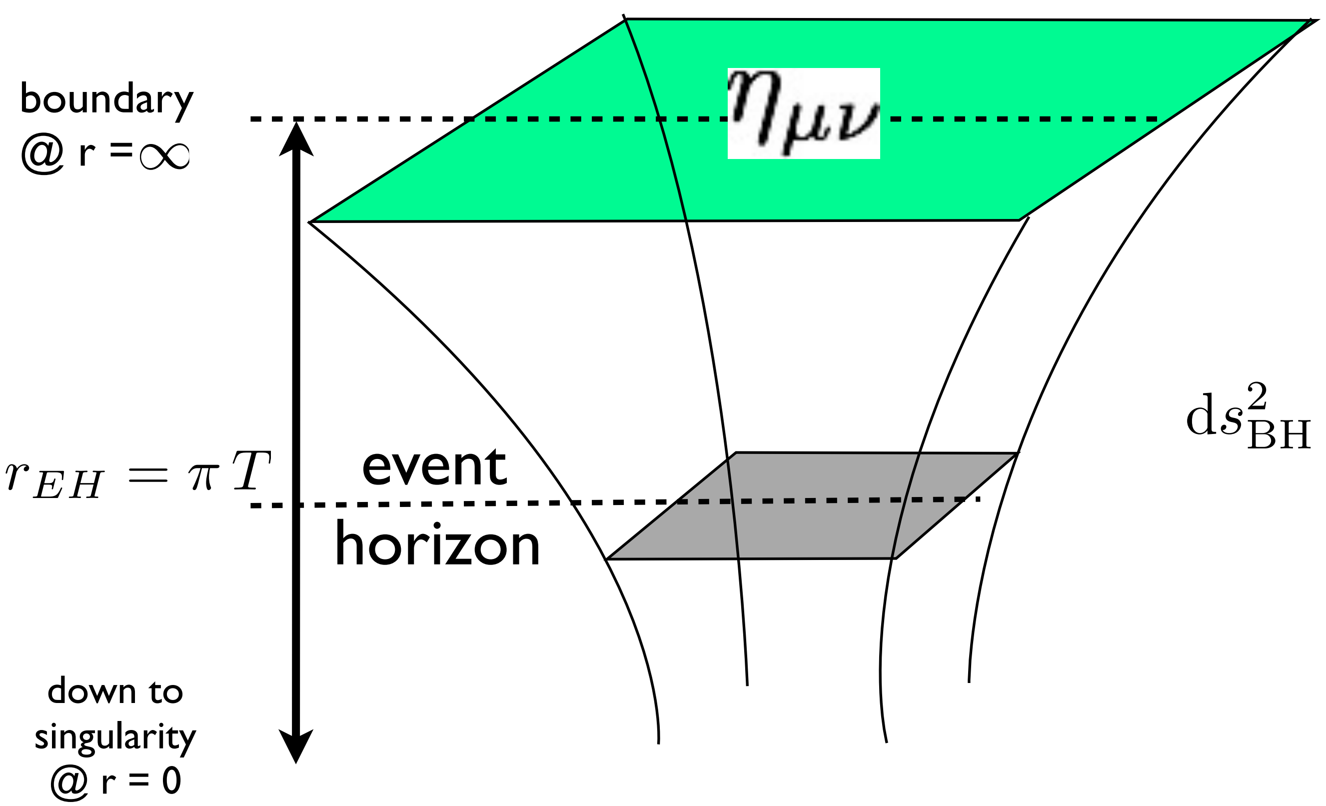
$$ds^2 = \frac{L^2}{z^2} \Big\{ dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu + \frac{2\pi^2}{N_c^2} \langle T_{\mu\nu} \rangle z^4 + \ldots \Big\}$$

Global equilibrium

=

Bulk black hole

AdS analogue of the Schwarzschild black hole is described by the metric



$$\mathrm{d}s^2_{\mathrm{BH}} = 2\mathrm{d}t\mathrm{d}r - r^2\left(1 - \frac{\pi^4 T^4}{r^4}\right)\mathrm{d}t^2 + r^2 \mathrm{d}\vec{x}^2$$

The plasma/black hole thermodynamics is given by

$$T_{\mu\nu} = \frac{1}{8}\pi^2 N_c^2 T^4 \,\mathrm{diag}\,(3,1,1,1)_{\mu\nu}\,,\; s\Big|_{\lambda\to\infty} = \mathrm{Area}_{\mathrm{EH}}/4G_N = \frac{1}{2}N_c^2\pi^2 V T^3 = \frac{3}{4}s\Big|_{\lambda\to 0}$$

Hydrodynamics and beyond

Modern hydrodynamics

hydrodynamics is an EFT of the slow evolution of conserved currents in collective media „close to equilibrium"

As any EFT it is based on the idea of the gradient expansion

DOFs: always local energy density ϵ and local flow velocity u^μ $(u_\nu u^\nu = -1)$

EOMs: conservation eqns $\nabla_\mu T^{\mu\nu} = 0$ for $T^{\mu\nu}$ *systematically* expanded in gradients

terms carrying 2 and more gradients

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu\,\} - \eta(\epsilon)\,\sigma^{\mu\nu} - \zeta(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu\,\}(\nabla\cdot u) + \ldots$$

perfect fluid stress tensor

microscopic input:

EoS $\epsilon = 3P\Big|_{\mathrm{CFT}}$

(famous) **shear viscosity**

bulk viscosity (vanishes for CFTs)

Excitations of strongly coupled plasmas

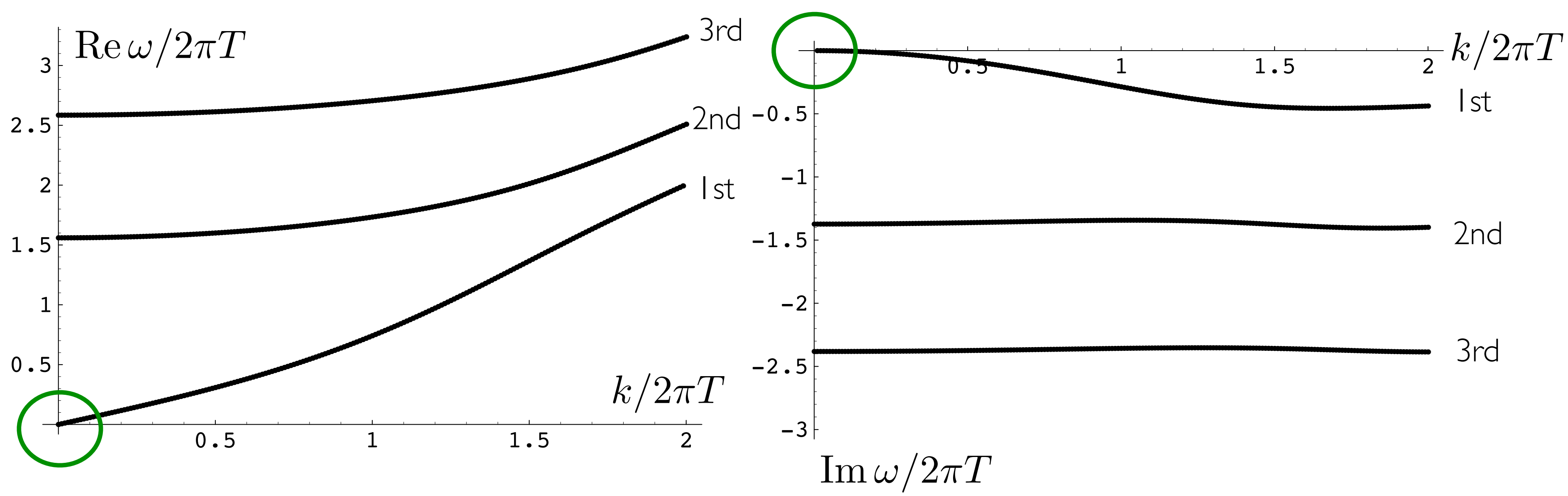
Kovtun & Starinets [hep-th/0506184]

Consider small amplitude perturbations ($\delta T_{\mu\nu}/{N_c}^2 \ll T^4$) on top of a holographic plasma

$$T_{\mu\nu} = \frac{1}{8}\pi^2 N_c^2 T^4 \, \mathrm{diag}\,(3,1,1,1)_{\mu\nu} + \delta T_{\mu\nu} \quad (\sim e^{-i\,\omega(k)\,t + i\,\vec{k}\cdot\vec{x}})$$

Dissipation leads to modes with complex $\omega(k)$, which in the sound channel look like

$$\left.\frac{\partial \omega}{\partial k}\right|_{k\to 0} = c_{sound}$$



There are two different kinds of modes:

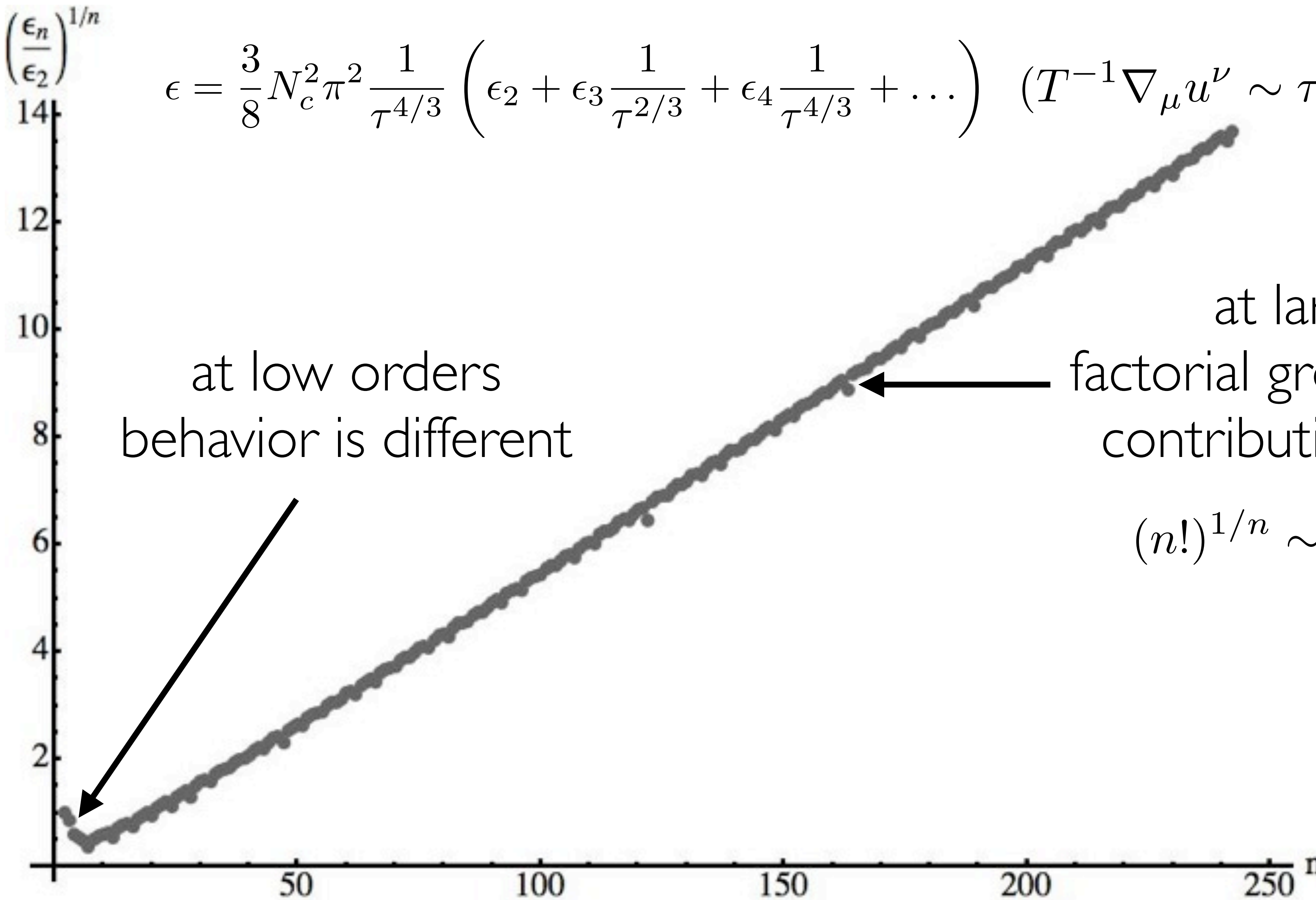
$\omega(k) \to 0$ **as** $k \to 0$: slowly evolving and dissipating modes (hydrodynamic sound waves)

all the rest: far-from-equilibrium (QNM) modes dampened over $t_{therm} = \mathcal{O}(1)/T$

The nature of hydrodynamics

1302.0697 [hep-th] PRL 110 (2013) 211602: MPH, R. A. Janik & P. Witaszczyk

$$\epsilon = \frac{3}{8} N_c^2 \pi^2 \frac{1}{\tau^{4/3}} \left(\epsilon_2 + \epsilon_3 \frac{1}{\tau^{2/3}} + \epsilon_4 \frac{1}{\tau^{4/3}} + \ldots \right) \quad (T^{-1} \nabla_\mu u^\nu \sim \tau^{-2/3})$$



at low orders behavior is different

at large orders factorial growth of gradient contributions with order

$$(n!)^{1/n} \sim \frac{(2\pi n)^{1/2n}}{e} \cdot n$$

First evidence that hydrodynamic expansion has zero radius of convergence!

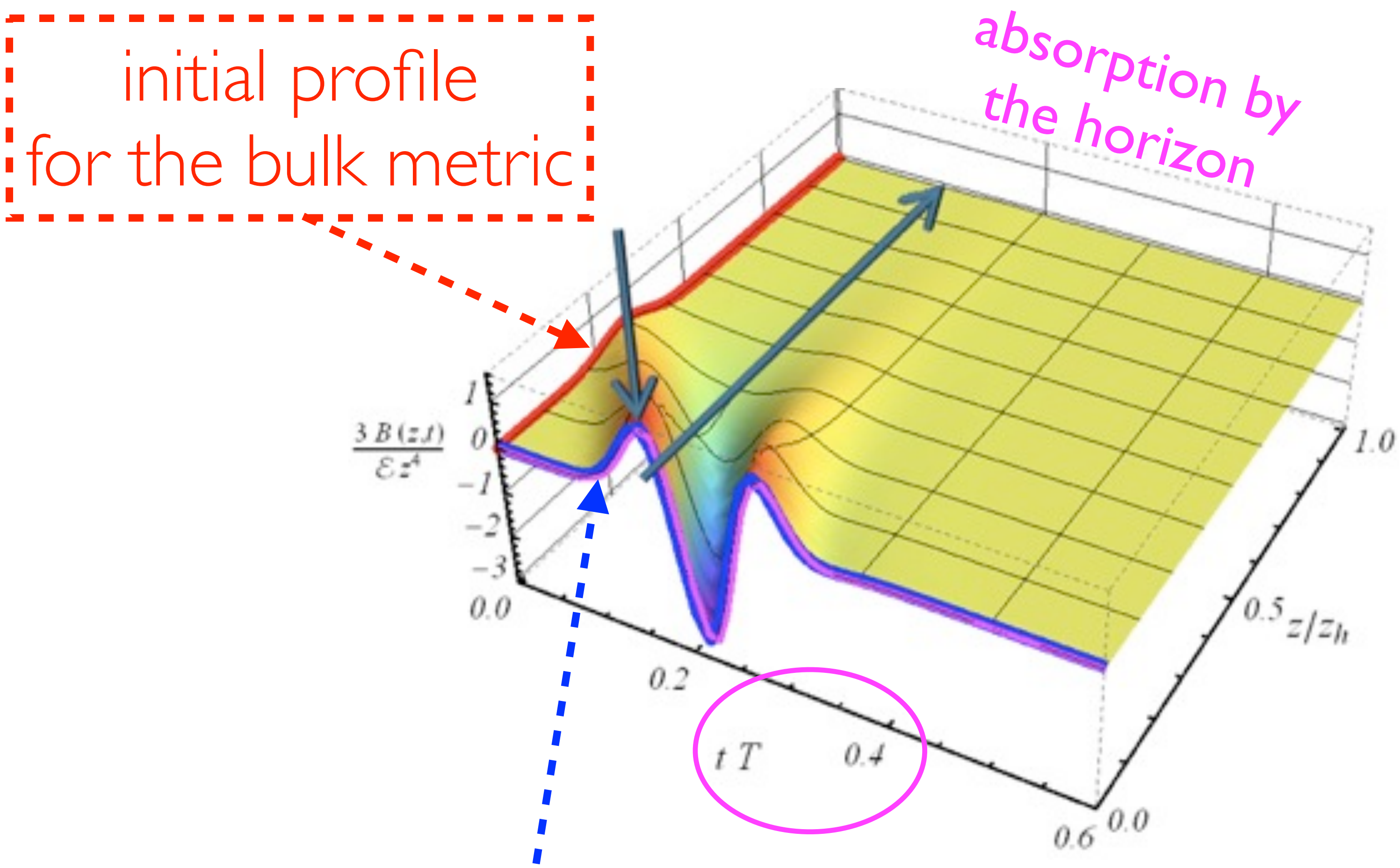
Holographic thermalization

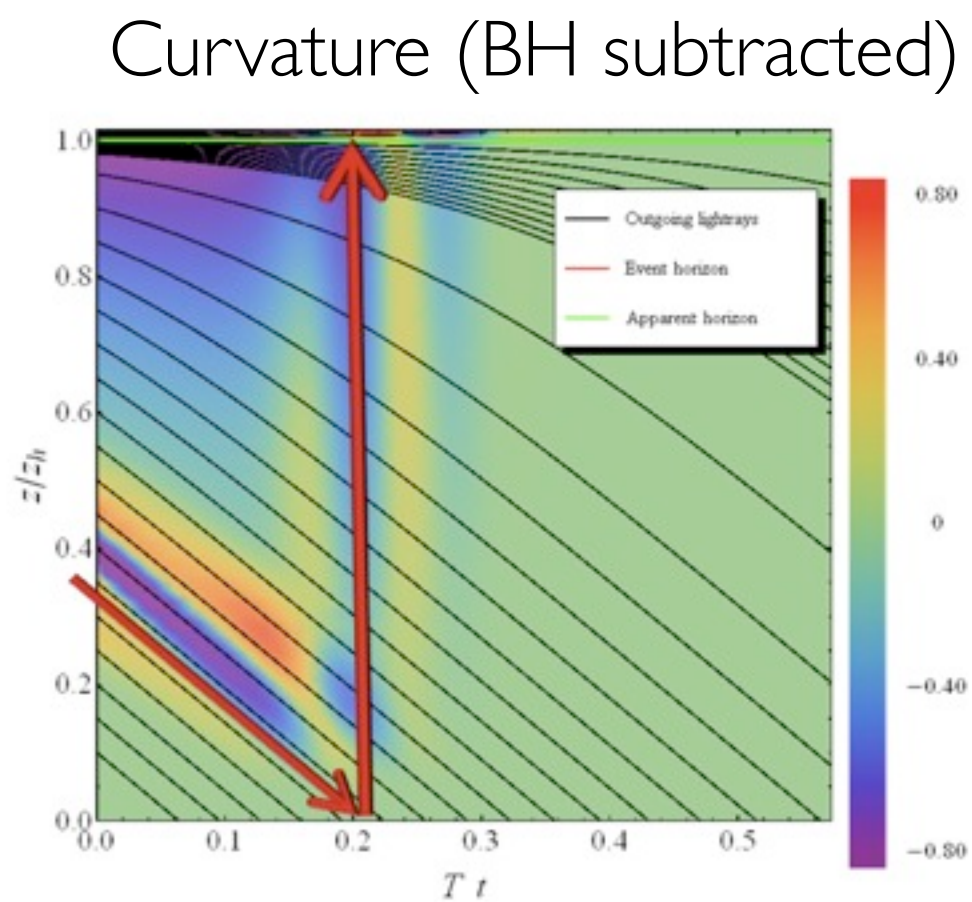
What is it?

0812.2053 [hep-th] P. Chesler & L. Yaffe

1202.0981 [hep-th] and **1304.5172 [hep-th]**
MPH, D. Mateos, W. van der Schee, D. Trancanelli / M. Triana

1309.1439 [hep-th] P. Chesler & L. Yaffe



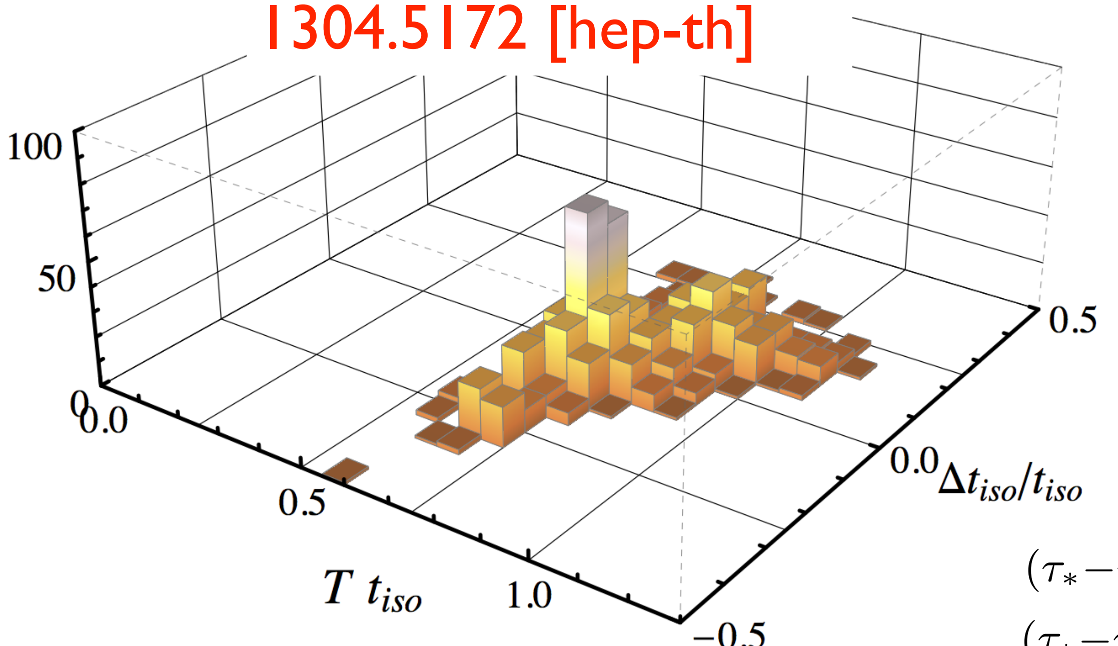


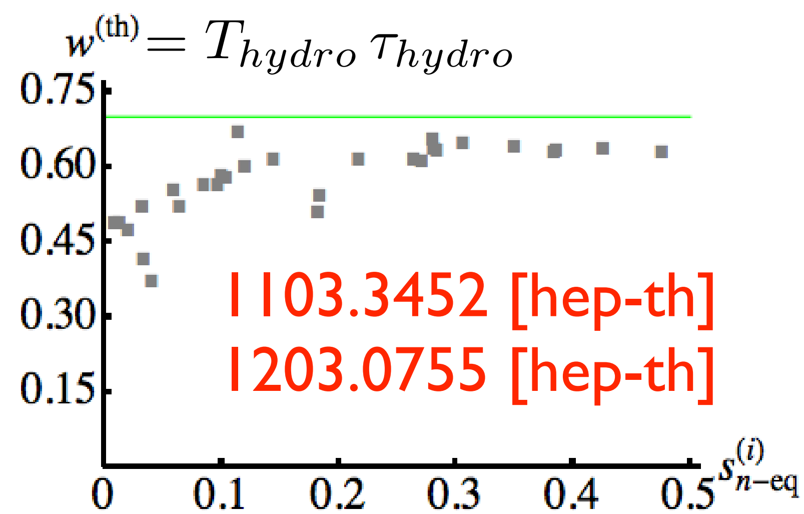
$$\langle T_{\mu\nu}\rangle = \mathrm{diag}\left\{\epsilon,\ \frac{1}{3}\epsilon - \frac{2}{3}\boxed{\Delta P(t)},\ \frac{1}{3}\epsilon + \frac{1}{3}\Delta P(t),\ \frac{1}{3}\epsilon + \frac{1}{3}\Delta P(t)\right\}$$

Fast „thermalization”

Omnipresent theme: <stress tensor> at strong coupling „thermalizes” over 1/T

1202.0981 [hep-th]
1304.5172 [hep-th]





1103.3452 [hep-th]
1203.0755 [hep-th]

0812.2053 [hep-th]
0906.4426 [hep-th]

$\tau_{\mathrm{iso}}\, T$	0.67	0.68	0.71	0.92	1.2	1.5	1.8			
$(\tau_* - \tau_i)\, T_*$	2.0	1.7	1.4	1.1	0.84	0.85	1.1	1.5	1.8	2.1
$(\tau_* - \tau_f)\, T_*$	0.00	0.05	0.11	0.19	0.24	0.24	0.20	0.11	0.04	0.00

Caveat: this has been checked only for conformally invariant QFTs.

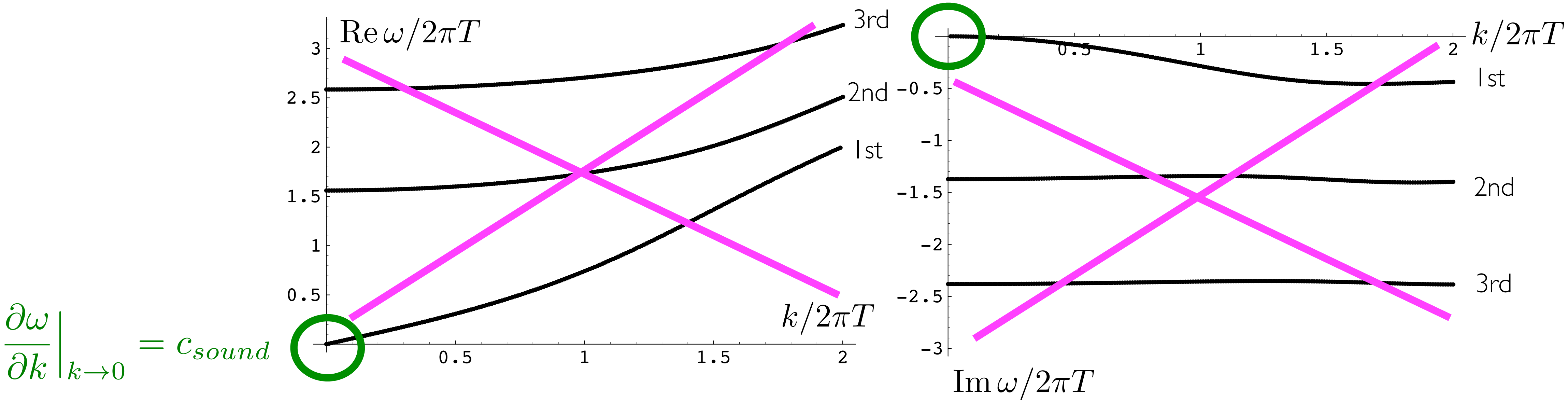
Transition to hydrodynamics

Hydrodynamization

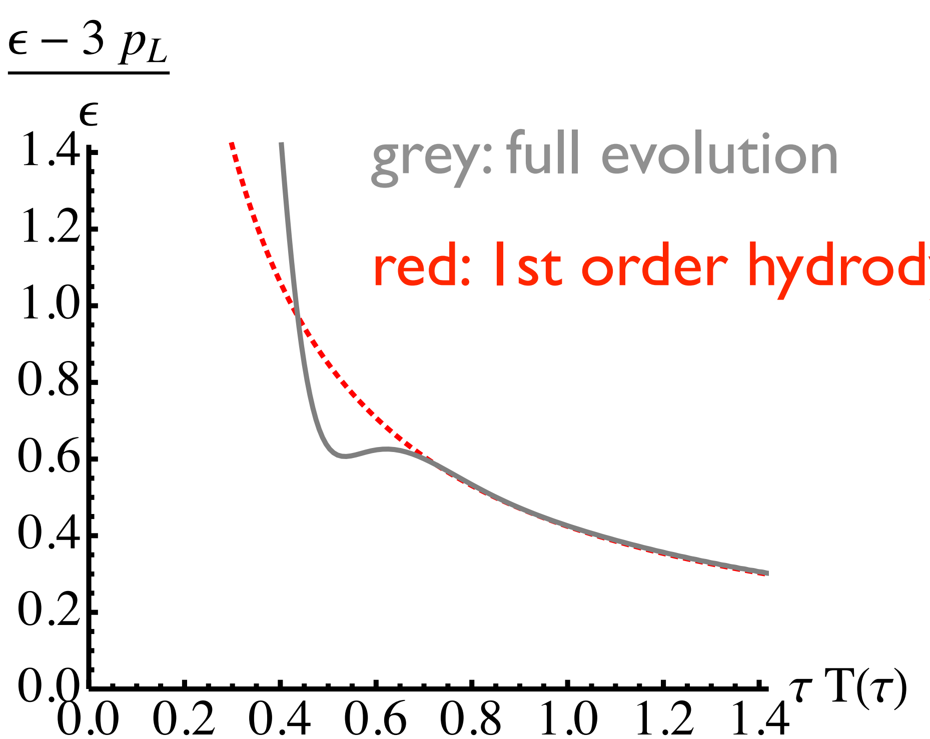
1103.3452 [hep-th] PRL 108 (2012) 201602:

MPH, R. A. Janik & P. Witaszczyk

For hydrodynamics to work all the other DOFs need to relax.



Surprising consequence, here demonstrated for the boost-invariant flow



Large anisotropy at the onset of hydrodynamics

$$\epsilon - 3\,p_L \approx 0.6\,\epsilon \text{ to } 1.0\,\epsilon$$

Thus

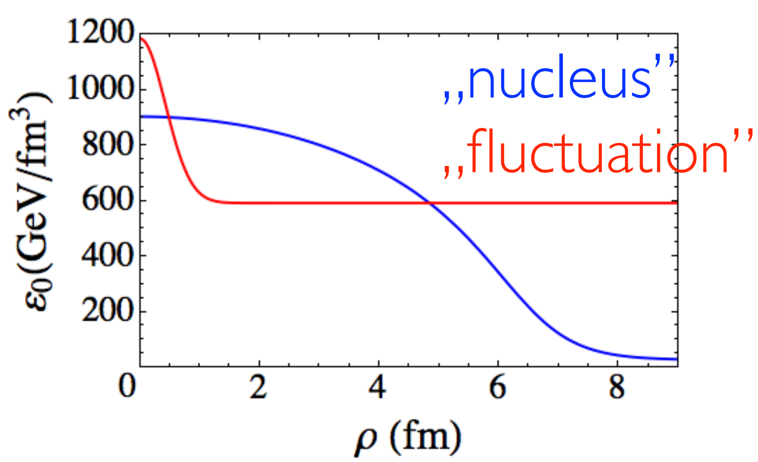
hydrodynamization $\neq$ isotropization, thermalization

see also Chesler & Yaffe **1011.3562 [hep-th]**
Chesler & Yaffe **0906.4426 [hep-th]**

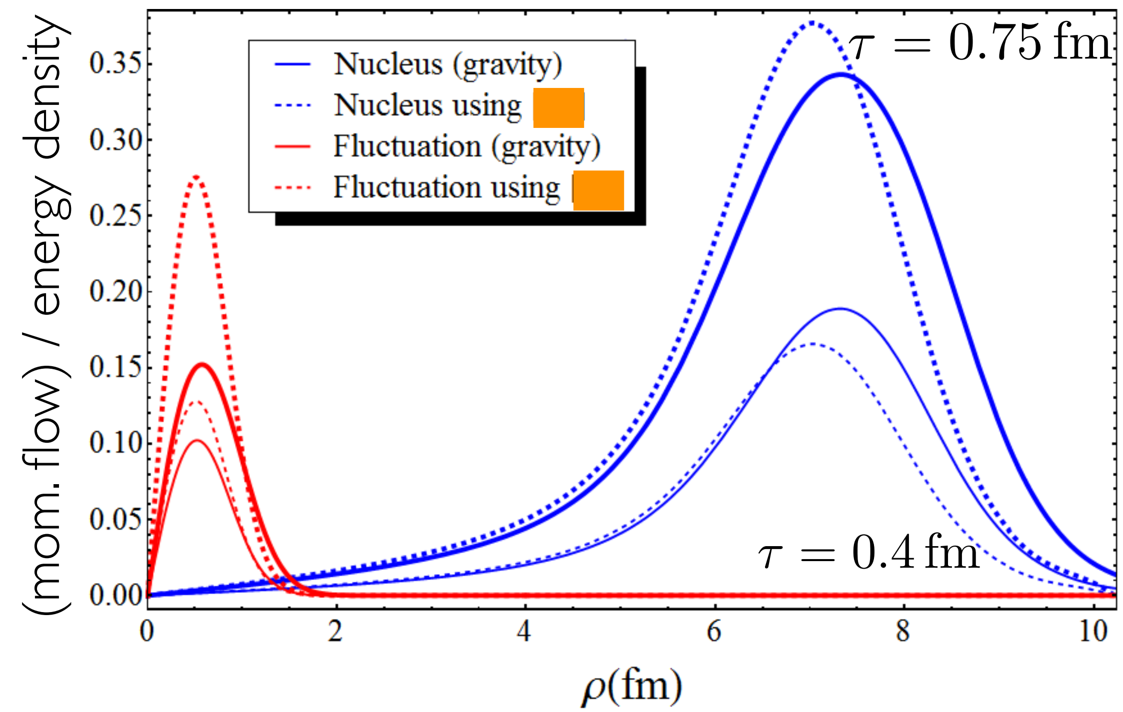
Holography and the pre-flow

1211.2218 [hep-th] W. van der Schee

Model: the boost-invariant flow with no initial transversal velocity and ϵ_0 at $\tau_{in} = 0.12\,\text{fm}$.



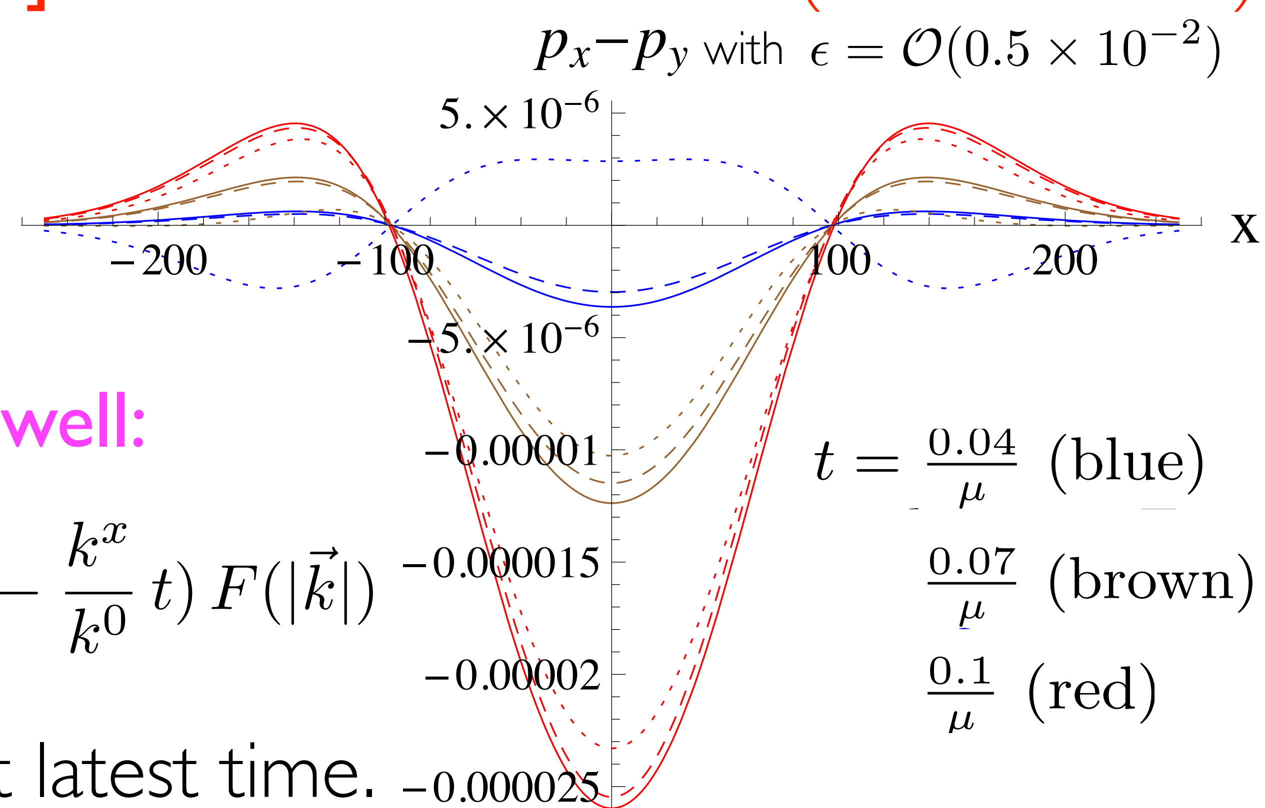
$$\boxed{\vec{s}/\varepsilon \approx -\frac{\vec{\nabla}_\perp \varepsilon_0}{2\varepsilon_0}(\tau - \tau_{in})}$$



Vredevoogd & Pratt's formula seems to work for not too large gradients.

1305.4919 [hep-th] V. Balasubramanian et al. (incl. B. Mueller)

Setup: vacuum of CFT$_{2+1}$ excited via scalar source localized in time and peaked in one spatial dimension.



Free streaming (dashed) works remarkably well:

$$T^{\alpha\beta}(t,\vec{x}) = \int d^2k \frac{k^\alpha k^\beta}{k^0} f(t,\vec{x},\vec{k}) \text{ with } f(t,\vec{x},\vec{k}) = n(x - \frac{k^x}{k^0}\,t)\, F(|\vec{k}|)$$

Hydrodynamics (dotted) at best applicable at latest time.

Holographic “heavy ion collisions”

see **W. van der Schee** talk at **parallel session 1B** // Thu **15:10-15:30** //

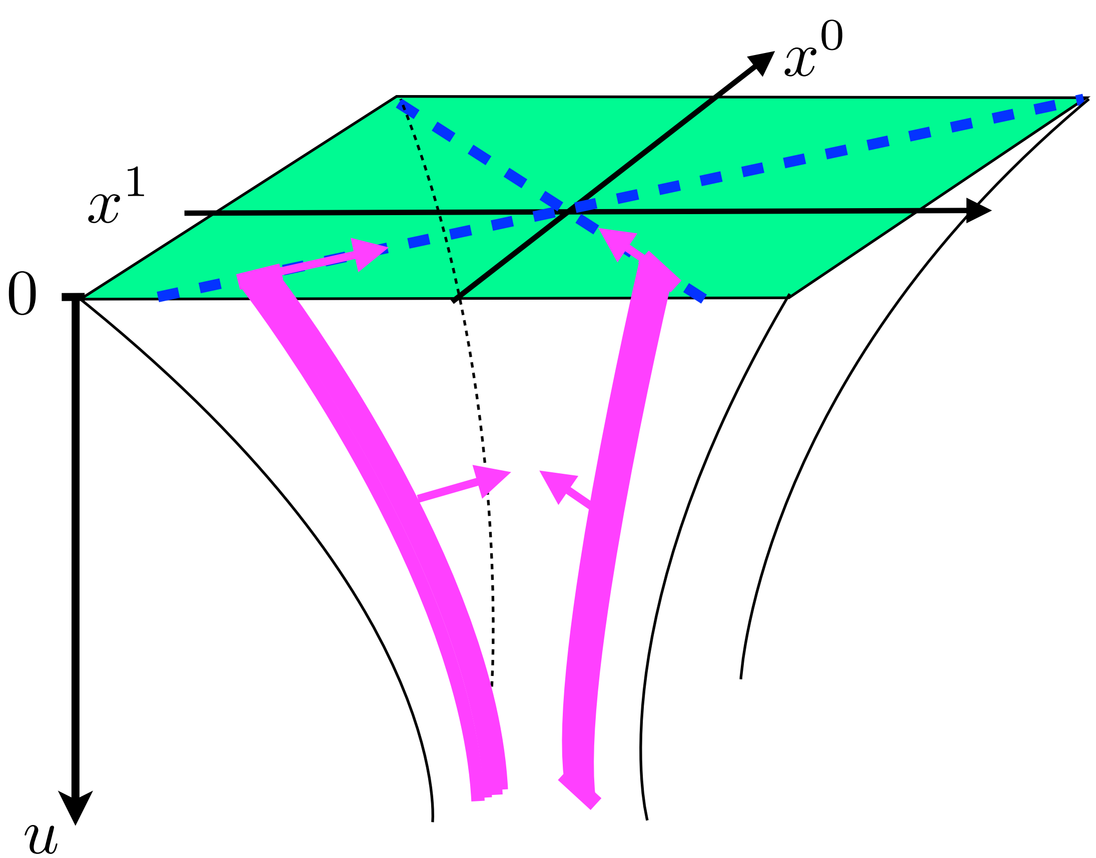
Towards a holographic „heavy ion collision”

[hep-th/0512162] Janik & Peschanski   1011.3562 [hep-th] P. Chesler & L. Yaffe

general issue: which holographic initial conditions are closest to the experiment?

practical viewpoint: collide two lumps of matter moving at relativistic speeds!

state of the art as of September 2013: colliding gravitational shock wave solutions



shock wave is dual to a lump of matter

- moving at the speed of light;
- having infinite transversal extent;
- having arbitrary longitudinal profile $h(t \pm z)$;

characterized by the following stress tensor

$$T^{tt} = T^{zz} = \mp T^{tz} = \frac{N_c^2}{2\pi^2} h(t \pm z)$$

We will specialize to $h(t \pm z) = \rho^4 \exp\left[-(t \pm z)^2/2\sigma^2\right]$. But, in a CFT, what matters is:

$e = \rho\,\sigma$ (in real HIC $e \sim \gamma^{-1/2}$ and $e_{CY} \approx 0.64$ corresponds to Pb at RHIC)

Dynamical crossover

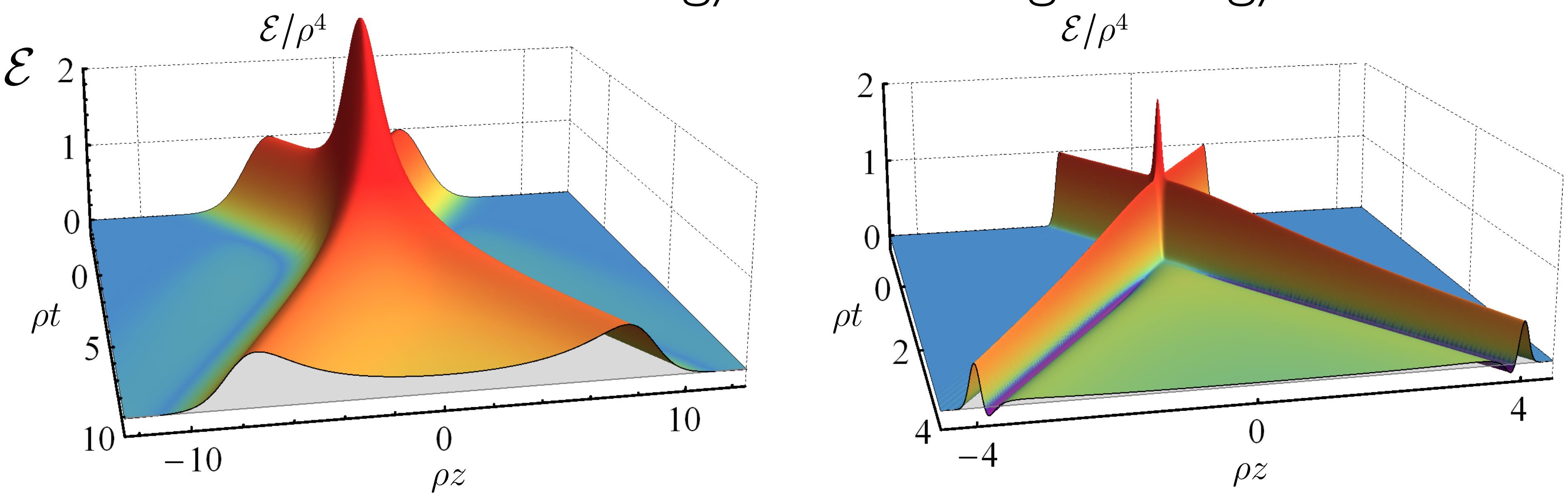
1305.4919 [hep-th]

Casalderrey-Solana, MPH, Mateos, van der Schee

$e_{left} = 2\,e_{CY}$ „low energy”

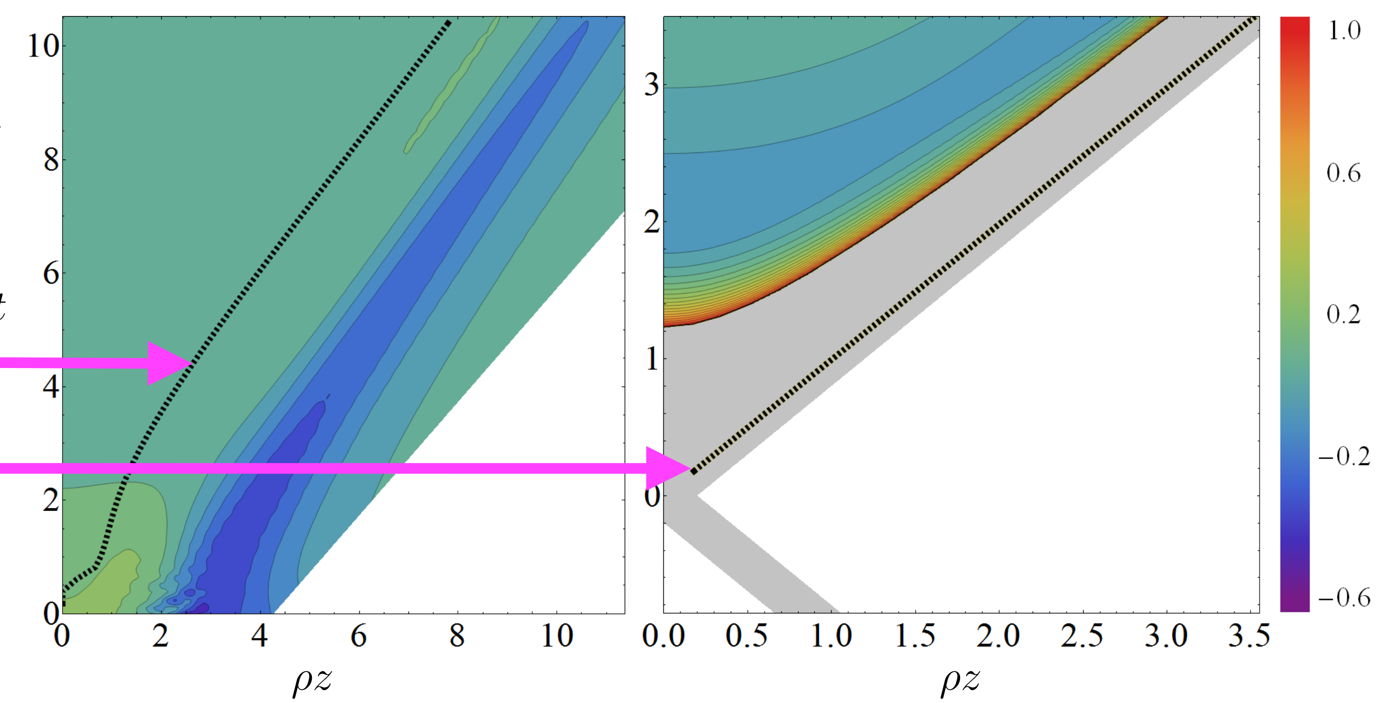
$e_{right} = 0.125\,e_{CY}$ „high energy”

$$T^{tt} = \frac{N_c^2}{2\pi^2}\,\mathcal{E}$$



$3\Delta\mathcal{P}_L^{\rm loc}/\mathcal{E}_{\rm loc}$

maximum of the energy flux



deviation from viscous hydro

shocks coalesce and explode hydro-dynamically (similar to the Landau picture)

hydro applicable only at mid-rapidities and late enough!!!

Dispels the myth that strong coupling necessarily leads to the full stopping*

Out-of-equilibrium energy loss

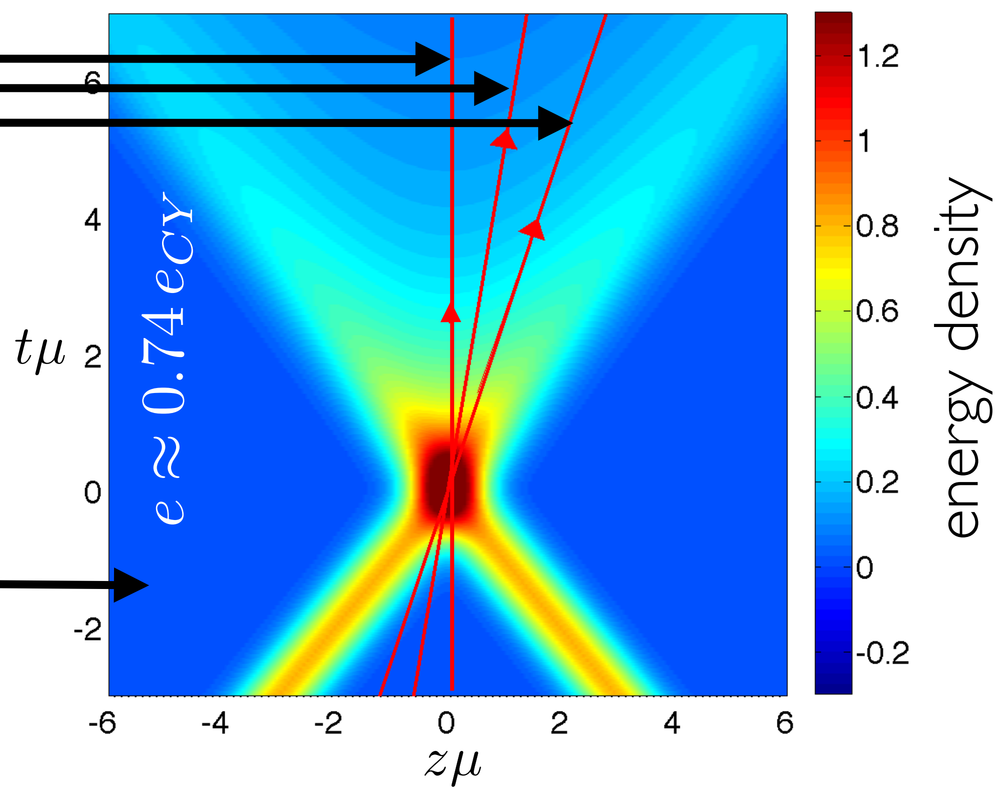
Heavy quark energy loss in non-equilibrium medium

Chesler, Lekaveckas & Rajagopal **1306.0564 [hep-th]**

Heavy quark is dragged through non-equilibrium matter with constant velocity.

Setup:

(projections of) heavy quark trajectory



low temperature heat bath

Question:

$$\left.\frac{d\vec{p}}{dt}\right|_{\mathrm{eq}} = \frac{\sqrt{\lambda}}{2\pi}(\pi T)^2 \frac{\vec{\beta}}{\sqrt{1-\beta^2}}$$

vs.

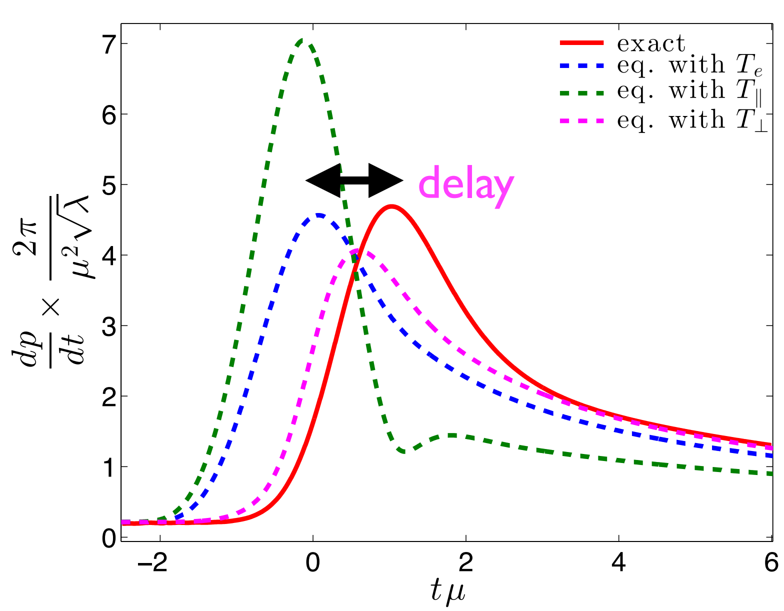
$$T^{\mu\nu}_{\text{rest frame}} = \frac{\pi^2 N_c^2}{8}\,\mathrm{diag}(3T_e^4, T_\perp^4, T_\perp^4, T_\parallel^4)$$

β_z and β_x

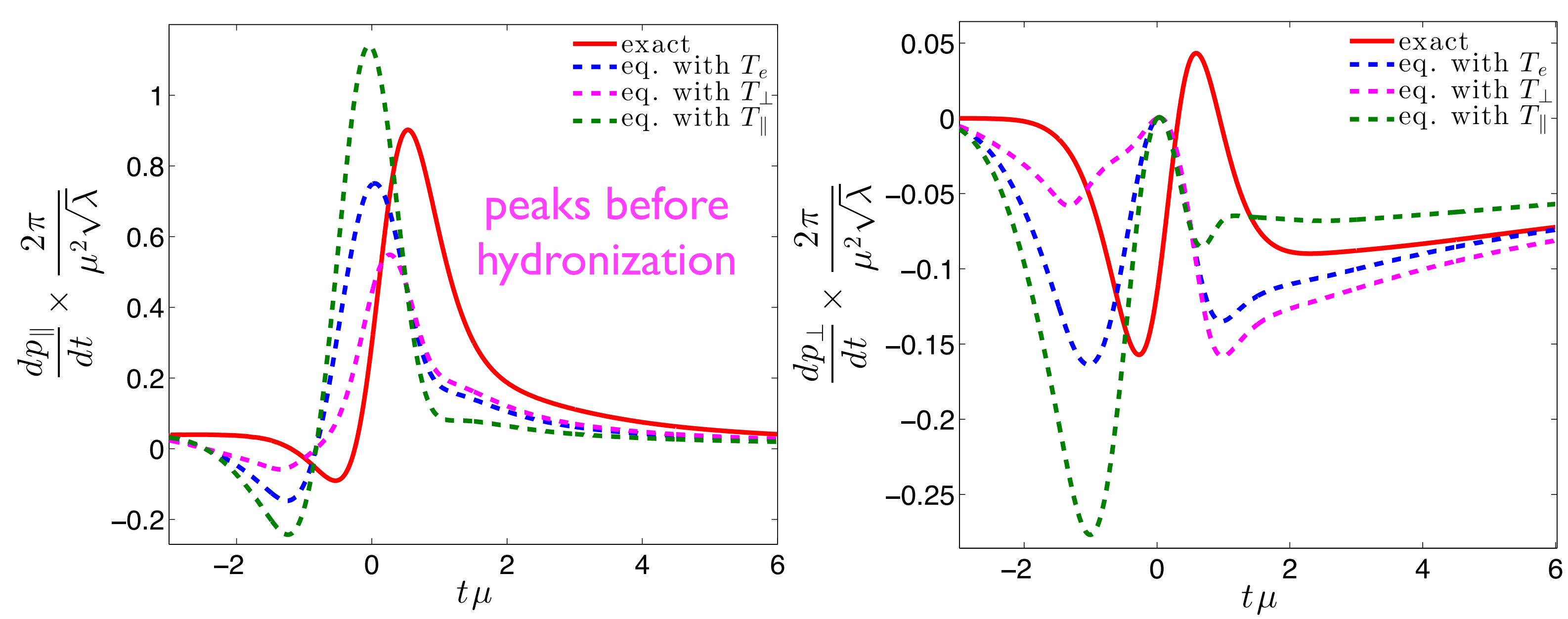
?

Results:

$\beta_z = 0$ and $\beta_x = 0.95$



$\beta_z = 0.4$ and $\beta_x = 0.2$



Outlook

Thermalization at strong coupling - summary

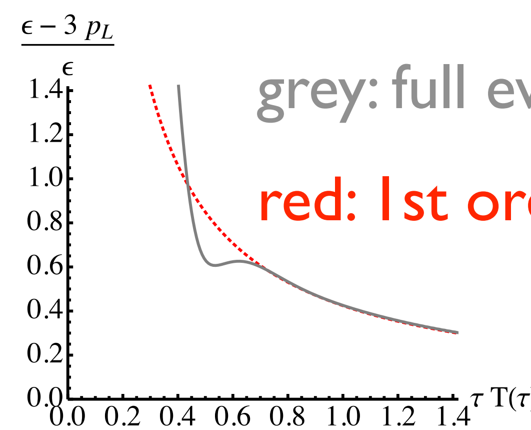
It occurs over 1/T*; applicability of hydro in HIC is then not <u>theoretically</u> outrageous.

$$\tau_{hydro} \times T_{hydro}\Big|_{\lambda=\infty} = \mathcal{O}(1)$$ (RHIC c=0-5%: $0.25\,\mathrm{fm} \times 500\,\mathrm{MeV} = 0.63$)

0801.4361 [nucl-th] W. Broniowski et al.

<u>A new way of thinking about the transition to hydrodynamics:</u>

hydrodynamization $\neq$ local thermalization



grey: full evolution

red: 1st order hydrodynamics

hydrodynamics is an asymptotic series