Initial-State Radiation in Proton-Nucleus Collisions

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Based on work done with Kang, Sterman, …

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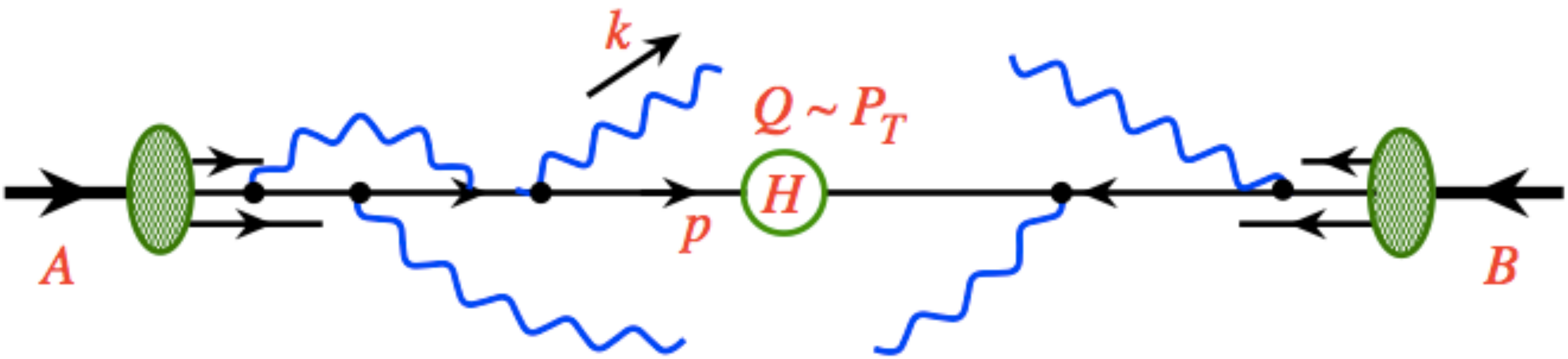
Initial-state radiation

❑ **Hadronic collisions:**



❑ **Gluon radiation:**

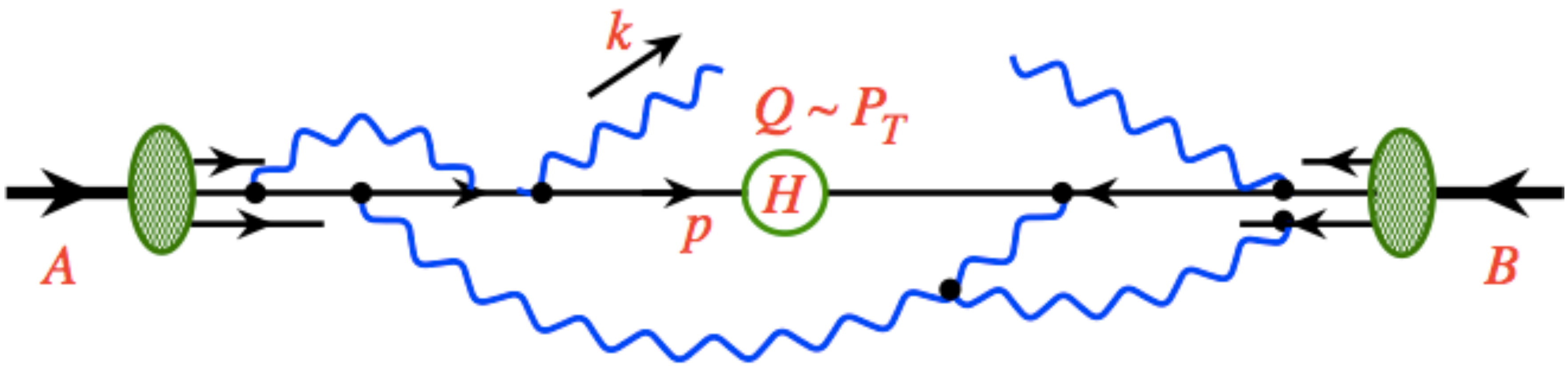
✧ Gluon radiation/shower is a consequence of the collision

✧ Radiation pattern depends on each event

✧ Treatment of radiation (approx.) depends on observables

Initial-state radiation

- ❑ **Hadronic collisions:**



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  - ✧ Radiation pattern depends on each event
  - ✧ Treatment of radiation (approx.) depends on observables

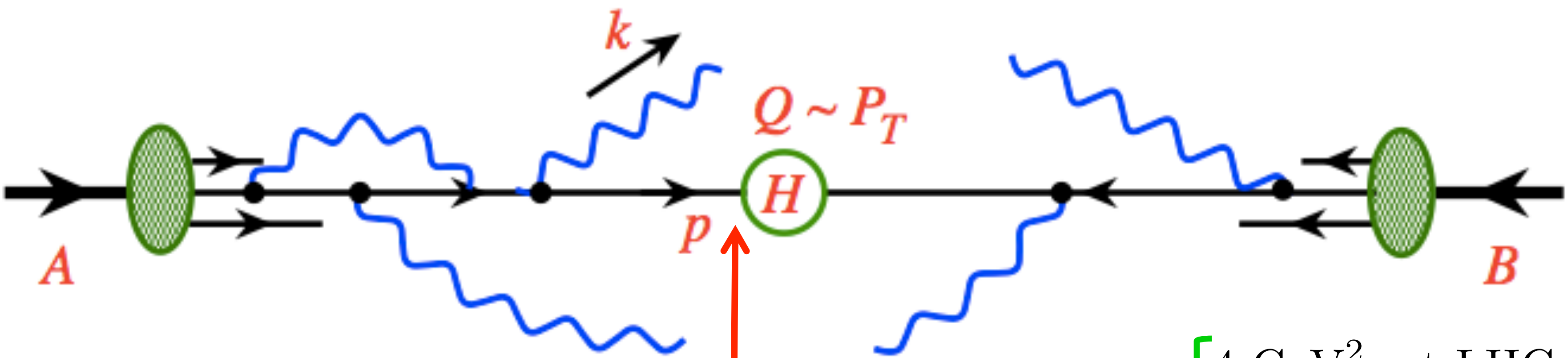
- ❑ **Long-range soft-gluon interaction:**

  "Talk" between hadrons before the "hard" collision

  Leading power – by unitarity, higher power – suppressed

Events with a single hard scale

❑ Approximations:

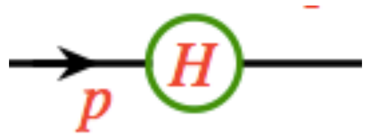


✧ **Active parton virtuality (from shower):**

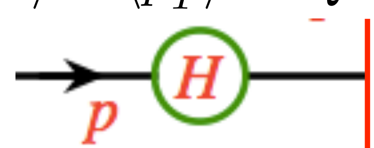
$$\langle p^2\rangle \sim \begin{cases} 4\ \text{GeV}^2 & \text{at LHC} \\ 1\ \text{GeV}^2 & \text{at RHIC} \end{cases}$$

$$\langle p^2\rangle \sim \langle p_T^2\rangle \sim (1/\text{fm})^2 \log(S/Q^2)\log(Q^2/(1/\text{fm})^2) \propto \log(1/x_p)$$

✧ **On-shell approximation:** $\langle p^2\rangle \sim \langle p_T^2\rangle \ll Q^2$



$\approx$



$\Big|_{p^2=0} + \mathcal{O}(1/Q)$

Events with a single hard scale

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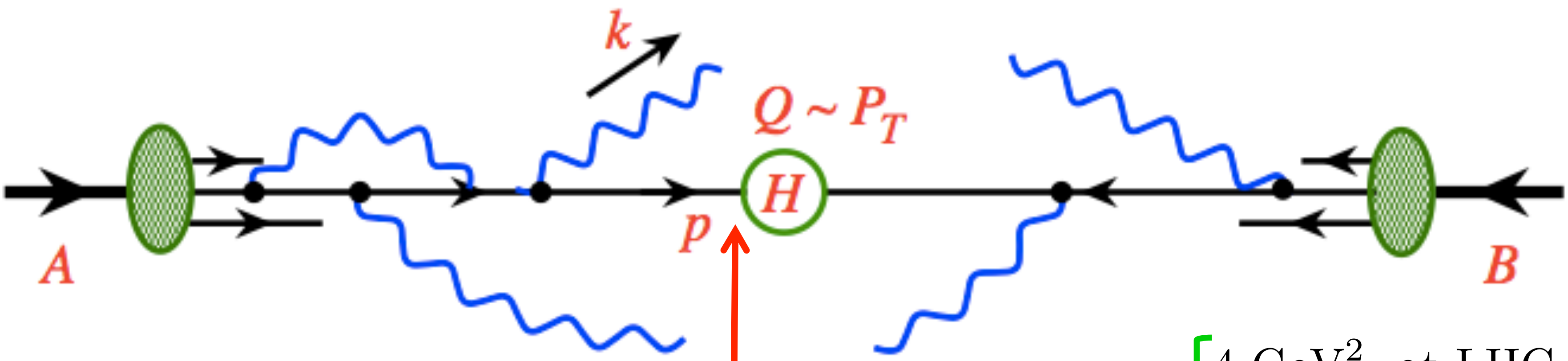

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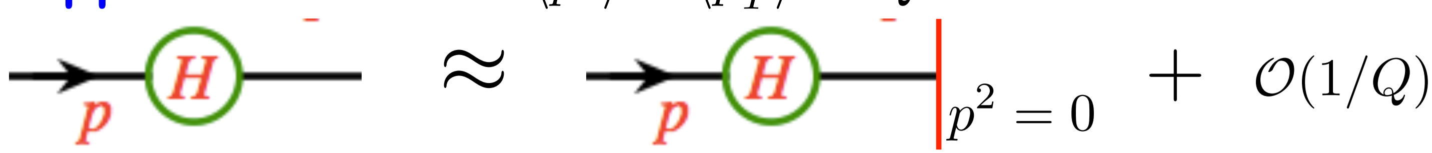


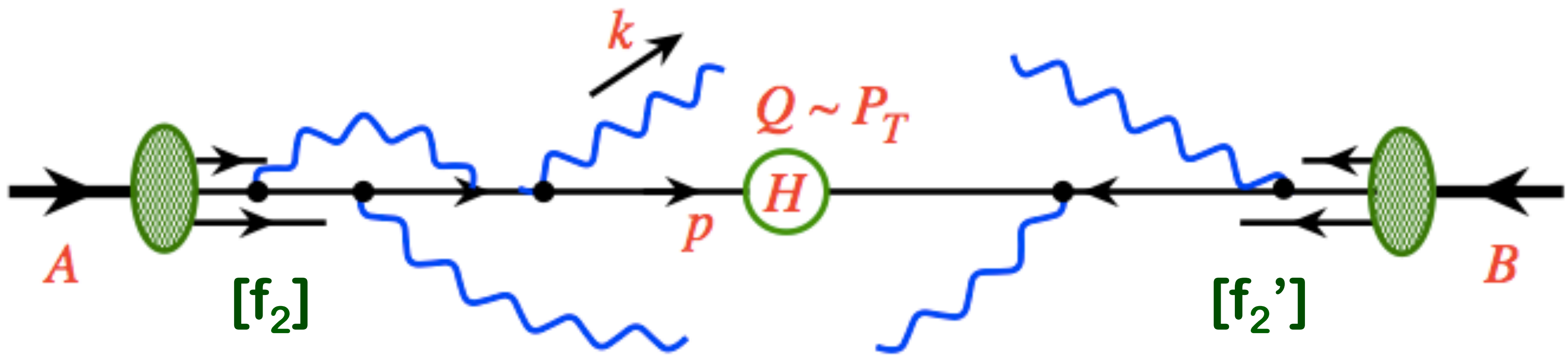
❑ Initial-state radiation:

✧ **Perturbative singularity:** $\log(Q^2/k_T^2)|_{k_T^2 \to 0} \implies$ **PDFs**

✧ **Rest of k–phase-space:** $\implies \alpha_s^n$ **corrections**

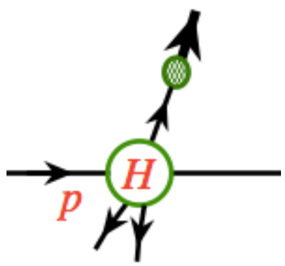
Events with a single hard scale

❑ **Factorization (Approx.):**



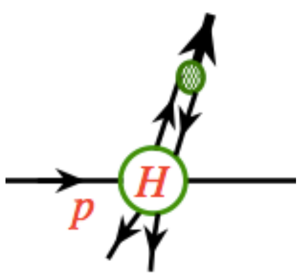
✧ **Leading power:**

$$d\sigma_{AB}(Q) \approx \hat{\sigma}(Q) \otimes f_2 \otimes f_2' [\otimes D_2]$$



✧ **1st power corrections:**

$$+ \frac{\langle p^2 \rangle}{Q^2} \hat{\sigma}_4 \otimes f_2 \otimes f_2' [\otimes D_4] + \ldots$$



Events with a single hard scale

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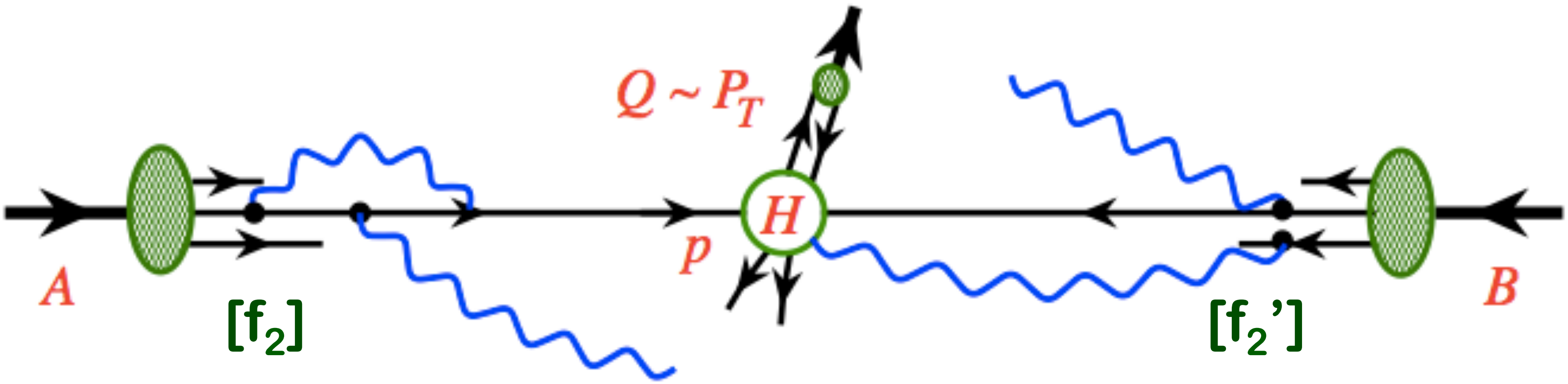

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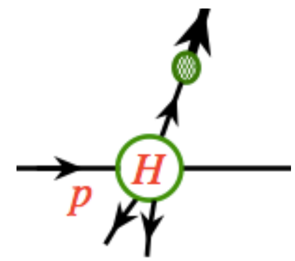

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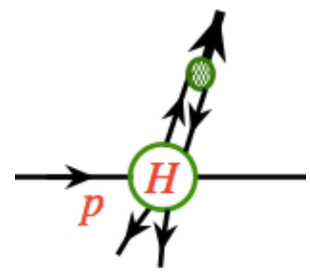


❑ **No-factorization beyond $1/Q^2$!**

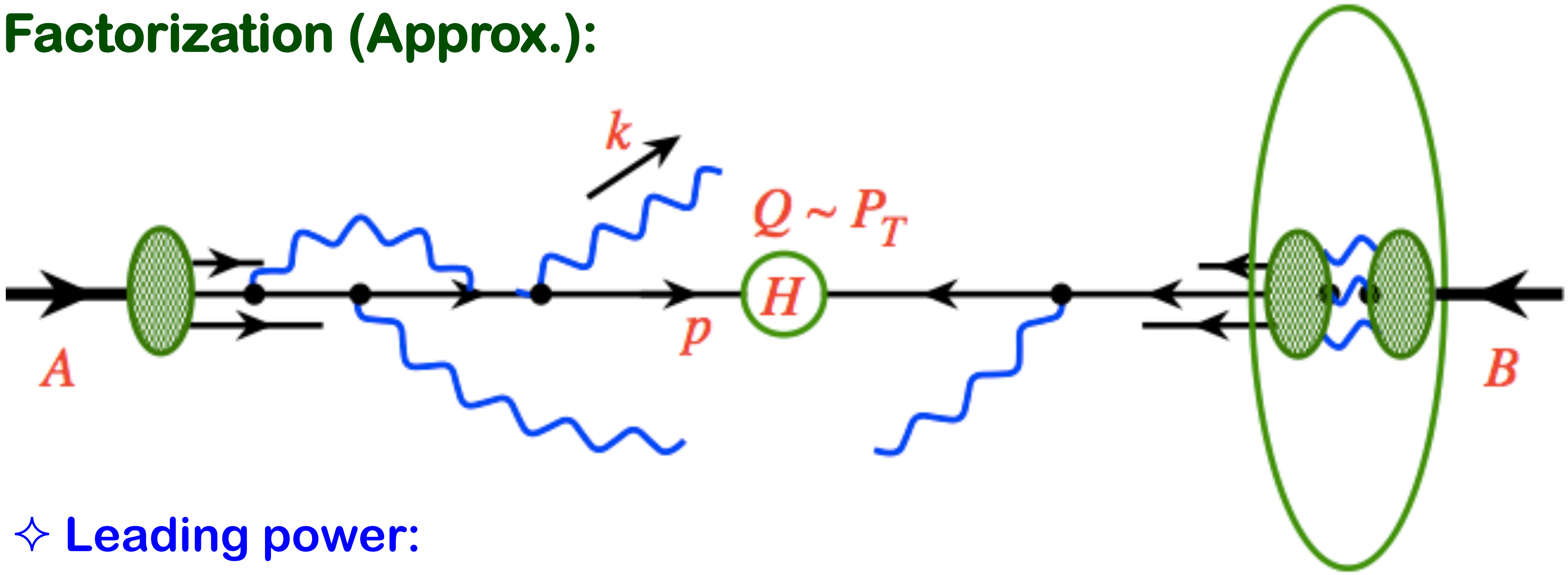
Venugopalan's talk

When $Q^2 \sim P_T^2 \sim \langle p^2 \rangle_{\text{shower}}$ $[\text{or } Q_s^2|_{\text{multiple scattering}}]$, every power is important!

More wave feature than particle feature $\Longrightarrow$ more collective behavior!

Single scale pA collisions

❑ **Factorization (Approx.):**



✧ **Leading power:**

$$d\sigma_{AB}(Q) \approx \hat{\sigma}(Q) \otimes f_2 \otimes f_2' [\otimes D_2] \qquad f_2' \Longrightarrow$$ **nPDFs**

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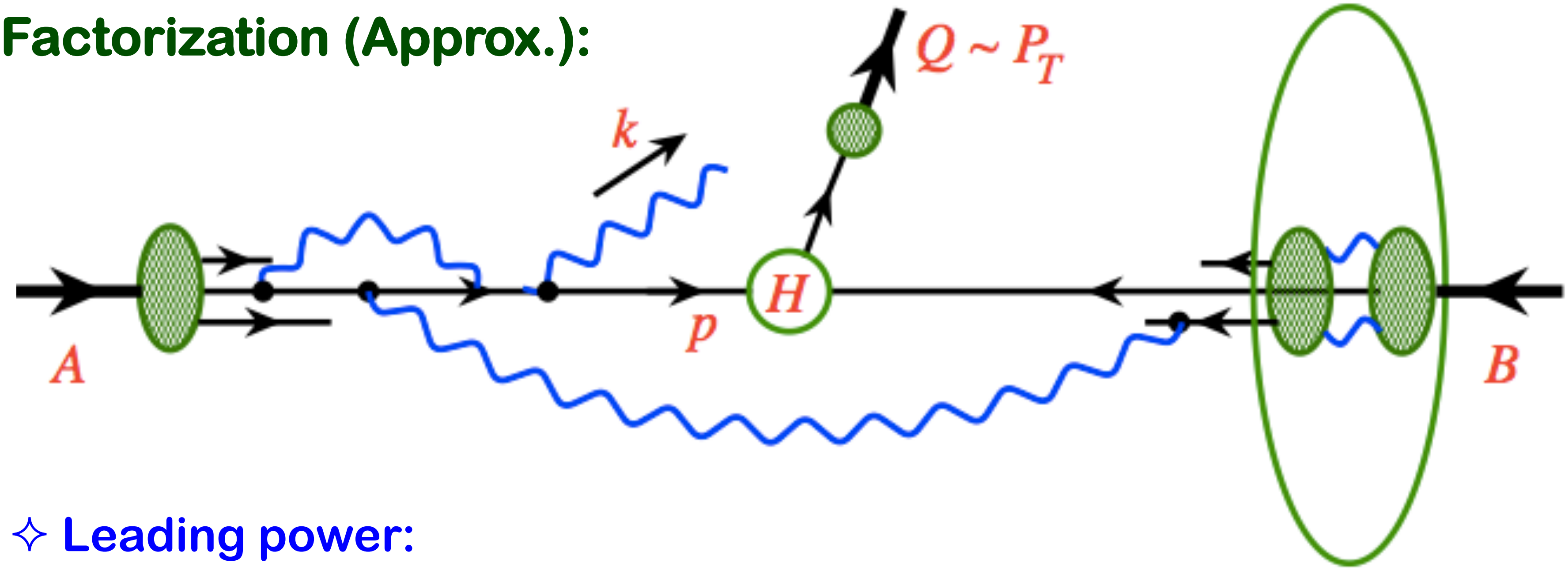

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✧ **Initial-state multiple scattering – power suppressed:**

Resummation of $A^{1/3}$ enhanced power corrections – broadening of $\langle p_T \rangle$

Single scale pA collisions

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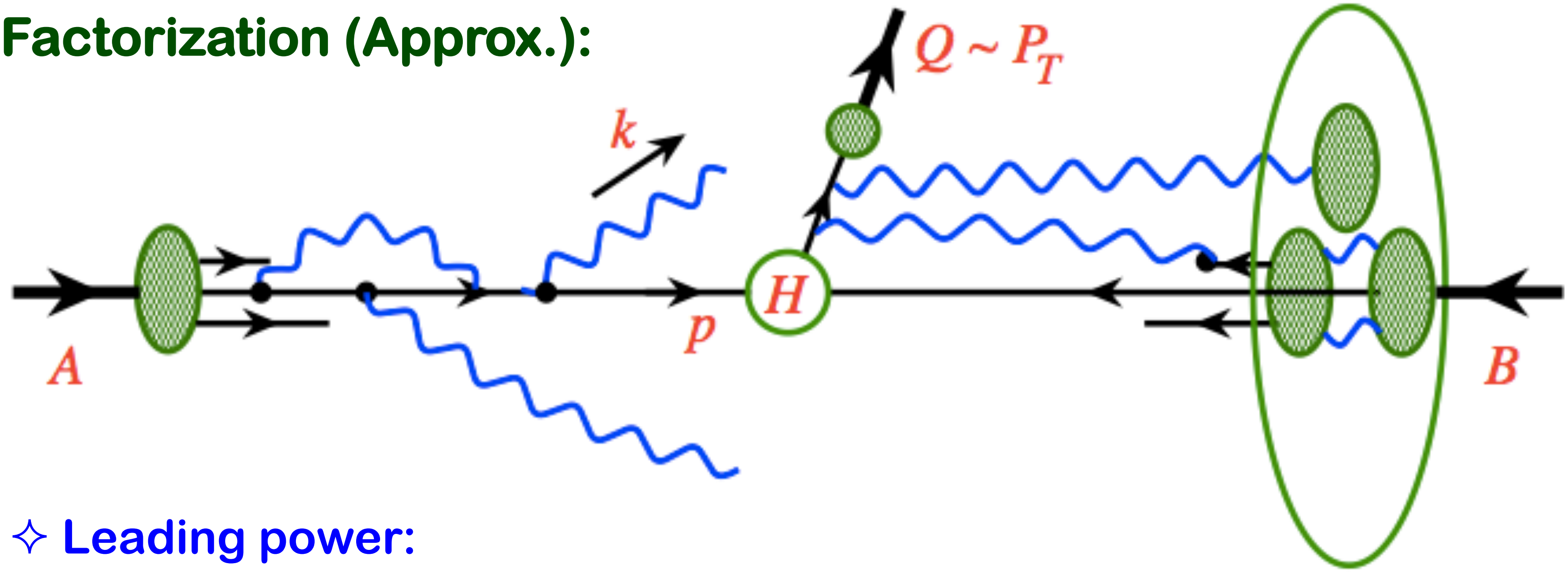

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✧ **Initial-state multiple scattering – power suppressed:**

Resummation of $A^{1/3}$ enhanced power corrections – broadening of $\langle p_T \rangle$

✧ **Final-state multiple scattering – Jet quenching in pA:**

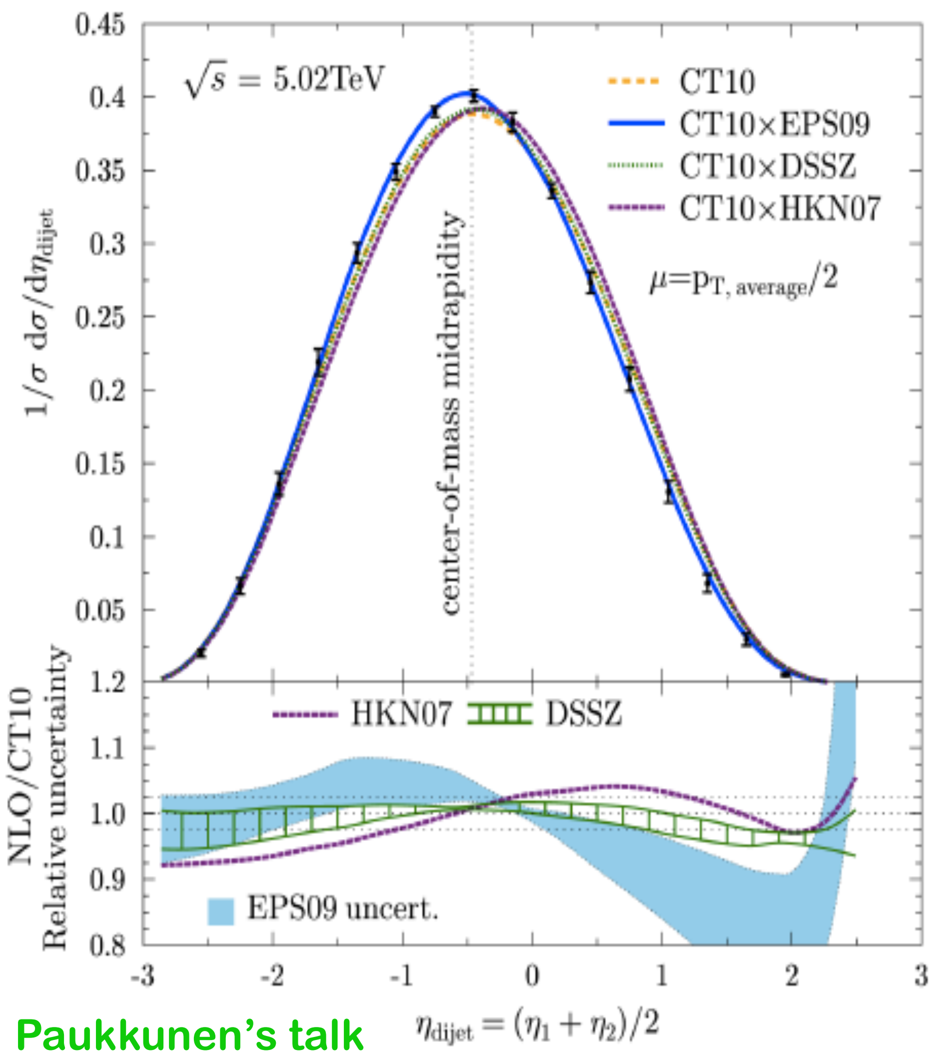
"High" density medium – very short path – same as pp

"Cold" nuclear medium – power suppressed – smaller effect at LHC

nPDFs and shadowing

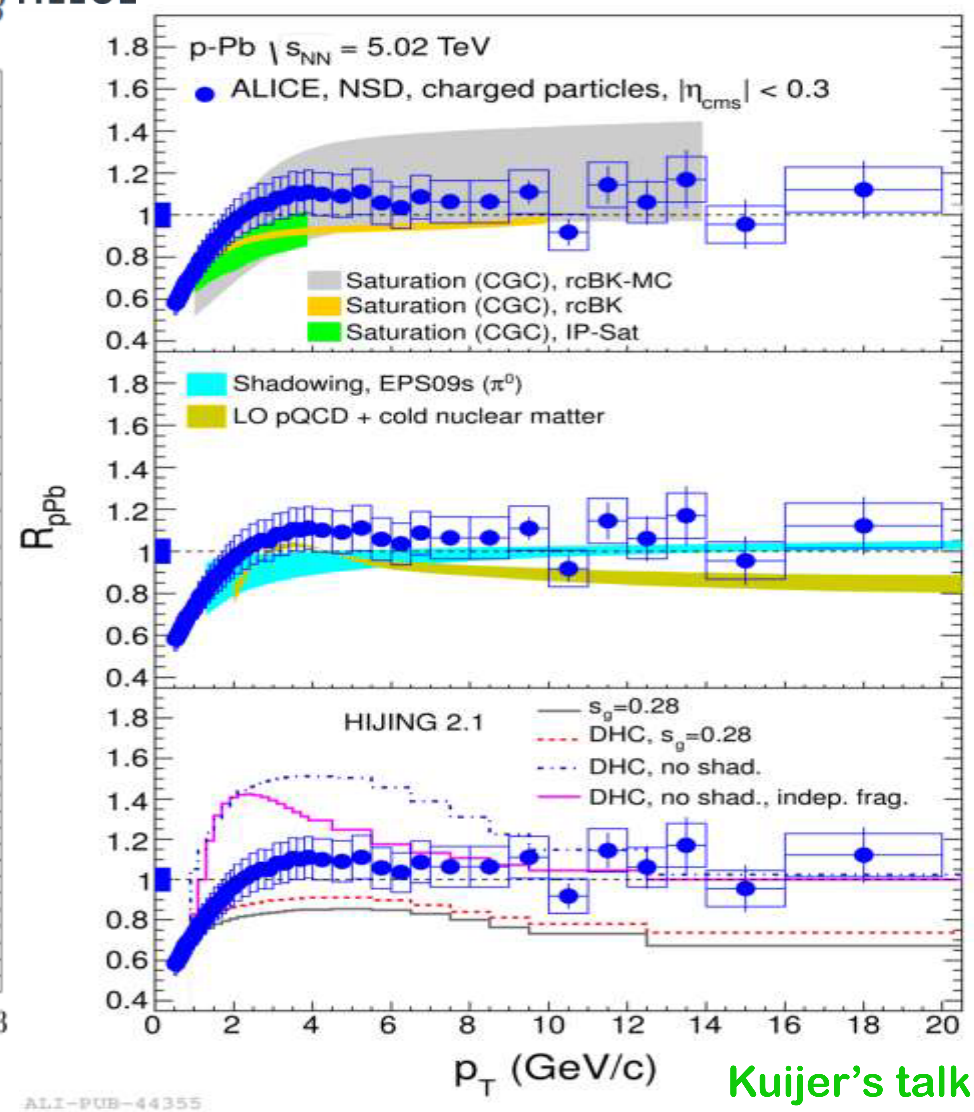
"Inclusive" dijets in pPb

Eskola, Paukkunen, Salgado, arXiv:1308.6733



Paukkunen's talk

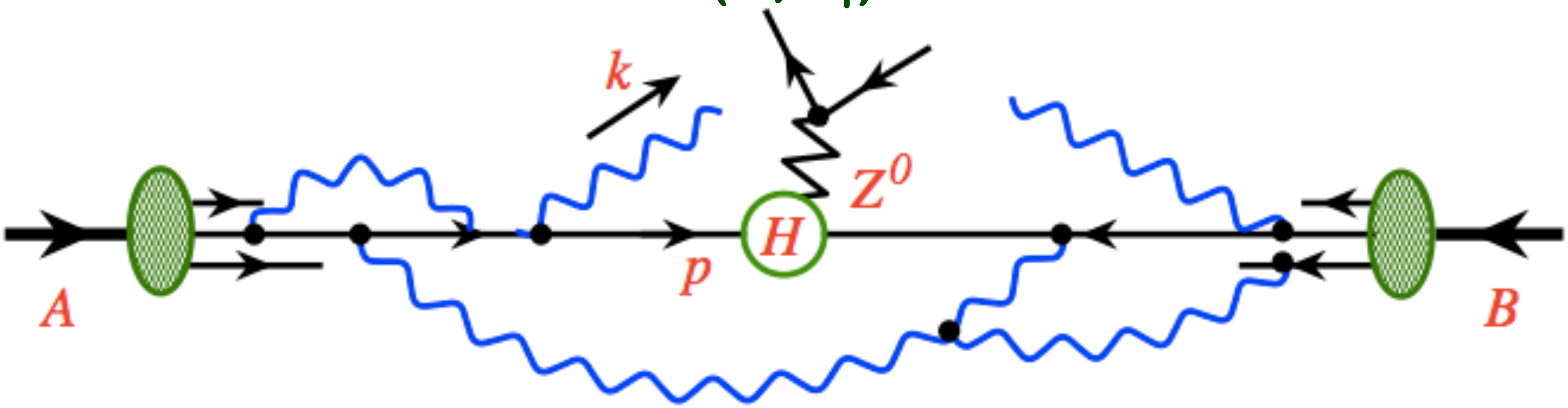
"Inclusive" charged particles



Kuijer's talk

Better probe of initial-state radiation

- ❑ **Cross section of two scales (M, P_T):**



Better probe of initial-state radiation

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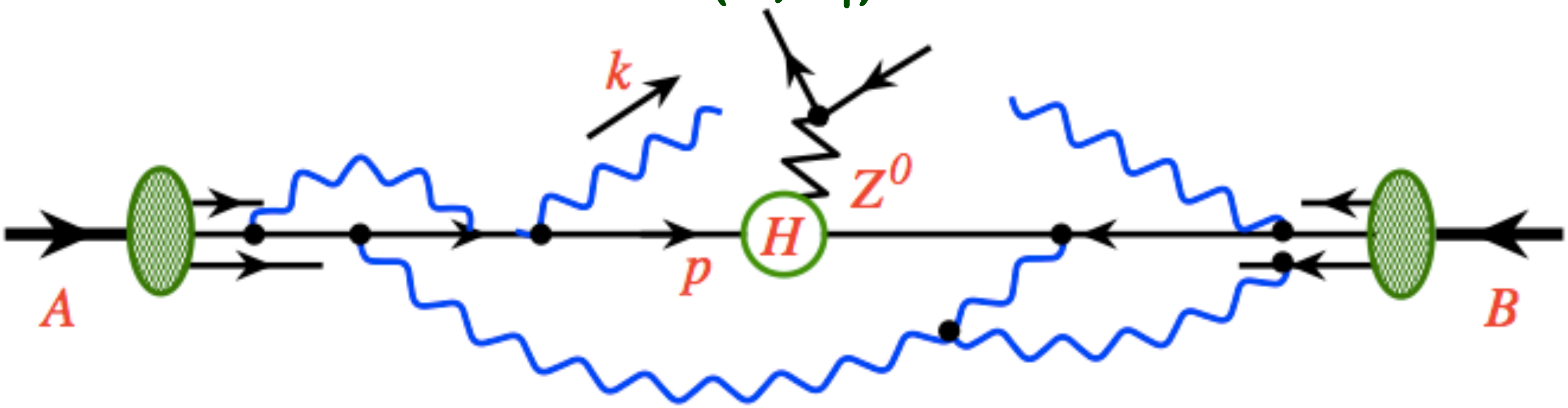


✧ **Z^0-p_T distribution:**

Initial-state radiation (gluon shower) determines the low-p_T distribution

✧ **Each radiation is too soft to be calculated**

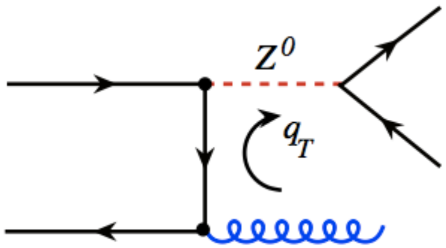
$\Longrightarrow$ **Resummation of all radiations in Fourier space of P_T**

✧ **Precision tests in pp and ppbar collisions!**

✧ **Z^0 in pA might be the best probe of gluon antishadowing!**

Early approach to resummation

- **LO Differential Q_T-distribution as $Q_T \to 0$:**



$$\frac{d\sigma}{dy\,dQ_T^2}\bigg|_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

$$\Longrightarrow \int_0^{Q^2} \frac{d\sigma}{dy\,dQ_T^2}\bigg|_{\text{real+virutal}} dQ_T^2 \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} + O(\alpha_s) \qquad \text{with } Q^2 \approx M_W^2$$

- **Integrated Q_T-distribution (DDT formula):**

Effect of gluon emission

$$\int_0^{Q_T^2} \frac{d\sigma}{dy\,dp_T^2}\bigg|_{\text{real+virutal}} dp_T^2 \equiv \left[\int_0^{Q^2} - \int_{Q_T^2}^{Q^2}\right] \frac{d\sigma}{dy\,dp_T^2}\bigg|_{\text{real+virutal}} dp_T^2$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \left[1 - \int_{Q_T^2}^{Q^2} 2C_F \frac{\alpha_s}{\pi} \frac{\ln\left(Q^2/p_T^2\right)}{p_T^2} dp_T^2\right] = \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \left[1 - C_F \frac{\alpha_s}{\pi} \ln^2\left(Q^2/Q_T^2\right)\right]$$

$$\approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times \exp\left[-C_F \frac{\alpha_s}{\pi} \ln^2\left(Q^2/Q_T^2\right)\right]$$

Assume this exponentiates

Resummed Q_T distribution

- Differentiate the integrated Q_T-distribution:

$$\frac{d\sigma}{dy\,dQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2}\times\exp\left[-C_F\left(\frac{\alpha_s}{\pi}\right)\ln^2\left(Q^2/Q_T^2\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- compare to the explicit LO calculation:

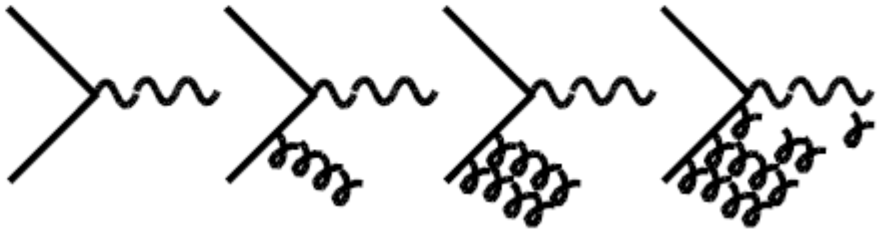
$$\left.\frac{d\sigma}{dy\,dQ_T^2}\right|_{\rm LO} \approx \left(\frac{d\sigma}{dy}\right)_{\rm Born} \times 2C_F\left(\frac{\alpha_s}{\pi}\right)\frac{\ln\left(Q^2/Q_T^2\right)}{Q_T^2} \Rightarrow \infty$$

Q_T-spectrum (as $Q_T \to 0$) is completely changed!

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2\,!} - \frac{(\alpha_s L^2)^3}{3\,!} + \ \ldots$$

$$L \propto \ln\left(Q^2/Q_T^2\right)$$



Soft gluon emission treated as uncorrelated

Still a wrong Q_T-distribution

- **Experimental fact:**

$$\frac{d\sigma}{dy\,dQ_T^2} \Rightarrow \text{finite [neither } \infty \text{ nor } 0!] \text{ as } Q_T \to 0$$

- **Double Leading Logarithmic Approximation (DLLA):**

  - Radiated gluons are both soft and collinear with strong ordering in their transverse momenta
  - Ignores the overall vector momentum conservation
  - Double logs ~ random work ~ zero probability to be at $Q_T = 0$

DLLA over suppress small Q_T region

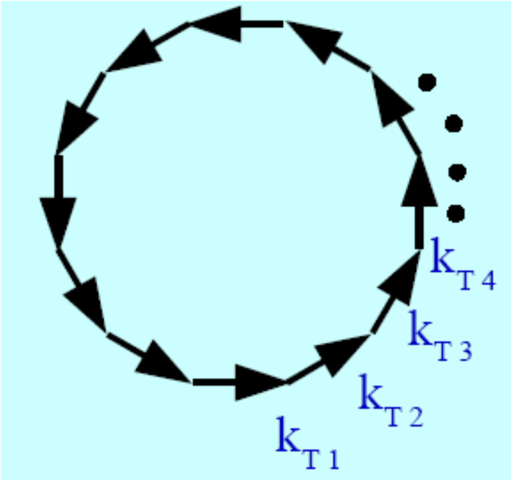
Resummation of uncorrelated soft gluon emission leads to too strong suppression at Q_T=0

Importance of momentum conservation

- **Vector momentum conservation:**

  Particle can receive many finite k_T kicks via soft gluon radiation, but, yet still has Q_T=0

  → Need vector sum!



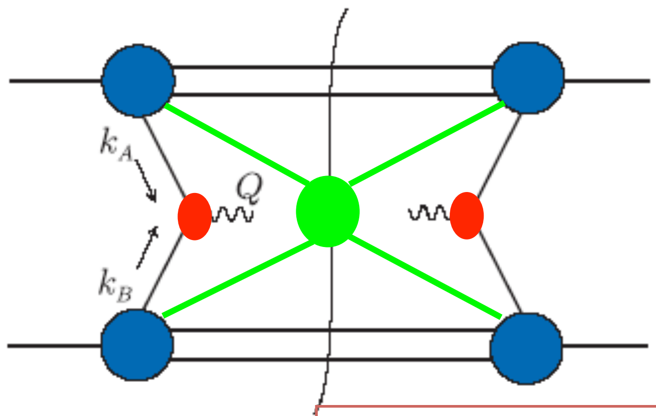
- **Subleading logarithms are equally important at Q_T=0**

- **Solution:**

  Impose 4-momentum conservation at each step of soft gluon resummation

CSS "b"-space resummation formalism

- **Leading order K_T-factorized cross section:**



$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a d\xi_b \int \frac{d^2k_{A_T} d^2k_{B_T} d^2k_{s,T}}{(2\pi)^6}$$
$$\times P_{f/A}(\xi_a, k_{A_T}) P_{\bar{f}/B}(\xi_b, k_{B_T}) H_{f\bar{f}}(Q^2) S(k_{s,T})$$
$$\times \delta^2(\vec{Q}_T - \vec{k}_{A_T} - \vec{k}_{B_T} - \vec{k}_{s,T})$$

$$\delta^2(\vec{Q}_T - \prod_i \vec{k}_{i,T}) = \frac{1}{(2\pi)^2} \int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T} \prod_i e^{-i\vec{b}\cdot\vec{k}_{i,T}}$$

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2} \int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T} \tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2, Q^2)$$

No large log's

resummed

$$= \frac{1}{(2\pi)} \int_0^\infty db\, J_0(bQ_T)\, b\tilde{W}_{AB}(b,Q) + \left[\frac{d\sigma_{AB}^{(\text{Pert})}}{dQ^2 dQ_T^2} - \frac{d\sigma_{AB}^{(\text{Asym})}}{dQ^2 dQ_T^2} \right]$$

The Q_T-distribution is determined by the b-space function: $b\, W_{AB}(b, Q)$

Resummation = CS + RG equations

- b-space distribution:

$$W_{AB}(b,Q) = \sum_{ij} W_{ij}(b,Q)\hat{\sigma}_{ij}(Q)$$

- Collins-Soper equation:

$$\frac{\partial}{\partial \ln Q^2}\tilde{W}_{ij}(b,Q) = \left[K(b\mu,\alpha_s) + G(Q/\mu,\alpha_s)\right]\tilde{W}_{ij}(b,Q) \quad (1)$$

- Evolution kernels satisfy RG equation:

$$\frac{\partial}{\partial \ln \mu^2}K(b\mu,\alpha_s) = -\frac{1}{2}\gamma_K(\alpha_s(\mu)) \quad (2)$$

$$\frac{\partial}{\partial \ln \mu^2}G(Q/\mu,\alpha_s) = \frac{1}{2}\gamma_K(\alpha_s(\mu)) \quad (3)$$

- Solution - resummation:

$$W_{ij}(b,Q) = W_{ij}(b,1/b)\,\mathrm{e}^{-S_{ij}(b,Q)}$$

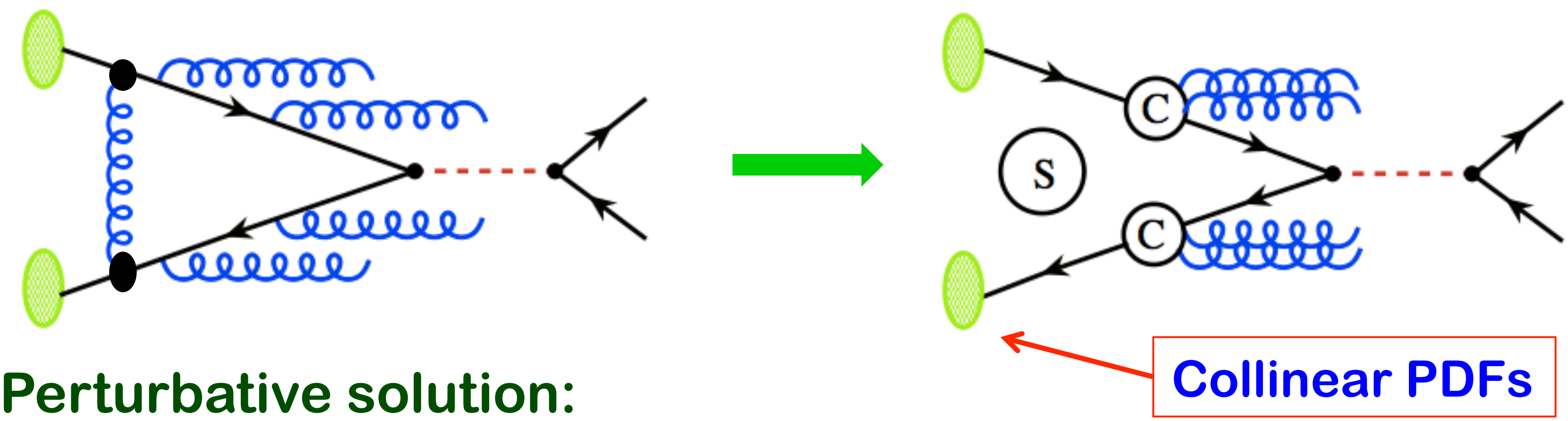
Sudakov form factor
All large logs

Boundary condition – perturbative if b is small!

Perturbative solution at small "b"

- Boundary condition – collinear factorization:

$$W_{ij}(b,Q) = \sum_{a,b} \sigma_{ij\to Z} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right]$$



- Perturbative solution:

$$W_{AB}^{\text{pert}}(b,Q) = \sum_{a,b,i,j} \sigma_{ij\to Z} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right] \times \mathrm{e}^{-S_{ij}(b,Q)}$$

- Extrapolation to large-b?

  - Non-perturbative
  - Predictive power?

$$\sigma^{\text{Resum}} \propto \int_0^\infty db\, J_0(q_T\, b) b\, W(b,Q)$$

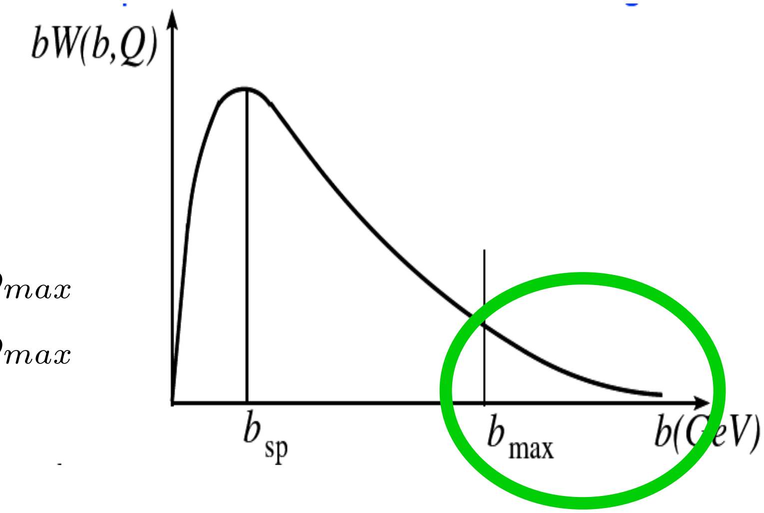
Phenomenology – uncertainty?

Qiu, Zhang, 2001

- Resummed cross section:

$$\frac{d\sigma^{\rm resum}_{AB\to Z}}{dq_T^2} \propto \int_0^\infty db\, J_0(q_T b)\, b\, W(b,Q)$$

$$W(b,Q) = \begin{cases} W^{\rm pert}(b,Q) & b \le b_{max} \\ W^{\rm pert}(b_{max},Q) F^{NP}_{QZ}(b,Q,b_{max}) & b > b_{max} \end{cases}$$



- Resummed cross section:

$$F^{NP}_{QZ}(b,Q;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b^2_{max}}{c^2}\right) \left[g_1 \left((b^2)^\alpha - (b^2_{max})^\alpha \right) + g_2 \left(b^2 - b^2_{max} \right) \right] - \bar{g}_2 \left(b^2 - b^2_{max} \right) \right\}$$

Leading twist

Intrinsic power corrections

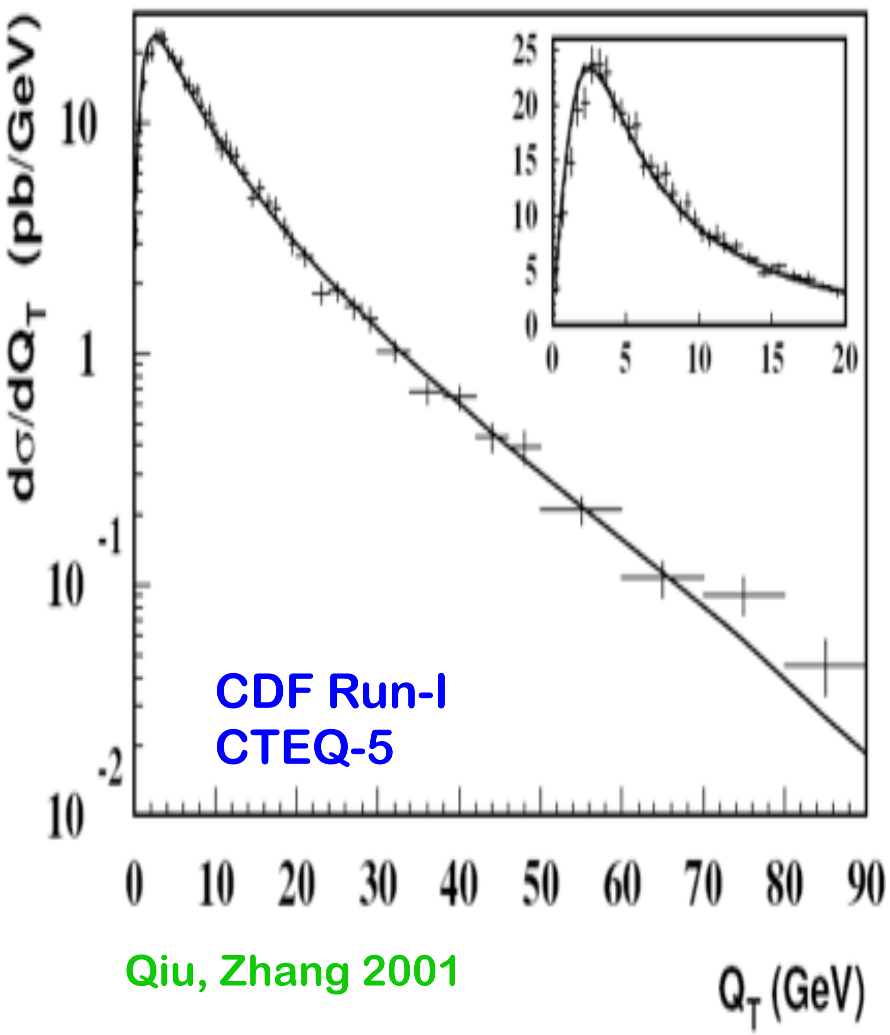
Dynamical power corrections

- Predictive power:

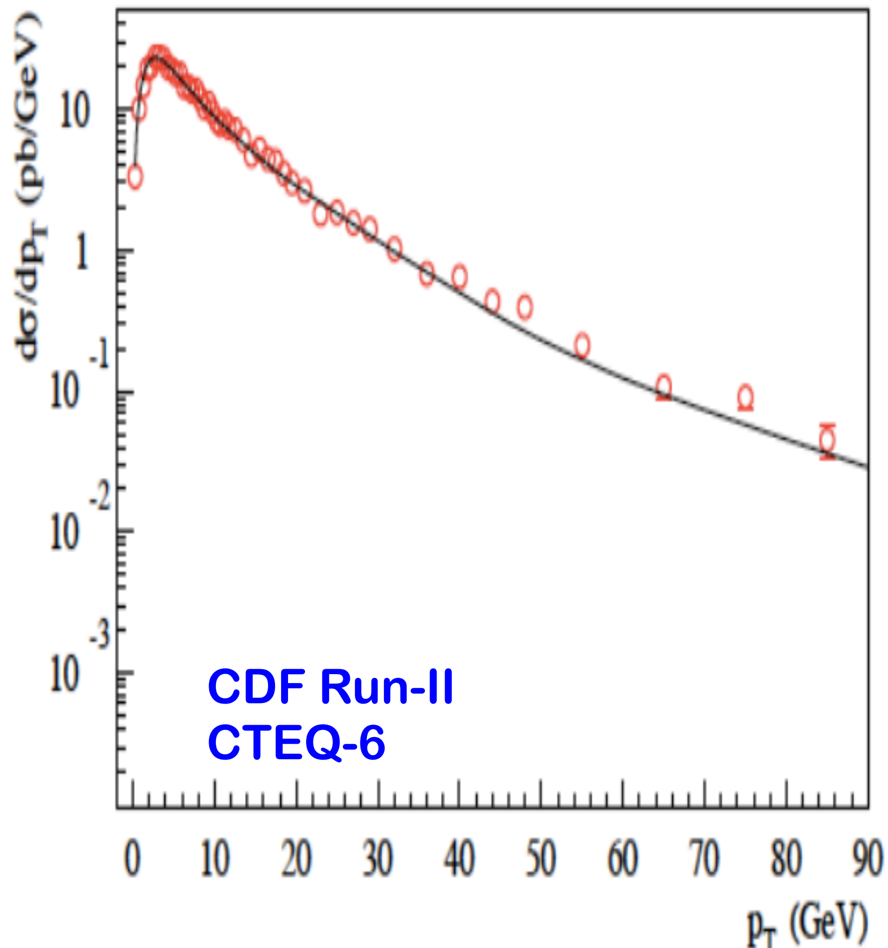
  - ✧ Larger Q
  - ✧ Larger S

→ Smaller b_{sp} → Better prediction

Phenomenology – Tevatron



Qiu, Zhang 2001

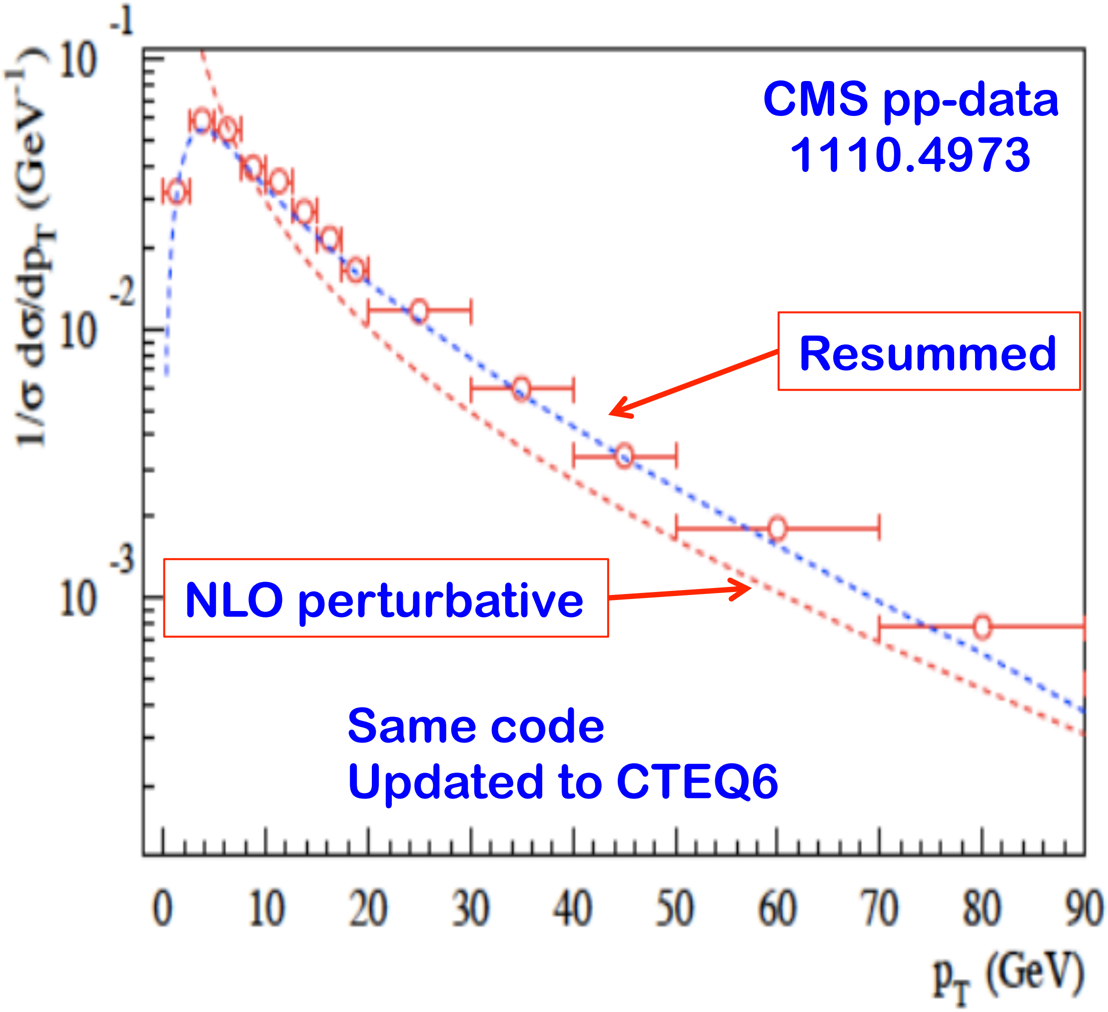


Kang, Qiu 2012

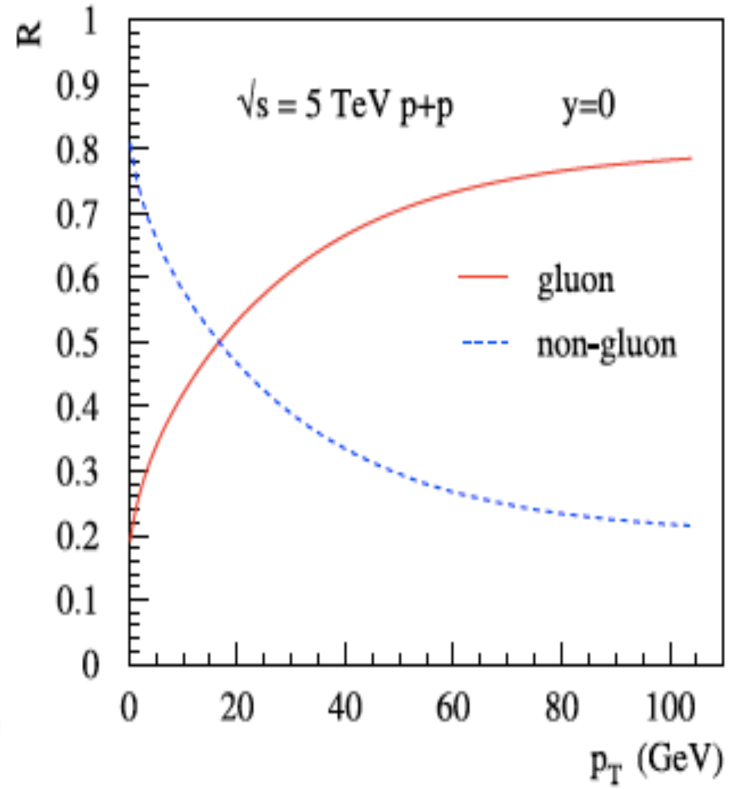
No free fitting parameter!

Phenomenology – Z^0 @ LHC

Kang, Qiu, 2012



Dominated by gluons!



Effectively NO non-perturbative uncertainty!

Nuclear dependence in pPb

- Resummed partonic hard parts are insensitive to A:

$$W_{AB}^{\text{pert}}(b,Q) = \sum_{a,b,i,j} \sigma_{ij\to Z} \left[\phi_{a/A} \otimes C_{a\to i}\right] \otimes \left[\phi_{b/B} \otimes C_{b\to j}\right] \times \mathrm{e}^{-S_{ij}(b,Q)}$$

Only source of A-dependence is from nPDFs

- Large-b non-perturbative parts are sensitive to A:

$$W^{\text{Nonpert}}(b,Q) = W^{\text{pert}}(b_{max},Q)\, F_{QZ}^{NP}(b,Q,b_{max})$$

$$F_{QZ}^{NP}(b,Q;b_{max}) = \exp\Big\{ -\ln(\frac{Q^2 b_{max}^2}{c^2}) \Big[g_1 \left((b^2)^\alpha - (b_{max}^2)^\alpha\right) + g_2 \left(b^2 - b_{max}^2\right)\Big] - \bar{g}_2 \left(b^2 - b_{max}^2\right) \Big\}$$

Leading twist

Intrinsic power corrections

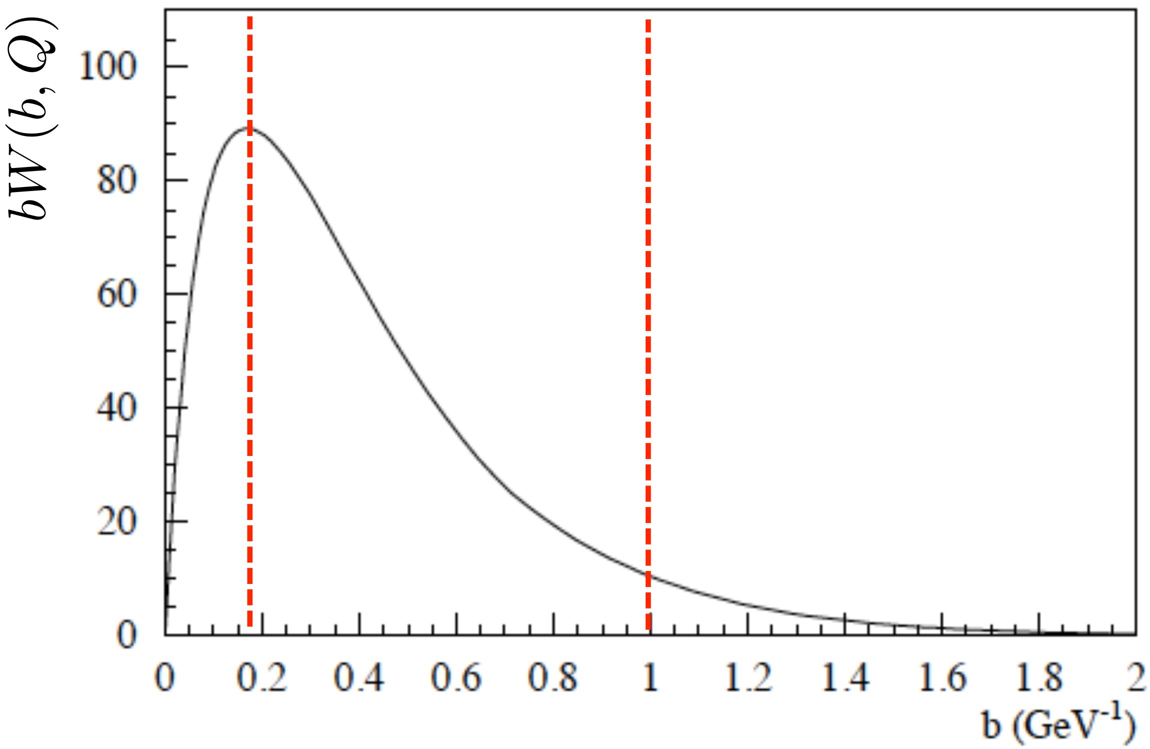
Dynamical power corrections

- g_2 = Power correction to evolution of K and G:

$$g_2 \propto Q_s^2 \quad \Longrightarrow \quad g_2 \Rightarrow g_2\, A^{1/3}$$

Expectations – pPb @ LHC

- Strong gluon shower ➡ $\langle p_T^2 \rangle \gg Q_s^2$



Small contribution from large-b

Small A-dependent power corrections

- Ideal for nPDFs:

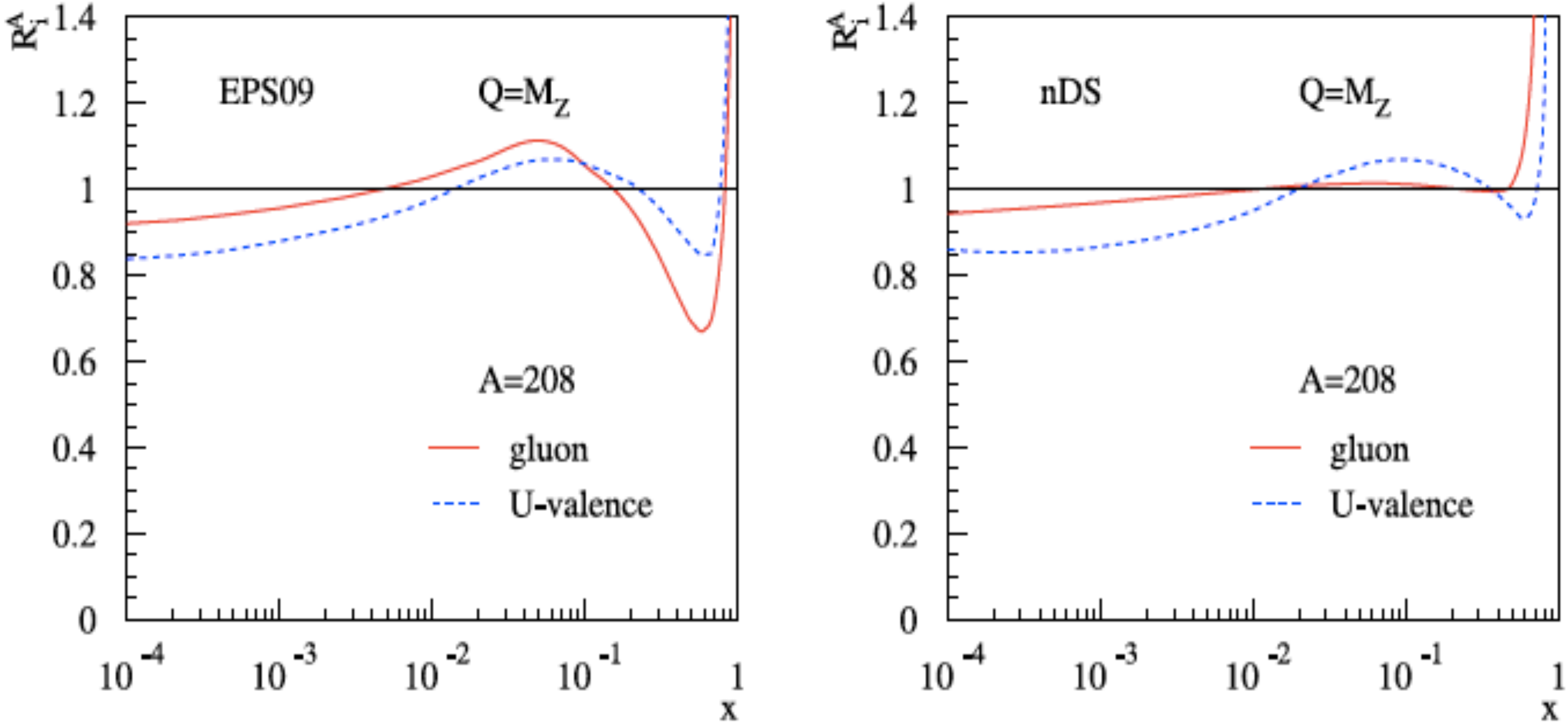
$$\mu = \frac{C_2}{b} \approx \frac{1}{b} > b_{max}^{-1} > 1 - 2 \text{ GeV}$$

Cover nPDFs for a much wider range of Q (or μ)!

Strong shadowing and antishadowing effect for Z^0 – production!

Nuclear PDFs – the "gluon"

❑ NLO global fitted nPDFs:

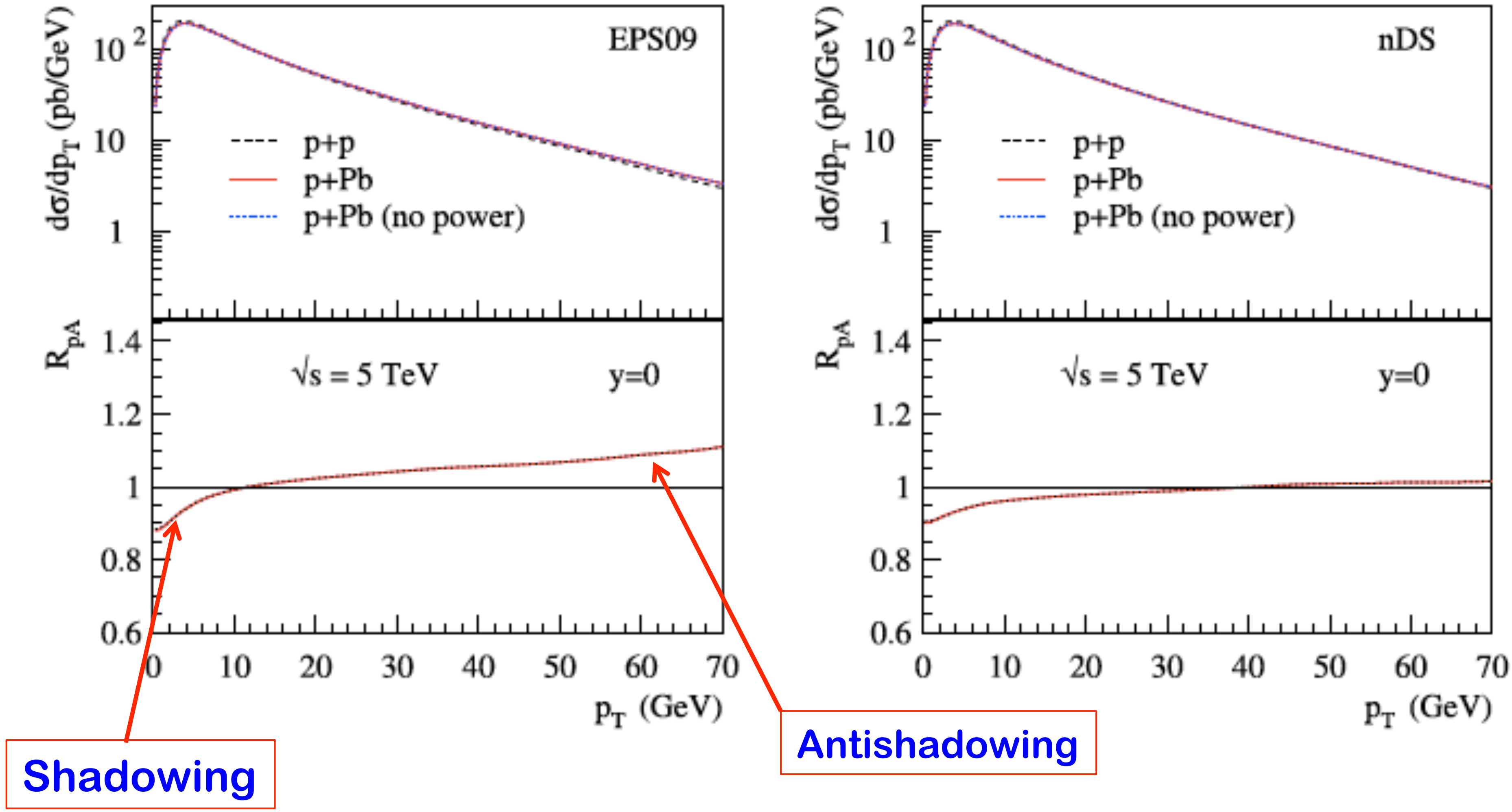


✧ Huge differences in nuclear gluon distributions

Large antishadowing in EPS09, but, almost none in nPDFs of nDS

Predictions – pPb @ LHC

❑ “Discovering” the antishadowing?



$A^{1/3}$-type power correction has almost “NO” effect! (large Q!)

Summary

- Initial-state radiation pattern depends on observables/events
- Observables with two scales (large + small) are sensitive to initial-state gluon radiation/shower
- Z^0 production is an excellent benchmark for testing QCD
- R_{pA} of Z^0 production vs p_T is an ideal probe of nuclear gluons (sensitive to both shadowing and and antishadowing)

Theoretical calculation is insensitive to non-perturbative physics other than nPDFs (stronger gluon shower)!

Thank you!