Initial-State Radiation in Proton-Nucleus Collisions

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Based on work done with Kang, Sterman, ...
Initial-state radiation

- **Hadronic collisions:**

  - Gluon radiation/shower is a consequence of the collision
  - Radiation pattern depends on each event
  - Treatment of radiation (approx.) depends on observables
Initial-state radiation

- Hadronic collisions:

- Gluon radiation:
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  - Treatment of radiation (approx.) depends on observables

- Long-range soft-gluon interaction:
  - “Talk” between hadrons before the “hard” collision

  Leading power – by unitarity, higher power – suppressed
Events with a single hard scale

- **Approximations:**

  - Active parton virtuality (from shower):
    \[ \langle p^2 \rangle \sim \langle p_T^2 \rangle \sim (1/\text{fm})^2 \log(S/Q^2) \log(Q^2/(1/\text{fm})^2) \propto \log(1/x_p) \]

  - On-shell approximation:
    \[ \langle p^2 \rangle \sim \langle p_T^2 \rangle \ll Q^2 \]
    \[ p^2 = 0 + O(1/Q) \]

- **Diagram**:

  - Event with a single hard scale with parton interactions and virtuality contributions.
Events with a single hard scale

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  - Active parton virtuality (from shower):
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    \]
  
  - On-shell approximation:
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    \langle p^2 \rangle \sim \langle p_T^2 \rangle \ll Q^2
    \]
    \[
    \Rightarrow \quad p^2 = 0 + \mathcal{O}(1/Q)
    \]

- **Initial-state radiation:**
  
  - Perturbative singularity:
    \[
    \log(Q^2/k_T^2)|_{k_T^2 \rightarrow 0} \quad \Rightarrow \quad \text{PDFs}
    \]
  
  - Rest of $k$-phase-space:
    \[
    \Rightarrow \quad \alpha_s^n \quad \text{corrections}
    \]
Events with a single hard scale

- **Factorization (Approx.):**

  $d\sigma_{AB}(Q) \approx \hat{\sigma}(Q) \otimes f_2 \otimes f_2' \otimes D_2$

- **Leading power:**

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- **1st power corrections:**

  $+ \frac{\langle p^2 \rangle}{Q^2} \hat{\sigma}_4 \otimes f_2 \otimes f_2' \otimes D_4 + \ldots$
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    \[
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    \]

- **No-factorization beyond 1/Q^2!**

  When \( Q^2 \sim P_T^2 \sim \langle p^2 \rangle_{\text{shower}} \) [or \( Q_s^2 \) \text{multiple scattering}], every power is important!

  More wave feature than particle feature \( \implies \) more collective behavior!

Venugopalan’s talk
Single scale pA collisions

- **Factorization (Approx.):**

\[ d\sigma_{AB}(Q) \approx \hat{\sigma}(Q) \otimes f_2 \otimes f_2' [\otimes D_2] \]

\[ f_2' \implies \text{nPDFs} \]
Single scale pA collisions

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  - **Leading power:**

  \[ d\sigma_{AB}(Q) \approx \hat{\sigma}(Q) \otimes f_2 \otimes f'_{2} \left[ \otimes D_2 \right] \]

  \[ f'_{2} \rightarrow \text{nPDFs} \]

- **Initial-state multiple scattering – power suppressed:**

  Resummation of A^{1/3} enhanced power corrections – broadening of \( <p_T> \)
Single scale pA collisions

- **Factorization (Approx.):**

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  - **Initial-state multiple scattering – power suppressed:**
    
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  - **Final-state multiple scattering – Jet quenching in pA:**
    
    "High" density medium – very short path – same as pp
    "Cold" nuclear medium – power suppressed – smaller effect at LHC
nPDFS and shadowing

“Inclusive” dijets in pPb

Eskola, Paukkunen, Salgado, arXiv:1308.6733

\[ \sqrt{s} = 5.02 \text{ TeV} \]

\[ 1/\sigma \frac{d\sigma}{d\eta_{\text{dijet}}} \]

\[ \eta_{\text{dijet}} = (\eta_1 + \eta_2)/2 \]

Paukkunen’s talk

“Inclusive” charged particles

Kuijer’s talk
Better probe of initial-state radiation

- Cross section of two scales \((M, P_T)\):
Better probe of initial-state radiation

Cross section of two scales \((M, P_T)\):

- \(Z^0-p_T\) distribution:
  - Initial-state radiation (gluon shower) determines the low-\(p_T\) distribution
  - Each radiation is too soft to be calculated
  - Resummation of all radiations in Fourier space of \(P_T\)
- Precision tests in \(pp\) and \(ppbar\) collisions!
- \(Z^0\) in \(pA\) might be the best probe of gluon antishadowing!
Early approach to resummation

- **LO Differential $Q_T$-distribution as $Q_T \to 0$**:

  \[
  \frac{d\sigma}{dydQ_T^2} \text{ LO} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{1\ln \left( \frac{Q^2}{Q_T^2} \right)}{Q_T^2} \Rightarrow \infty
  \]

  \[
  \int_0^{Q^2} \frac{d\sigma}{dydQ_T^2} \text{ real+virtual} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} + O(\alpha_s) \quad \text{with} \quad Q^2 \approx M_W^2
  \]

- **Integrated $Q_T$-distribution (DDT formula)**:

  \[
  \int_0^{Q^2} \frac{d\sigma}{dydQ_T^2} \text{ real+virtual} = \left[ \int_0^{Q^2} \frac{d\sigma}{dydQ_T^2} \text{ real+virtual} \right] \frac{d\sigma}{dydQ_T^2} \text{ real+virtual} \\
  \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times \left[ 1 - \int_0^{Q^2} 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{1\ln \left( \frac{Q^2}{p_T^2} \right)}{p_T^2} dp_T^2 \right] = \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times \left[ 1 - C_F \left( \frac{\alpha_s}{\pi} \right) \ln^2 \left( \frac{Q^2}{Q_T^2} \right) \right]
  \]

  Assume this exponentiates
Resummed $Q_T$ distribution

- Differentiate the integrated $Q_T$-distribution:

$$ \frac{d\sigma}{dydQ_T^2} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{1n(Q^2/Q_T^2)}{Q_T^2} \times \exp \left[ -C_F \left( \frac{\alpha_s}{\pi} \right) 1n^2 \left( \frac{Q^2}{Q_T^2} \right) \right] \Rightarrow 0 \quad \text{as } Q_T \to 0 $$

- Compare to the explicit LO calculation:

$$ \frac{d\sigma}{dydQ_T^2} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{1n(Q^2/Q_T^2)}{Q_T^2} \Rightarrow \infty $$

$Q_T$-spectrum (as $Q_T \to 0$) is completely changed!

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$ e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ldots $$

$L \propto 1n\left( \frac{Q^2}{Q_T^2} \right)$

Soft gluon emission treated as uncorrelated
Still a wrong $Q_T$-distribution

- **Experimental fact:**
  \[
  \frac{d\sigma}{dydQ_T^2} \Rightarrow \text{finite [neither } \infty \text{ nor } 0!] \text{ as } Q_T \to 0
  \]

- **Double Leading Logarithmic Approximation (DLLA):**
  - Radiated gluons are both soft and collinear with strong ordering in their transverse momenta
  - Ignores the overall vector momentum conservation
  - Double logs ~ random work ~ zero probability to be at $Q_T = 0$

  DLLA over suppress small $Q_T$ region

Resummation of uncorrelated soft gluon emission leads to too strong suppression at $Q_T=0$
importance of momentum conservation

- **Vector momentum conservation:**
  
  Particle can receive many finite $k_T$ kicks via soft gluon radiation, but, yet still has $Q_T=0$

  Need vector sum!

- **Subleading logarithms are equally important at $Q_T=0**

- **Solution:**
  
  Impose 4-momentum conservation at each step of soft gluon resummation
The Q_T-distribution is determined by the b-space function:  \( b W_{AB}(b, Q) \)
Resummation = CS + RG equations

- **b-space distribution:**

\[ W_{AB}(b, Q) = \sum_{ij} W_{ij}(b, Q) \hat{\sigma}_{ij}(Q) \]

- **Collins-Soper equation:**

\[ \frac{\partial}{\partial \ln Q^2} \tilde{W}_{ij}(b, Q) = \left[ K(b\mu, \alpha_s) + G(Q/\mu, \alpha_s) \right] \tilde{W}_{ij}(b, Q) \quad (1) \]

- **Evolution kernels satisfy RG equation:**

\[ \frac{\partial}{\partial \ln \mu^2} K(b\mu, \alpha_s) = -\frac{1}{2} \gamma_K(\alpha_s(\mu)) \quad (2) \]

\[ \frac{\partial}{\partial \ln \mu^2} G(Q/\mu, \alpha_s) = \frac{1}{2} \gamma_K(\alpha_s(\mu)) \quad (3) \]

- **Solution - resummation:**

\[ W_{ij}(b, Q) = W_{ij}(b, 1/b) \ e^{-S_{ij}(b,Q)} \]

**Sudakov form factor**  
All large logs

**Boundary condition – perturbative if b is small!**
Perturbative solution at small “b”

- Boundary condition – collinear factorization:

\[
W_{ij}(b, Q) = \sum_{a,b} \sigma_{ij \rightarrow Z} \left[ \phi_{a/A} \otimes C_{a \rightarrow i} \right] \otimes \left[ \phi_{b/B} \otimes C_{b \rightarrow j} \right]
\]

- Perturbative solution:

\[
W_{AB}^{pert}(b, Q) = \sum_{a,b,i,j} \sigma_{ij \rightarrow Z} \left[ \phi_{a/A} \otimes C_{a \rightarrow i} \right] \otimes \left[ \phi_{b/B} \otimes C_{b \rightarrow j} \right] \times e^{-S_{ij}(b,Q)}
\]

- Extrapolation to large-b?

- Non-perturbative
- Predictive power?

\[
\sigma^{\text{Resum}} \propto \int_{0}^{\infty} db \ J_{0}(q_{T}, b) b \ W(b, Q)
\]
Resummed cross section:

\[
\frac{d\sigma_{AB \rightarrow Z}}{dq_T^2} \propto \int_0^\infty db \ J_0(q_T b) \ b \ W (b, Q)
\]

\[W (b, Q) = \begin{cases} 
W^{\text{pert}} (b, Q) & b \leq b_{\text{max}} \\
W^{\text{pert}} (b_{\text{max}}, Q) F_{QZ}^{NP} (b, Q, b_{\text{max}}) & b > b_{\text{max}}
\end{cases}\]

Predictive power:

- Larger Q
- Larger S
  \[\Rightarrow\] Smaller \(b_{sp}\)
  \[\Rightarrow\] Better prediction
Phenomenology – Tevatron

CDF Run-I
CTEQ-5
Qiu, Zhang 2001

CDF Run-II
CTEQ-6
Kang, Qiu 2012

No free fitting parameter!
Phenomenology – $Z^0$ @ LHC

**Effectively NO non-perturbative uncertainty!**

Kang, Qiu, 2012
Nuclear dependence in pPb

- Resummed partonic hard parts are insensitive to A:

\[ W_{AB}^{\text{Pert}}(b, Q) = \sum_{a,b,i,j} \sigma_{i,j \rightarrow Z} \left[ \phi_{a/A} \otimes C_{a \rightarrow i} \right] \otimes \left[ \phi_{b/B} \otimes C_{b \rightarrow j} \right] \times e^{-S_{ij}(b, Q)} \]

Only source of A-dependence is from nPDFs

- Large-b non-perturbative parts are sensitive to A:

\[ W^{\text{Nonpert}}(b, Q) = W^{\text{pert}}(b_{\text{max}}, Q) F^{NP}_{QZ}(b, Q, b_{\text{max}}) \]

\[ F^{NP}_{QZ}(b, Q; b_{\text{max}}) = \exp \left\{ -\ln\left( \frac{Q^2 b_{\text{max}}^2}{c^2} \right) \left[ g_1 \left( b^2 \right)^{\alpha} - \left( b_{\text{max}}^2 \right)^{\alpha} \right] + g_2 \left( b^2 - b_{\text{max}}^2 \right) \right\} \]

- \( g_2 = \text{Power correction to evolution of K and G} \):

\[ g_2 \propto Q_s^2 \quad \Rightarrow \quad g_2 \propto A^{1/3} \]
Expectations – pPb @ LHC

- Strong gluon shower $\langle p_T^2 \rangle >> Q_s^2$

- Ideal for nPDFs:
  \[ \mu = \frac{C'_2}{b} \approx \frac{1}{b} > b^{-1}_{max} > 1 - 2 \text{ GeV} \]

  **Cover nPDFs for a much wider range of Q (or $\mu$)!**

  *Strong shadowing and antishadowing effect for $Z^0$ – production!*
Nuclear PDFs – the “gluon”

NLO global fitted nPDFs:

- Huge differences in nuclear gluon distributions

*Large antishadowing in EPS09, but, almost none in nPDFs of nDS*
Predictions – pPb @ LHC

“Discovering” the antishadowing?

A\(^{1/3}\)-type power correction has almost “NO” effect! (large Q!)
Initial-state radiation pattern depends on observables/events

Observables with two scales (large + small) are sensitive to initial-state gluon radiation/shower

$Z^0$ production is an excellent benchmark for testing QCD

$R_{p_A}$ of $Z^0$ production vs $p_T$ is an ideal probe of nuclear gluons (sensitive to both shadowing and antishadowing)

Theoretical calculation is insensitive to non-perturbative physics other than nPDFs (stronger gluon shower)!

Thank you!