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Tilted initial state: reasons and consequence
Nuclei colliding at ultra-relativistic energies with nonzero impact parameter have a large initial angular momentum, which is usually ignored in the initial conditions assumed for hydro calculations.

**RHIC:** \( J \approx 5 \cdot 10^4 \)

\( \text{Au Au, } \sqrt{s_{NN}} = 200 \text{ GeV} \)

**LHC:** \( J \approx 1.5 \cdot 10^6 \)

\( \text{Pb+Pb, } s_{NN}^{1/2} = 5.5 \text{ TeV} \)

Becattini, Piccinini, Rizzo, *PRC 77, 024906 (2008)*
Nuclei colliding at ultra-relativistic energies with nonzero impact parameter have a large initial angular momentum.

Symmetry axis = z-axis. Transverse plane divided into streaks.

Myers, Gosset, Kapusta, Westfall
Initial state from effective string rope model

Au+Au at 100+100 GeV/nucl, b=0.25*2R
Initial state from effective string rope model

There are similar models, with different string decay mechanism, which produce initial state in $(\tau, z)$ coordinates:
Mishustin, Kapusta,
PRL 88 (2002) 112501;
Mishustin, Lyakhov,

Au+Au at 100+100 GeV/nucl, $b=0.25*2R$

Initial State in $(t, z)$ coordinate

FIG. 2. Schematic space-time representation of a symmetric (a) and asymmetric (b) slab-slab collision in the $t-z$ plane. The projectile and target slab trajectories are shown by thick solid lines which start at the origin and terminate at points $P$ and $T$, respectively. The quark-gluon plasma is produced at the portion of the hyperbola $\tau = \tau_0$ between these points (thick solid line). Horizontal solid lines represent strings. For symmetric collisions $t_0^* = z_0^* = t^* = z^*$. 
3+1D hydro

Bjorken flow: \( u^\mu(\tau_0, x, y, \eta_\parallel) = (\cosh \eta_\parallel, 0, 0, \sinh \eta_\parallel) \)

“Tilted” initial state:

\[
\rho_0(\tau_0) = \rho_0 [2(N_+(x, y)f_+(\eta_\parallel) + N_-(x, y)f_-(\eta_\parallel))(1 - \alpha) + 2\alpha N_{\text{bin}}(x, y)f(\eta_\parallel)] / N_0.
\] 

(13)

where \( N_+ \) and \( N_- \) are the densities of participants from the two colliding nuclei

Matter is redistributed to the edges
Which potential output such an initial state may have?

- Directed flow, $v_1$
- Elliptic flow, $v_2$
- Vorticity $\Rightarrow$ particle polarization
- HBT
Directed flow starts to deviate from straight line behavior:
not just a simple Bounce-off – there is some matter accelerated by pressure

Antiflow or 3rd flow component

Was proposed as a possible QGP signal in
Csernai, Röhrich, PL B458 (1999) 454
Strong antiflow in $v_1$ is observed at RHIC

STAR Coll.

PHOBOS Coll.

FIG. 2: Directed flow of charged particles in Au+Au collisions as a function of $\eta$, averaged over centrality (0-40%), shown separately for four beam energies. Note the different vertical axis scales between the upper and lower panels. The boxes represent systematic uncertainties at 90% C.L., and $\langle N_{\text{part}} \rangle$ gives the average number of participants for each data sample.
Strong antiflow in $v_1$ is observed at RHIC.


$Au+Au$, $\sqrt{s_{NN}} = 130$ GeV; $b = 0.7 \cdot 2R_{Au}$
Pb+Pb at 1.38 + 1.38 ATeV
Initial energy density [GeV/fm^3] in the reaction plane
b = 0.5 \_b_{\text{max}}
t=4 fm/c after the first touch of the colliding nuclei
Pb+Pb reaction at LHC
1.38 + 1.38 A*TeV collision energy

Csernai, Magas, Stöcker, Strottman,

Anti-flow (v1) at LHC

Peaks in opposite directions!!!
What happened?
PIC-hydro

Pb+Pb
1.38+1.38 A TeV
b = 0.7b_max

MIT Bag EoS

FO: T~200 MeV but calculated much longer, until pressure is zero for 90% of the cells.

Structure and asymmetries of init. state are maintained in nearly perfect expansion.
At LHC energies
“tilted” initial state $\rightarrow$ “rotating” initial state
$V_1$ at LHC has the same negative slope as at RHIC, but smaller magnitude.

$V_1$ consistent with ideal hydrodynamic model calculations for dipole-like energy fluctuations in the overlap zone:

Gardim et al., PRC83, 064901 (2011);
Retinskaya, Luzum, Ollitrault, PRL108, 252302 (2012)
Kelvin-Helmholtz instability in high-energy heavy-ion collisions

L.P. Csernai\textsuperscript{1,2,3}, D.D. Strottman\textsuperscript{2,3}, and Cs. Anderlik\textsuperscript{4}

PHYSICAL REVIEW C 85, 054901 (2012)

**FIG. 1:** (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a \(1.38A + 1.38A \) TeV peripheral, \(b = 0.7b_{\text{max}}\), Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are \(dx = dy = dz = 0.4375\text{fm} \) and the time-step is \(0.04233\text{ fm/c}\). The number of randomly placed marker particles in each fluid cell is \(8^3\). The axis-labels indicate the cell numbers in the \(x\) and \(z\) (beam) direction. The initial development of a KH type instability is visible from \(t = 1.5\) up to \(t = 7.41\text{ fm/c}\) corresponding from 35 to 175 calculation time steps.)
Elliptic flow from rotating initial state

Euler equation

\[(e + P)(u \cdot \partial)u^\mu = g^{\mu\nu} \partial_\nu P - (u \cdot \partial P)u^\mu\]

\[t = 0 : v_x = v_y = 0, \gamma_0^2 = 1/(1 - v_{z0}^2); P = e/3\]

\[e_0\gamma_0^3 \frac{\partial u_i}{\partial t} \bigg|_{t=0} = -\frac{1}{4} \frac{\partial e \gamma^2}{\partial x_i} \bigg|_{t=0} + \frac{1}{4} 2e_0\gamma_0^4 v_{z0} \frac{\partial v_{z0}}{\partial x_i} \bigg|_{t=0}\]

\[\Omega_1\]

\[\Omega_2\]

\[\frac{\partial v_{z0}}{\partial x} < 0\]

\[v_{z0} < 0 \text{ for } x > 0\]

\[v_{z0} > 0 \text{ for } x < 0\]
Elliptic flow from rotating initial state

Euler equation

\[ e_0 \gamma_0^3 \frac{\partial u_i}{\partial t} \big|_{t=0} = -\frac{1}{4} \frac{\partial e \gamma^2}{\partial x_i} \big|_{t=0} + \frac{1}{4} 2e_0 \gamma_0^4 v_{z0} \frac{\partial v_{z0}}{\partial x_i} \big|_{t=0} \]

\[ \Omega_1 \]
\[ \Omega_2 \]

\[ \left| \frac{\partial v_{z0}}{\partial x} \right| > \left| \frac{\partial v_{z0}}{\partial y} \right| \]

\( \Omega_2 \) helps to produce elliptic flow!
Simple toy model initial state
Becattini, Piccinini, Rizzo, PRC 77, 024906 (2008)

Landau disc
but with initial flow velocity $v_z$

$$J = -\int d^3\vec{r} \, x \, T^{0z} = -\int d^3\vec{r} \, x \, (e + P) \gamma^2 v_z(x)$$

$J = 0$

FIG. 4. Initial longitudinal velocity profile for the limiting case of sudden thermalization in the very thin overlap region of the colliding ultrarelativistic nuclei.

FIG. 5. Initial longitudinal velocity profile along the reaction plane $y = 0$ for two different impact parameters for the collision of two hard-sphere nuclei with 7-fm radius.
Simple toy model initial state

Becattini, Piccinini, Rizzo, PRC 77, 024906 (2008)

Landau disc
but with initial flow velocity $v_z$

FIG. 4. Initial longitudinal velocity profile for the limiting case of sudden thermalization in the very thin overlap region of the colliding ultrarelativistic nuclei.

FIG. 6. Ratio of the term proportional to the vorticity and the term proportional to energy density gradient along $x$ in Eq. (16) as a function of $x$ for $y = 0$ and $y = 2$ fm for the collision of two hard-sphere nuclei with 7-fm radius at an impact parameter $b = 6$ fm.
Elliptic flow from spinning system

\[ v_2^{(J)} = \frac{\int d^3 x \frac{K_1 \left( m_T \sqrt{1 - |\omega \times x|^2 / T} \right)}{\sqrt{1 - |\omega \times x|^2}} I_2 \left( \frac{p_T z \omega}{T} \right)}{\int d^3 x \frac{K_1 \left( m_T \sqrt{1 - |\omega \times x|^2 / T} \right)}{\sqrt{1 - |\omega \times x|^2}} I_0 \left( \frac{p_T z \omega}{T} \right)} \]

FIG. 7. Elliptic flow coefficient \( v_2^{(J)} \) as a function of \( p_T \) for hadrons originated from a spherical spinning plasma at a chemical freeze-out \( T = 165 \text{ MeV} \) and a radius of 10.1 fm for \( \omega / T = 0.03 \). The elliptic flow would simply vanish if \( J = 0 \).
FIG. 5: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at $t=3.56$ fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$. The average vorticity in the reaction plane is 0.0538 / 0.10685 for the classical / relativistic weighted vorticity respectively.

Csernai, Magas, Wang, PRC 87 (2013) 034906
Vorticity ⇒ particle polarization

- T. Liang, X. N. Wang, PRL 94 (2005) 102301
  [Erratum-ibid. 96 (2006) 039901]


- J. H. Gao et al., PRC 77 (2008) 044902

- Becattini, Piccinini, Rizzo, PRC 77, 024906 (2008)

- Becattini, Csernai, Wang, arXiv:1304.4427, accepted to PRC
Detecting rotation: Lambda polarization

\[ \Pi(p) = \frac{\hbar c}{8m} \int \frac{dV}{n_F} \left( \nabla \times \beta \right) \]

\[ \beta^\mu(x) = \left( \frac{1}{T(x)} \right) u^\mu(x) \]

\[ \Pi_0(p) = \Pi(p) - \frac{P}{\varepsilon(\varepsilon + m)} \Pi(p) \cdot P \]

Becattini, Csernai, Wang, arXiv:1304.4427
Detecting initial rotation: Azimuthally sensitive HBT
UrQMD simulation

Pb+Pb

$E_{lab} = 8\, AGeV$

Graef, Lisa, Bleicher, arXive 1302.3408

**Figure 3.** Shape of the freeze out region from pions frozen out at different times (colored surface). The contour lines depict the position of the spectators in each timestep. The vector field shows the direction of movement at each position and time. The black arrowheads contribute to the directed flow while the magenta arrows contribute to the antiflow. The inlay shows the freeze out luminosity of pions versus freeze out time. The shaded region in the inlay highlights the luminosity corresponding to the timestep in the overall picture.
Detecting initial rotation: Differential HBT

L.P. Csernai, S. Velle, arXive 1305.0385
L.P. Csernai, S. Velle, D.J. Wang, arXive 1305.0396

\[ \Delta C(k_{\pm}, q_{out}) \equiv C(k_{+}, q_{out}) - C(k_{-}, q_{out}) = \frac{4 \exp(-R^2 q^2) \sinh\left(\frac{2u_x b k}{T_0}\right) (1-\epsilon^2) \left[1 - \cosh\left(\frac{u_x b q}{T_0}\right) \cos(\alpha d x)\right]}{\left[(1+\epsilon^2) \cosh\left(\frac{2u_x b k}{T_0}\right) + (1-\epsilon^2)^2\right] - 4\epsilon^2 \sinh^2\left(\frac{2u_x b k}{T_0}\right)}. \]

**FIG. 14.** (color online) Two moving sources in the reaction \([(x-z)]\) plane, separated in the \(x-\) direction. The sources are moving in the directions indicated by the (red) arrows. The two "tilted" detector directions are indicated by the (blue) arrows labeled with \(k_+\) and \(k_-\).

**FIG. 3.** (color online) The flow velocity dependence of the differential correlation function at the final time.
Conclusions

Initial state with large initial angular momentum at LHC energies results in rotating “fireball”

The effect of such a rotation may be seen in:

- Directed flow, $v_1$
- Elliptic flow, $v_2$
- Particle polarization
- HBT
Back up
Effective string rope model

Transparency in the first moment
+ stretching chromoelectric field
+ energy-momentum conservation \( \rightarrow \)
  deceleration of the colliding partons
\[ y_{CM} \text{ is fluctuating around “0” with Gaussian distribution} \]

\[ f(y_{CM}) = \frac{1}{\sqrt{2\pi\delta y^2}} e^{-y_{CM}^2 / \delta y^2} \]

FIG. 5. Participant center-of-mass rapidity distribution for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for three different centralities.

Drastic effect: no peaks!
Similar results within quark-gluon string model with parton rearrangement