Collective excitations in anisotropic quark-gluon plasma

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Outline

- Motivation
- General dispersion equation
- Isotropic system
- Weakly anisotropic system
- Finite prolateness or oblateness
- Extremely prolate system
- Extremely oblate system
- Conclusions

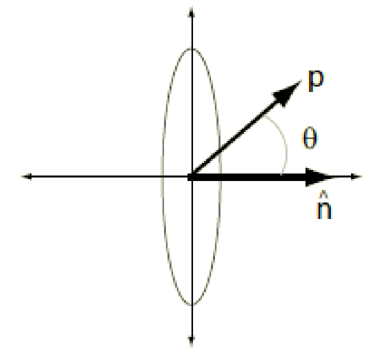
Motivation

- Spectrum of collective excitations is an important characteristics of any many body system.
- Anisotropic plasma is qualitatively different than the isotropic one.
- QGP from relativistic heavy-ion collisions is anisotropic.
- Existing analyses of collective excitations are not complete.

Momentum distribution

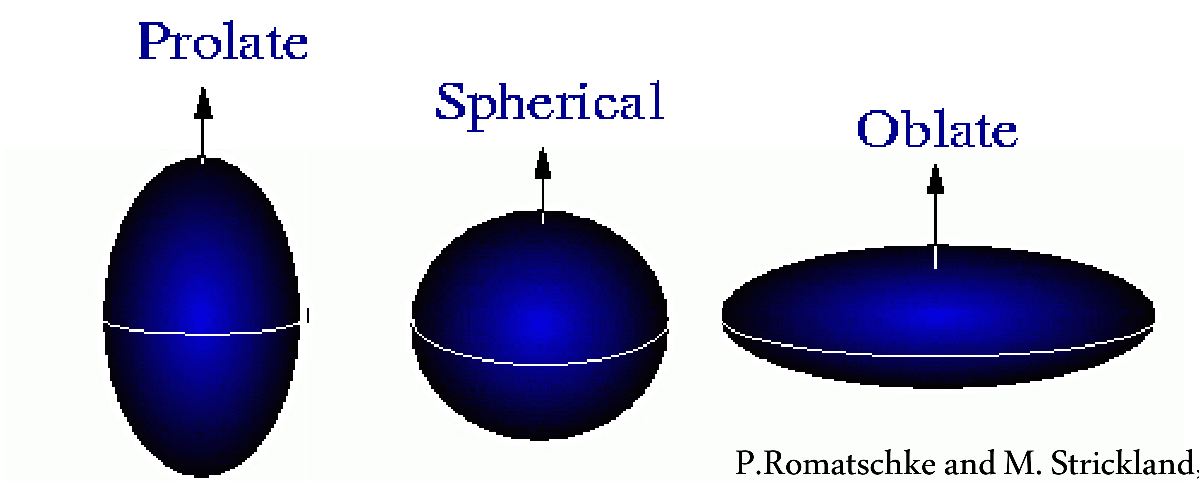
The anisotropic momentum distribution is obtained from an isotropic one by rescaling it in one direction

$$f_\xi(\mathbf{p}) = \sqrt{1+\xi}\; f_{\rm iso}\left(\sqrt{\mathbf{p}^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$



$\xi \in (-1, \infty)$

$\xi < 0$ $\quad\quad$ $\xi = 0$ $\quad\quad$ $\xi > 0$



P.Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003)

General dispersion equation

Gluon polarization tensor can be written down, as

$$\Pi^{ij}(\omega,\mathbf{k}) = \frac{g^2}{2}\int \frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{|\mathbf{p}|}\left[\delta^{ij} + \frac{k^i v^j + v^i k^j}{\omega - \mathbf{k}\cdot\mathbf{v}} + \frac{(\mathbf{k}^2-\omega^2)v^i v^j}{(\omega-\mathbf{k}\cdot\mathbf{v})^2}\right]$$

The dielectric tensor is related to the retarded gluon polarization tensor

$$\varepsilon^{ij}(\omega,\mathbf{k}) = \delta^{ij} - \frac{1}{\omega^2}\Pi^{ij}(\omega,\mathbf{k})$$

Definition of the matrix sigma

$$\Sigma^{ij} \equiv (\omega^2 - \mathbf{k}^2)\delta^{ij} + k^i k^j - \Pi^{ij}$$

inverse gluon propagator in temporal axial gauge

The dispersion equation

$$\det\left[\Sigma(\omega,\mathbf{k})\right] = 0$$

$\omega(\mathbf{k})$ - collective mode in a plasma system

How to invert matrix Σ ?

Method to inverse the matrix Σ

Inversion of the matrix Σ which depends on **k** and **n**

$$\Sigma = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2}$$

$$C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j$$

$$n_T^i = \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right) n^j$$

$$\Sigma^{-1} = \bar{\alpha} A + \bar{\beta} B + \bar{\gamma} C + \bar{\delta} D$$

$$\Sigma \Sigma^{-1} = \mathbf{1} \quad \Rightarrow \quad \bar{\alpha}, \bar{\beta}, \bar{\gamma}, \bar{\delta}$$

The coefficients α, β, γ, δ are determined by the following contractions:

$$k^i \Sigma^{ij} k^j = k^2 \beta, \qquad n_T^i \Sigma^{ij} k^j = n_T^2 k^2 \delta,$$

$$n_T^i \Sigma^{ij} n_T^j = n_T^2 (\alpha + \gamma), \qquad \mathrm{Tr}\Sigma = 2\alpha + \beta + \gamma,$$

P. Romatschke and M. Srickland, Phys. Rev. D **68**, 036004 (2003)

Collective mode in isotropic QGP

In isotropic plasma the matrix Σ is decomposed as: $\Sigma^{ij} = \alpha_{\rm iso} A^{ij} + \beta_{\rm iso} B^{ij}$

$$\alpha_{iso}(\omega,\mathbf{k}) = \omega^2 - k^2 - \frac{m_D^2\omega^2}{2k^2}\left[1 - \left(\frac{\omega}{2k} - \frac{k}{2\omega}\right)\ln\left(\frac{\omega+k}{\omega-k}\right)\right]$$

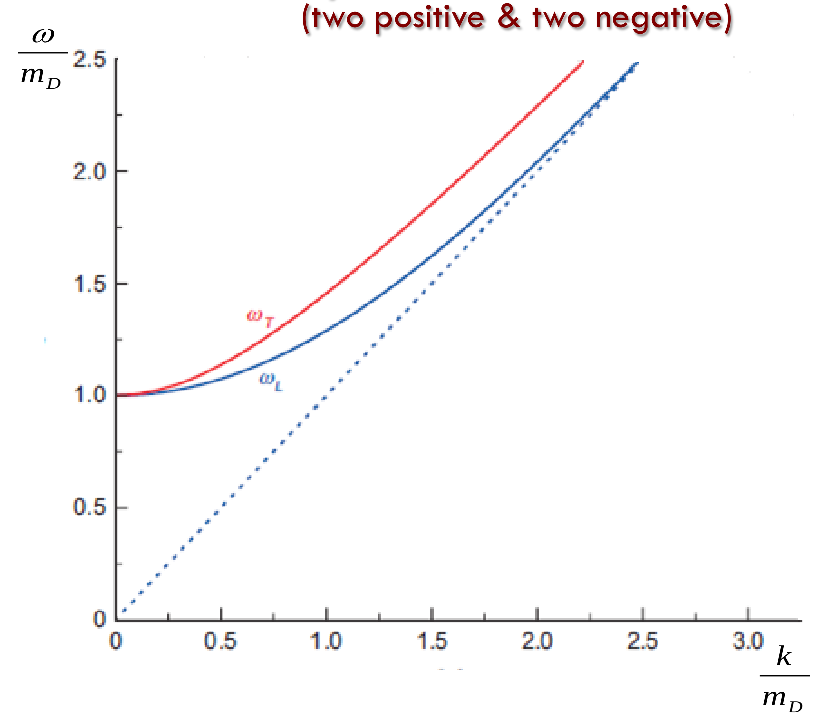
and

$$\beta_{iso}(\omega,\mathbf{k}) = \omega^2 + \frac{m_D^2\omega^2}{k^2}\left[1 - \frac{\omega}{2k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right]$$

$$m_D = g^2\int\frac{d^3p}{(2\pi)^3}\frac{f_\xi(\mathbf{p})}{|\mathbf{p}|}$$ Debye mass

Only four real solutions

(two positive & two negative)



Weakly anisotropic system

Weakly anisotropic distribution $|\xi| << 1$

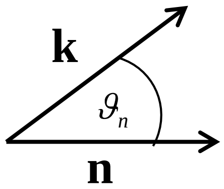
$$f_\xi(\mathbf{p}) \approx \left(1+\frac{\xi}{2}\right) f_{\text{iso}}(p) + \frac{\xi}{2}\frac{d\ f_{\text{iso}}(p)}{d\ p}\, p(\mathbf{v}\cdot\mathbf{n})^2$$

Coefficients α, β, γ, δ are found in analytic form

$$\alpha(\omega,\mathbf{k}) = \left(1+\frac{\xi}{2}\right)\alpha_{iso}(\omega,\mathbf{k}) + \xi\frac{m_D^2}{8}\left\{\frac{8}{3}\cos^2\vartheta_n + \frac{2}{3}\left(5-19\cos^2\vartheta_n\right)\frac{\omega^2}{k^2} - 2\left(1-5\cos^2\vartheta_n\right)\frac{\omega^4}{k^4}\right.$$

$$\left. + \left[1-3\cos^2\vartheta_n - \left(2-8\cos^2\vartheta_n\right)\frac{\omega^2}{k^2} + \left(1-5\cos^2\vartheta_n\right)\frac{\omega^4}{k^4}\right]\frac{\omega}{k}\ln\left(\frac{\omega+k}{\omega-k}\right)\right\},$$

$$\beta(\omega,\mathbf{k}) = \ldots,$$
$$\gamma(\omega,\mathbf{k}) = \ldots,$$
$$\delta(\omega,\mathbf{k}) = \ldots$$



Weakly anisotropic system

Dispersion equations:

1) $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

2) $\delta^2(\omega,\mathbf{k})k^2 n_T^2-\left(\beta(\omega,\mathbf{k})-\omega^2\right)\left(\alpha(\omega,\mathbf{k})+\gamma(\omega,\mathbf{k})-\omega^2+k^2\right)=0$

In the limit of weak anisotropy, we have three dispersion equations because $\delta^2=O(\xi^2)$

1) $\alpha(\omega,\mathbf{k})-\omega^2+k^2=0$

2) $\beta(\omega,\mathbf{k})-\omega^2=0$

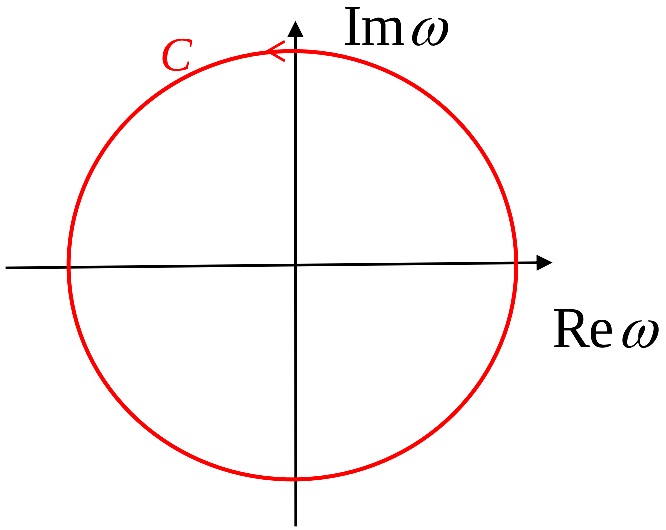
3) $\alpha(\omega,\mathbf{k})+\gamma(\omega,\mathbf{k})-\omega^2+k^2=0$

Nyquist analysis

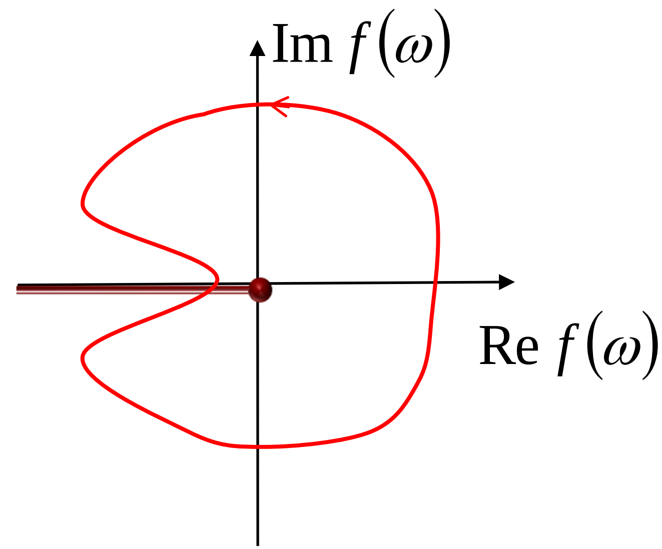
Nyquist analysis allows one to find the number of solution of the equation

$$f(\omega)=0$$

$$\oint_C \frac{d\omega}{2\pi i}\frac{f'(\omega)}{f(\omega)}=\frac{\ln f(\omega)}{2\pi i}\Bigg|_{\omega_0^+}^{\omega_0^-}=number\ of\ \ solutions\ in\ C$$



$$f(\omega)\,analytic\ \ in\ C$$



Equation $\alpha(\omega, \mathbf{k}) - \omega^2 + k^2 = 0$

$k^2 - \xi \frac{m_D^2}{3} \cos^2 \vartheta_n \geq 0$ **2 solutions**

$k^2 - \xi \frac{m_D^2}{3} \cos^2 \vartheta_n < 0$ **4 solutions**

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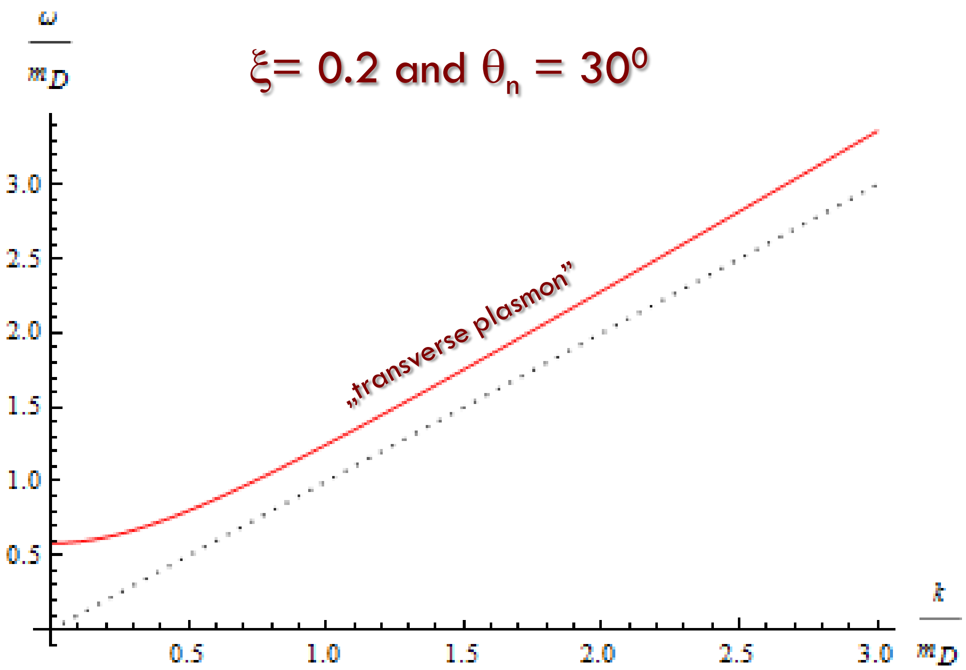
$$\omega^2(\mathbf{k})\underset{k<<m_D}{\approx}\frac{m^2}{3}\left(1+\frac{\xi}{10}\right)+\frac{6}{5}\left[1+\frac{\xi}{14}\left(\frac{4}{15}+\cos^2\vartheta_n\right)\right]k^2$$



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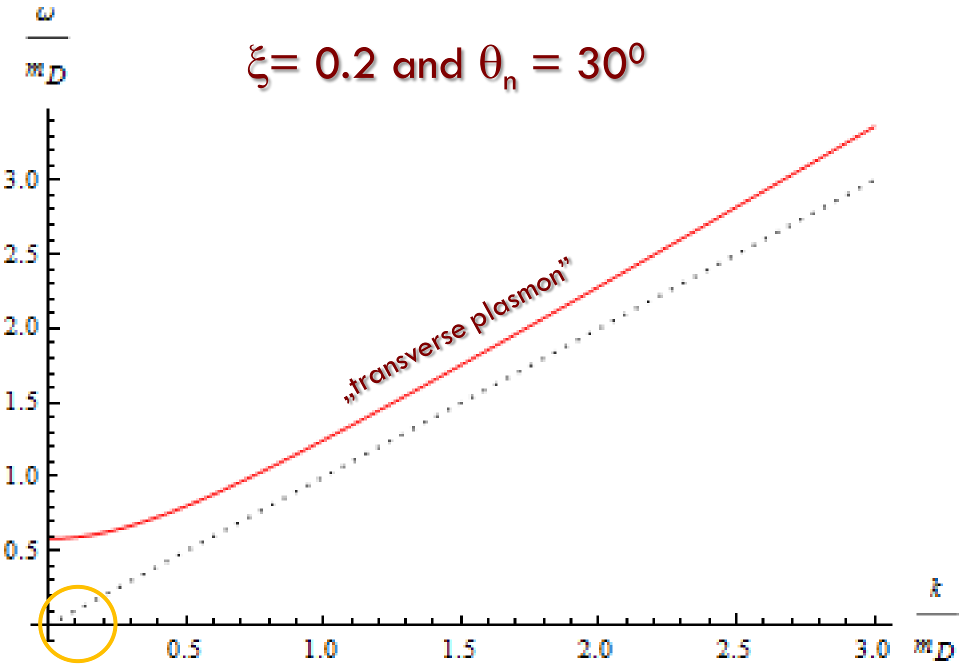
$$\omega(\mathbf{k})=\pm i\gamma(\mathbf{k})$$

$$\gamma(\mathbf{k})\underset{k>>\gamma}{\approx}\frac{1}{2}\left(\sqrt{\frac{\lambda^2}{k^2}+4\left(\eta-k^2\right)}-\frac{\lambda}{k}\right)$$

$\xi = 0.2$ and $\theta_n = 30^0$



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where

$$\lambda\equiv\frac{\pi}{4}\left(1+\frac{3}{2}\xi\cos^2\vartheta_n\right)m_D^2$$

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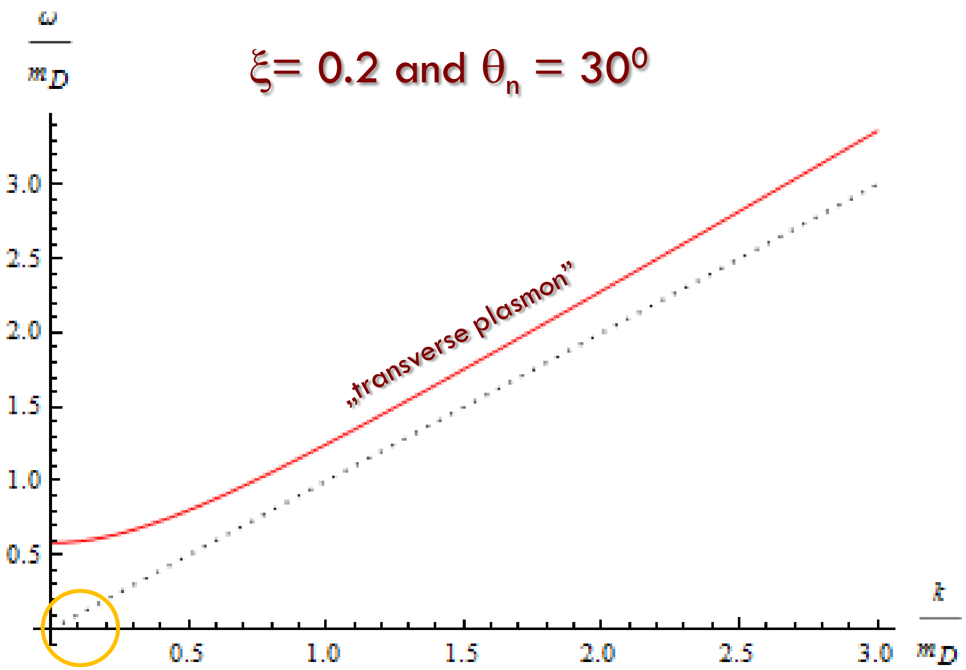

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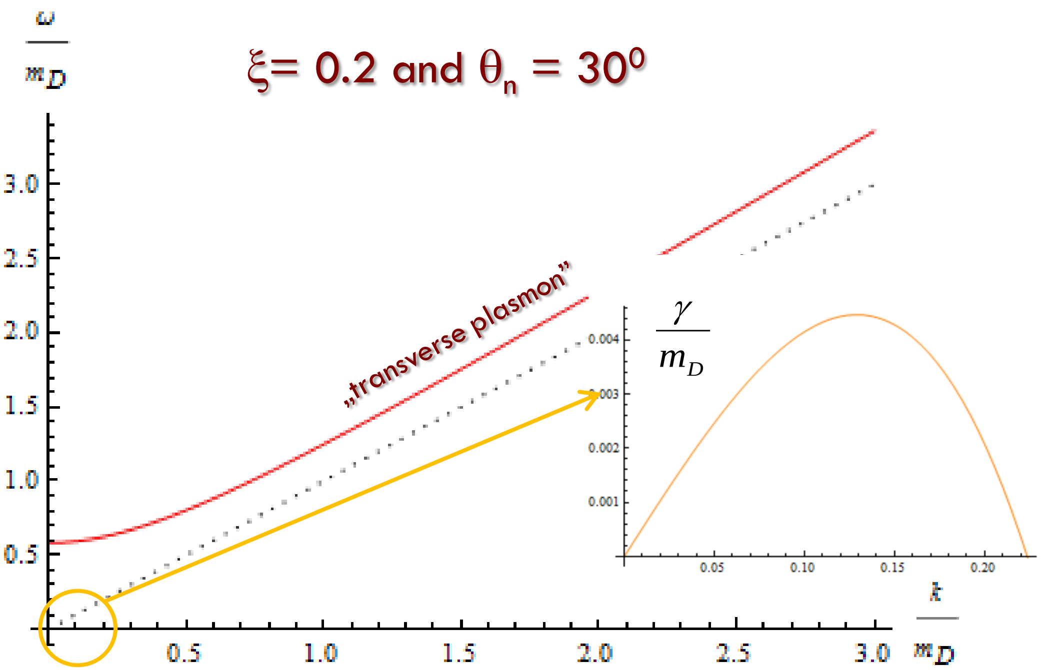

Equation $\beta(\omega, \mathbf{k}) - \omega^2 = 0$

There are always two solutions

$$\omega^2(\mathbf{k}) \underset{k<<m_D}{\approx} \frac{m_D^2}{3}\left(1+\frac{\xi}{5}\left(1+\cos^2\vartheta_n\right)\right)+\frac{3}{5}\left[1+\frac{\xi}{280}\left(102-61\cos^2\vartheta_n\right)\right]k^2$$

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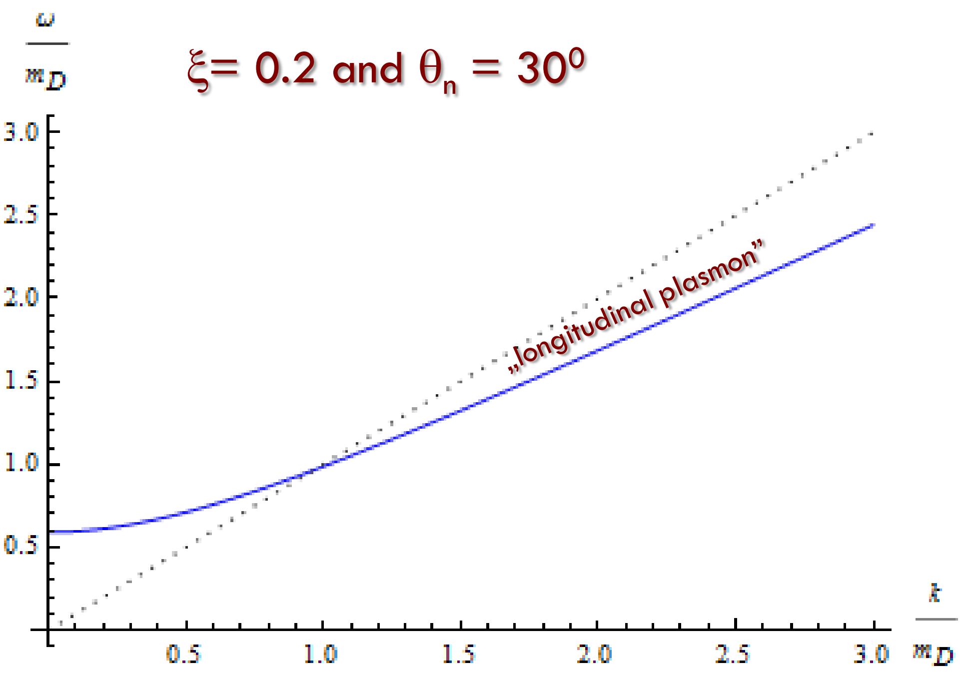

Equation $\alpha(\omega,\mathbf{k})+\gamma(\omega,\mathbf{k})-\omega^2+k^2=0$

$k^2+\xi\frac{m_D^2}{3}(1-2\cos^2\vartheta_n)\geq 0$ **2 solutions** $k^2+\xi\frac{m_D^2}{3}(1-2\cos^2\vartheta_n)<0$ **4 solutions**

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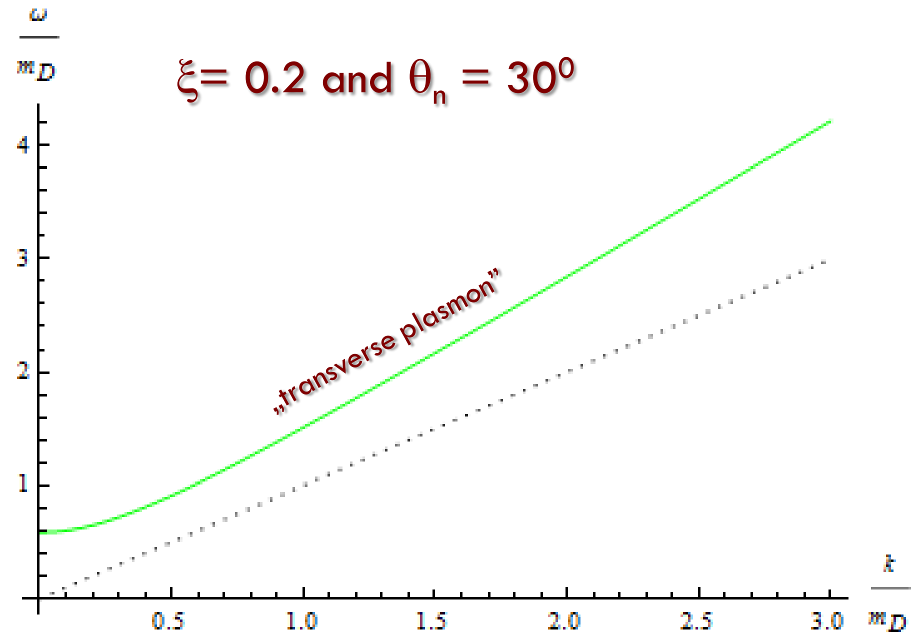

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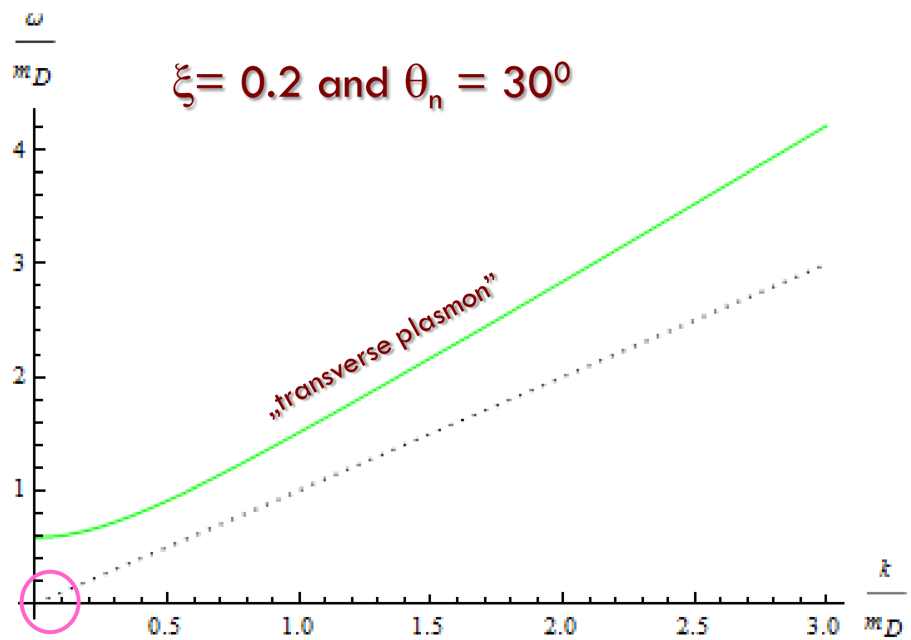

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where

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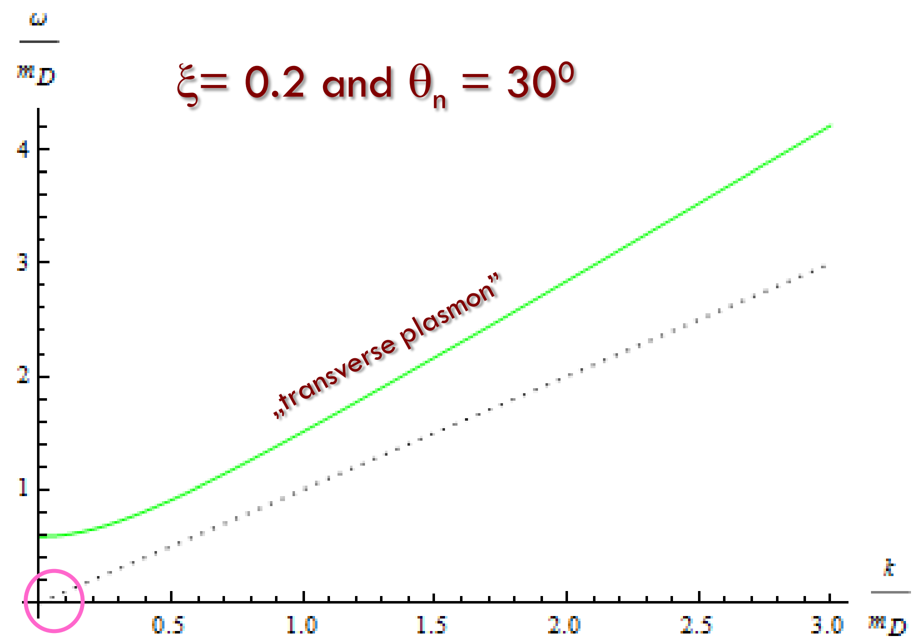

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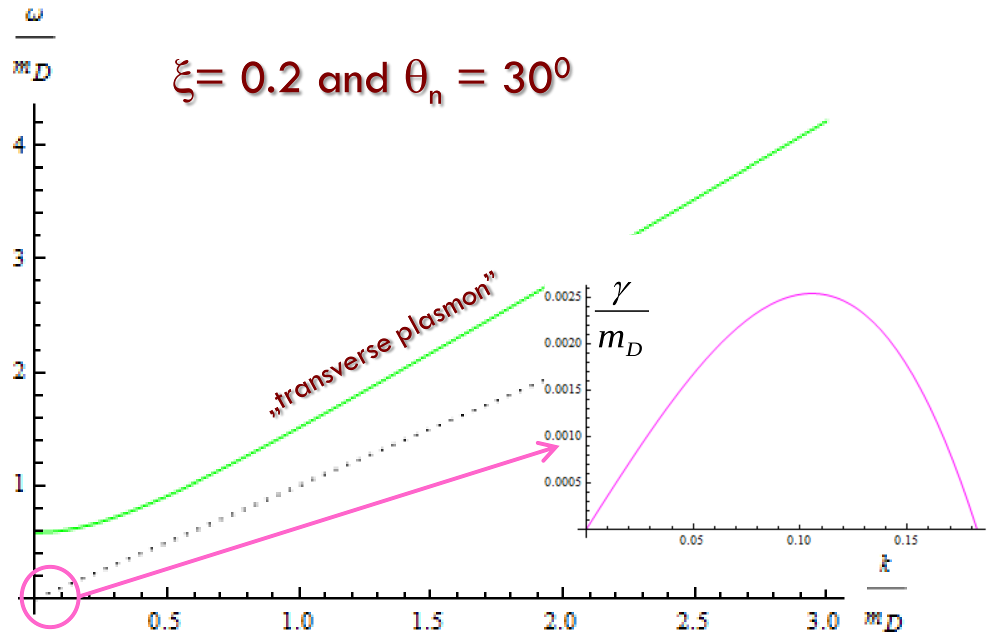
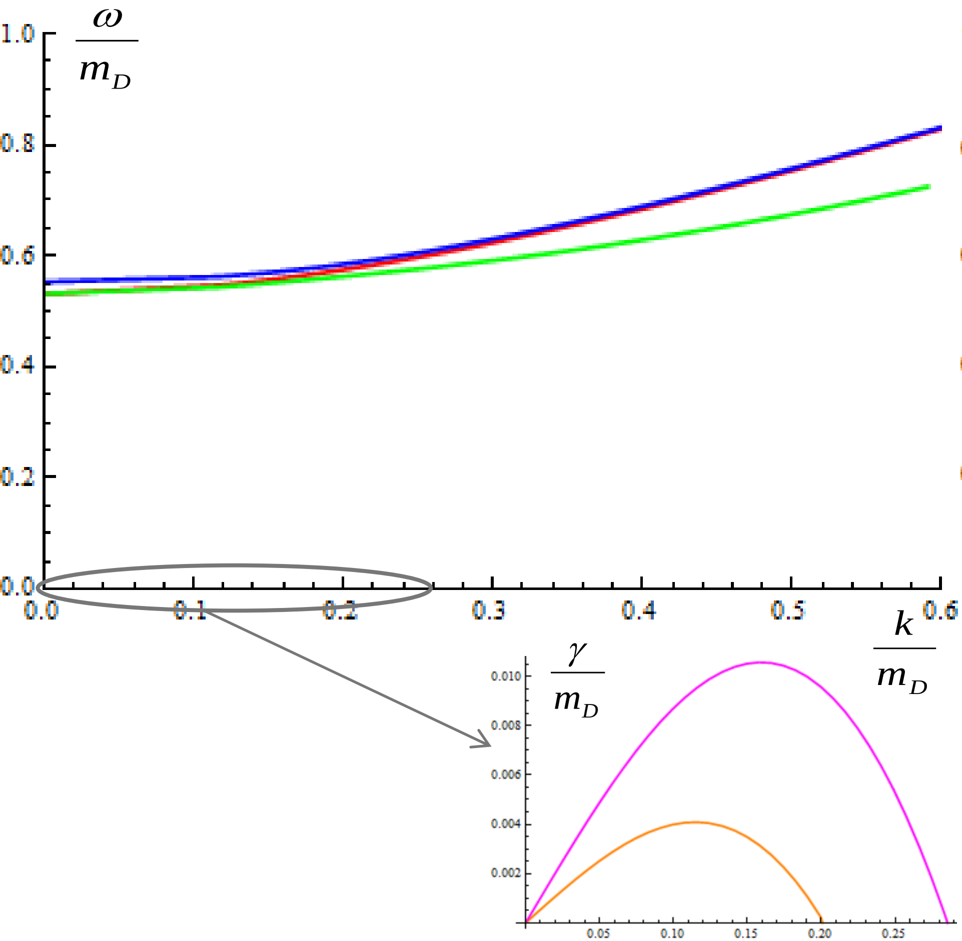
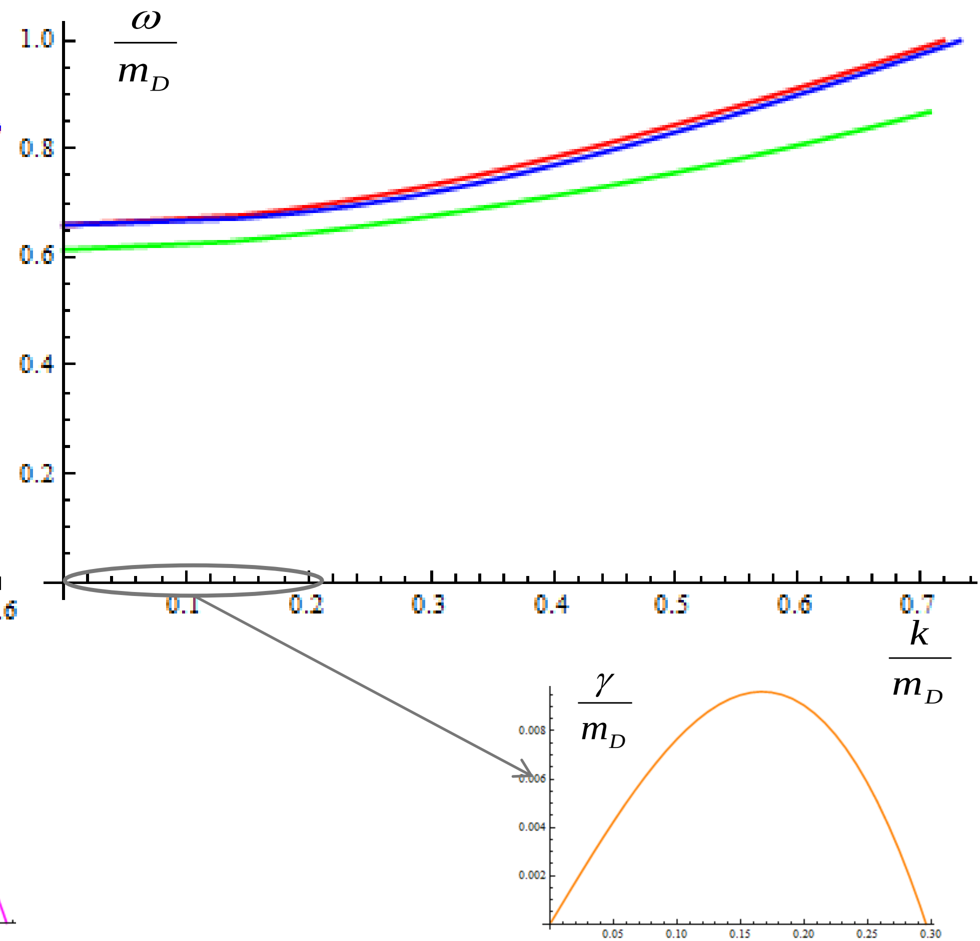


Finite prolateness or oblateness

$\xi = 0.5$ and $\theta_n = 30^0$



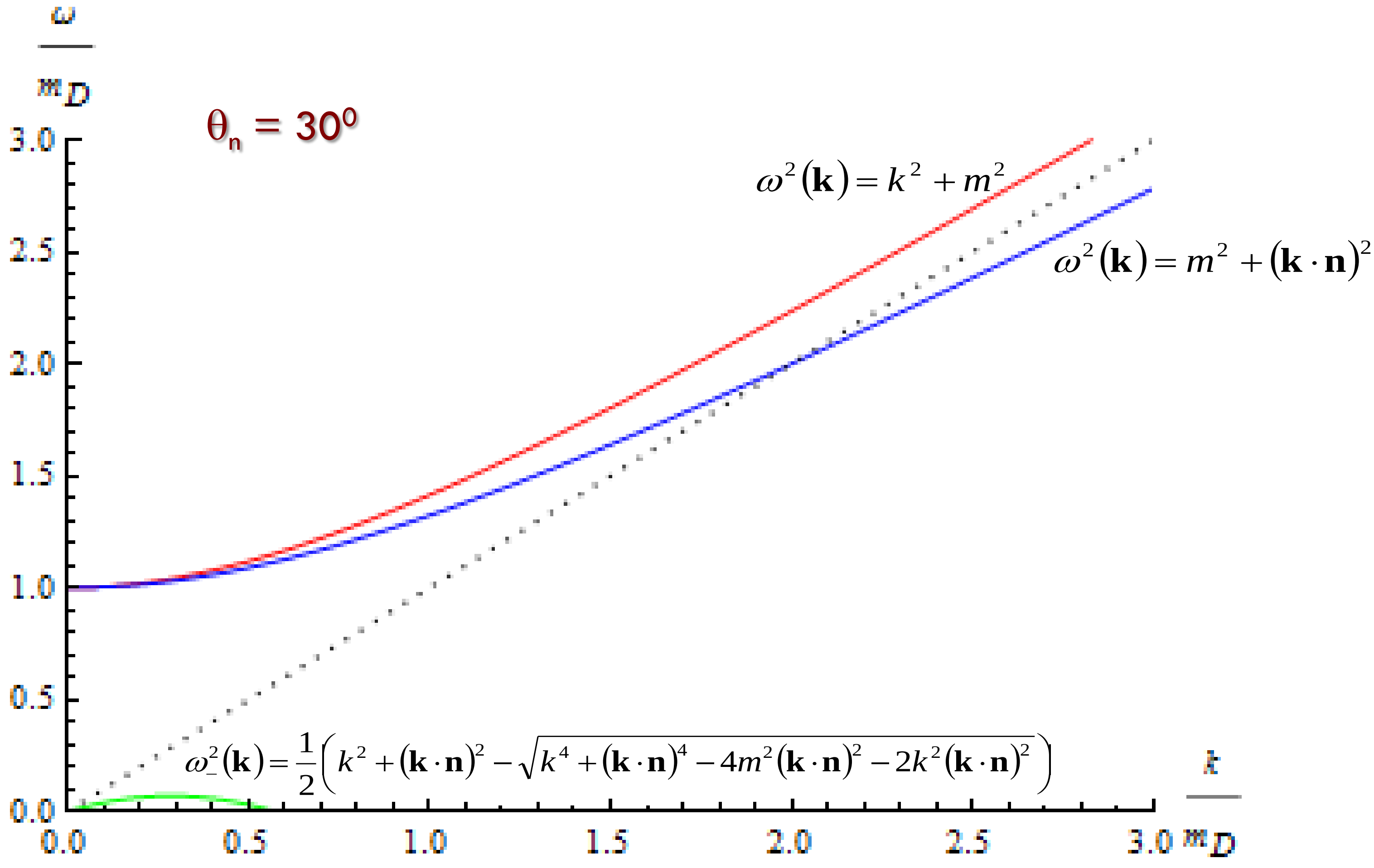
$\xi = -0.5$ and $\theta_n = 50^0$



Extremly prolate QGP

Extremly prolate distribution: $f(\mathbf{p}) \sim \delta(p_T)$

6 solutions

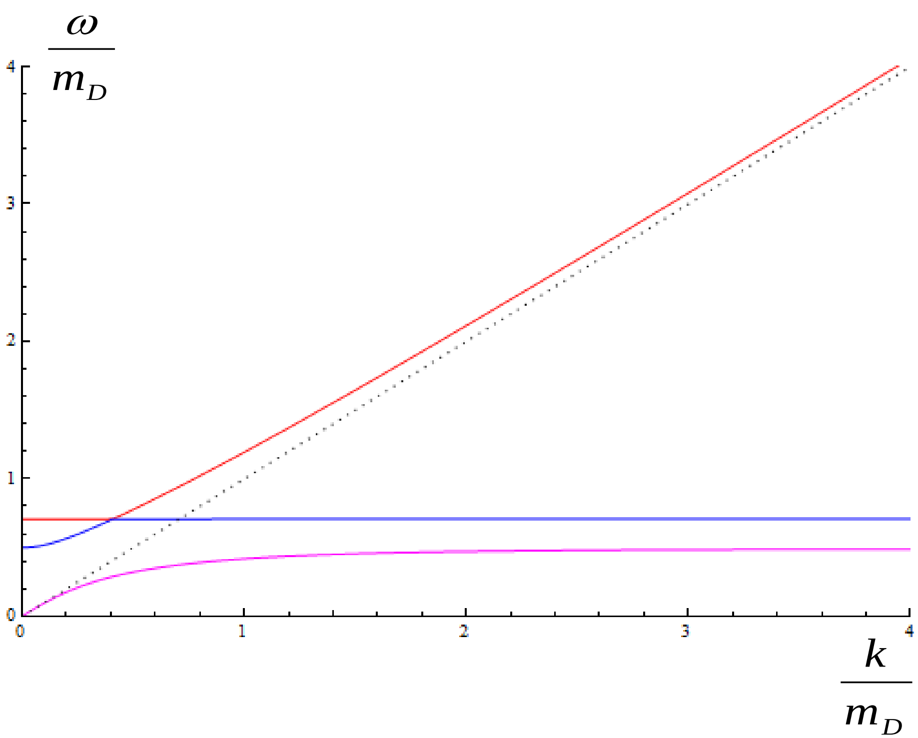


Extremly oblate QGP $\xi \to \infty$

Extremly oblate distribution: $f(\mathbf{p}) \sim \delta(p_L)$

8 or 10 solutions

ξ= ∞ and $\theta_n = 0^0$



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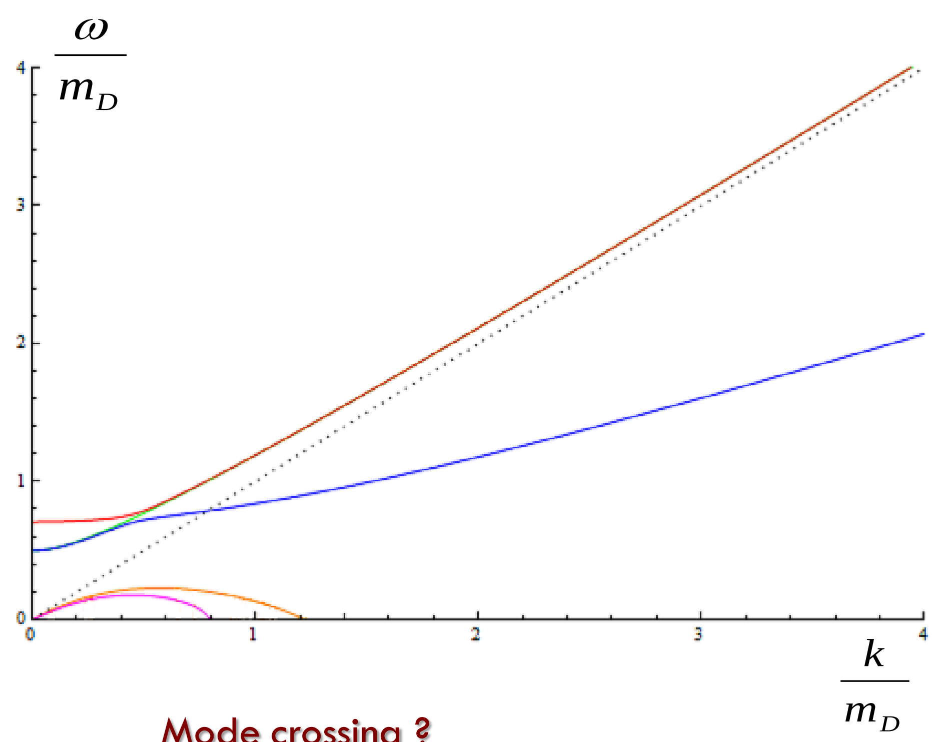
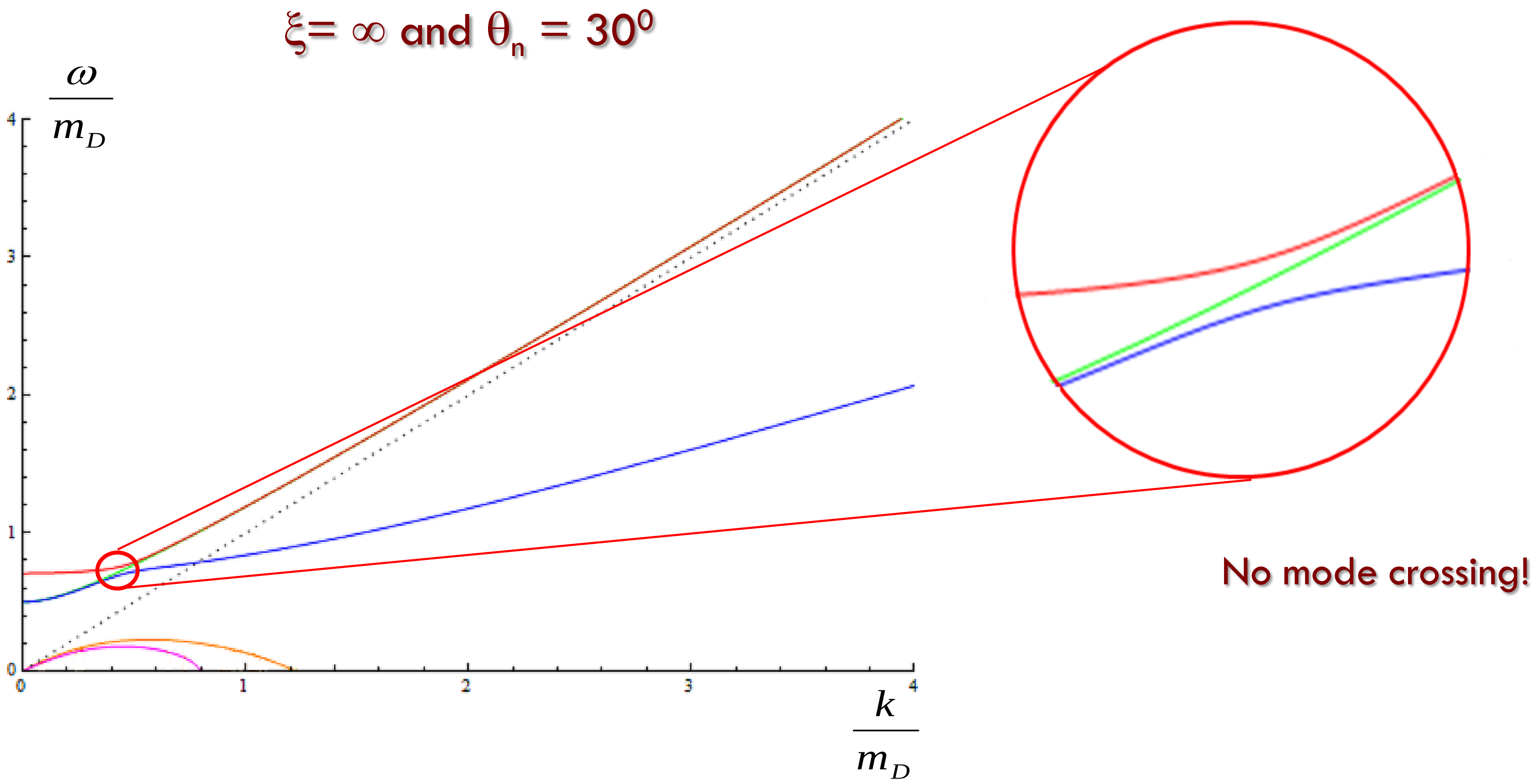


Mode crossing ?

Extremly oblate QGP $\xi \to \infty$

Extremly oblate distribution: $f(\mathbf{p}) \sim \delta(p_L)$



Conclusions

- Systematical analysis of the complete mode spectrum is performed.
- The number of modes is found in every case.
- Analytical and numerical solutions are found.
- Complete spectrum of modes is needed to compute various plasma characteristics e.g. the energy loss in anisotropic QGP.