Generalized medium-induced gluon radiation and its coherence properties

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Motivation

- Production of QGP is expected in heavy-ion collisions:
  - Jet Quenching experimentally confirmed:
    - Several observations @ RHIC and LHC
      - Suppression of high-p_T hadrons, strong dijet asymmetry with an almost unchanged azimuthal correlation, jet energy loss with a mild dependency in p_T and jet radius, ...

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  - Most models claim to describe some or all of jet quenching observables at the same time (Q-Pythia, as an example, 1211.1161)

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Qualitative agreement with data.
But there is space for improvement!

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- Different level of sophistication between Monte Carlo codes and analytical expressions:

$\boxed{\text{Monte Carlo}}$

Include energy conservation by construction

↓

Make further assumptions without a firm theoretical model
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Previous works:
[1204.2929]: Single-gluon emission spectrum in the hard limit;
[1209.4585]: 3-gluon splitting vertex with improved kinematics;
[1006.1367,1109.5619]: SCET

Jet Quenching

- Jet Quenching:
  - High-energy particles propagating through an extended colored medium experience energy loss phenomena
    - Ex: Medium-induced radiation, elastic scatterings with medium constituents.

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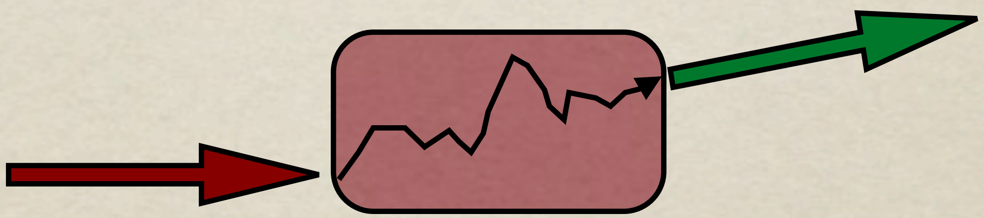

    - Propagators described by a Green's Function:

Initial/Final transverse coordinates

Path-Integral

Longitudinal particle momentum

$$G(x_{0+}, \mathbf{x}_{0\perp}; L_+, \mathbf{x}_\perp | p_+) = \int_{\mathbf{r}_\perp(x_{0+})=\mathbf{x}_{0\perp}}^{\mathbf{r}_\perp(L_+)=\mathbf{x}_\perp} \mathcal{D}\mathbf{r}_\perp(\xi) \exp\left\{ \frac{ip_+}{2} \int_{x_{0+}}^{L_+} d\xi \left(\frac{d\mathbf{r}_\perp}{d\xi} \right)^2 \right\} \times W(x_{0+}, L_+; \mathbf{r}_\perp(\xi)),$$

Wilson Line

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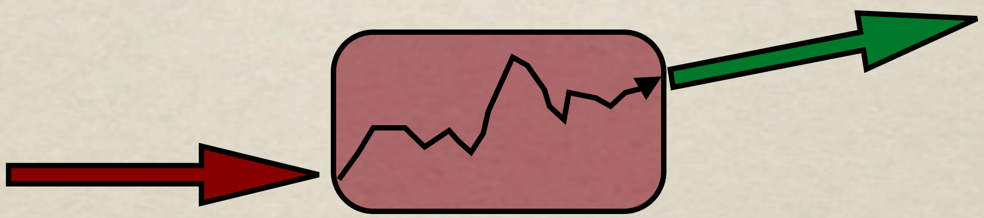

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Color Rotation

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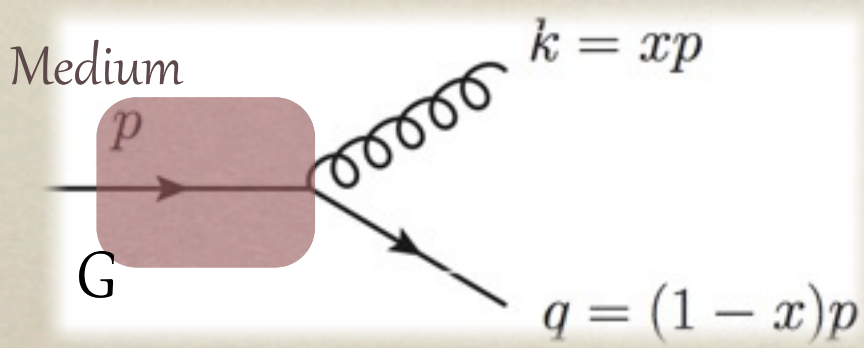
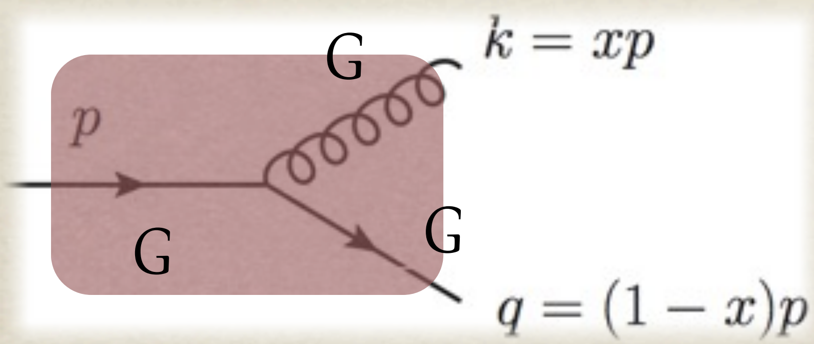
Brownian Motion

Medium-induced gluon radiation

- What we want to describe?
  - Gluon emission off a quark inside a medium:





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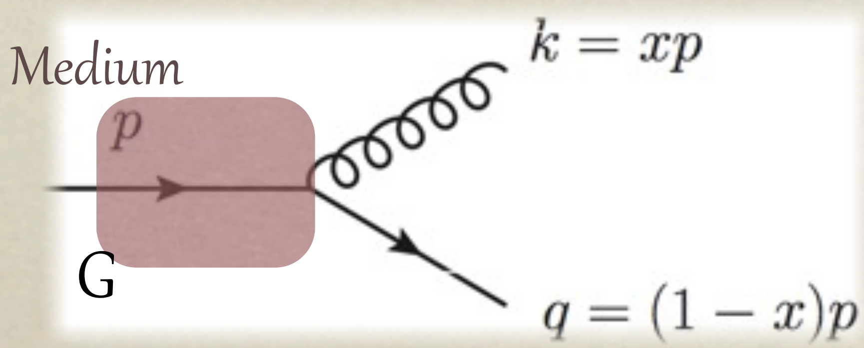
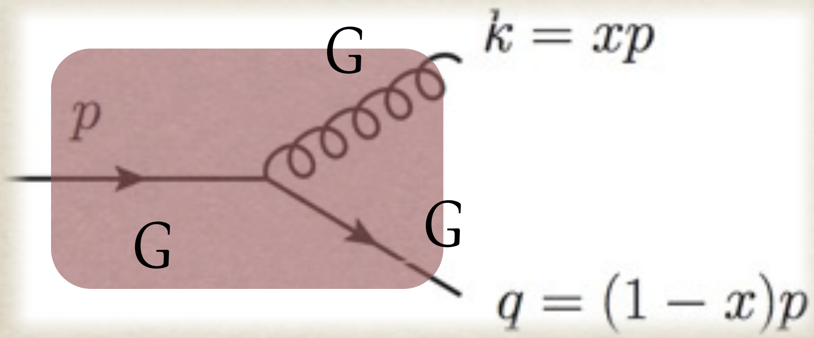
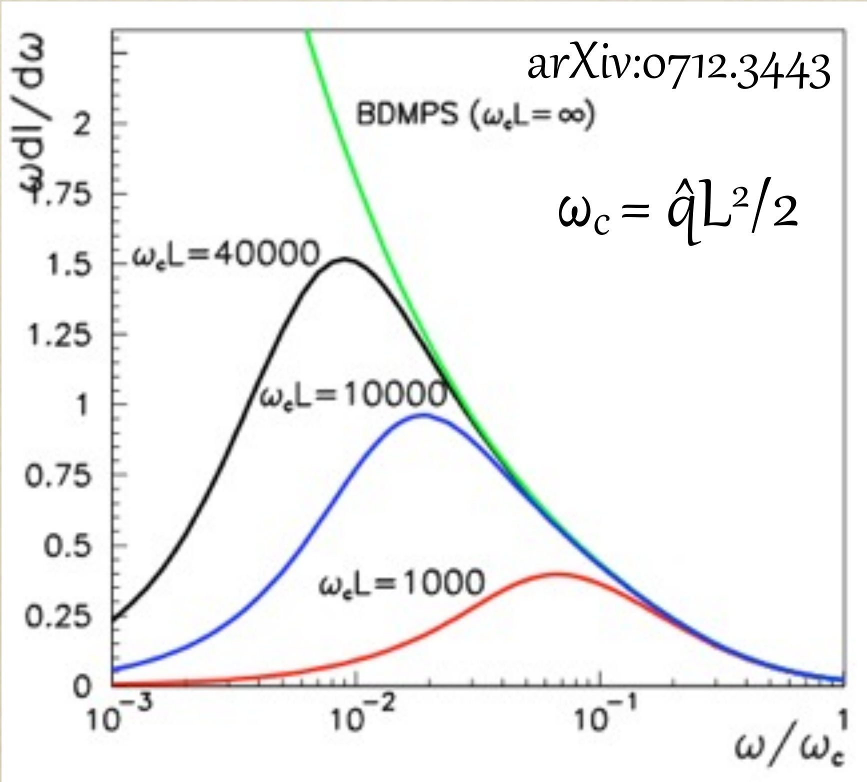




- Splitting g→gg already computed in (arXiv:1209.4585).
  - But, assuming small t_{form}...
    $t_{form} << L \Rightarrow \omega << \omega_c$



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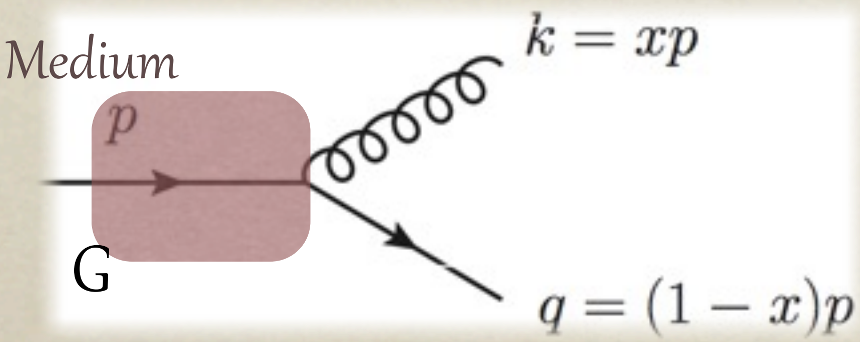

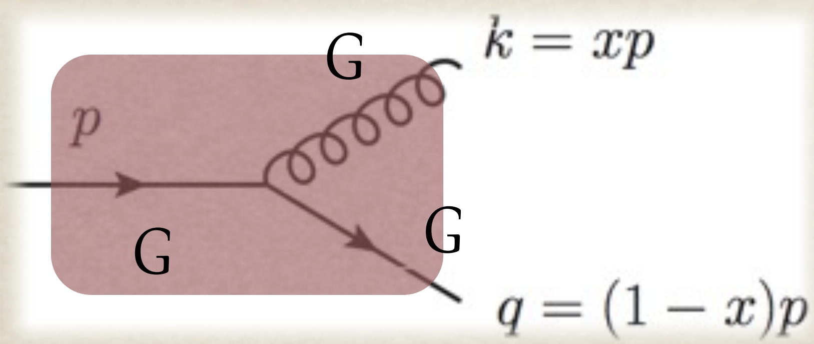

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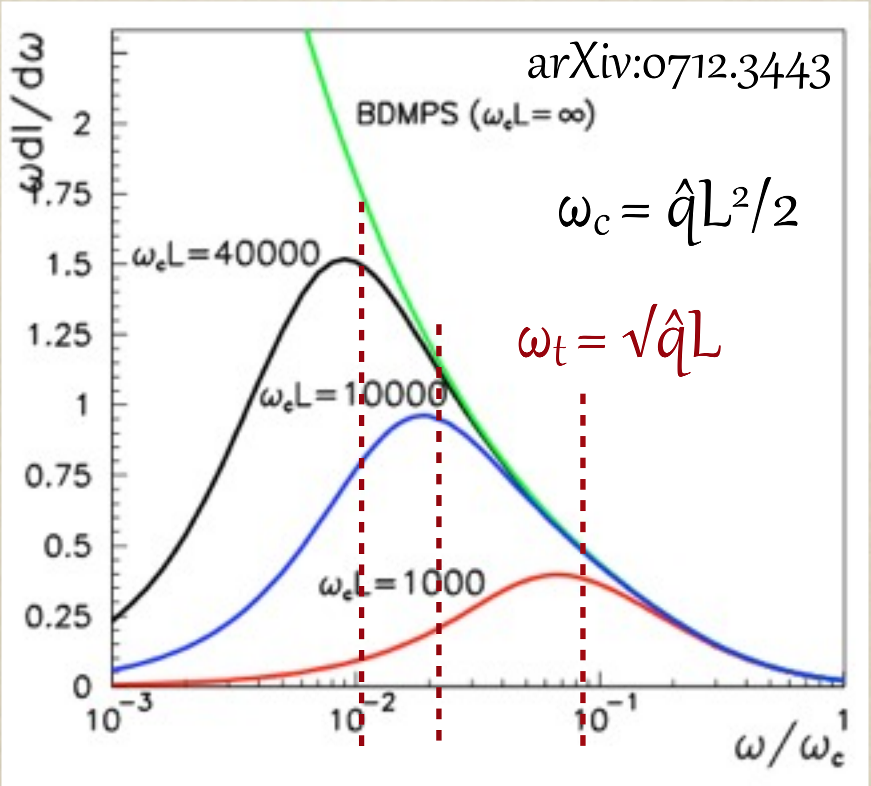

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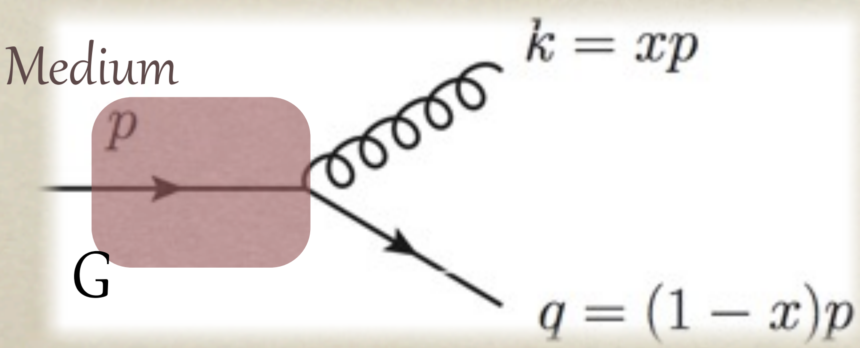

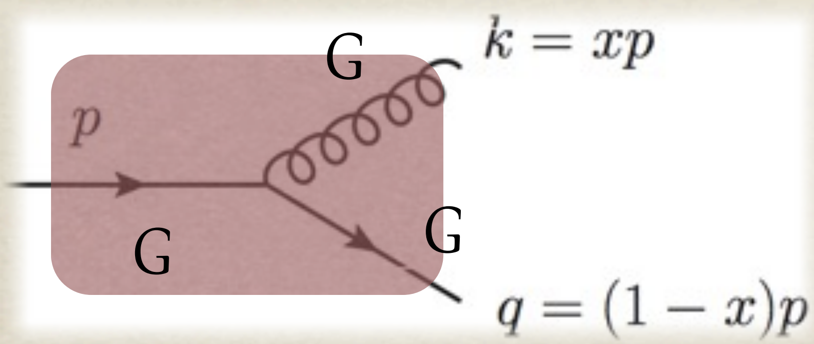

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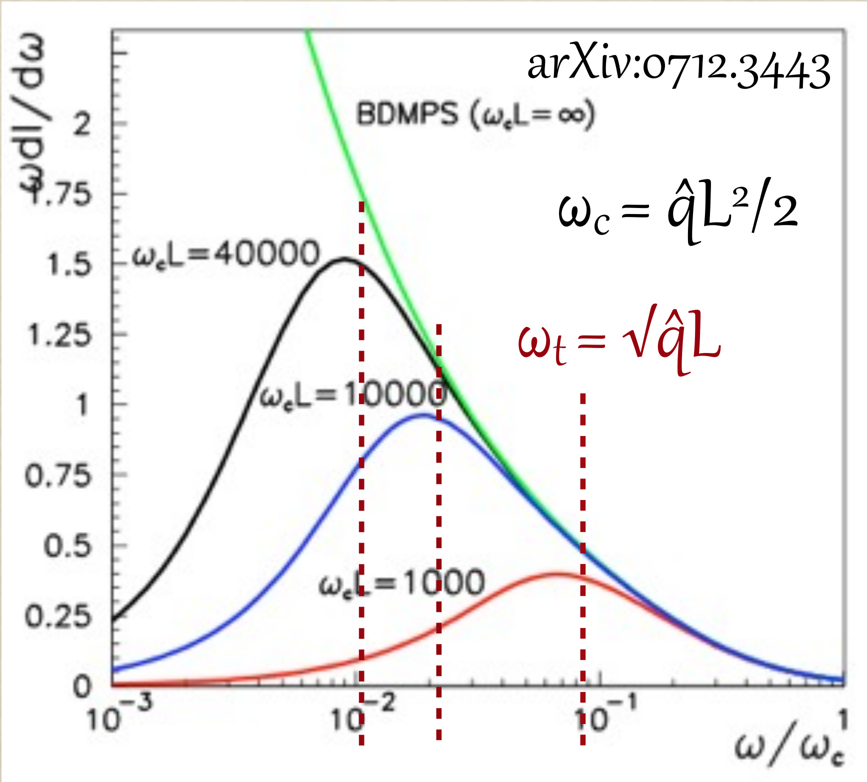
$t_{form} << L \Rightarrow \omega << \omega_c$

$\omega_t << \omega_c ? \Rightarrow \hat{q}L^3 >> 1$?

Already satisfied for:

$\hat{q} = 1\ GeV^2 fm^{-1}$ and $L = 1\ fm$



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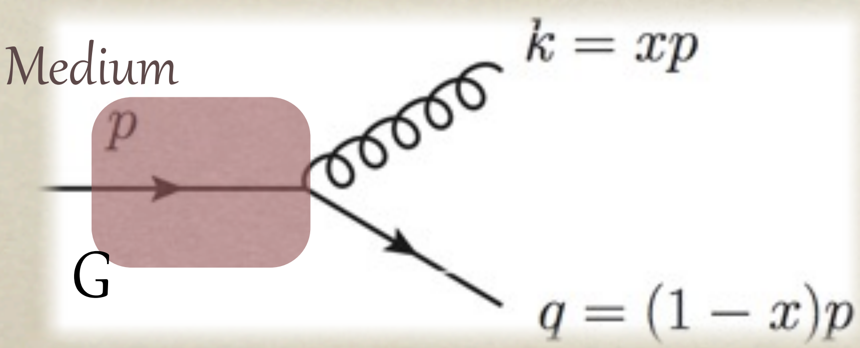

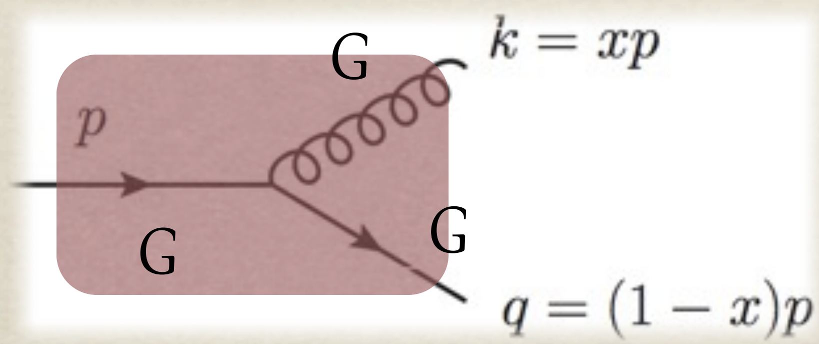

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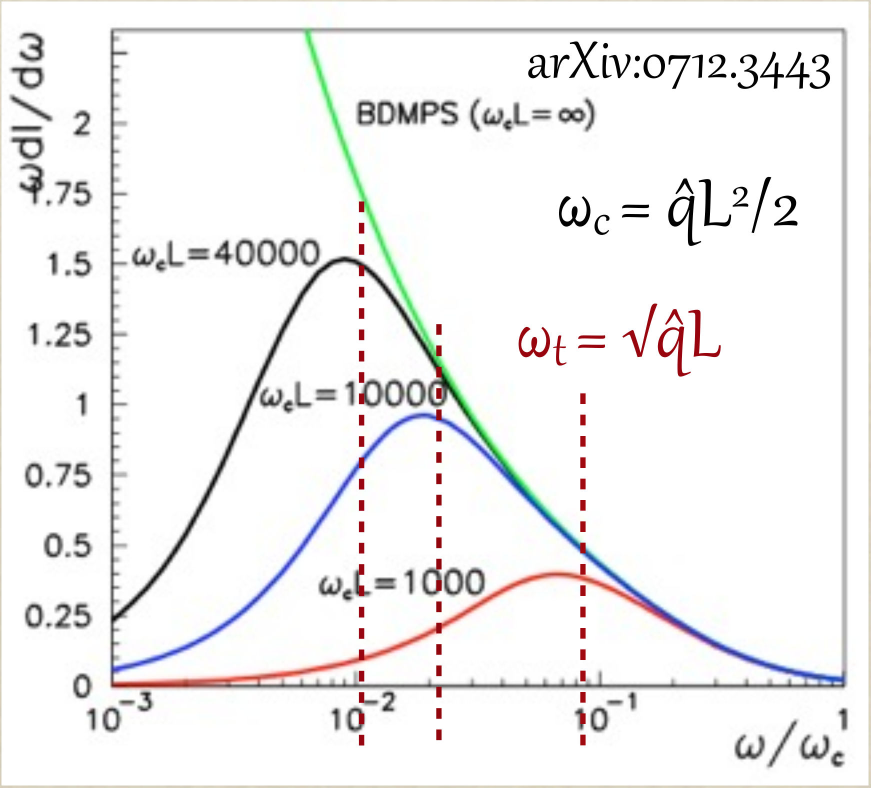
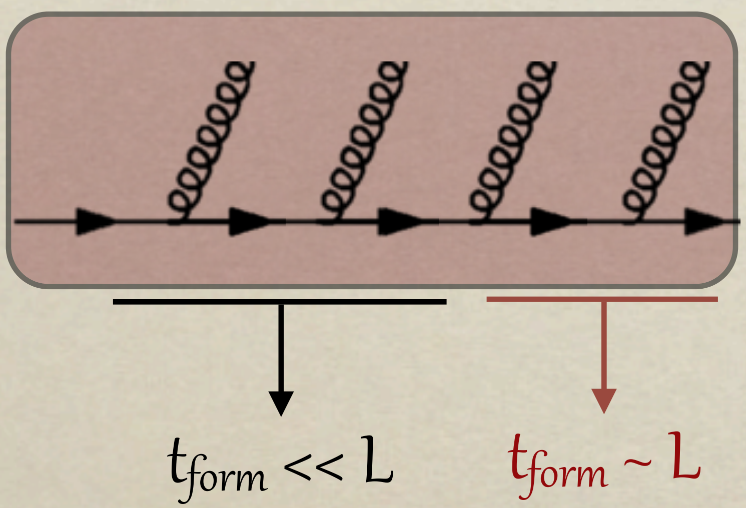


Nevertheless...

Medium-induced gluon radiation

- Improvement in the approximation $t_{form} << L$:
  - Approximation valid for a fixed L, but how about the showering nearer the medium border?



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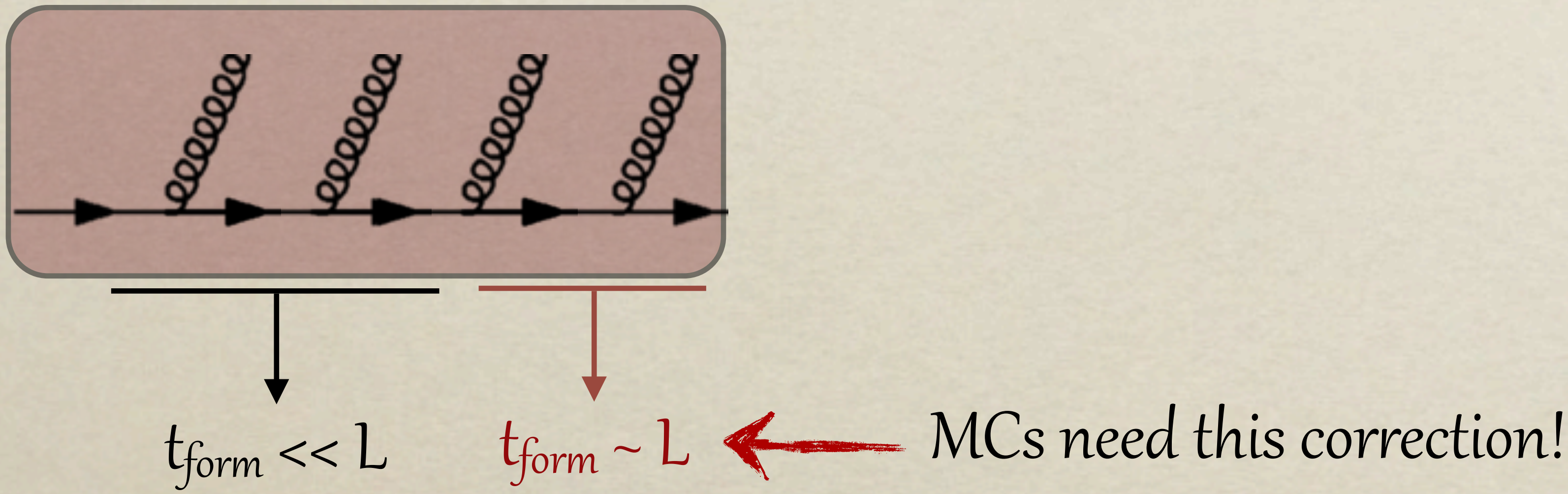

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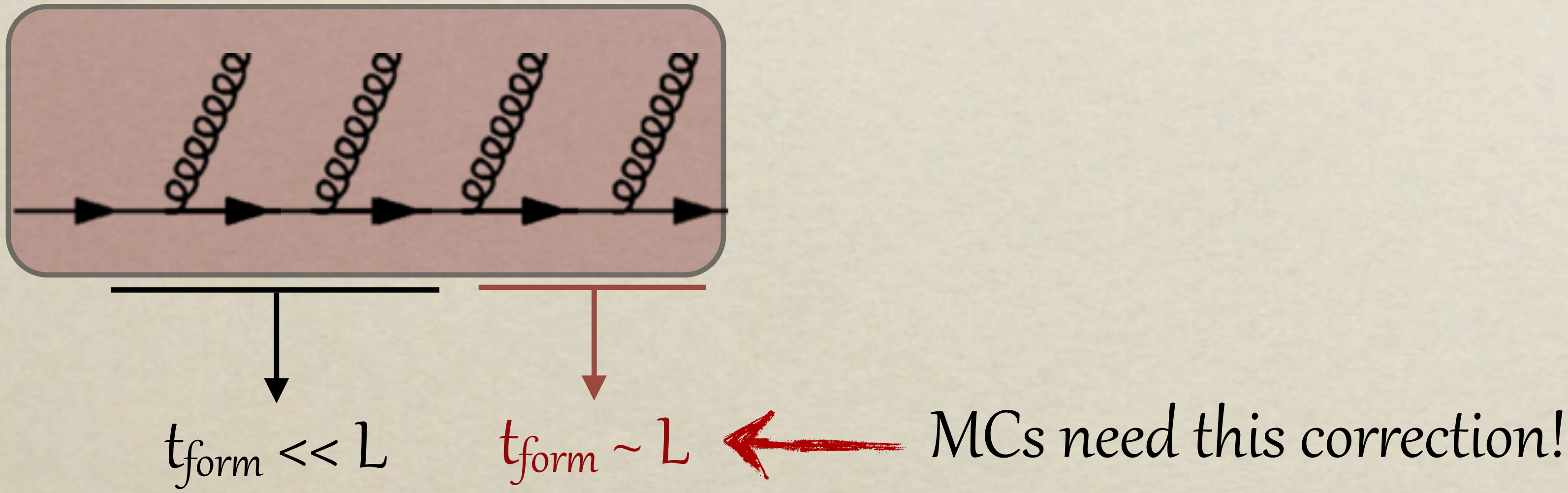

- It is interesting to study $q \to qg$:
  - Not constraining to $t_{form} << L$;
  - Do not have a symmetry in the final state:
    - $S_{g \leftarrow g}(x) = S_{g \leftarrow g}(1\text{-}x)$ but not the $S_{g \leftarrow q}(x)$;

Dirac Structure

- Total Spectrum (M M†):

$$\left\langle \overline{|M_{tot}|^2} \right\rangle = \left\langle \overline{|M_q^2|} \right\rangle + \left\langle \overline{|M_g^2|} \right\rangle + 2\mathrm{Re} \left\langle \overline{\{M_g M_q^\dagger\}} \right\rangle$$

  - Dirac structure:

$$\left\langle \overline{|M_q|^2} \right\rangle \propto \frac{2g^2 x(1-x)}{[(1-x)\mathbf{k}_\perp - z\mathbf{q}_\perp]^2} P_{g\leftarrow q}(x)$$

$$\left\langle \overline{M_g M_q^\dagger} \right\rangle \propto \frac{1}{2(k \cdot q)p_+} \left\{ \mathbf{k}_\perp^2 \frac{(1-x)(2-x)}{x^2} + \mathbf{q}_\perp^2 \frac{1}{1-x} - \mathbf{q}_\perp \cdot \mathbf{k}_\perp \frac{3-x}{x} \right\}$$

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Only term with factorization of the Altarelli-Parisi

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Extra-term with respect to the splitting function

When $k_T = -q_T$, we recover the results from the hard gluon spectrum (arXiv:1204.2929)!

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  - Assumed a frozen color profile at the beginning of the calculation.

    $\Rightarrow$Have to average over all possible configurations

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- No assumption on the t_{form}, but working in the large Nc limit and multiple soft scattering approximation:

Color Structure

Decomposition into three regions:

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- Medium averages are performed locally:

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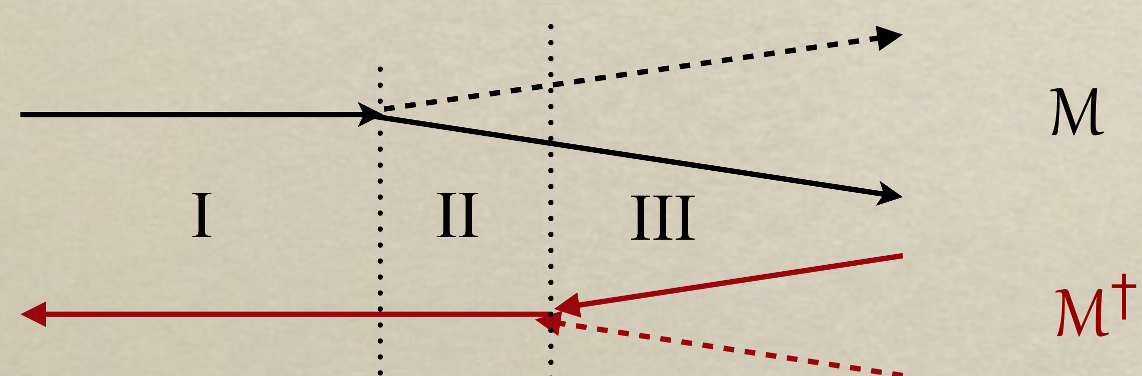
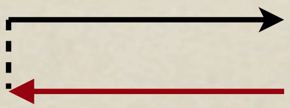
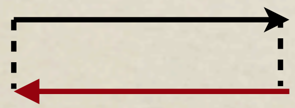
- No assumption on the t_{form}, but working in the large Nc limit and multiple soft scattering approximation:

$\left\langle \overline{|M_g|^2} \right\rangle \propto$

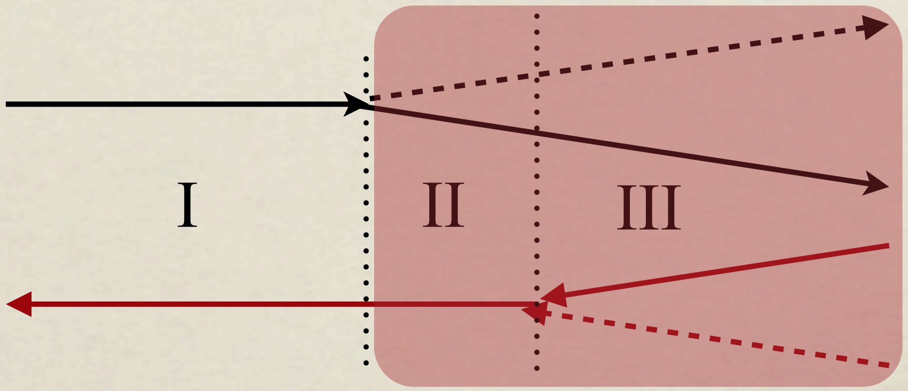


Color Structure

- Region I:

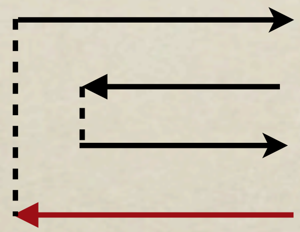
   $=$  $\times\ \delta_{ij}$

  - Brownian motion of the initial quark.

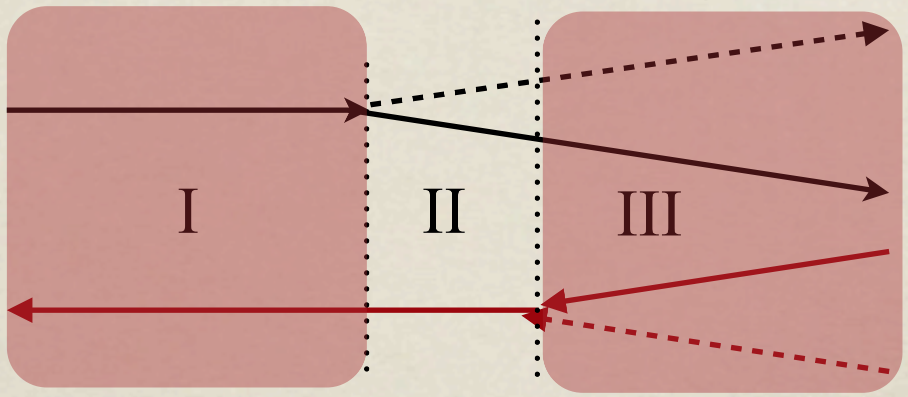


Color Structure

- Region II:



$$= \left\langle \left[W(\mathbf{r_2})W^\dagger(\mathbf{w_2})\right]_{ij} \left[W(\mathbf{w_2})W^\dagger(\bar{\mathbf{s}}_2)\right]_{kl} \right\rangle$$



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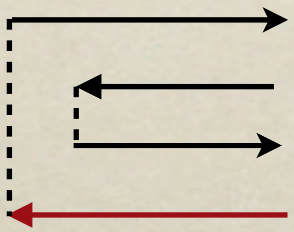

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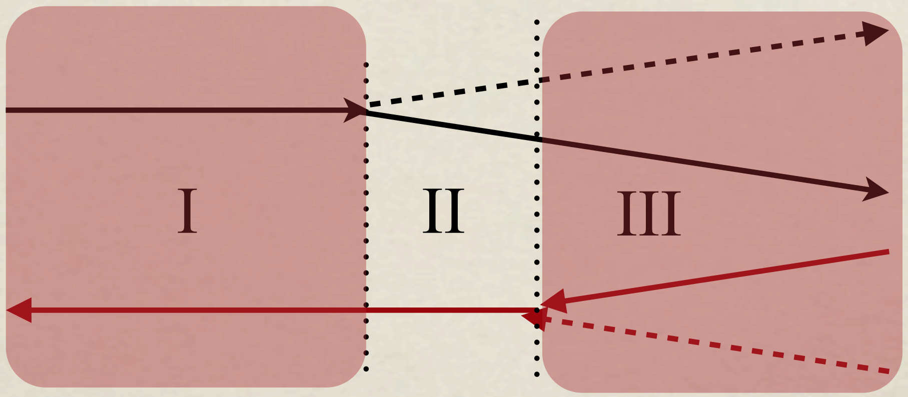

  - 4-point function: expand the Wilson lines in an infinitesimal interval.

$$W(\mathbf{x}_\perp)_{ij} = V(\mathbf{x}_\perp)_{i\alpha}\left\{\delta_{\alpha j}\left(1-\frac{C_F}{2}B(0)\right) - iT^a_{\alpha j}A^a(\mathbf{x}_\perp)\right\}$$

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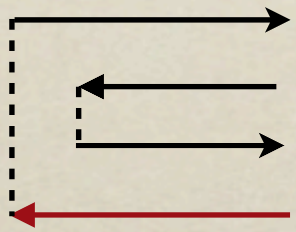

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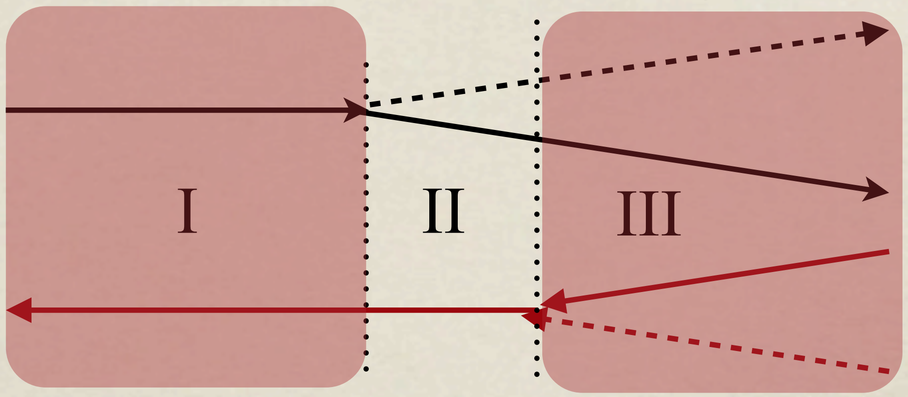

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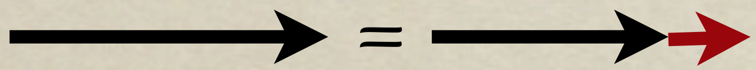

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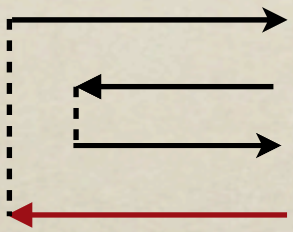

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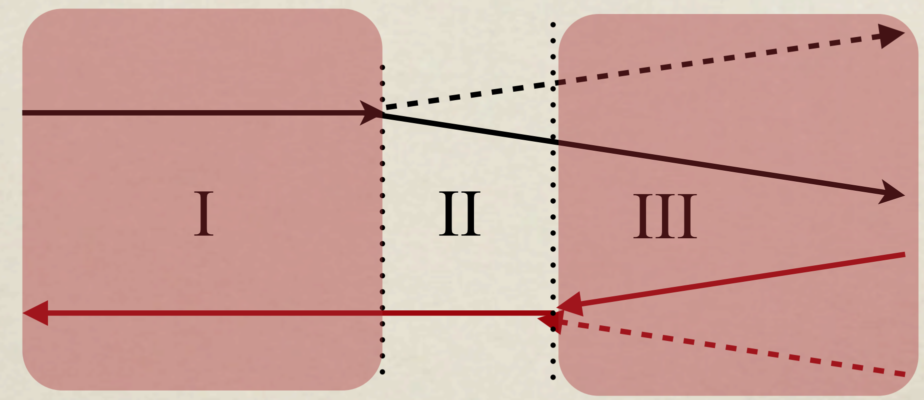

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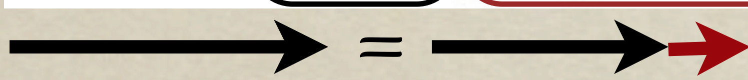

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$$= \delta_{ij}\delta_{kl}\exp\left\{\frac{1}{2N}v(\mathbf{r_q}-\mathbf{r_{\bar{q}}}) - \frac{N}{2}\left[v(\mathbf{r_q}-\mathbf{r_g}) + v(\mathbf{r_g}-\mathbf{r_{\bar{q}}})\right]\right\}$$
$$+ \frac{1}{N}\delta_{il}\delta_{jk}\exp\left\{-C_F v(\mathbf{r_q}-\mathbf{r_{\bar{q}}})\right\} -$$
$$- \frac{1}{N}\delta_{il}\delta_{jk}\exp\left\{\frac{1}{2N}v(\mathbf{r_q}-\mathbf{r_{\bar{q}}}) - \frac{N}{2}\left[v(\mathbf{r_q}-\mathbf{r_g}) + v(\mathbf{r_g}-\mathbf{r_{\bar{q}}})\right]\right\}$$

$$v(\mathbf{x}-\mathbf{y}) = B(\mathbf{0}) - B(\mathbf{x}-\mathbf{y}) \equiv \frac{1}{2}\int dx_+ \sigma(\mathbf{x}-\mathbf{y})n(x_+).$$

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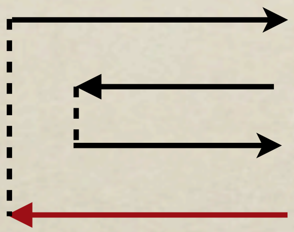

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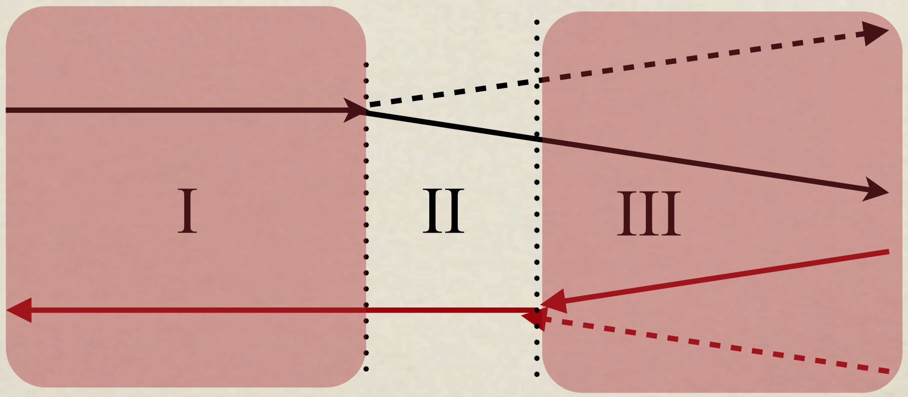
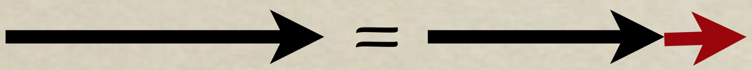


- 4-point function: expand the Wilson lines in an infinitesimal interval.

$$W(\mathbf{x}_\perp)_{ij} = \boxed{V(\mathbf{x}_\perp)_{i\alpha}} \boxed{\left\{\delta_{\alpha j}\left(1 - \frac{C_F}{2}B(0)\right) - iT^a_{\alpha j}A^a(\mathbf{x}_\perp)\right\}}$$

$$B(\mathbf{x}_\perp - \mathbf{y}_\perp) \propto \langle A^a(\mathbf{x}_\perp)A^a(\mathbf{y}_\perp)\rangle$$



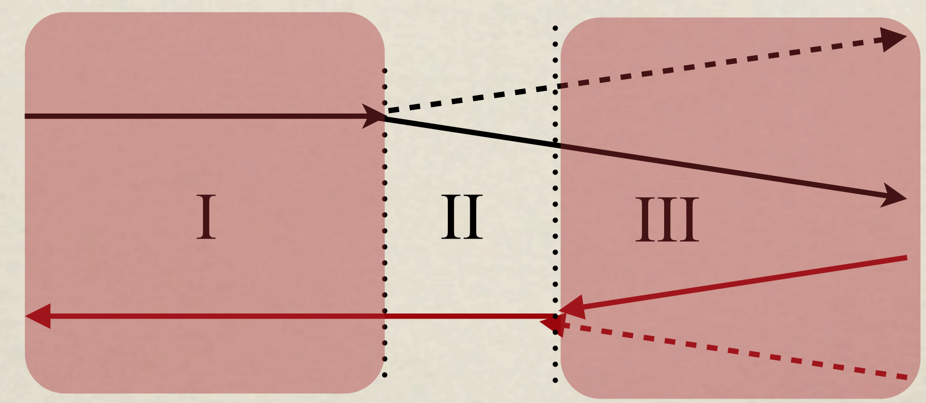
$$\begin{aligned}
&\left\langle \left[W(\mathbf{r_q})W^\dagger(\mathbf{r_g})\right]_{ij} \left[W(\mathbf{r_g})W^\dagger(\mathbf{r_{\bar q}})\right]_{kl} \right\rangle \\
&= \delta_{ij}\delta_{kl}\exp\left\{\frac{1}{2N}v(\mathbf{r_q} - \mathbf{r_{\bar q}}) \boxed{- \frac{N}{2}\left[v(\mathbf{r_q} - \mathbf{r_g}) + v(\mathbf{r_g} - \mathbf{r_{\bar q}})\right]}\right\} \\
&+ \frac{1}{N}\delta_{il}\delta_{jk}\exp\left\{-C_F v(\mathbf{r_q} - \mathbf{r_{\bar q}})\right\} - \\
&- \frac{1}{N}\delta_{il}\delta_{jk}\exp\left\{\frac{1}{2N}v(\mathbf{r_q} - \mathbf{r_{\bar q}}) - \frac{N}{2}\left[v(\mathbf{r_q} - \mathbf{r_g}) + v(\mathbf{r_g} - \mathbf{r_{\bar q}})\right]\right\}
\end{aligned}$$

$$v(\mathbf{x} - \mathbf{y}) = B(\mathbf{0}) - B(\mathbf{x} - \mathbf{y}) \equiv \frac{1}{2}\int dx_+ \sigma(\mathbf{x} - \mathbf{y})n(x_+).$$

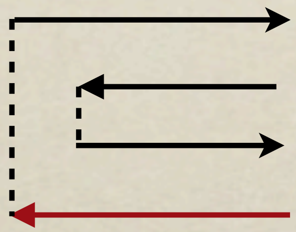
Leading term in the large N_c limit:

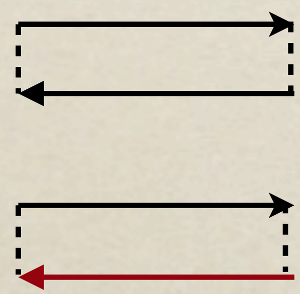
$$\left\langle \left[W_q W_g^\dagger\right]_{ij} \left[W_g W_q^\dagger\right]_{kl} \right\rangle \propto \mathrm{Tr}\left\langle W_q W_g^\dagger \right\rangle \mathrm{Tr}\left\langle W_g W_q^\dagger \right\rangle \delta_{ij}\delta_{kl}$$

Color Structure



- Region II:

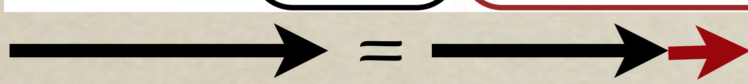


=  $\times\ \delta_{ij}\,\delta_{kl} + \mathcal{O}(N^{-1})$

- 4-point function: expand the Wilson lines in an infinitesimal interval.

$$W(\mathbf{x}_\perp)_{ij} = \boxed{V(\mathbf{x}_\perp)_{i\alpha}} \boxed{\left\{\delta_{\alpha j}\left(1-\frac{C_F}{2}B(0)\right) - iT^a_{\alpha j}A^a(\mathbf{x}_\perp)\right\}}$$

$$B(\mathbf{x}_\perp - \mathbf{y}_\perp) \propto \langle A^a(\mathbf{x}_\perp)A^a(\mathbf{y}_\perp)\rangle$$



$$\begin{aligned}
&\left\langle \left[W(\mathbf{r_q})W^\dagger(\mathbf{r_g})\right]_{ij}\left[W(\mathbf{r_g})W^\dagger(\mathbf{r_{\bar{q}}})\right]_{kl}\right\rangle \\
&= \delta_{ij}\delta_{kl}\exp\left\{\frac{1}{2N}v(\mathbf{r_q}-\mathbf{r_{\bar{q}}}) \boxed{- \frac{N}{2}\left[v(\mathbf{r_q}-\mathbf{r_g})+v(\mathbf{r_g}-\mathbf{r_{\bar{q}}})\right]\Big\}} \\
&+ \frac{1}{N}\delta_{il}\delta_{jk}\exp\left\{-C_F v(\mathbf{r_q}-\mathbf{r_{\bar{q}}})\right\} - \\
&- \frac{1}{N}\delta_{il}\delta_{jk}\exp\left\{\frac{1}{2N}v(\mathbf{r_q}-\mathbf{r_{\bar{q}}}) - \frac{N}{2}\left[v(\mathbf{r_q}-\mathbf{r_g})+v(\mathbf{r_g}-\mathbf{r_{\bar{q}}})\right]\right\}
\end{aligned}$$

$$v(\mathbf{x}-\mathbf{y}) = B(\mathbf{0}) - B(\mathbf{x}-\mathbf{y}) \equiv \frac{1}{2}\int dx_+ \sigma(\mathbf{x}-\mathbf{y})n(x_+).$$

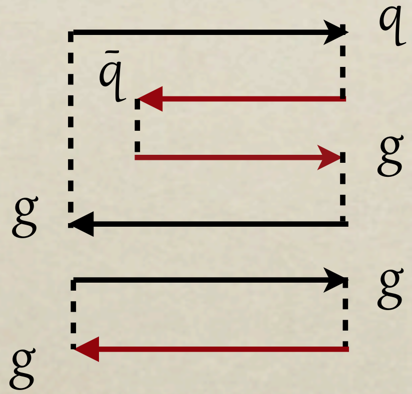
Leading term in the large N_c limit:

$$\left\langle \left[W_q W_g^\dagger\right]_{ij}\left[W_g W_q^\dagger\right]_{kl}\right\rangle \propto \mathrm{Tr}\left\langle W_q W_g^\dagger\right\rangle \mathrm{Tr}\left\langle W_g W_q^\dagger\right\rangle \delta_{ij}\delta_{kl}$$

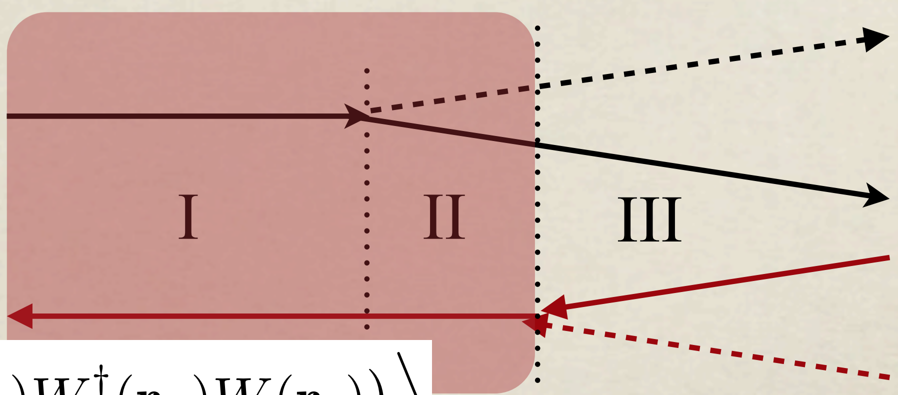
- Coherent propagation of the final quark and emitted gluon.

Color Structure

- Region III:

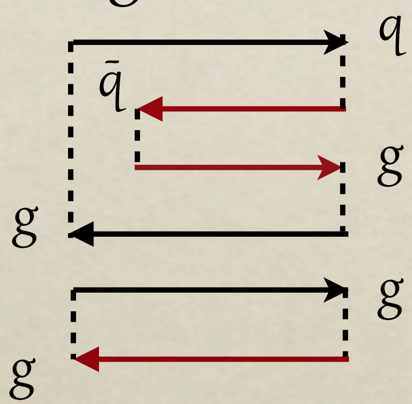


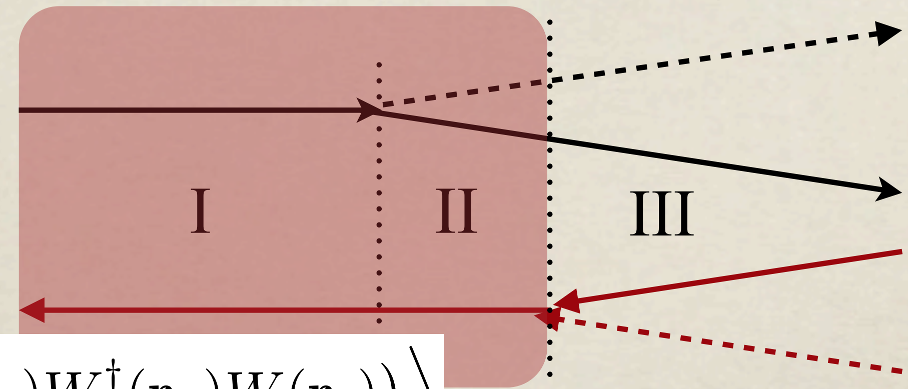
$$= \left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\right)\mathrm{Tr}\left(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\right)\right\rangle$$



Color Structure

- Region III:



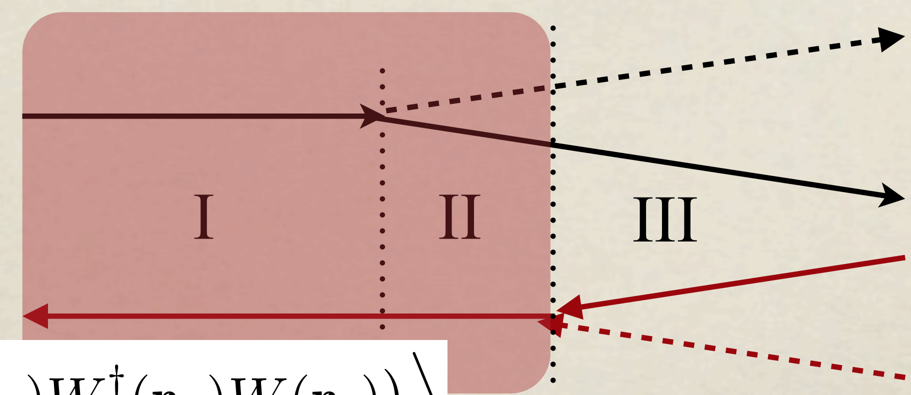


$$= \Big\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\Big\rangle$$

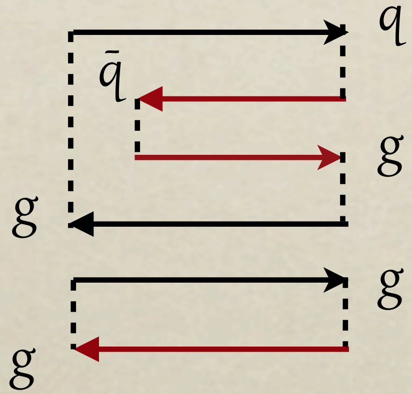
- 6-point function: factorization into a 4-point function and a 2-point function

$$\Big\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\Big\rangle$$
$$\underset{N\to\infty}{\simeq} \Big\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\Big\rangle\Big\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\Big\rangle$$

Color Structure



- Region III:



$$= \left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\right\rangle$$

- 6-point function: factorization into a 4-point function and a 2-point function

$$\begin{aligned} &\left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\right\rangle \\ \underset{N\to\infty}{\simeq} &\left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\right\rangle\left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\right\rangle \end{aligned}$$

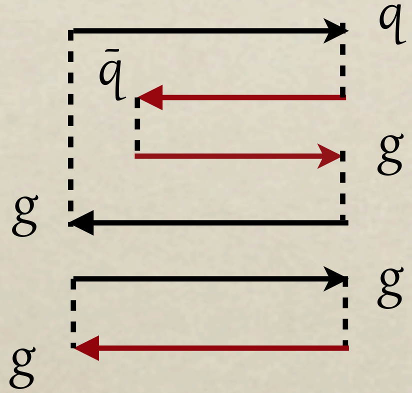
- 4-point function result:

$$\begin{aligned} &\mathrm{Tr}\left\langle W(\mathbf{r}_{\mathbf{q}})W^\dagger(\mathbf{r}_{\mathbf{g}})W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{x}_{\bar{\mathbf{q}}})\right\rangle \\ &\quad = N\left\{-\frac{O_3-O_2}{O_2-O_1}\mathrm{e}^{-NO_2/2}+\frac{O_3-O_1}{O_2-O_1}\mathrm{e}^{-NO_1/2}\right\} \end{aligned}$$

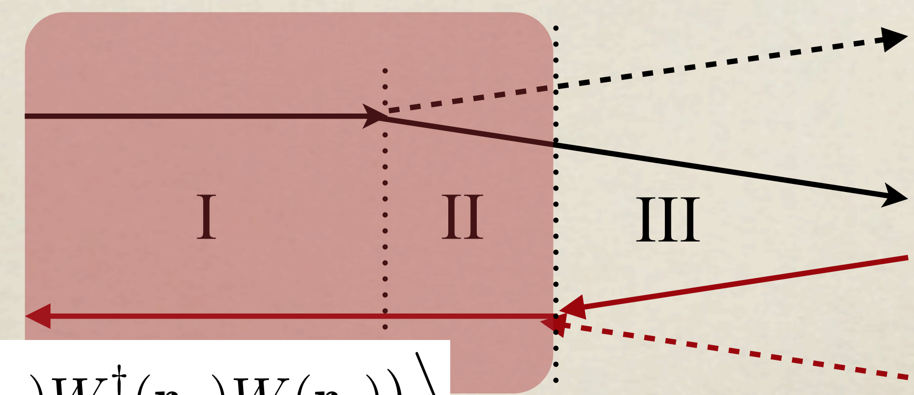
$$\begin{aligned} O_1 &= v(\mathbf{r}_{\mathbf{q}}-\mathbf{r}_{\bar{\mathbf{q}}})+v(\mathbf{r}_{\mathbf{g}}-\mathbf{r}_{\bar{\mathbf{g}}}), \\ O_2 &= v(\mathbf{r}_{\mathbf{q}}-\mathbf{r}_{\mathbf{g}})+v(\mathbf{r}_{\bar{\mathbf{q}}}-\mathbf{r}_{\bar{\mathbf{g}}}), \\ O_3 &= v(\mathbf{r}_{\mathbf{q}}-\mathbf{r}_{\bar{\mathbf{g}}})+v(\mathbf{r}_{\mathbf{g}}-\mathbf{r}_{\bar{\mathbf{q}}}). \end{aligned}$$

Color Structure

- Region III:



$$= \left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\right\rangle$$



- 6-point function: factorization into a 4-point function and a 2-point function

$$\left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\right\rangle$$
$$\underset{N\to\infty}{\simeq} \left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\big)\right\rangle \left\langle \mathrm{Tr}\big(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\big)\right\rangle$$

- 4-point function result:

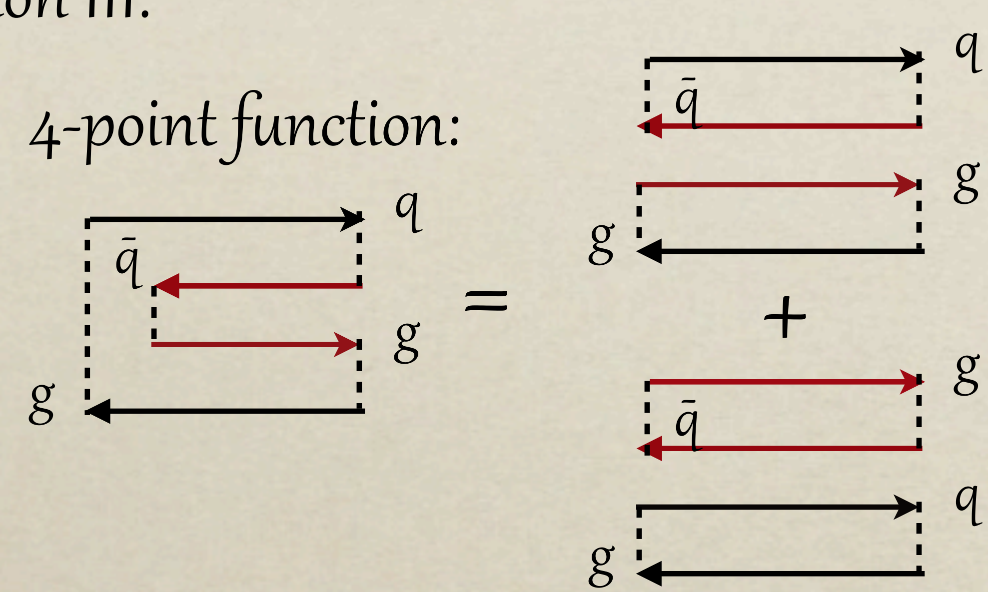
$$\mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{x}_{\bar{\mathbf{q}}})\right\rangle = N\left\{-\frac{O_3-O_2}{O_2-O_1}\mathrm{e}^{-NO_2/2}+\frac{O_3-O_1}{O_2-O_1}\mathrm{e}^{-NO_1/2}\right\}$$

$$\begin{aligned} O_1 &= v(\mathbf{r_q}-\mathbf{r}_{\bar{\mathbf{q}}})+v(\mathbf{r_g}-\mathbf{r}_{\bar{\mathbf{g}}}), \\ O_2 &= v(\mathbf{r_q}-\mathbf{r_g})+v(\mathbf{r}_{\bar{\mathbf{q}}}-\mathbf{r}_{\bar{\mathbf{g}}}), \\ O_3 &= v(\mathbf{r_q}-\mathbf{r}_{\bar{\mathbf{g}}})+v(\mathbf{r_g}-\mathbf{r}_{\bar{\mathbf{q}}}). \end{aligned}$$

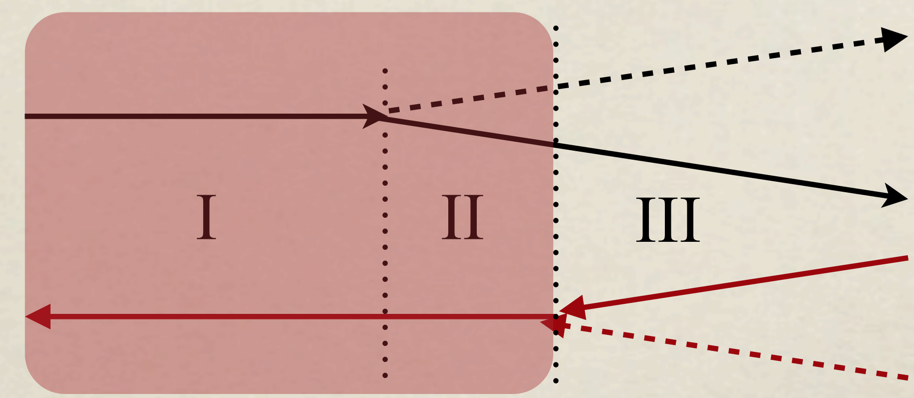
$O \propto v(x_i\text{-}x_j) \propto (x_i\text{-}x_j)^2 = e^{2\ln(xi\text{-}xj)}$ ⇒ Neglect the norm (Dipole approximation)

Color Structure

- Region III:
  - 4-point function:



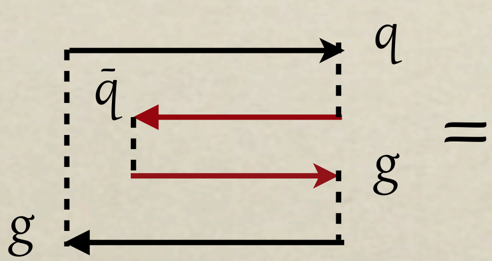
Factorization into two 2-point functions

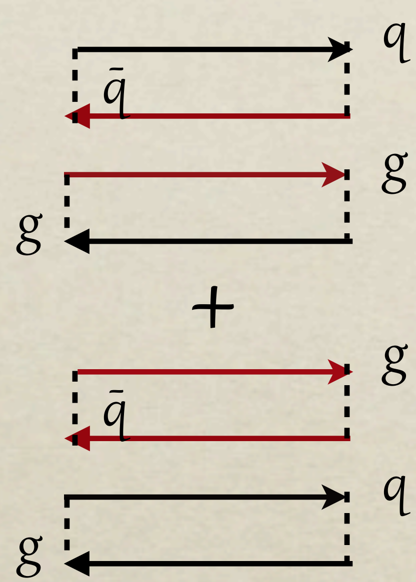


$$\mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})W(\mathbf{r_{\bar{g}}})W^\dagger(\mathbf{r_{\bar{q}}})\right\rangle$$
$$= \mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_{\bar{q}}})\right\rangle \mathrm{Tr}\left\langle W(\mathbf{r_{\bar{g}}})W^\dagger(\mathbf{r_g})\right\rangle$$
$$- \mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})\right\rangle \mathrm{Tr}\left\langle W(\mathbf{r_{\bar{g}}})W^\dagger(\mathbf{r_{\bar{q}}})\right\rangle$$

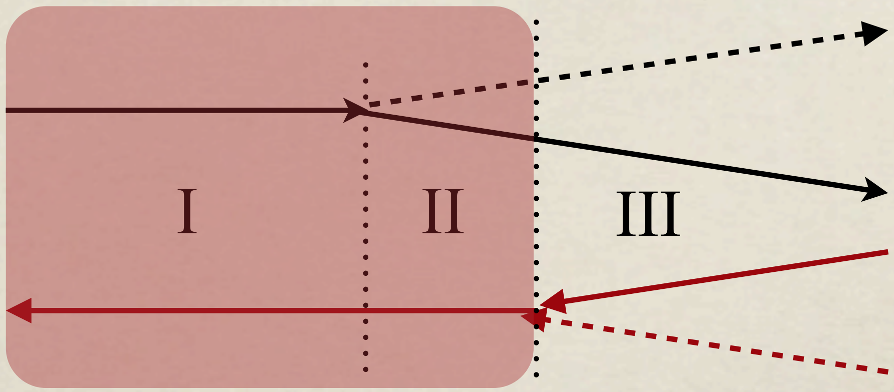
Color Structure

- Region III:
    - 4-point function:





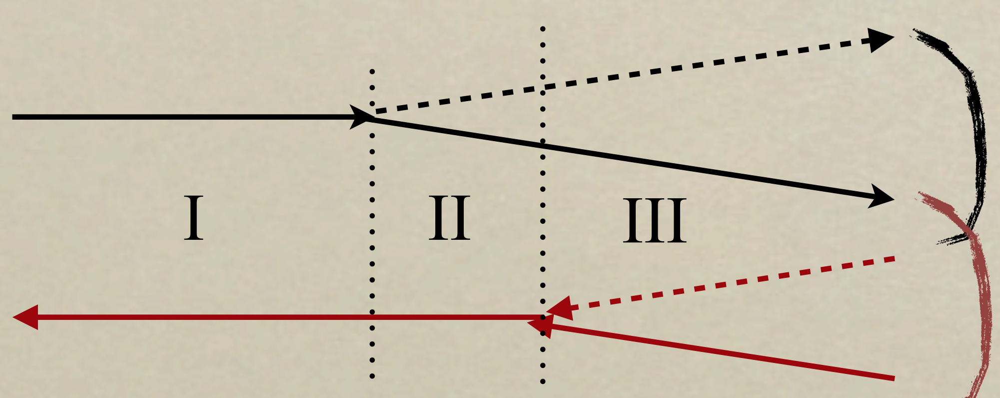
Factorization into two 2-point functions



$$
\begin{aligned}
&\mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})W(\mathbf{r_{\bar g}})W^\dagger(\mathbf{r_{\bar q}})\right\rangle \\
&= \mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_{\bar q}})\right\rangle \mathrm{Tr}\left\langle W(\mathbf{r_{\bar g}})W^\dagger(\mathbf{r_g})\right\rangle \\
&- \mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})\right\rangle \mathrm{Tr}\left\langle W(\mathbf{r_{\bar g}})W^\dagger(\mathbf{r_{\bar q}})\right\rangle
\end{aligned}
$$

- Independent + Coherent propagation of both final particles.

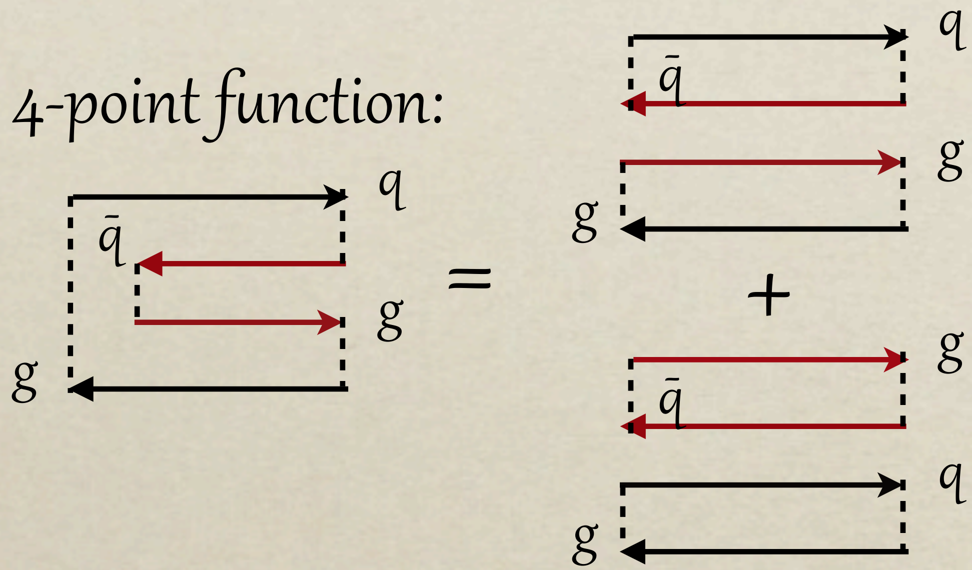
Also present when t_{form}<<L



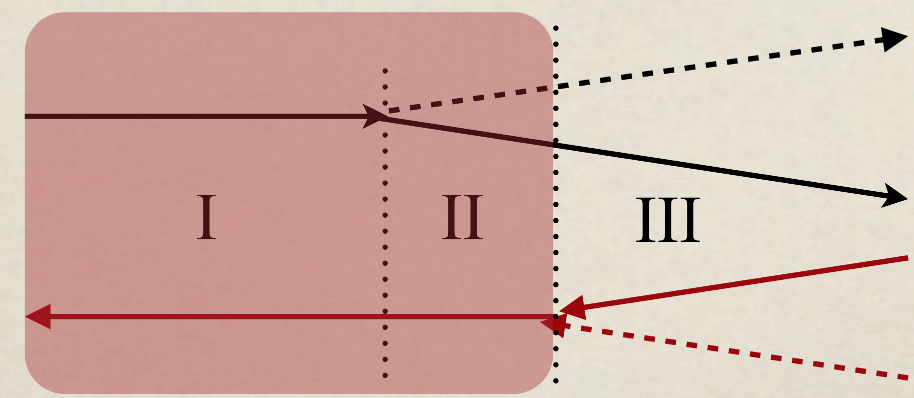
Quark and gluon factorize

Color Structure

- Region III:
  - 4-point function:



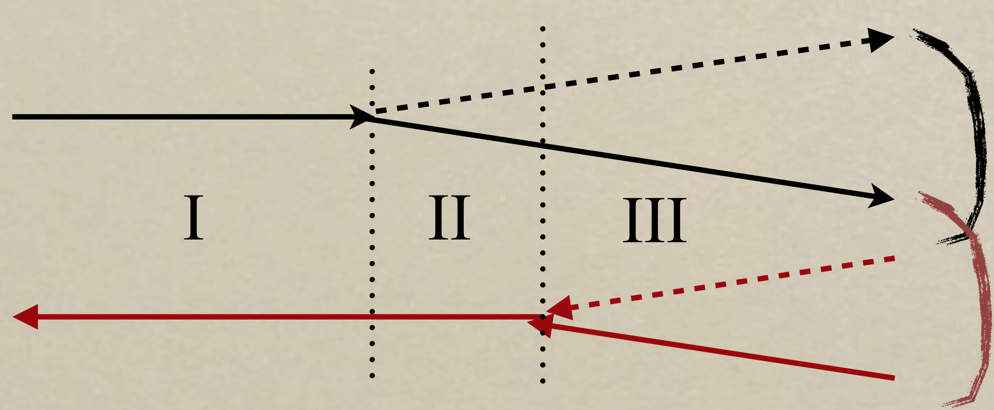
Factorization into two 2-point functions



$$\begin{aligned}&\mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})W(\mathbf{r_{\bar{g}}})W^\dagger(\mathbf{r_{\bar{q}}})\right\rangle\\ &=\mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_{\bar{q}}})\right\rangle \mathrm{Tr}\left\langle W(\mathbf{r_{\bar{g}}})W^\dagger(\mathbf{r_g})\right\rangle\\ &-\mathrm{Tr}\left\langle W(\mathbf{r_q})W^\dagger(\mathbf{r_g})\right\rangle \mathrm{Tr}\left\langle W(\mathbf{r_{\bar{g}}})W^\dagger(\mathbf{r_{\bar{q}}})\right\rangle\end{aligned}$$

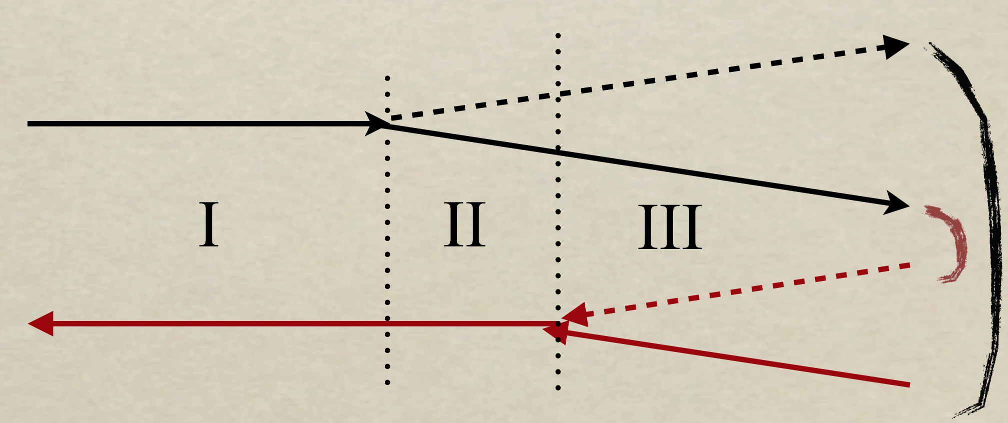
  - Independent + Coherent propagation of both final particles.

Also present when $t_{form} << L$



Quark and gluon factorize

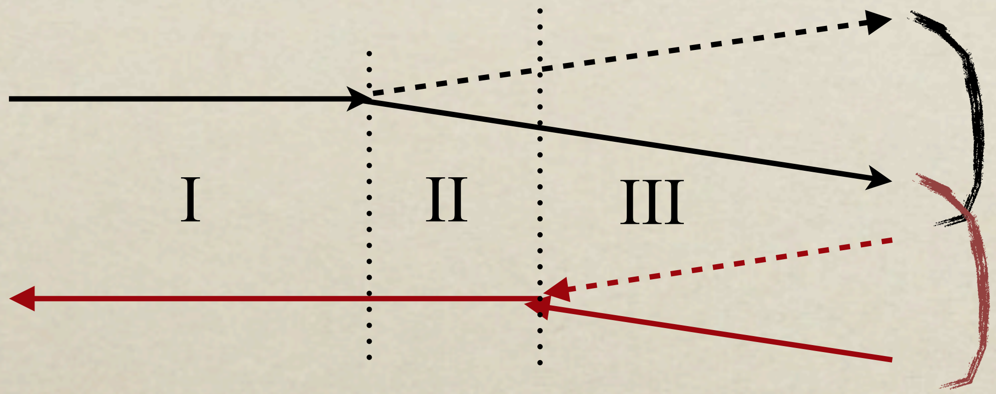
Additional term

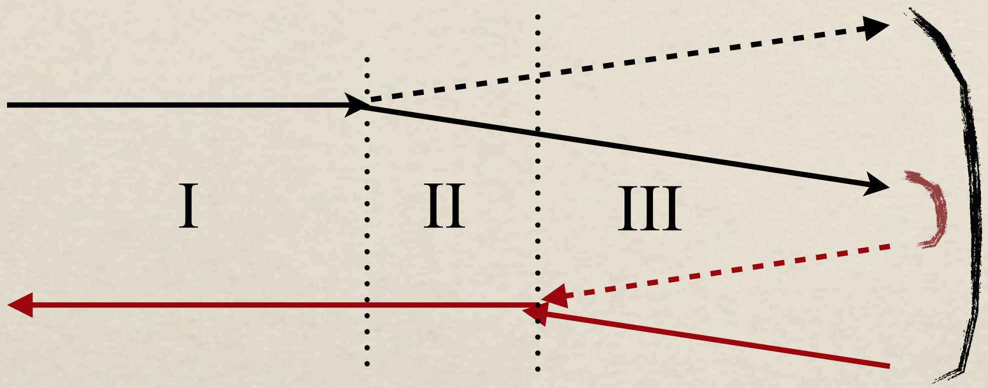


Correlation between final partons

Color Structure

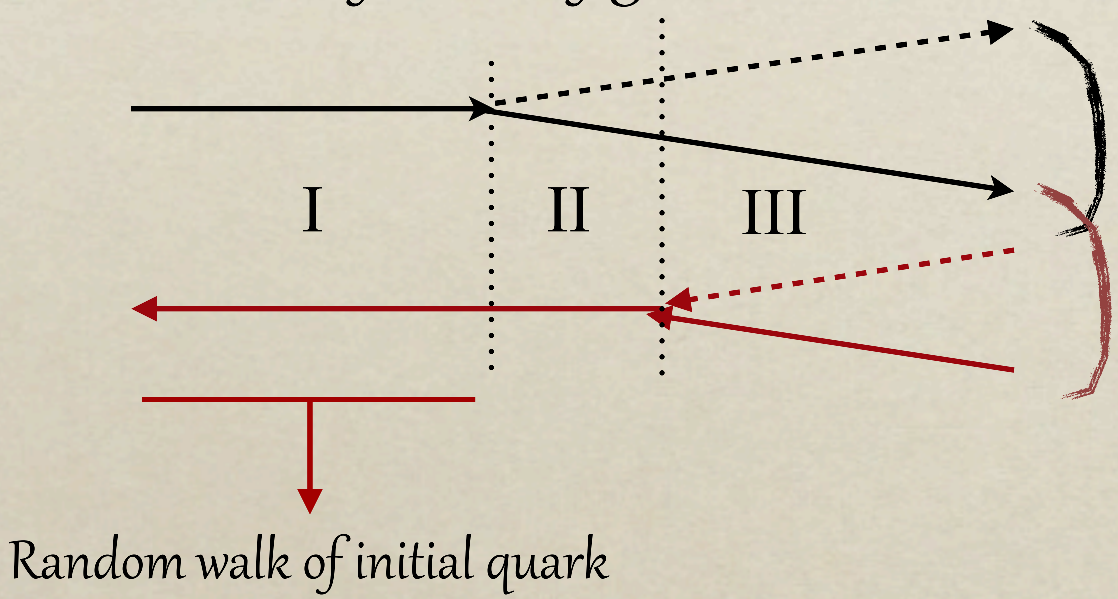
- Color final configurations:

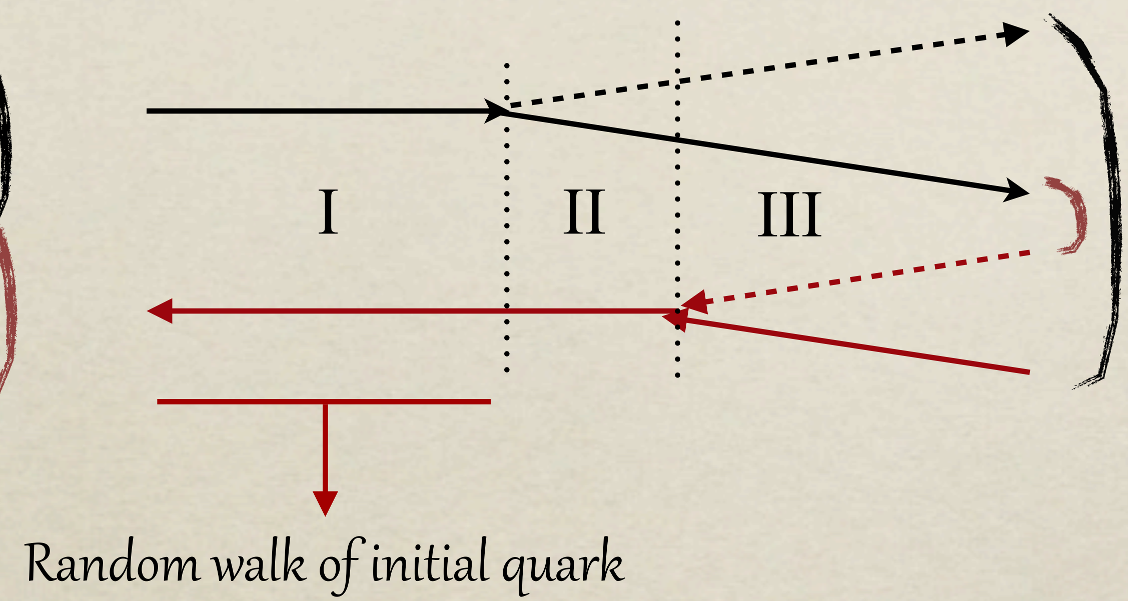




Color Structure

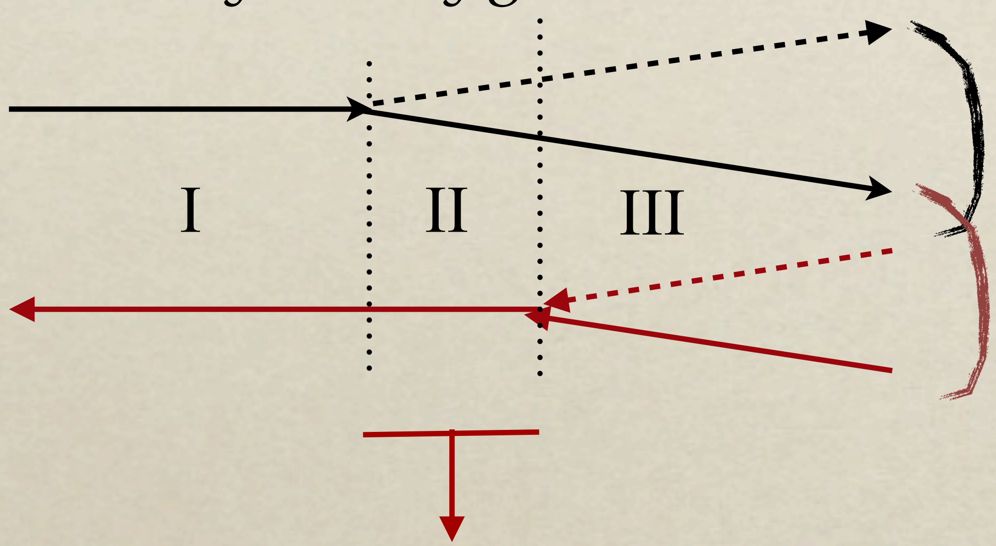
- Color final configurations:



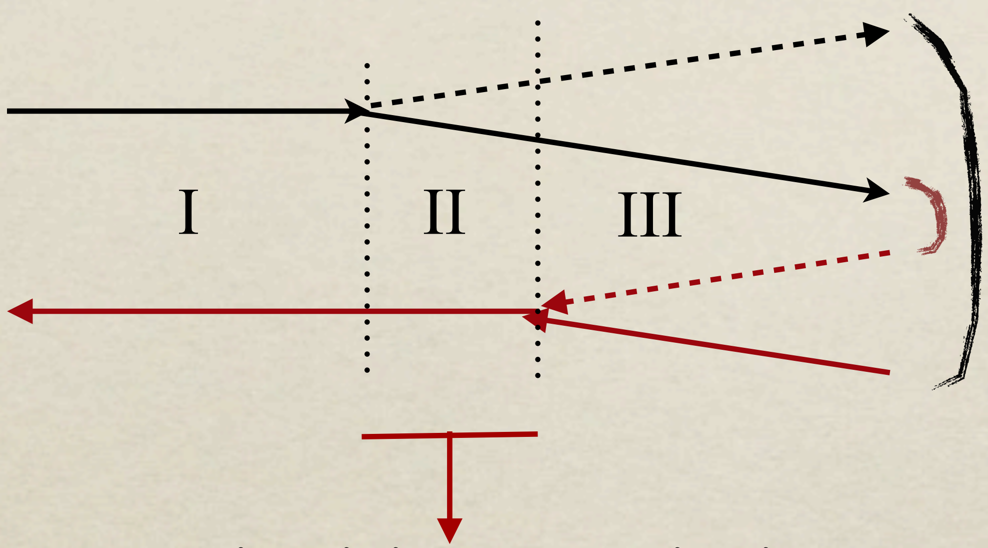


Color Structure

- Color final configurations:



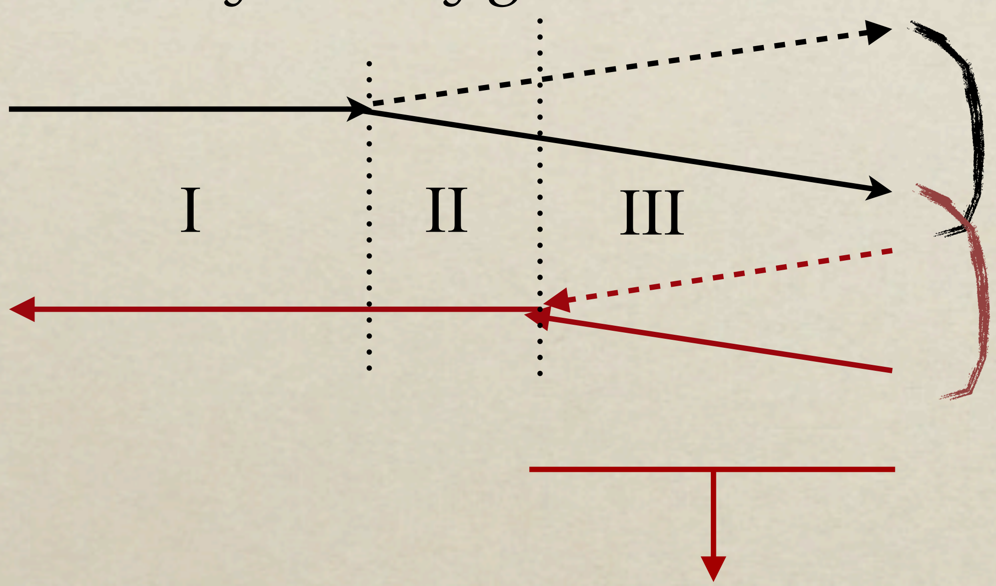
Quark and gluon are correlated
(gluon formation time)



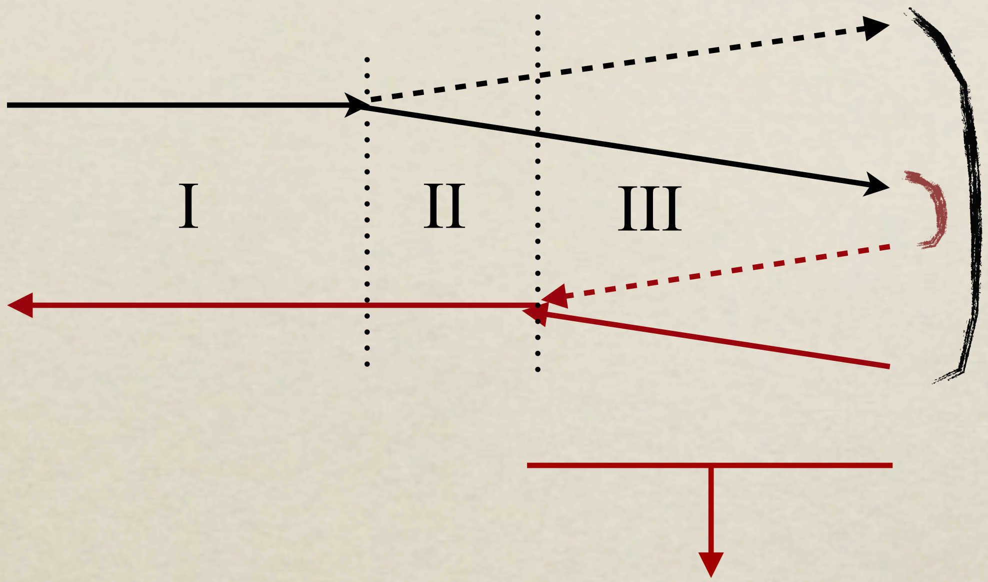
Quark and gluon are correlated
(gluon formation time)

Color Structure

- Color final configurations:



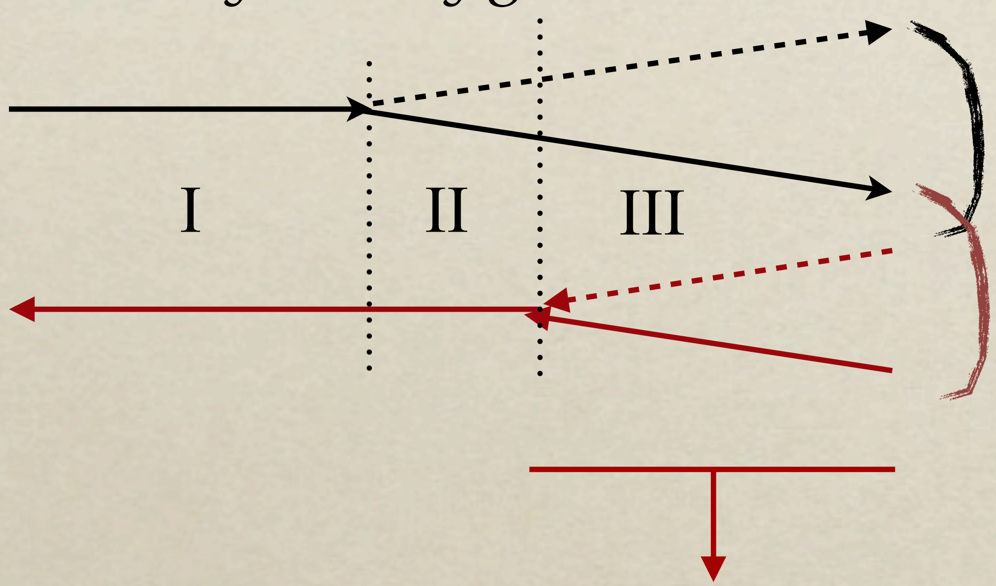
Independent broadening of final partons



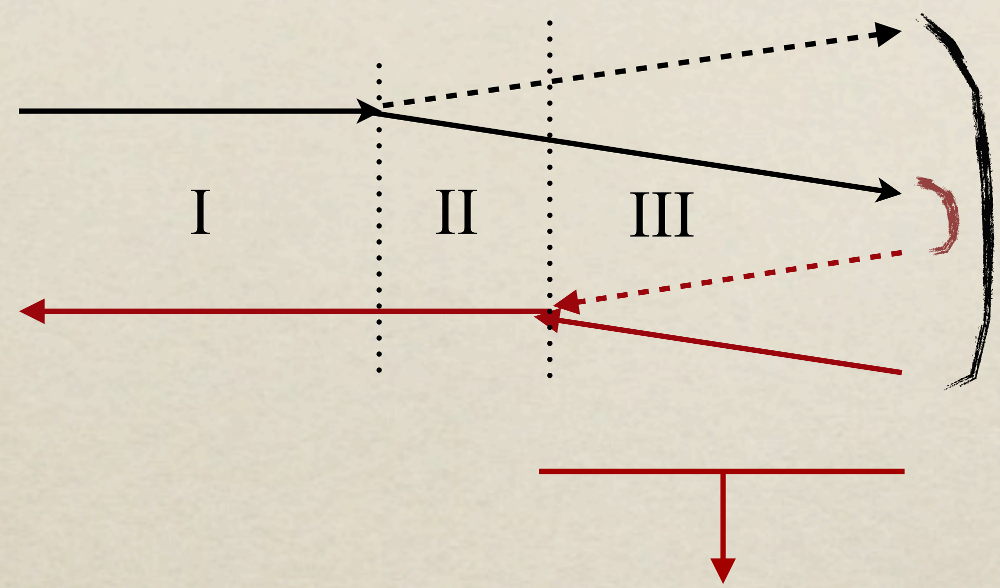
Quark and gluon act coherently
(coherent propagation controlled by dipole distance)

Color Structure

- Color final configurations:



Independent broadening of final partons

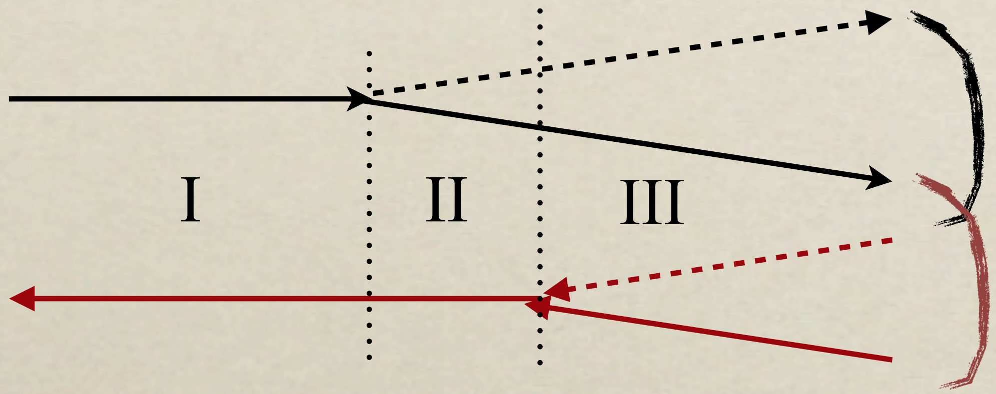


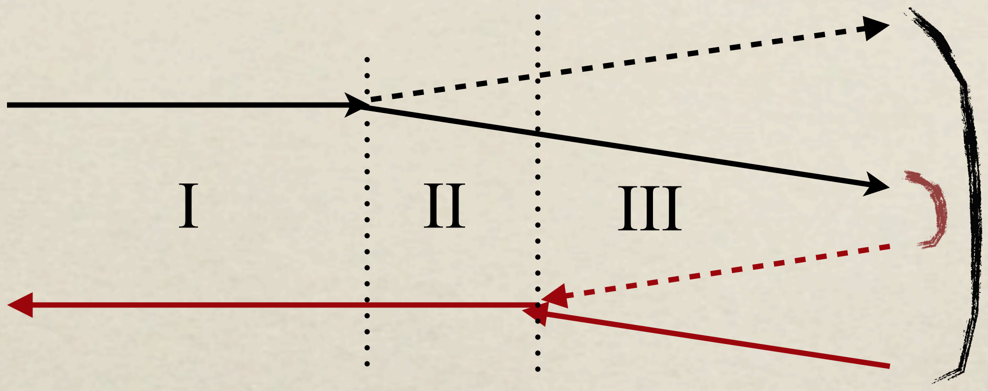
Quark and gluon act coherently
(coherent propagation controlled by dipole distance)

$$\left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\right)\right\rangle \left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\right)\right\rangle$$
$$\sim \exp\left\{-\frac{N}{2}\left[(\mathbf{r_q}-\mathbf{r}_{\bar{\mathbf{q}}})^2+2(\mathbf{r_g}-\mathbf{r}_{\bar{\mathbf{g}}})^2\right]\right\}\left[1-\exp\left\{-N\left(\mathbf{r_g}-\mathbf{r}_{\bar{\mathbf{q}}}\right)\cdot\left(\mathbf{r}_{\bar{\mathbf{g}}}-\mathbf{r_q}\right)\right\}\right]$$

Color Structure

- Color final configurations:





Independent broadening of final partons

Quark and gluon act coherently
(coherent propagation controlled by dipole distance)

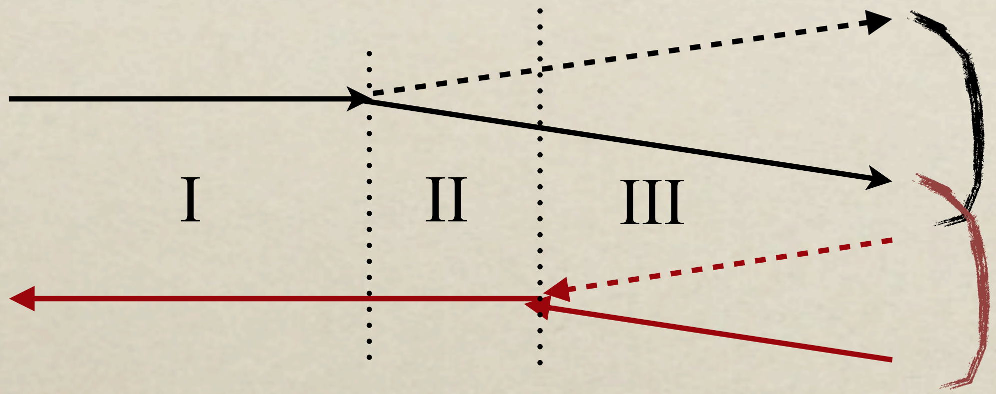
$$\left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\right)\right\rangle\left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\right)\right\rangle$$

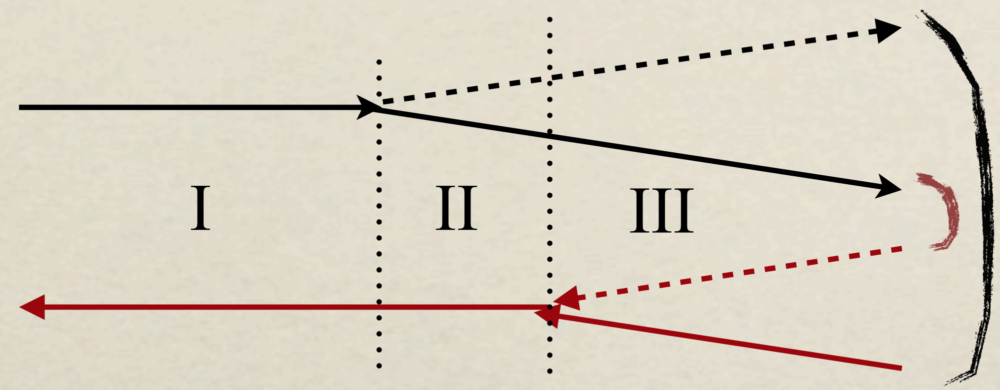
$$\sim \exp\left\{ \underline{-\frac{N}{2}\left[(\mathbf{r_q}-\mathbf{r_{\bar{q}}})^2+2(\mathbf{r_g}-\mathbf{r_{\bar{g}}})^2\right]} \right\}\left[1-\exp\left\{-N\left(\mathbf{r_g}-\mathbf{r_{\bar{q}}}\right)\cdot\left(\mathbf{r_{\bar{g}}}-\mathbf{r_q}\right)\right\}\right]$$

Independent dipoles

Color Structure

- Color final configurations:





Independent broadening of final partons

Quark and gluon act coherently
(coherent propagation controlled by dipole distance)

$$\left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_{\bar{\mathbf{g}}})W(\mathbf{r}_g)\right)\right\rangle\left\langle \mathrm{Tr}\left(W^\dagger(\mathbf{r}_g)W(\mathbf{r}_{\bar{\mathbf{g}}})W^\dagger(\mathbf{r}_{\bar{\mathbf{q}}})W(\mathbf{r}_q)\right)\right\rangle$$

$$\sim \exp\left\{-\frac{N}{2}\left[(\mathbf{r_q}-\mathbf{r_{\bar q}})^2+2(\mathbf{r_g}-\mathbf{r_{\bar g}})^2\right]\right\}\boxed{\left[1-\exp\left\{-N\left(\mathbf{r_g}-\mathbf{r_{\bar q}}\right)\cdot\left(\mathbf{r_{\bar g}}-\mathbf{r_q}\right)\right\}\right]}$$

Independent dipoles

Δ_{med} parameter
(For an eikonal antenna, $\Delta_{med} \sim r^2$)

Generalization of a QCD in-medium antenna

Summary

- Extension beyond eikonal approximation:
  - Able to extend previous work by allowing all particles undergo Brownian motion in the transverse plane;
  - Results in the large N_c limit, but with no constrains on the formation time;
  - Able to solve the 6-point function up to the first 3 terms in an expansion in N_c.
  - Dirac and color structure with the interference term included in calculations;
  - Found a generalization of a non-eikonal in-medium antenna.

Conclusions/Prospects

- Study of Jet Quenching is important since it provides a way of probing matter created in heavy-ion collisions.
- Further understanding the mechanisms of interaction and propagation with the medium:
  - Already several efforts up to now:
    - Medium-induced hard gluon radiation, Medium-induced gluon branching (small t_{form}), (Massive) Medium antenna, SCFT, lattice QCD...
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  - And even more coming soon...

Thanks!