Relativistic viscous hydrodynamic modeling of the Quark-Gluon Plasma with ECHO-QGP

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outline:

Viscous hydro with ECHO-QGP

Tests (numerical/analytic/physical)

Freeze-out: Spectra and elliptic flow

Future Directions

Ref.: Euro. Phys. J. C **73**, 2524 (2013)[arXiv:1305.7052 [nucl-th]]

ECHO-QGP

- ECHO-QGP is a $(3+1)$-D relativistic hydro code for modeling QGP
- ECHO-QGP (Eulerian Conservative High-Order Code for QGP): Relativistic viscous hydrodynamics for heavy-ion collisions with ECHO-QGP, L. Del Zanna, V. Chandra, G. Inghirami, V. Rolando, A. Beraudo, A. De Pace, G. Pagliara, A. Drago, F. Becattini, Eur. Phys. J. C **73**, 2524 (2013)[`arXiv:1305.7052 [nucl-th]`]
- Based on ECHO Code: Eulerian Conservative High-Order astro-physical code for general relativistic magneto-hydrodynamics, L. Del Zanna *et al.*,Astron. Astrophys. **473**, 11 (2007).
- ECHO-QGP webpage:
  www.astro.unifi.it/echo-qgp
- For queries on ECHO-QGP:
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ECHO-QGP Salient features

- ECHO-QGP: Both viscous and ideal hydro runs are possible in $(2+1)$-D and $(3+1)$-D
- Bjorken/Cartesian coordinate
- Finite baryon density
- Optical Glauber as well as Monte Carlo Glauber initial conditions
- Analytic/tabulated EOSs
- Upgrade to perform event-by-event hydro is in progress. Results are coming soon!
- Inclusion of magnetic field in the near future!
- ECHO-QGP code will be made public in the near future!

ECHO-QGP:viscous hydro

- Dynamics of any relativistic fluid is described by number current, N^μ, and energy momentum tensor, $T^{\mu\nu}$, satisfying the evolution laws (conservation)

(1) $$d_\mu N^\mu = 0,\ d_\mu T^{\mu\nu} = 0.$$

d_μ is the covariant derivative: $d_\mu A^\alpha_\beta = \partial_\mu A^\alpha_\beta + \Gamma^\alpha_{\lambda\mu} A^\lambda_\beta - \Gamma^\lambda_{\mu\beta} A^\alpha_\lambda$, where $\Gamma^\lambda_{\alpha\beta} := \frac{1}{2} g^{\lambda\gamma}(\partial_\alpha g_{\gamma\beta} + \partial_\beta g_{\alpha\gamma} - \partial_\gamma g_{\alpha\beta})$

- In the presence of dissipation:,

(2) $$N^\mu = n u^\mu + V^\mu$$

(3) $$T^{\mu\nu} = e u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + w^\mu u^\nu + w^\nu u^\mu.$$

$n = -u_\mu N^\mu$: charge density, $V^\mu = \Delta^\mu_\alpha N^\alpha$: particle diffusion flux

$e = u_\mu T^{\mu\nu} u_\nu$: energy density, $P + \Pi = \frac{1}{3}\Delta_{\mu\nu} T^{\mu\nu}$: isotropic pressure

$w^\mu = -\Delta^\mu_\alpha T^{\alpha\beta} u_\beta$: energy-momentum flow orthogonal to fluid velocity u^μ

$\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$ (with the metric signature $(-,+,+,+)$).

- We choose Landau frame for our investigations
- Even in this case, it is convenient to evolve the continuity equation (in the limit $V^\mu = 0$) for numerical reasons,

(4) $$Dn + n\theta = 0,$$

(n must be interpreted just as a tracer responding to the evolution of the fluid velocity through the expansion scalar $\theta = d_\mu u^\mu$.

- In conservation form,

(5) $$d_\mu N^\mu = |g|^{-\frac{1}{2}} \partial_\mu (|g|^{\frac{1}{2}} N^\mu) = 0$$

(6) $$d_\mu T^\mu_{\ \nu} = |g|^{-\frac{1}{2}} \partial_\mu (|g|^{\frac{1}{2}} T^\mu_{\ \nu}) - \Gamma^\mu_{\ \nu\lambda} T^\lambda_{\ \mu} = 0,$$

- ECHO-QGP equations for viscous stress:

(7) $$\partial_0 (|g|^{\frac{1}{2}} N^0 \pi^{\mu\nu}) + \partial_k (|g|^{\frac{1}{2}} N^k \pi^{\mu\nu}) = |g|^{\frac{1}{2}} n[-\frac{1}{\tau_\pi}(\pi^{\mu\nu} + 2\eta\sigma^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\theta + \mathcal{I}_0^{\mu\nu} + \mathcal{I}_1^{\mu\nu} + \mathcal{I}_2^{\mu\nu}],$$

where, $\mathcal{I}_0^{\mu\nu} = -u^\alpha(\Gamma^\mu_{\lambda\alpha}\pi^{\lambda\nu} + \Gamma^\nu_{\lambda\alpha}\pi^{\mu\lambda})$, $\mathcal{I}_1^{\mu\nu} = (\pi^{\lambda\mu}u^\nu + \pi^{\lambda\nu}u^\mu)Du_\lambda$ and $\mathcal{I}_2^{\mu\nu} = -\lambda(\pi^{\mu\lambda}\omega^\nu_{\ \lambda} + \pi^{\nu\lambda}\omega^\mu_{\ \lambda})$, $\sigma^{\mu\nu}$: NS Tensor, $\omega^{\mu\nu}$: fluid vorticity

- ECHO-QGP equations can be written as,

$$\partial_0 \mathbf{U} + \partial_k \mathbf{F}^k = \mathbf{S}, \tag{8}$$

where,

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} N \equiv N^0 \\ S_i \equiv T^0_i \\ E \equiv -T^0_0 \\ N\Pi \\ N\pi^{ij} \end{pmatrix}, \quad \mathbf{F}^k = |g|^{\frac{1}{2}} \begin{pmatrix} N^k \\ T^k_i \\ -T^k_0 \\ N^k\Pi \\ N^k\pi^{ij} \end{pmatrix} \tag{9}$$

are respectively the set of *conservative variables* and *fluxes*, while the source terms are given by

$$\mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{\mu\nu} \partial_i g_{\mu\nu} \\ -\frac{1}{2} T^{\mu\nu} \partial_0 g_{\mu\nu} \\ n[-\frac{1}{\tau_\pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta] \\ n[-\frac{1}{\tau_\pi}(\pi^{ij} + 2\eta\sigma^{ij}) - \frac{4}{3}\pi^{ij}\theta + \mathcal{I}^{ij}_0 + \mathcal{I}^{ij}_1 + \mathcal{I}^{ij}_2] \end{pmatrix}. \tag{10}$$

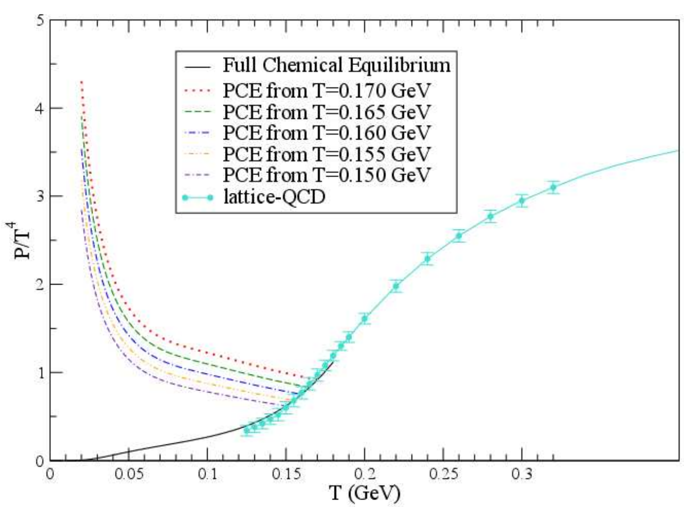
- We evolve the six spatial components of $\pi^{\mu\nu}$ invoking orthogonality. Tracelessness: as a check during the evolution!
- Solve for Local Rest Frame (LRF) energy density, components of fluid velocity and components of $\pi^{\mu\nu}$ and Π
- First and second order transport coefficients:
  $\eta/s = 0.08, 0.16, \qquad \zeta/s = 2(\eta/s)(c_s^2 - 1/3)$
  $\tau_\pi = \tau_\Pi = 3\frac{\eta}{sT}$
- Initialization for $\pi^{\mu\nu}$ and Π (in Bjorken Coordinates with their NS values and choosing LRF):

$$2\pi^{xx} = 2\pi^{yy} = -\tau^2\pi^{\eta\eta} = \frac{4}{3}\frac{\eta}{\tau}, \qquad \Pi = -\zeta/\tau$$

- For the purpose of stability of the code viscous effects are modulated for temperature much below the freeze-out (in 3D)
- For details on the numerical methods and algorithms in ECHO: L. Del Zanna *et al.*, Astron. Astroph. **473**, 11 (2007); **390**, 1177 (2002) .

EOS and initial conditions

- Three EOSs have been employed in ECHO-QGP:
  a) EOS for the ultra-relativistic gas of Quarks and Gluons: EOS-I:
  $P = e/3 = \frac{g\pi^2}{90}T^4, \quad c_s^2 = \frac{1}{3}$, where $g = 37$ for a non-interacting QGP with 3 light flavors.
  b) EOS-LS (improved pQCD+Hadron resonance gas) by Laine and Schröder, Phys. Rev. **D 73**, 085009 (2006).
  c) EOS-PCE (Lattice QCD+Hadron resonance gas with partial chemical equilibration)
  Details on EOS-PCE: [A. Beraudo's talk during IS2013, Ref.: arXiv:1306.6188 [hep-ph]]



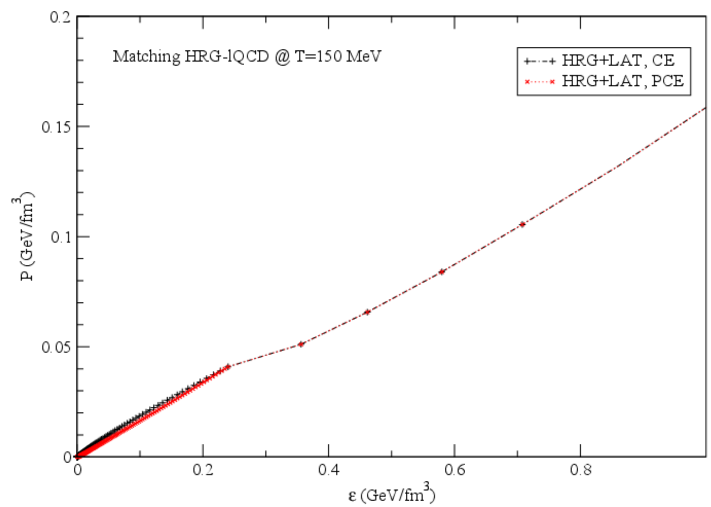


Figure 1: HRG+LAT and HRG+PCE+LAT equations of state. Matching with lattice in the case of PCE is done at $T = 150$ MeV.

Initial Conditions

- Optical Glauber: In the 3D case the initialization is performed using the model for the density distribution as:

$$
\begin{aligned}
e(\tau_0, x, \eta_s; b) &= \tilde{e}_0\, \theta(Y_b - |\eta_s|)\, f^{\mathrm{pp}}(\eta_s)\, [\alpha n_{\mathrm{coll}}(x; b) + \\
&\quad (1-\alpha) \left(\frac{Y_b - \eta_s}{Y_b} n^{A}_{\mathrm{part}}(x; b) + \frac{Y_b + \eta_s}{Y_b} n^{B}_{\mathrm{part}}(x; b) \right)
\end{aligned}
$$

(12)

where, $f^{\mathrm{pp}}(\eta_s) = \exp\left[-\theta(|\eta_s| - \Delta_\eta/2) \frac{(|\eta_s| - \Delta_\eta/2)^2}{2\sigma_\eta^2} \right]$, Refs: Hirano *et. al*, Phys. Lett. **B 636**, 299 (2006); Adil and Gyulassy, Phys. Rev. **C 72**, 034907 (2005).

- Glauber MC:

$$
e(\tau_0, x) = \frac{K}{2\pi\sigma} \left\{ (1-\alpha) \sum_{i=1}^{N_{\mathrm{part}}} \exp\left[-\frac{(x - x_i^{\mathrm{part}})^2}{2\sigma^2} \right] + \alpha \sum_{i=1}^{N_{\mathrm{coll}}} \exp\left[-\frac{(x - x_i^{\mathrm{coll}})^2}{2\sigma^2} \right] \right\}.
$$

For details on tuning the model for Au-Au and Pb-Pb collisions: Holopainen, Neimi, Eskola, Phys. Rev. C 83, 034901 (2001).
The rapidity dependence in 3D with Glauber MC can be inserted *aposteriori* as in the optical Glauber case

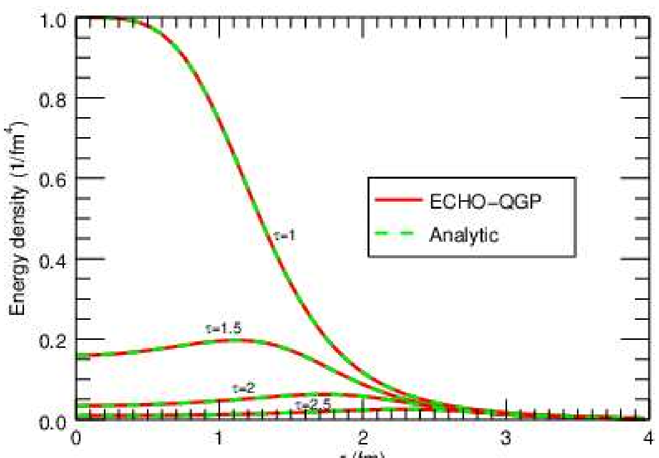
Numerical/Analytic tests: $(2+1)$-D test in Bjorken Coordinates

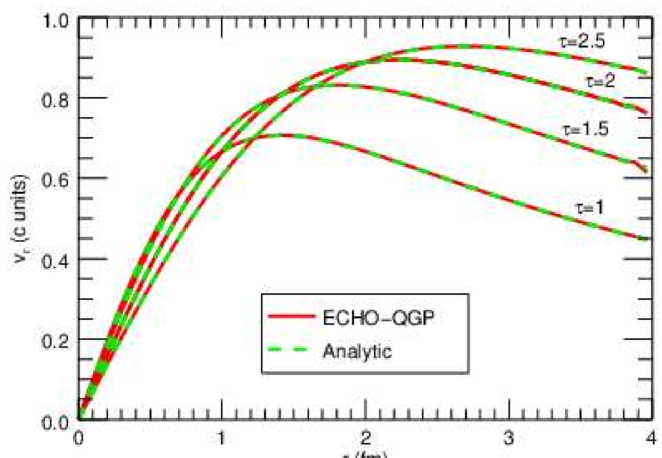
- Gubser's analytic solution (for $P = e/3$ and for the case of azimuthal symmetry.), S. S. Gubser, Phys. Rev. D 82, 085027 (2010):

$$e = \frac{\hat{e}_0}{\tau^{4/3}}(2q)^{8/3}\left[1 + 2q^2(\tau^2 + r^2) + q^4(\tau^2 - r^2)^2\right]^{-4/3},$$

$$u^\tau = \cosh[k(\tau, r)], \quad u^\eta = 0, u^x = \frac{x}{r}\sinh[k(\tau, r)],$$

$$u^y = \frac{y}{r}\sinh[k(\tau, r)], \text{ where } \mathrm{k}(\tau, \mathrm{r}) = \mathrm{arctanh}\left(\frac{2\mathrm{q}^2\tau\mathrm{r}}{1 + \mathrm{q}^2\tau^2 + \mathrm{q}^2\mathrm{r}^2}\right),$$



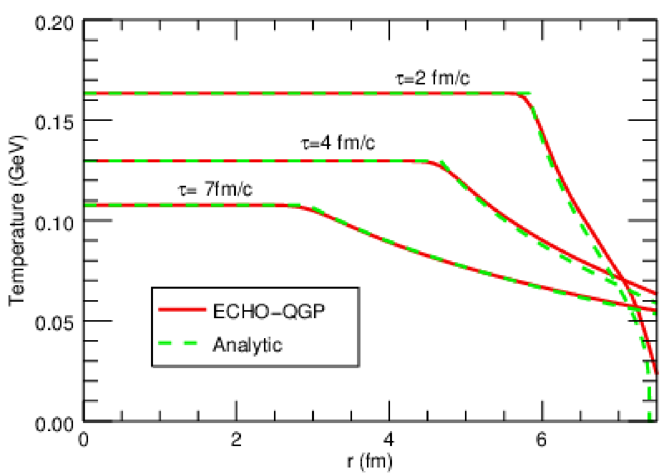


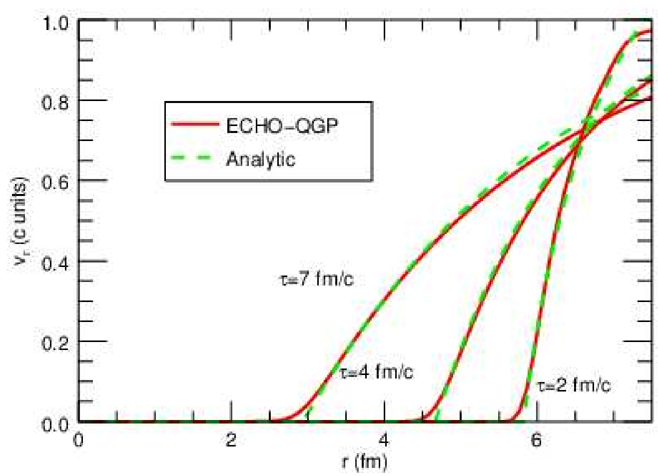
$(2+1)$-D test: Bjorken coordinates

- (for $P = e/3$ and for the case of azimuthal symmetry): Initial profile is of Woods-Saxon form:

$$e(r, \tau_0) = \frac{e_0}{1 + \exp\left[(r - R)/\sigma\right]},$$

$R = 6.4$ fm, $\sigma = 0.02$ fm and an initial temperature $T_0 = 0.2$ GeV at $r = 0$, where $r = \sqrt{x^2 + y^2}$.
Analytical Profile: Baym *et. al*. Nucl. Phys. A. **407**, 541(1983).





Spherically Symmetric 3D profile (Cartesian coordinates)

- Initial profile is of Woods-Saxon form: $e(r,\tau_0) = \frac{e_0}{1+\exp[(r-R)/\sigma]}$, $R=6.4$ fm, $\sigma=0.5$ fm and an initial temperature $T_0=0.307$ GeV at $r=0$, $P_0=4.0$ Gev/fm^3, where $r=\sqrt{x^2+y^2+z^2}$.

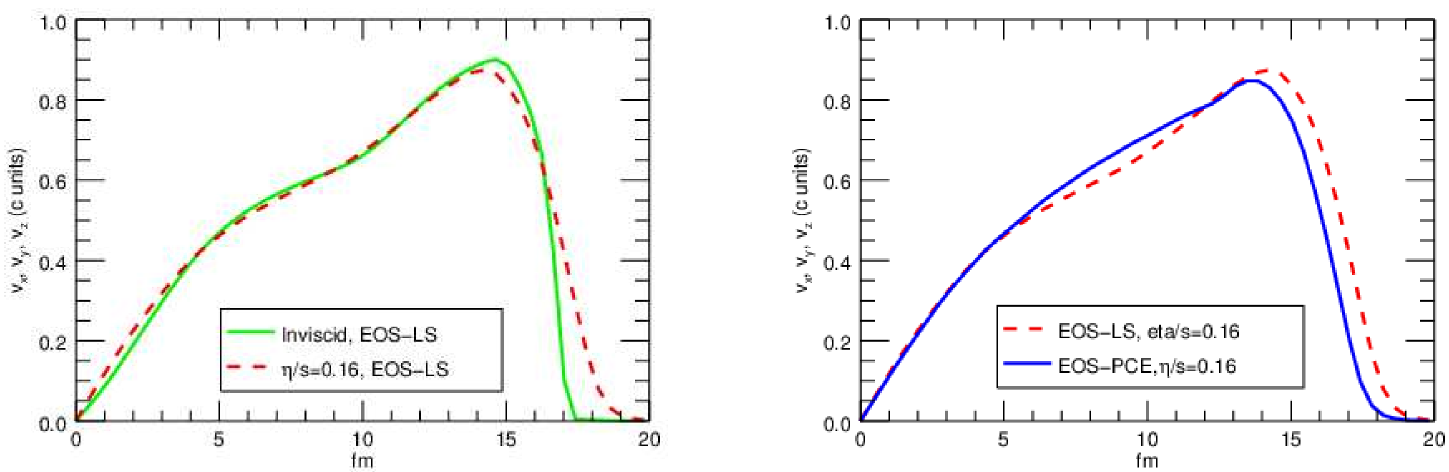


Figure 2: Grid size: -20 fm to 20 fm in each direction with 101 cells.

- Other 1D and 2D analytic tests in viscous case are also performed (see EPJC **73**, 2524 (2013))

Physical tests: Anisotropies (spatial and momentum)

Au-Au collisions at $\sqrt{s_{\rm N}} = 200$ GeV with optical Glauber initial conditions

- Spatial anisotropy, $e_x := \frac{\langle y^2 - x^2 \rangle_e}{\langle y^2 + x^2 \rangle_e}$
  $\langle .. \rangle_e$ denotes a spatial average over the transverse plane, with the local energy-density.
- The momentum anisotropy, $e_p := \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$
  Here, $\langle .. \rangle$ denotes a spatial averaging (over the transverse plane) with weight factor unity.

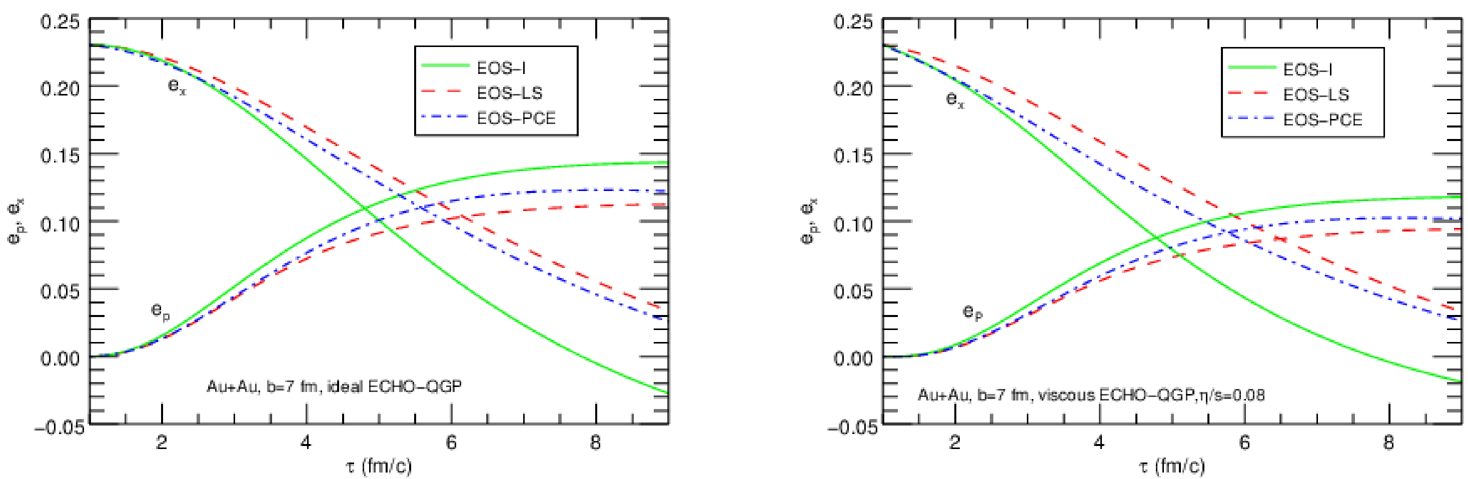


Figure 3: Left: Ideal, Right:Viscous. e_x, e_p for different EOSs at $b = 7$ fm employed in $(2{+}1)$-D simulations

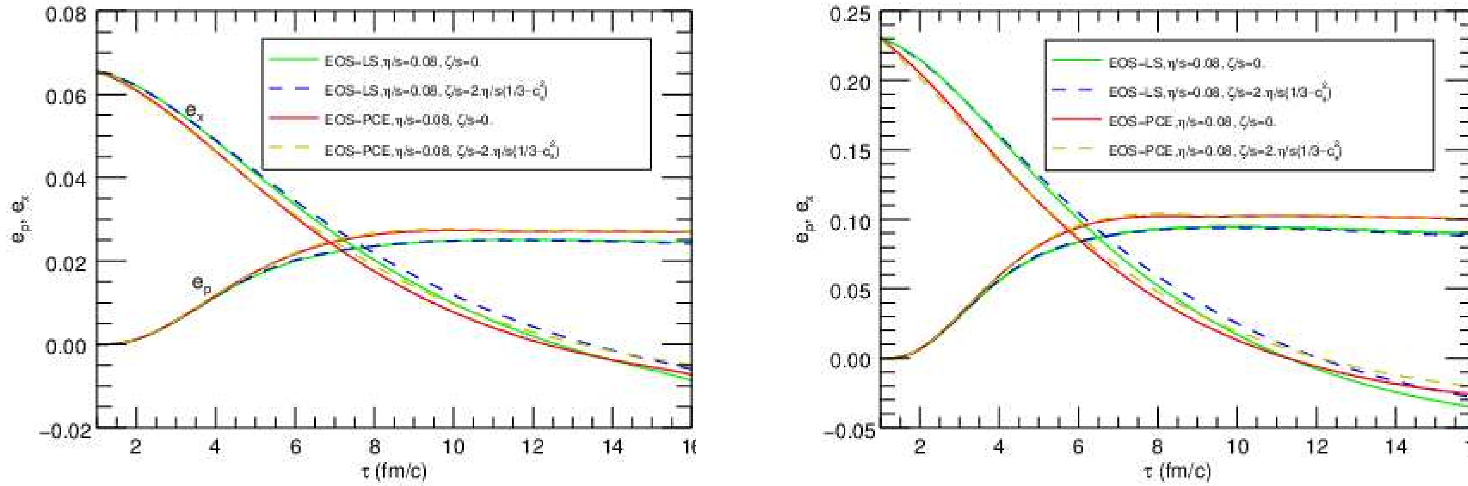


Figure 4: $(2{+}1)$-D simulation in the presence of both η and ζ for $b = 3, 7$ fm.

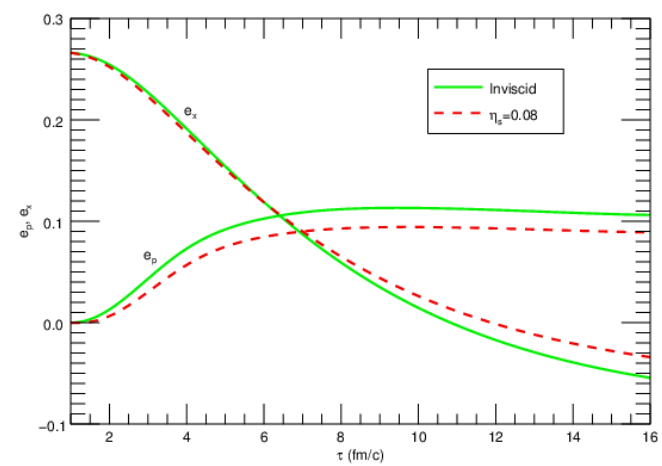


Figure 5: RHIC-type $(3{+}1)$-D simulations, using EOS-LS for $b = 7$ fm at mid-rapidity.

Temporal profile with MC Glauber initial conditions

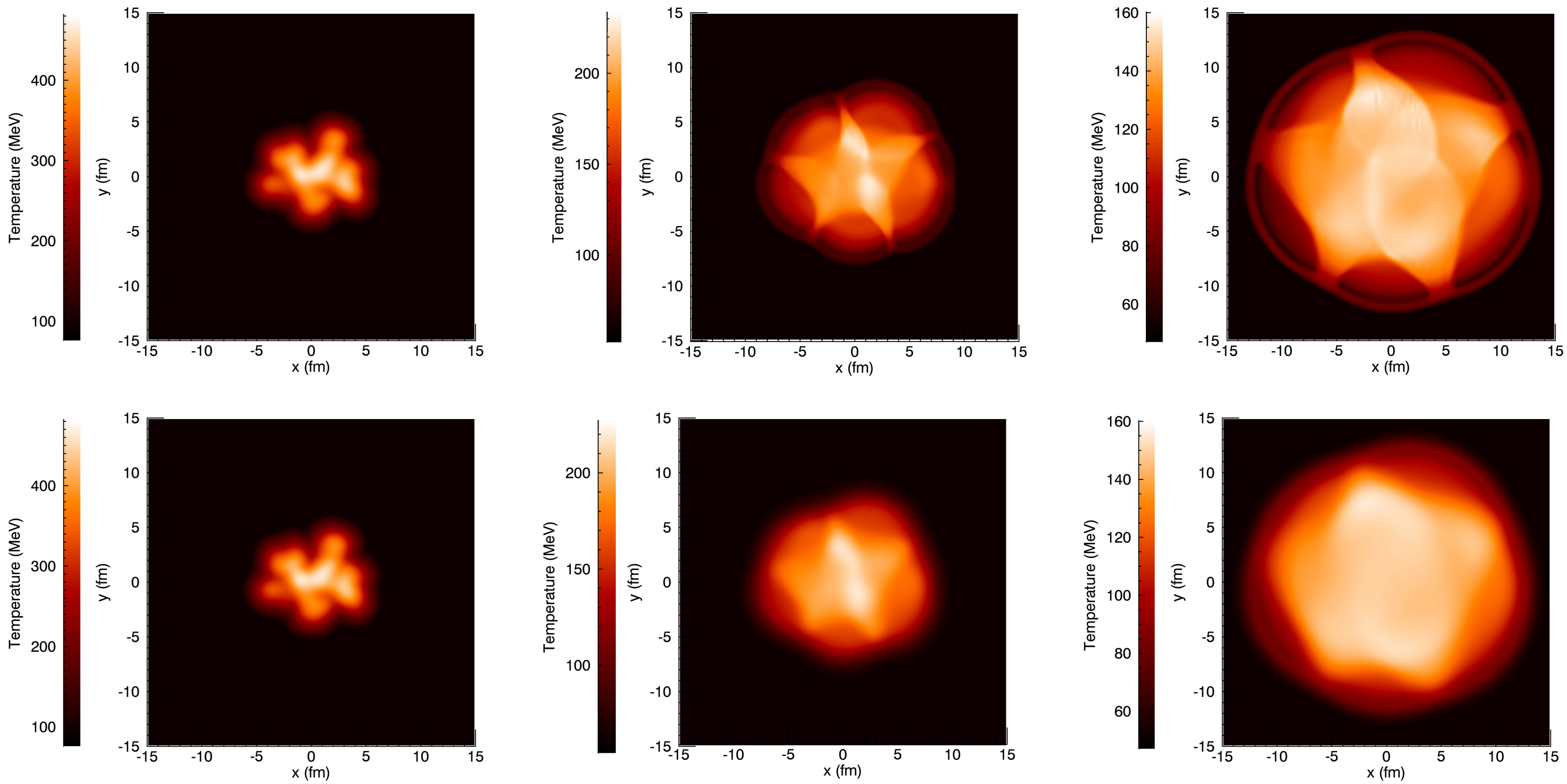


Figure 6: Upper panel: Ideal hydro, Lower panel: viscous hydro. Temperature scans at various times – at $\tau = 1$, 5, and 10 fm/c. ECHO-QGP simulations with Glauber-MC initial conditions. We choose, $\sigma = 0.6$ fm, $K = 19\ \mathrm{GeV/fm^2}$, and $\alpha = 0.2$. Grid size: -15 fm to -15 fm with 151 cells, in each direction at $\eta_s = 0$

ECHO-QGP Freeze-out

- In ECHO-QGP freeze-out is performed using a freeze-out routine based on Cooper-Frye prescription:

$$E\frac{d^3N_i}{dp^3} = \frac{d^3N}{dy p_T dp_T d\phi} = \frac{g}{(2\pi)^3}\int_\Sigma \frac{-p^\mu d^3\Sigma_\mu}{\exp\left[-\frac{u^\mu p_\mu + \mu}{T_{\rm freeze}}\right] \pm 1},$$

where, $f_0(x,p) = \frac{1}{\exp\left[-\frac{u^\mu p_\mu + \mu}{T_{\rm freeze}}\right] \pm 1}$

- Viscous corrections enters as: $f(x,p) = f_(x,p) + \delta f_\pi + \delta f_\Pi$.
- Shear viscous corrections based on quadratic momentum ansatz: $\delta f_\pi = f_0(1 \pm f_0)\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2T^2(e+P)}$
- Bulk-viscous correction will be included in the future!

Particle spectra and elliptic flow

- Ideal $(3+1)$-D hydro:

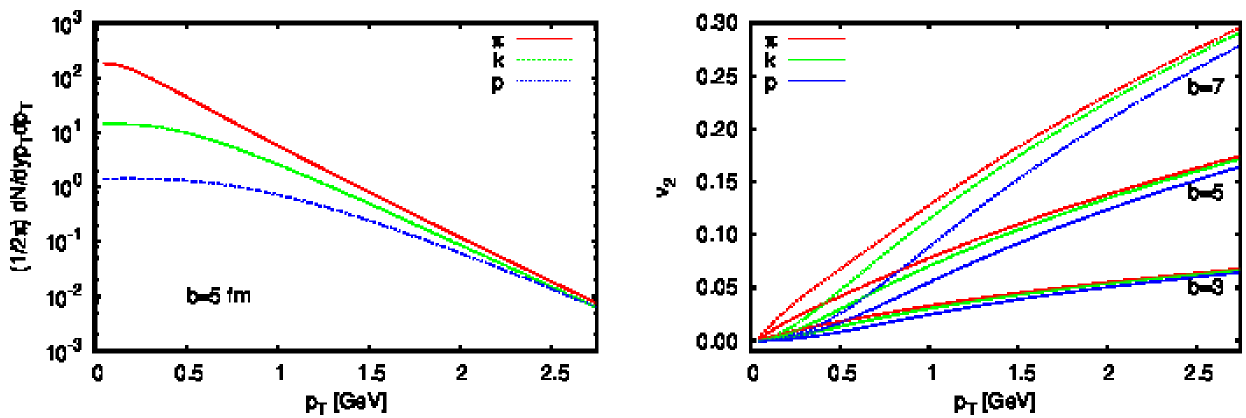


Figure 7: $\mathrm{p_T}$ spectra and v_2 of primary pions, kaons and protons as obtained in $(3{+}1)$-D ideal hydrodynamics

Ideal vs Viscous in $(2+1)$-D

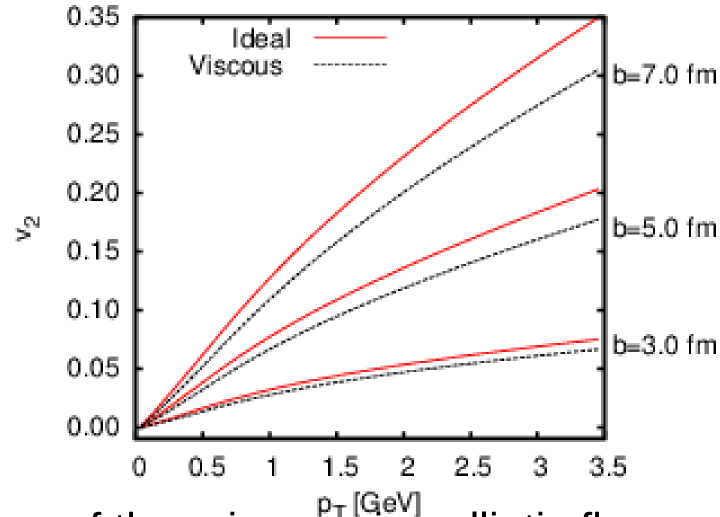


Figure 8: p_T and b dependence of the primary pion elliptic flow coefficient v_2 within $(2+1)$-D ideal and viscous hydrodynamics

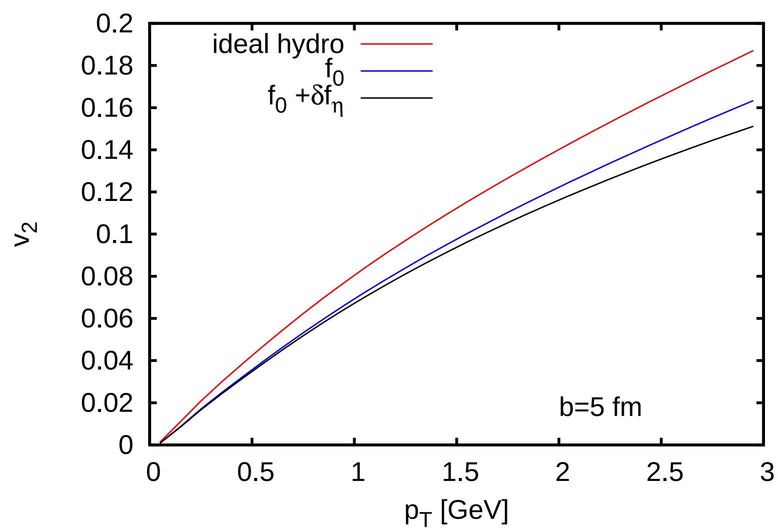


Figure 9: Shear viscous correction to v_2

Future directions

- Event-by-Event hydro for $d - A$ and $p - A$ collisions
- Flow analysis with Event-by-Event Hydro
- CGC Initial conditions/KLN MC initial conditions
- Investigations on the role of Vorticity in HIC
- Bulk viscous corrections to the particle spectra
- Inclusion of magnetic field

Thank you!

Back-ups: Mildly relativistic shear flow in $(1+1)$-D

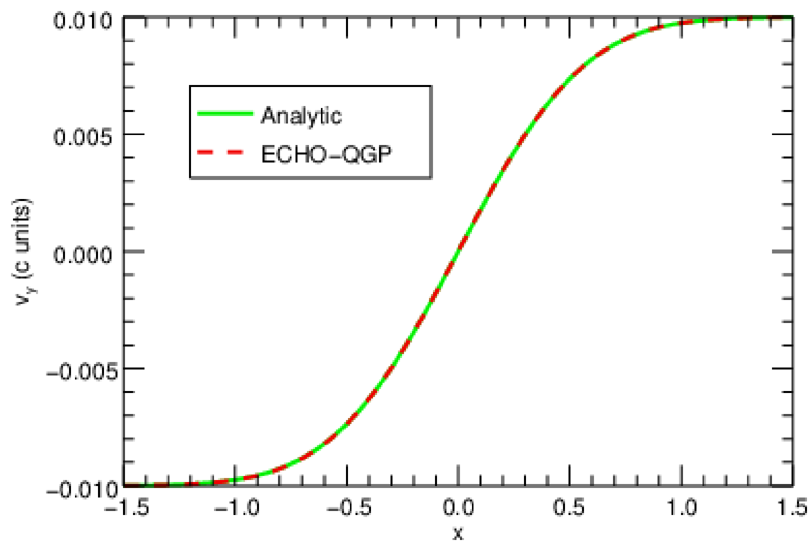


Figure 10: Spatial dependence of the velocity shown along with the analytic result at $t = 10$ fm/c. The grid is made by 301 cells, ranging from $x = -1.5$ to 1.5 fm.

- Choose $e + P \sim constt$, velocity pofile: $v^y \equiv v^y(x)$, EOS-I. Only shear (no bulk), always preserving $\gamma \approx 1$ and NS limit

2D shock tubes

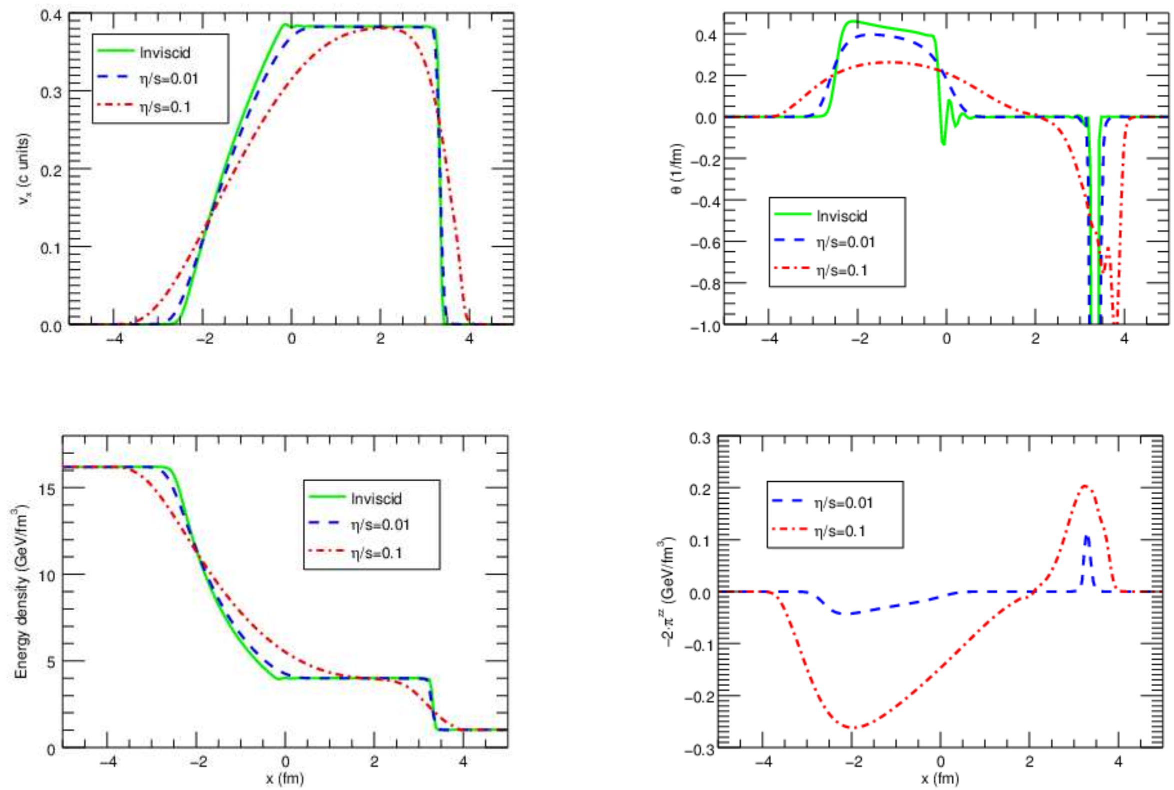


Figure 11: The velocity component v_x, the expansion rate θ, the energy density e, and $-2\pi^{zz}$ as a function of x for $\eta/s = 0,\ 0.01,\ 0.1$ at $t = 4$ fm/c. The grid is made by 201×201 regularly spaced cells, with x and y coordinates ranging from -5 to 5 fm using Minkowski coordinates.

Boost-invariance along z-axis

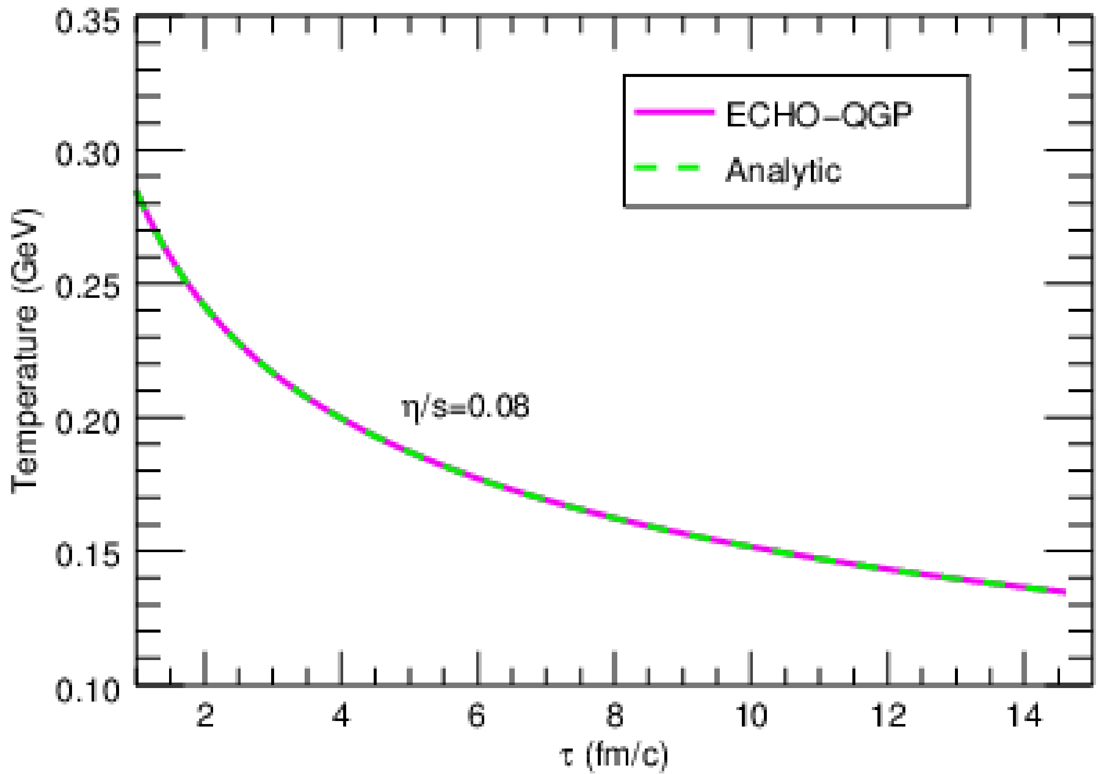


Figure 12: EOS-I, $\eta/s = 0.08$, $\tau_0 = 1.0$ NS expressions for the viscous fluxes.

$(1+1)$-D Analytic test

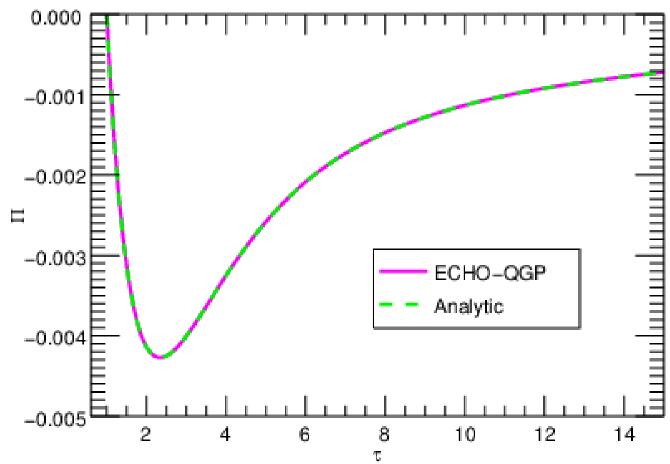


Figure 13: Comparison between the evolution of $\Pi(\tau)$ computed by ECHO-QGP and the semi-analytic solution in EJPC **73**, 2524 (2013). The parameters are $\tau_0 = 1$ fm/c, $\zeta = 0.01\,\mathrm{GeV/fm^2}$, and $\tau_\pi = 1$ fm/c.

Back-ups: Procedure

- Assuming for simplicity a metric in which $g_{00} = -1$ and $g_{0i} = 0$ (both conditions are met either by flat space and Bjorken coordinates).
- First, the orthogonality conditions $\pi^{\mu\nu} u_\nu = 0$ yield the relations

(13) $$\pi^{0i} = \pi^{ij} v_j, \quad \pi^{00} = \pi^{0i} v_i = \pi^{ij} v_i v_j,$$

where $v^i = u^i/\gamma$, $v_i = g_{ij} v^j$, and $\gamma = (1 - v_i v^i)^{-1/2}$.

- we rewrite the conservative variables as

$$N = n\gamma, \qquad S^i = (e + P + \Pi)\gamma^2 v^i + \pi^{0i},$$
$$E = (e + P + \Pi)\gamma^2 - (P + \Pi) + \pi^{00},$$

where we have substituted $S^i = g^{ij} S_j$ and $\pi^{00} = -\pi^0_0$.

- In LRF charge and energy densities would be given by

(14) $$e = E - g_{ij} S^i v^j, \quad n = N/\gamma,$$

then also the pressure $P = \mathcal{P}(e)$ or even $P = \mathcal{P}(e, n)$ can be worked out.

Echo-qgp algo

- An external cycle on v^i components is performed, starting from the values at the previous time-step. Then we define the quantities $\tilde{E} = E - \pi^{00}, \quad \tilde{S}^i = S^i - \pi^{0i}$;
- An inner cycle on P is performed, then we define $\tilde{P}(P) = P + \Pi, \quad v^2(P) = \tilde{S}^2/(\tilde{E} + \tilde{P})^2$ and $e(P) = (\tilde{E} + \tilde{P})(1 - v^2) - \tilde{P}, \quad n(P) = N\sqrt{1 - v^2}$. The cycle can be iterated via a Newton-Raphson procedure, trying to minimize the quantity $f(P) = \mathcal{P}[e(P), n(P)] - P$, and then $P_{\rm new} = P - f(P)/f'(P)$, where $f'(P) = \frac{\partial \mathcal{P}}{\partial e}\frac{de}{dP} + \frac{\partial \mathcal{P}}{\partial n}\frac{dn}{dP} - 1$, until $|P_{\rm new} - P| \to 0$ to a given tolerance.
- Once the pressure for the old choice of v^i components has been found, the new choice is provided by $v^i_{\rm new} = \tilde{S}^i/(\tilde{E} + \tilde{P})$, and the external loop is closed when a given tolerance in $v^i_{\rm new} - v^i$ terms is reached.