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BAMPS with the improved Gunion-Bertsch matrix element

C. Greiner,

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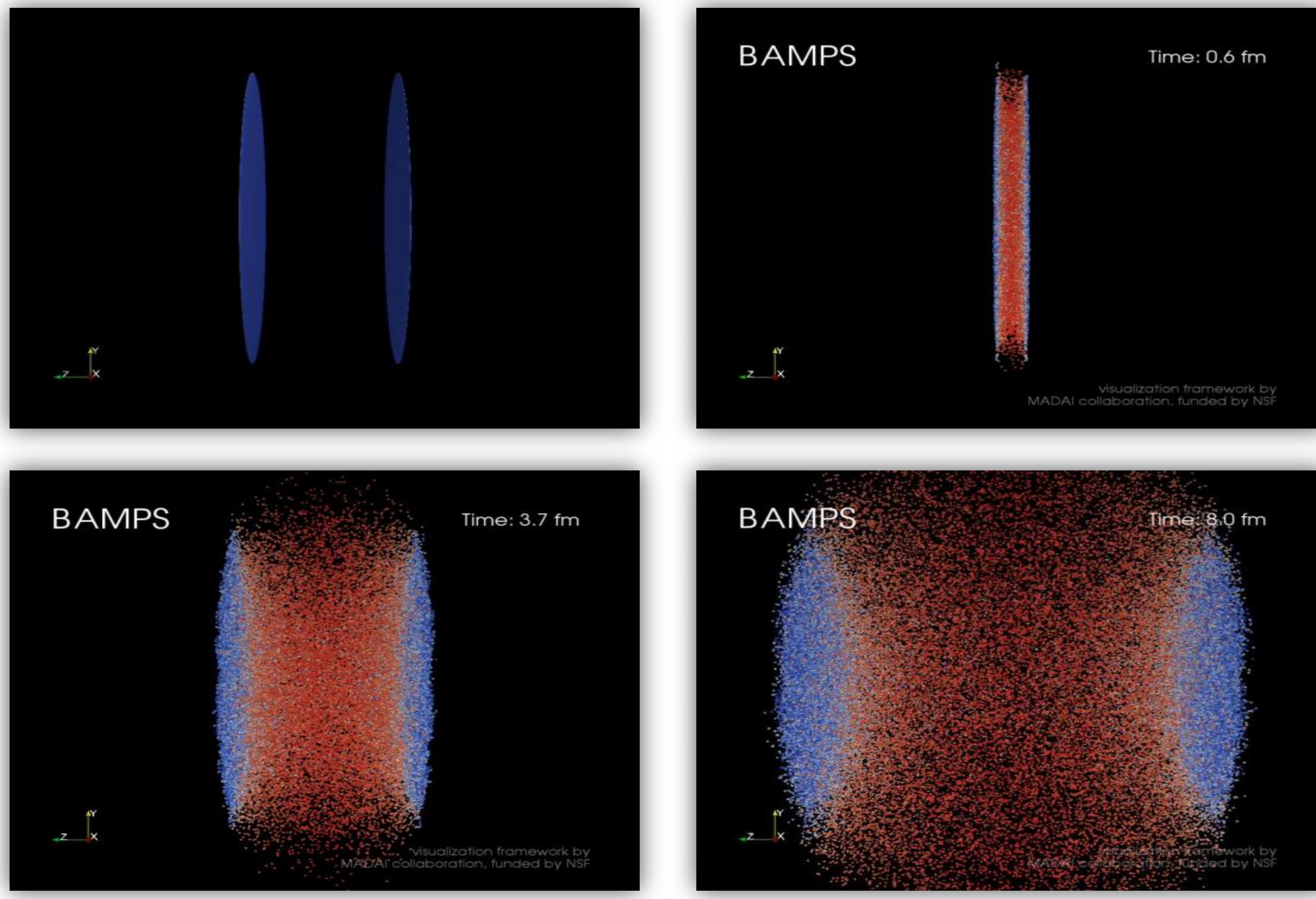
in collaboration with:

O. Fochler F. Senzel, J. Uphoff, and Zhe Xu

- BAMPS: Boltzmann parton transport including radiation
- Gunion-Bertsch ME (literature) and improved ME
- observables at RHIC and LHC (elliptic flow, jets, heavy quarks)

QCD thermalization using parton cascade



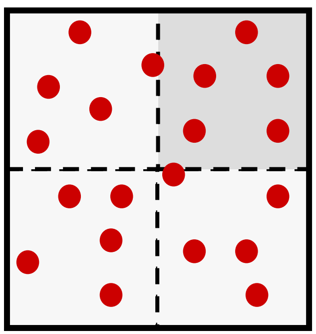
BAMPS: Boltzmann Approach of MultiParton Scatterings

A transport algorithm solving the Boltzmann-Equations for on-shell partons with pQCD interactions

$$p^{\mu}\partial_{\mu} f(x,p) = C_{gg\to gg}(x,p) + C_{gg\leftrightarrow ggg}(x,p)$$

↑

(Z)MPC, VNI/BMS, AMPT



collision probability:

$$\text{for } 2\leftrightarrow 2 \quad P_{22} = v_{rel}\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$

$$\text{for } 2\to 3 \quad P_{23} = v_{rel}\sigma_{23}\frac{\Delta t}{\Delta^3 x}$$

$$\text{for } 3\to 2 \quad P_{32} = \frac{I_{32}}{8E_1E_2E_3}\frac{\Delta t}{(\Delta^3 x)^2}$$

Xiong, Shuryak, PRC 49, 2203 (1994)
Dumitru, Gyulassy, PLB 494, 215 (2000)
Serreau, Schiff, JHEP 0111, 039 (2001)
Baier, Mueller, Schiff, Son, PLB 502, 51 (2001)

BAMPS:
Z. Xu and C. Greiner, PRC 71, 064901 (2005);
Z. Xu and C. Greiner, PRC 76, 024911 (2007)

screened partonic interactions in leading order pQCD

$$|M_{gg\to gg}|^2 = \frac{9g^4}{2}\frac{s^2}{(q_\perp^2+m_D^2)^2},$$

elastic part

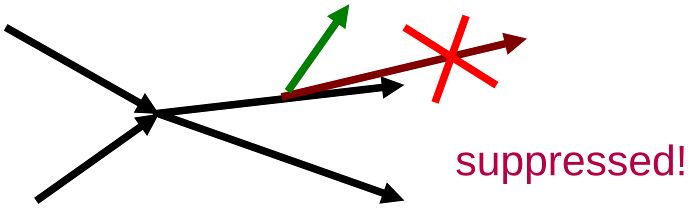
$$|M_{gg\to ggg}|^2 = \left(\frac{9g^4}{2}\frac{s^2}{(q_\perp^2+m_D^2)^2}\right)\left(\frac{12g^2q_\perp^2}{k_\perp^2\left((\vec{k}_\perp-\vec{q}_\perp)^2+m_D^2\right)}\right)\Theta_{LPM}(k_\perp\Lambda_g-\cosh y)$$

radiative part

J.F.Gunion, G.F.Bertsch, PRD 25, 746(1982)
T.S.Biro at el., PRC 48, 1275 (1993)
S.M.Wong, NPA 607, 442 (1996)

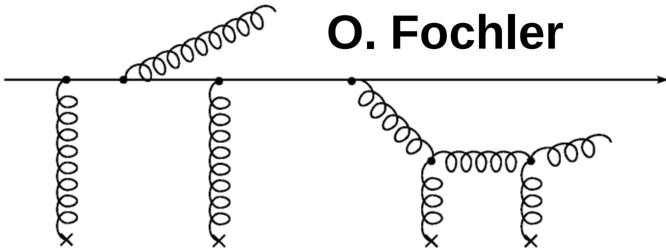
screening mass: $m_D^2 = m_D^2(x,t) = 16\pi\alpha_s \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}(3f_g + n_f f_q),$

LPM suppression: the formation time $\Delta\tau \approx \frac{1}{k_\perp}\cosh y < \Lambda_g$



Λ_g: mean free path

O. Fochler



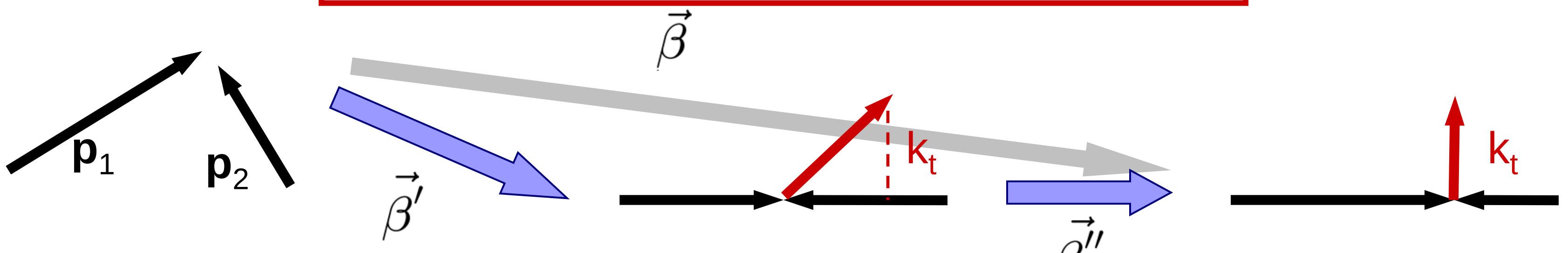
LPM - cutoff

- transport model: incoherent treatment of gg→ggg processes

➔ parent gluon must not scatter during formation time of emitted gluon

  - discard all possible interference effects (**Bethe-Heitler regime**)

$$|M_{gg\to ggg}|^2 \to |M_{gg\to ggg}|^2\,\Theta\left(\lambda-\tau\right)$$



lab frame

$\lambda = 1/<n\sigma>$

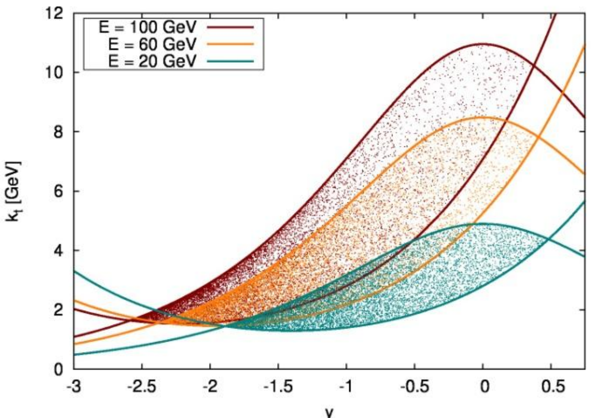
CM frame

$\beta'' = \tanh y$

$\tau = 1 / k_t$

total boost $\gamma = \gamma'\gamma''(1+\vec{\beta}'\vec{\beta}'') = \dfrac{\cosh y}{\sqrt{1-\beta'^2}}\left(1+\beta' \tanh y \cos\Theta\right)$

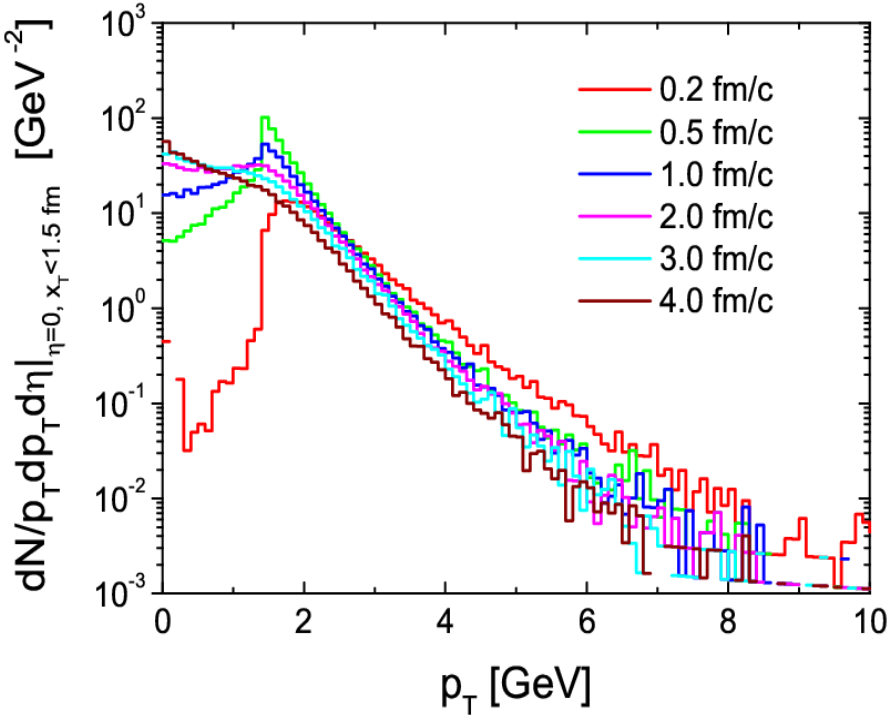
$$\Theta(\lambda-\tau) \to \Theta\left(k_\perp - \frac{\gamma}{\lambda}\right)$$



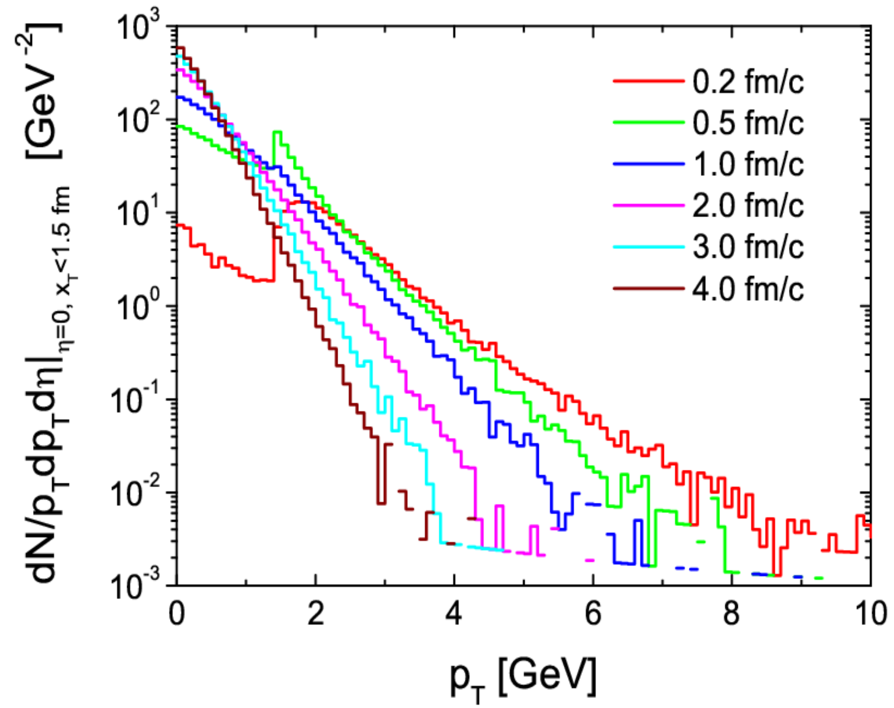
p_T spectra

at collision center: x_T<1.5 fm, Δz < 0.4 t fm of a central Au+Au at $s^{1/2}$=200 GeV
Initial conditions: minijets p_T>1.4 GeV; coupling α_s=0.3

simulation pQCD, **only 2-2**



simulation pQCD 2-2 + 2-3 + 3-2



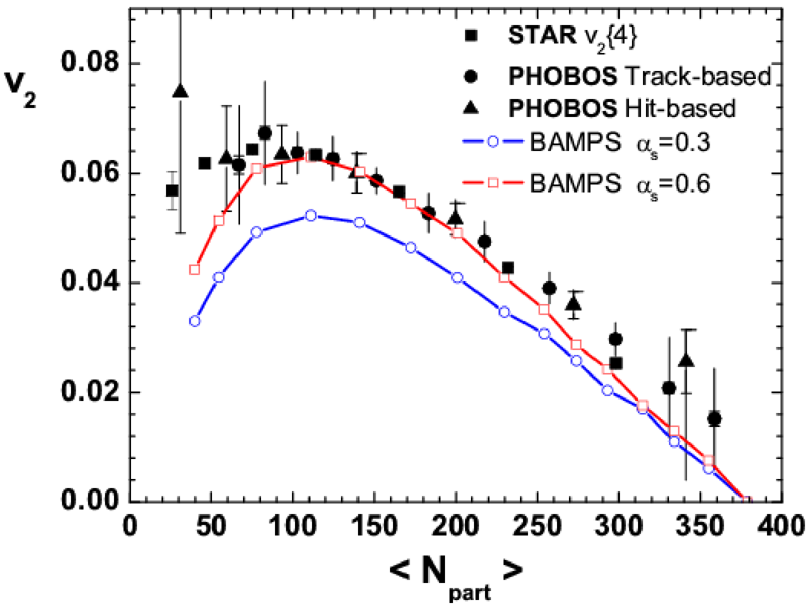
2-2: NO thermalization

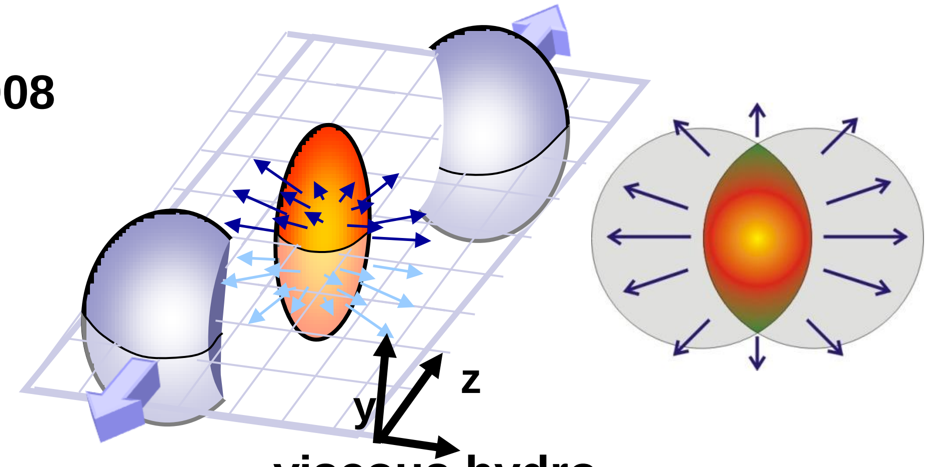
3-2 + 2-3: thermalization!
Hydrodynamic behavior!

Elliptic Flow and Shear Viscosity in 2-3 at RHIC

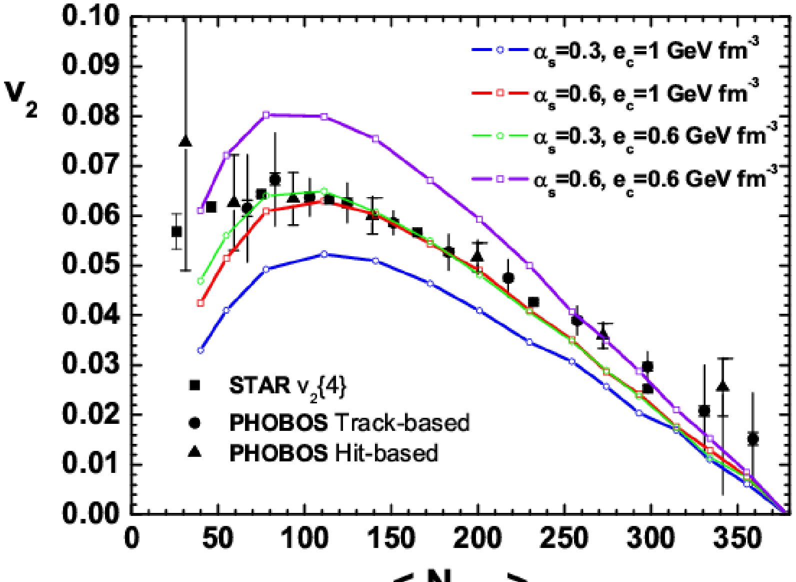
2-3 Parton cascade BAMPS

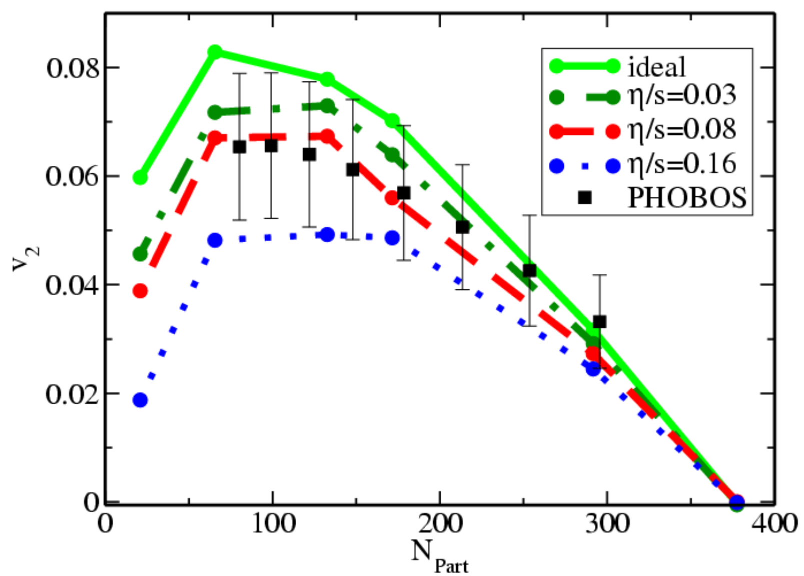
Z. Xu, CG, H. Stöcker, PRL 101:082302,2008





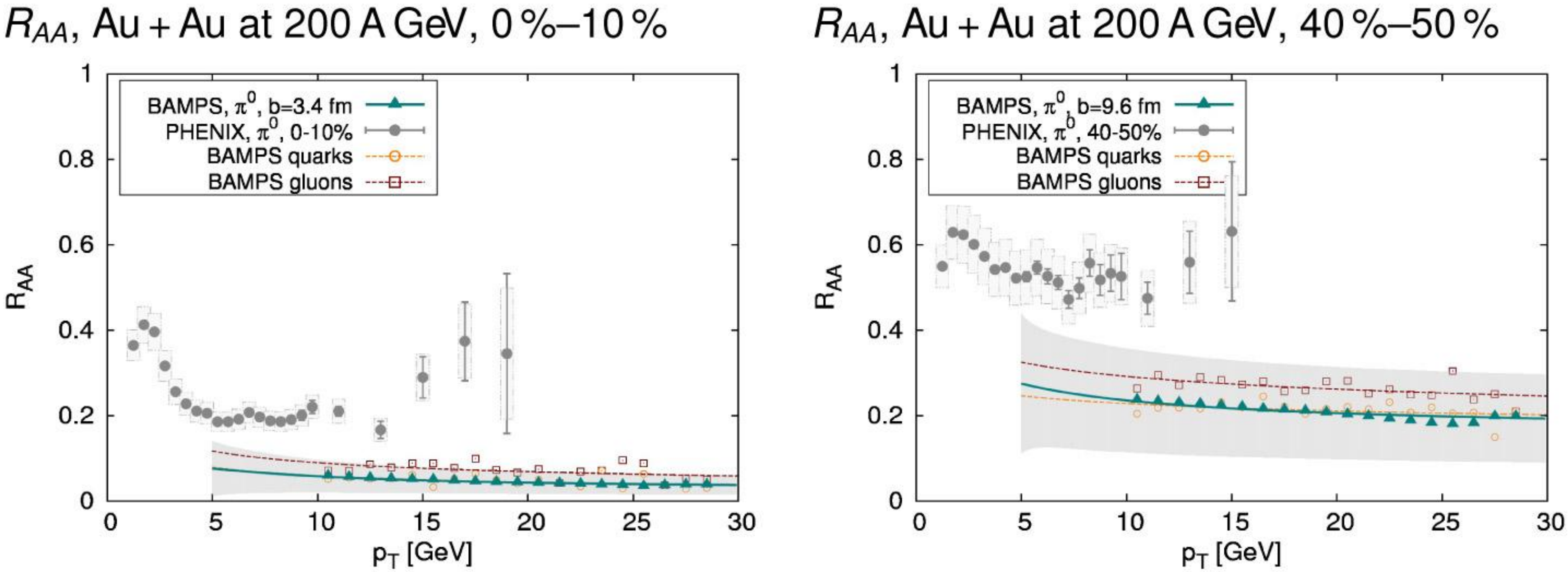
Romatschke, PRL 99, 172301,2007





η/s at RHIC: 0.08-0.2

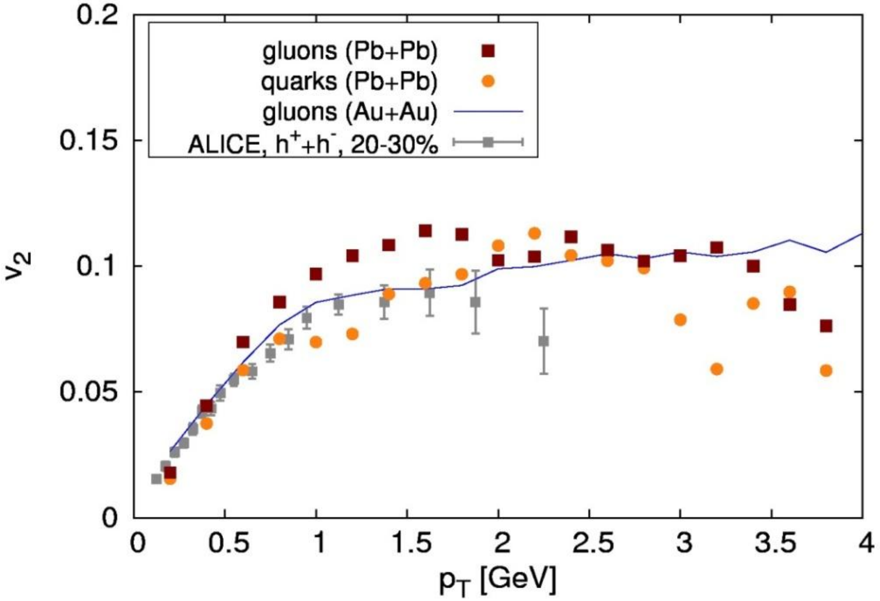
Jet Suppression in BAMPS Simulations at RHIC

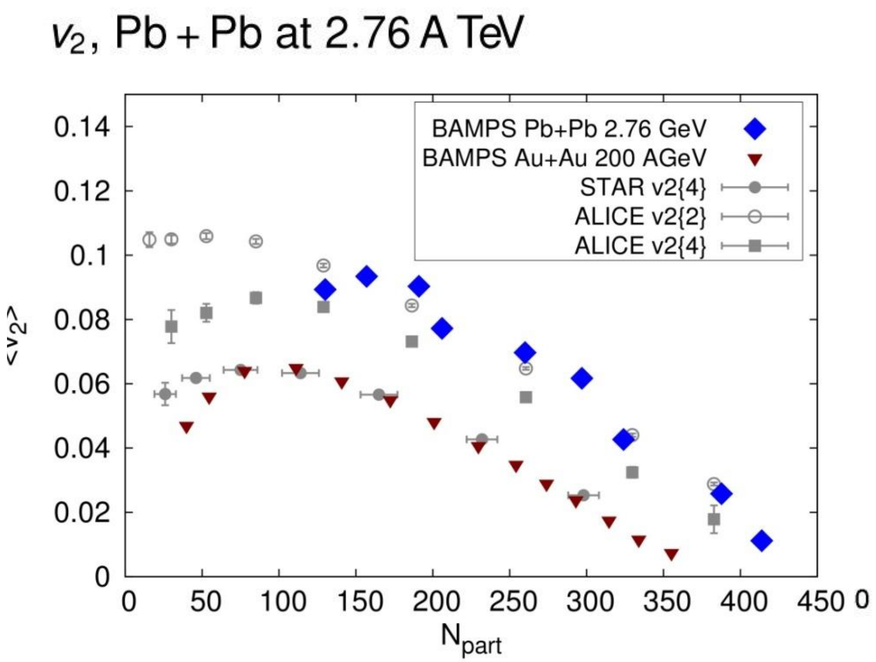


- Hadronization via AKK fragmentation functions
- Suppression in BAMPS is too strong
  - Strong mean energy loss in $2 \rightarrow 3$ processes
  - Sizeable conversion of quark jets into gluon jets
  - Small difference in the energy loss of quarks and gluons

O. Fochler et al, Phys. Rev. C 82 (2010)

Jet Suppression and Elliptic Flow at LHC

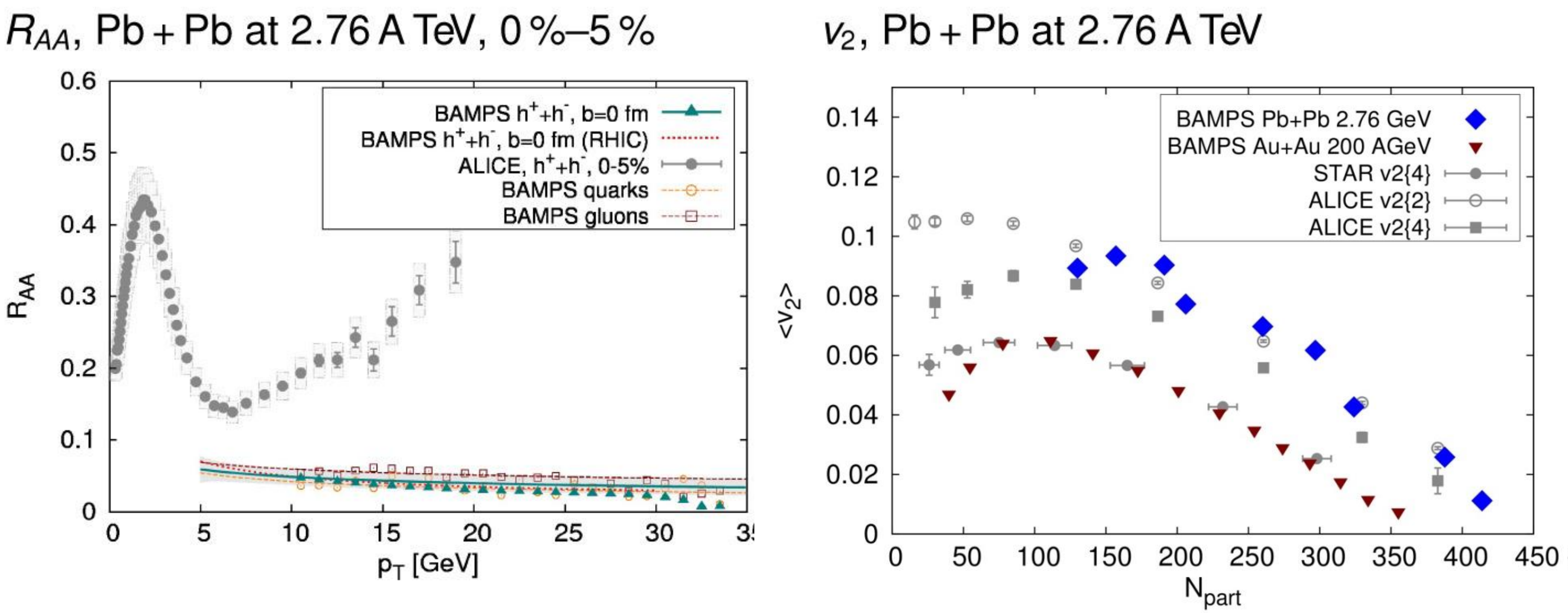




- PYTHIA initial conditions (Uphoff, OF et al. PRC 82 (2010)), $\alpha_s = 0.3$
- R_{AA} almost identical to RHIC, does not reproduce rise towards large p_T
- Integrated v_2 shows increase, drops below data at about 50 % centrality

O. Fochler et al,
J. Phys. G 38 (2011)

Jet Suppression and Elliptic Flow at LHC



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O. Fochler et al, J. Phys. G 38 (2011)

A Closer Look on the Gunion-Bertsch Approximation

- J.-W. Chen, J. Deng, H. Dong, Q. Wang claim:
  BAMPS results are off by a factor 6 due to miscounting of symmetry factors arXiv:1107:0522
- B. Zhang analyzes GB vs. exact and finds differences up to 50% arXiv:1208.1224

$$|\mathcal{M}_{GB}|^2 = \frac{72\pi^2\alpha_s^2 s^2}{\mathbf{q}_\perp^2} \frac{48\pi\alpha_s}{\mathbf{k}_\perp^2(\mathbf{k}_\perp - \mathbf{q}_\perp)^2}$$

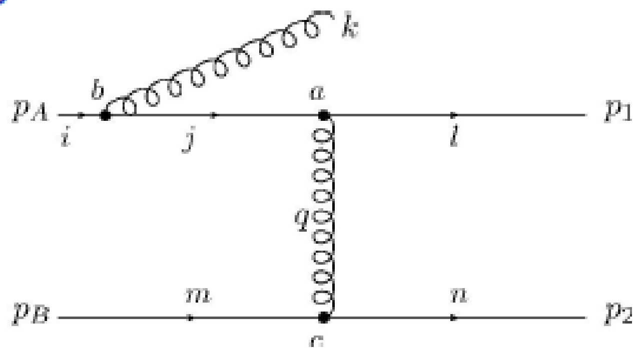
Gunion-Bertsch approximation
Phys.Rev.,D25 (1982)

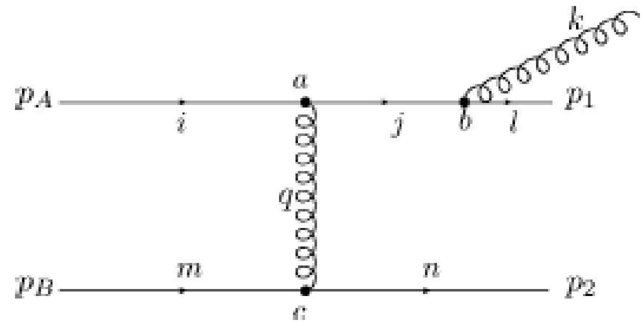
Exact result
Berends et al., PLB 103 (19
Ellis and Sexton, Nucl.Phys
(1986)

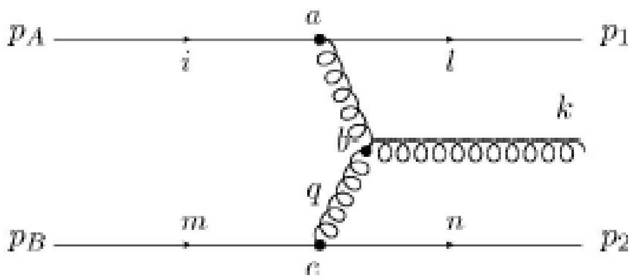
$$\begin{aligned}|M_{\text{exact}}|^2 = \frac{g^6}{2}\left[N^3/(N^2-1)\right] &[(12345)+(12354)+(12435)+(12453)+(12534)\\ &+(12543)+(13245)+(13254)+(13425)+(13524)+(14235)+(14325)]\\ &\times \frac{[(p_1p_2)^4+(p_1p_3)^4+(p_1p_4)^4+(p_1p_5)^4+(p_2p_3)^4]}{(p_1p_2)(p_1p_3)(p_1p_4)(p_1p_5)(p_2p_3)(p_2p_4)(p_2p_5)(p_3p_4)(p_3p_5)(p_4p_5)}\\ &+\frac{[(p_2p_4)^4+(p_2p_5)^4+(p_3p_4)^4+(p_3p_5)^4+(p_4p_5)^4]}{(p_1p_2)(p_1p_3)(p_1p_4)(p_1p_5)(p_2p_3)(p_2p_4)(p_2p_5)(p_3p_4)(p_3p_5)(p_4p_5)}\end{aligned}$$

Gunion-Bertsch – Some Details

Diagrams:







Rapidity of emitted gluon

$$y = \frac{1}{2} \ln \frac{k^+}{k^-} = \ln \frac{x\sqrt{s}}{k_\perp}$$

Kinematics: (light-cone coordinates)

$$p_A = (\sqrt{s}, 0, 0, 0) \qquad p_B = (0, \sqrt{s}, 0, 0)$$

$$k = (x\sqrt{s}, \frac{k_\perp^2}{x\sqrt{s}}, \mathbf{k}_\perp) \qquad q = (q^+, q^-, \mathbf{q}_\perp)$$

Momentum conservation gives

$$p_1 = p_A + q - k \qquad p_2 = p_B - q$$

- k = momentum of radiated gluon, q = exchanged momentum
- Gunion-Bertsch: $A^+ = 0$ gauge, lower lines do not contribute (much)
- Scalar QCD to simplify calculations

The Problems with Gunion-Bertsch

Gunion and Bertsch explicitly state the following approximations:
$k_\perp \ll \sqrt{s}$, $q_\perp \ll \sqrt{s}$, $xq_\perp \ll k_\perp$

So where are the problems?

- A missing $(1-x)^2$ term

$$|\mathcal{M}_{GB}|^2 \sim (1-x)^2 \frac{s^2}{\mathbf{q}_\perp^2} \frac{1}{\mathbf{k}_\perp^2 (\mathbf{k}_\perp - \mathbf{q}_\perp)^2}$$

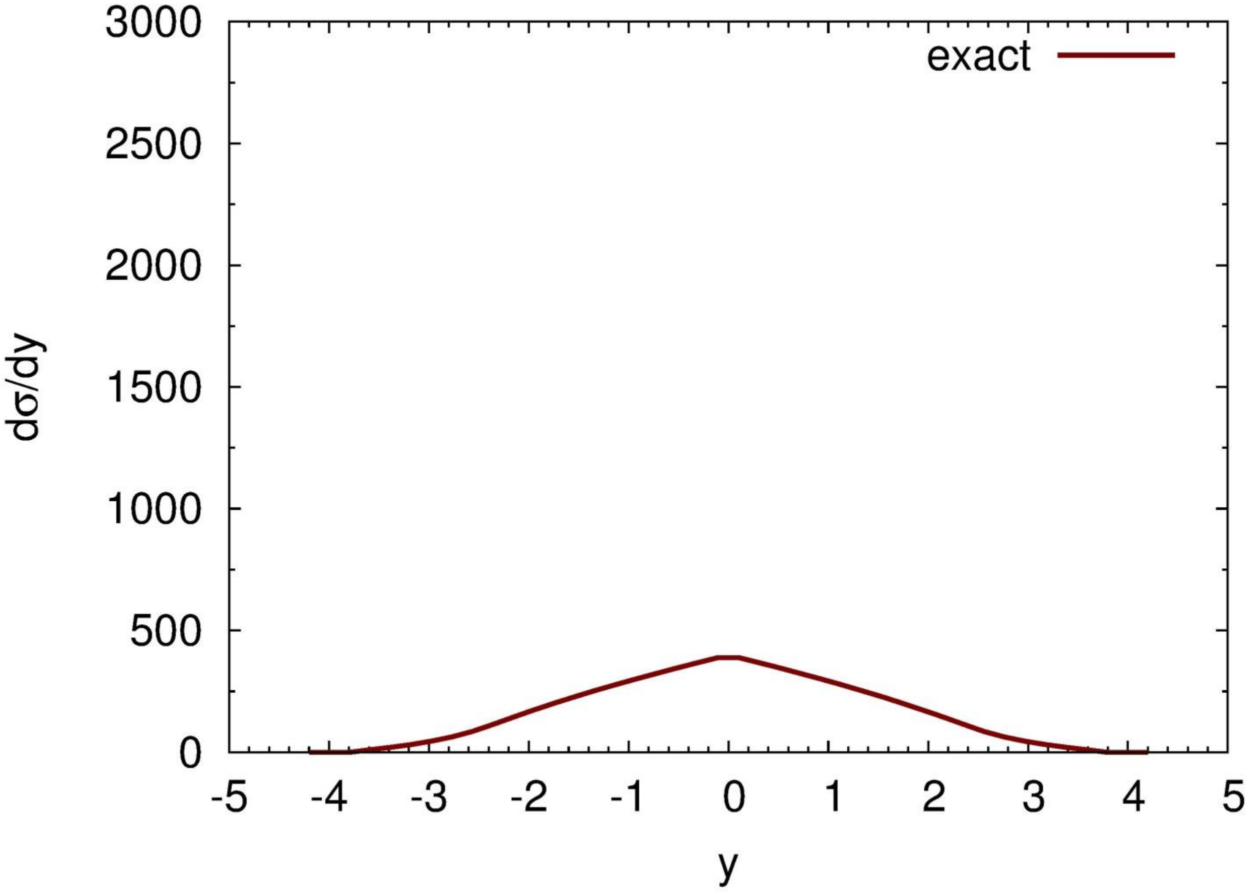
  x is the fraction of forward-momentum carried by the radiated gluon, $x = \frac{k_\perp}{\sqrt{s}} e^y$

- When not at midrapidity, $y = 0 \equiv x = \frac{k_\perp}{\sqrt{s}}$, constraints are needed to arrive at the GB result that break the symmetry and make it only valid for forward emission

$$k_\perp^2 \ll x^2 s \quad \equiv \quad k^+ \gg k^- \quad \equiv \quad y \gg 0$$

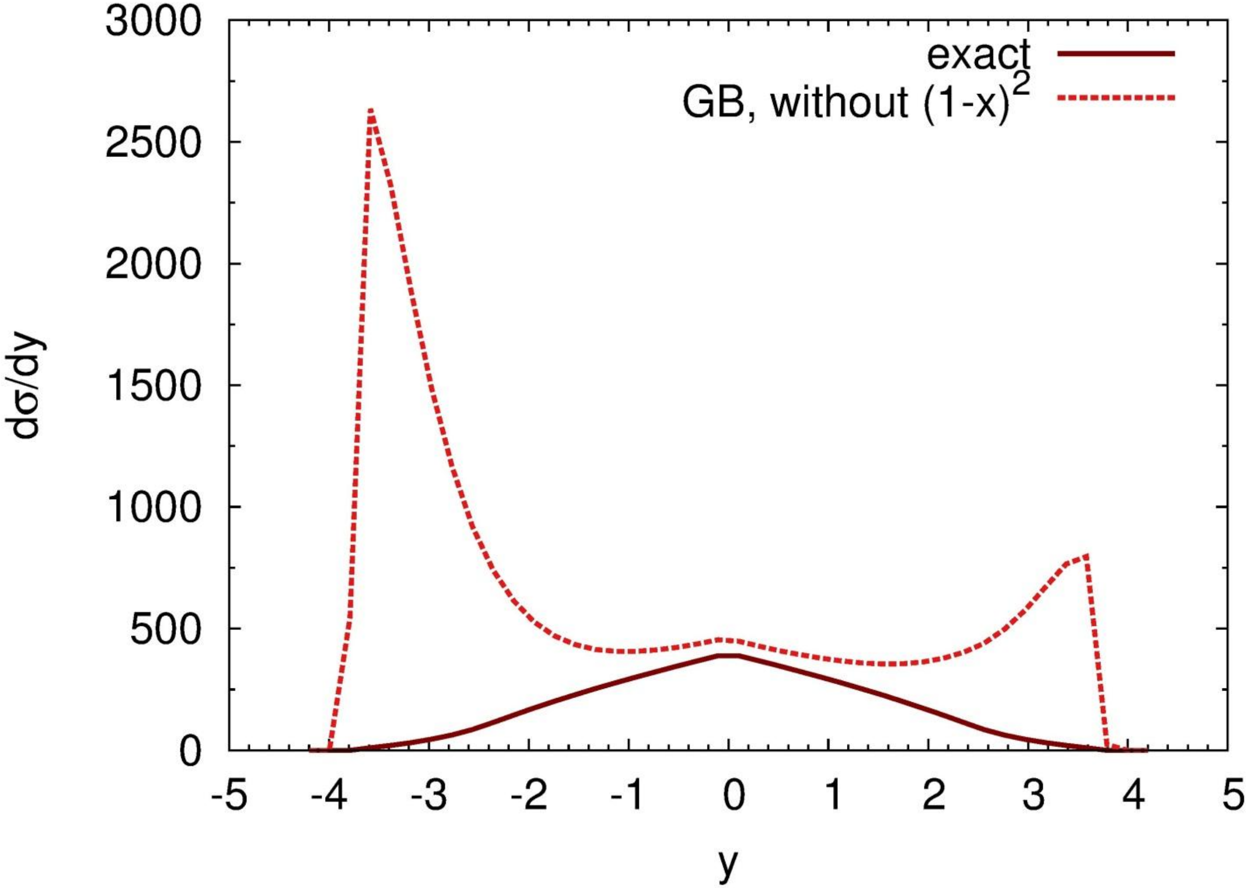
  Using $x = \frac{k_\perp}{\sqrt{s}} e^{|y|}$ takes this into account.

A Closer Look on the Gunion-Bertsch Approximation



- Infrared screening for both GB and exact: $\Theta(cut) = \Theta(p_i p_j - \lambda)$
- Integration both in GB coordinates and in standard phase space with numeric δ-functions

The Problems with Gunion-Bertsch



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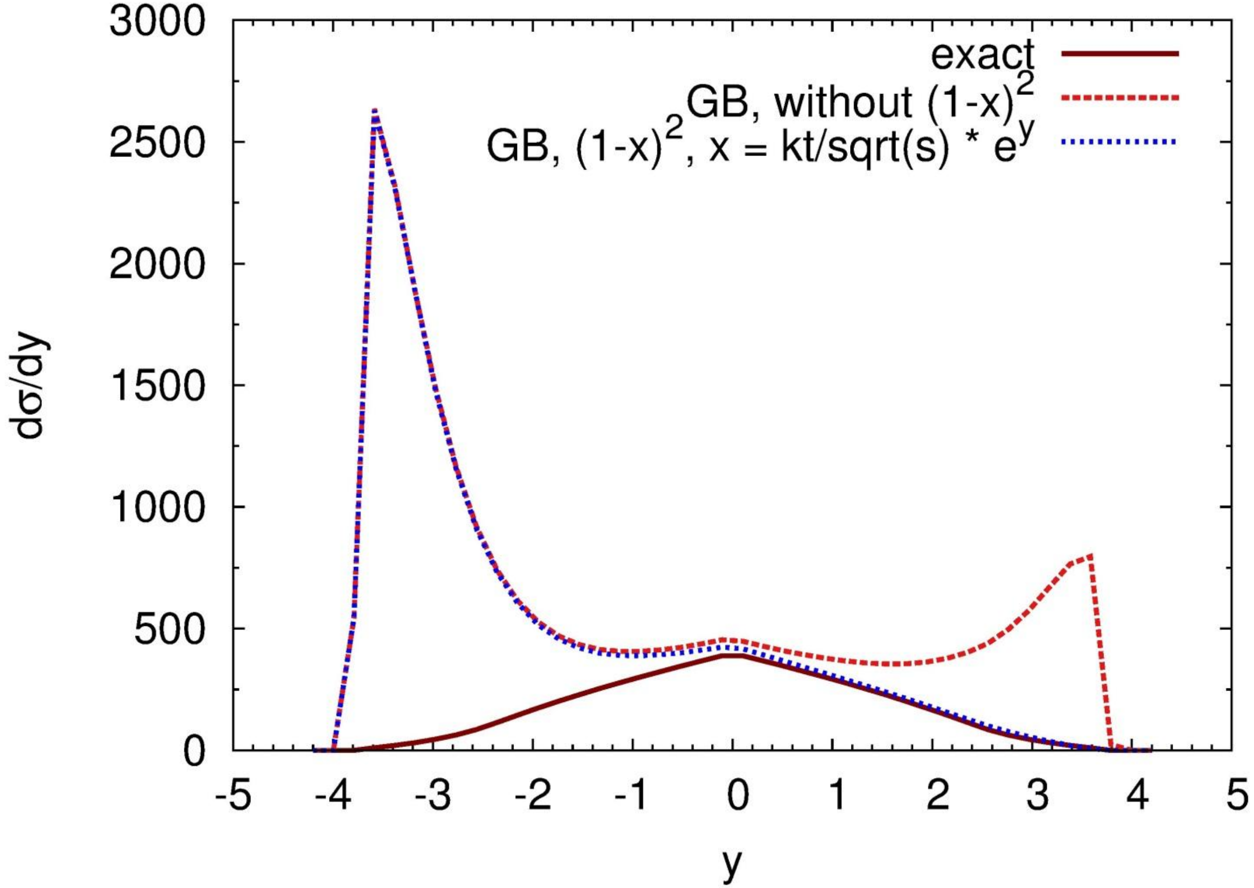

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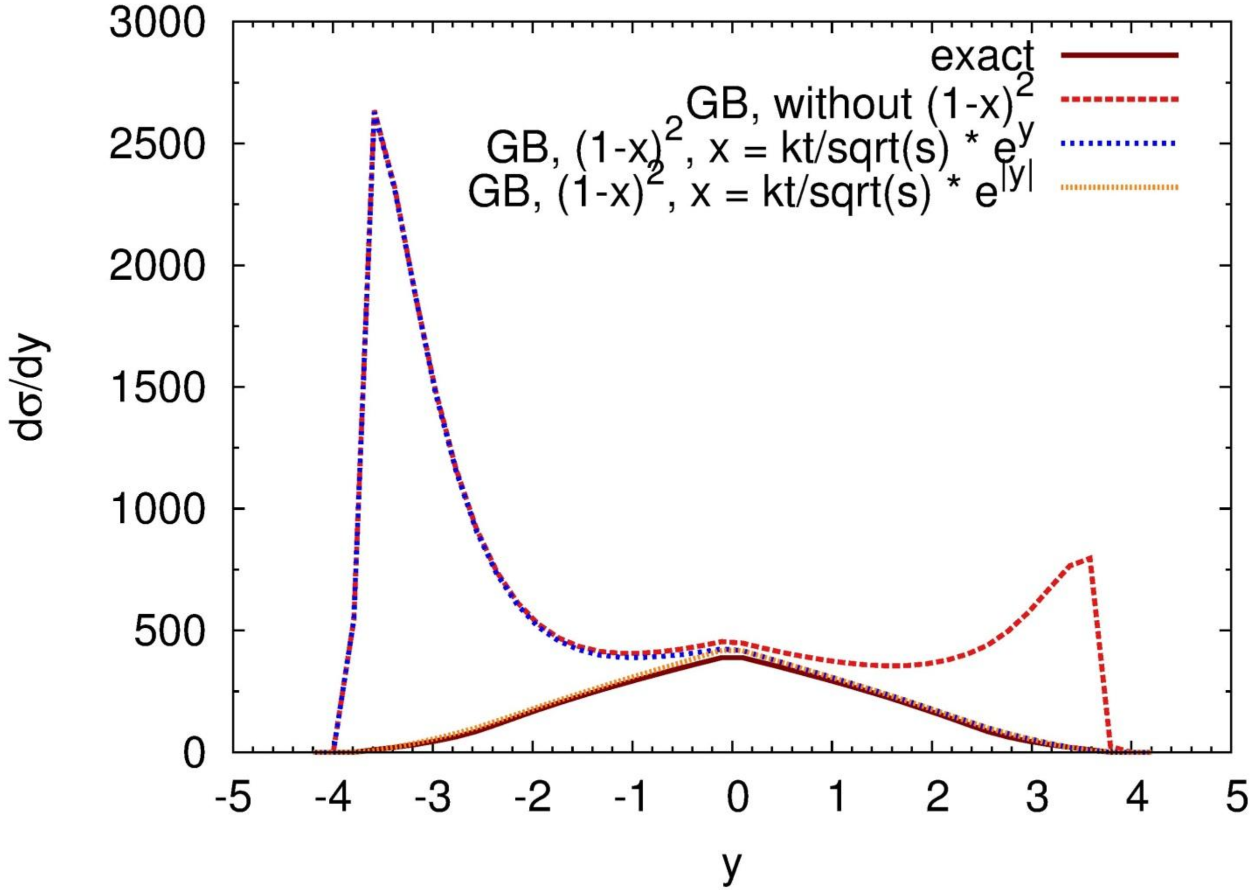
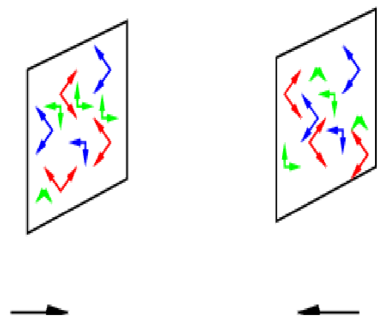


Fochler, Uphoff, Xu, CG, PRD 88 (2013) 014018

color glass condensate



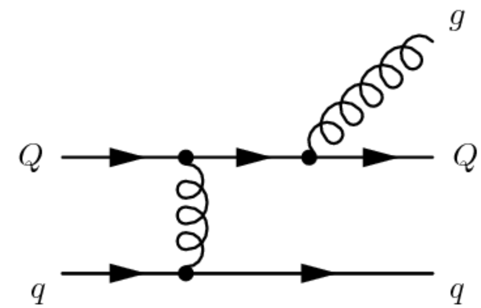
see eg
Kovchegov, Rischke, Phys.Rev. C56 (1997)

- Infrared screening for both GB and exact: $\Theta(cut) = \Theta(p_i p_j - \lambda)$
- Integration both in GB coordinates and in standard phase space with numeric δ-functions

Radiative pQCD processes

Exact matrix element

Kunszt, Pietarinen, Reya, Phys.Rev. D21 (1980)



$$\left|\overline{\mathcal{M}}\right|^2 = -16 \sum_{i,j=1}^{5} C_{ij} \frac{N_{ij}}{D_{ij}}$$

$$C=\frac{1}{3}\begin{pmatrix} 8 & 1 & 9 & -2 & -7 \\ & 8 & 9 & -7 & -2 \\ & & 18 & -9 & -9 \\ & & & 8 & 1 \\ & & & & 8 \end{pmatrix}$$

$$D=\begin{pmatrix} x_{31}{}^2 x_{54}{}^2, & 2x_{23}x_{31}x_{54}{}^2, & 4s_{12}x_{31}x_{54}{}^2, & 2s_{12}x_{31}x_{43}x_{54}, & 2s_{12}x_{31}x_{53}x_{54} \\ & x_{23}{}^2x_{54}{}^2, & 4s_{12}x_{23}x_{54}{}^2, & 2s_{12}x_{23}x_{43}x_{54}, & 2s_{12}x_{23}x_{53}x_{54} \\ & & 4s_{12}{}^2x_{54}{}^2, & 4s_{12}{}^2x_{43}x_{54}, & 4s_{12}{}^2x_{53}x_{54} \\ & & & s_{12}{}^2x_{43}{}^2, & 2s_{12}{}^2x_{43}x_{53} \\ & & & & s_{12}{}^2x_{53}{}^2 \end{pmatrix}$$

$$N_{11}=x_{31}(-x_{42}x_{53}-x_{43}x_{52}+2m_Q{}^2x_{54})+2m_Q{}^2(x_{41}x_{52}+x_{42}x_{51}+x_{42}x_{53}+x_{43}x_{52}+2m_Q{}^2x_{54}),$$

$$N_{12}=x_{12}[x_{41}(2x_{52}+x_{53})+x_{42}(2x_{51}+x_{53})+x_{43}(x_{51}+x_{52})+4m_Q{}^2x_{54}]$$
$$+x_{23}[x_{41}(-2x_{51}+x_{52})+x_{42}x_{51}+2m_Q{}^2x_{54}]$$
$$+x_{31}[x_{41}x_{52}+x_{42}(x_{51}-2x_{52})+2m_Q{}^2x_{54}]-4m_Q{}^2x_{43}x_{53},$$

$$N_{13}=x_{12}[-2x_{31}x_{54}+x_{41}(4x_{52}+3x_{53})+x_{42}(4x_{51}+3x_{53})+x_{43}(3x_{51}+3x_{52}+2x_{53})+8m_Q{}^2x_{54}]$$
$$+x_{23}[x_{41}(-6x_{51}+x_{52}-x_{53})+(x_{42}-x_{43})x_{51}+4m_Q{}^2x_{54}]$$
$$+x_{31}[x_{41}x_{52}+x_{42}(x_{51}-2x_{52}-3x_{53})-3x_{43}x_{52}]$$
$$+2m_Q{}^2[x_{41}(2x_{51}+4x_{52}+3x_{53})+x_{42}(4x_{51}-2x_{52}+5x_{53})+x_{43}(3x_{51}+5x_{52})+8m_Q{}^2x_{54}],$$

$$N_{14}=x_{51}(-x_{12}x_{43}+x_{23}x_{41}+x_{42}x_{43})+x_{31}[2x_{41}x_{52}+x_{42}(x_{51}-x_{54})+2x_{43}x_{52}+2m_Q{}^2x_{54}]$$
$$+x_{41}[2(x_{41}+x_{43})x_{52}+x_{42}(2x_{51}+x_{53})+2m_Q{}^2(x_{53}+2x_{54})]+2m_Q{}^2x_{43}(x_{53}+x_{54}-x_{51}-x_{52}),$$

$$N_{15}=N_{14}(4\leftrightarrow 5),\quad N_{22}=N_{11}(1\leftrightarrow 2),\quad N_{23}=N_{13}(1\leftrightarrow 2),\quad N_{24}=N_{14}(1\leftrightarrow 2),\quad N_{25}=N_{24}(4\leftrightarrow 5),$$

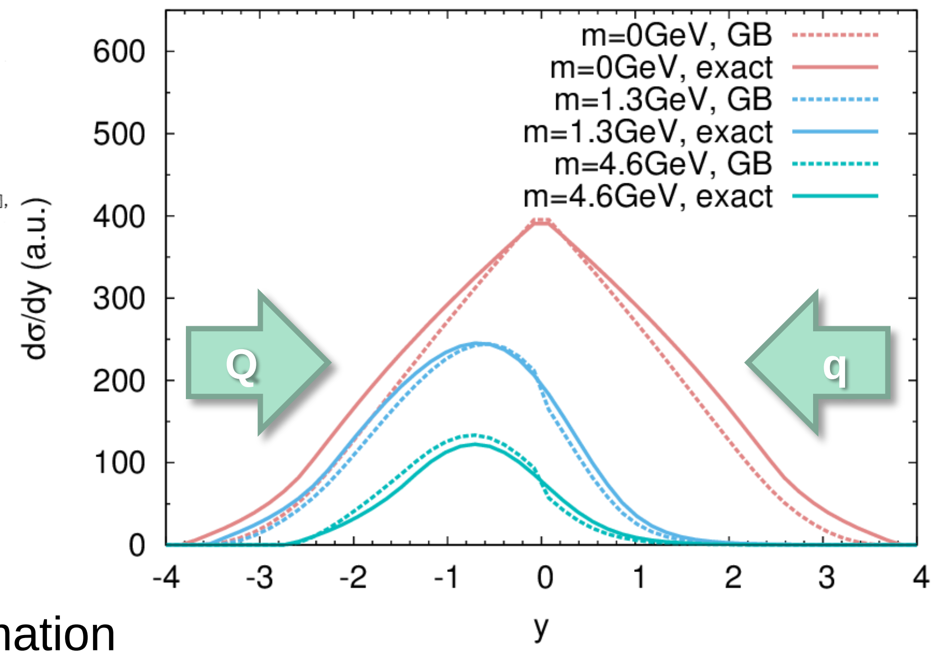
$$N_{33}=x_{12}[2x_{54}(x_{12}-x_{23}-x_{31})+x_{41}(-2x_{51}+6x_{52}+5x_{53})+x_{42}(-2x_{52}+6x_{51}+5x_{53})$$
$$+x_{43}(5x_{51}+5x_{52}+4x_{53})+28m_Q{}^2x_{54}]$$
$$+x_{23}[-2x_{31}x_{54}+x_{41}(-6x_{51}+x_{52}-3x_{53})+(x_{42}-3x_{43})x_{51}]$$
$$+x_{31}[x_{41}x_{52}+x_{42}(x_{51}-6x_{52}-3x_{53})-3x_{43}x_{52}]$$
$$+2m_Q{}^2[x_{41}(-4x_{51}+4x_{52}+7x_{53})+x_{42}(4x_{51}-4x_{52}+7x_{53})+x_{43}(7x_{51}+7x_{52}+2x_{53})+24m_Q{}^2x_{54}],$$

$$N_{34}=x_{12}[x_{54}(x_{23}+x_{31}+x_{42}+x_{41})-2x_{43}(x_{51}+x_{52})]$$
$$+x_{23}[2x_{31}x_{54}+x_{41}(x_{52}-x_{53}-x_{54})+3x_{51}(x_{42}+x_{43})+8m_Q{}^2x_{54}]$$
$$+x_{31}[x_{42}(x_{51}-x_{53}-x_{54})+3x_{52}(x_{41}+x_{43})+8m_Q{}^2x_{54}]$$
$$+x_{41}[3(x_{41}+x_{43})x_{52}+x_{42}(x_{51}+x_{52}+2x_{53})+2m_Q{}^2(x_{53}+5x_{54})]$$
$$+x_{42}[3(x_{42}+x_{43})x_{51}+2m_Q{}^2(x_{53}+5x_{54})]+2m_Q{}^2x_{43}(2x_{53}+2x_{54}-5x_{51}-5x_{52}),$$

$$N_{44}=x_{43}(-x_{23}x_{51}-x_{31}x_{52}-2m_Q{}^2x_{53}),$$

$$N_{45}=x_{54}[x_{23}(x_{41}+x_{51})+x_{31}(x_{42}+x_{52})+4m_Q{}^2(x_{53}+x_{54}+x_{43})]$$
$$+x_{41}[x_{52}(x_{43}+2x_{54})+x_{53}(x_{52}-2x_{42})]+x_{51}[-2x_{43}x_{52}+x_{42}(x_{53}+2x_{54}+x_{43})],$$

$$N_{35}=N_{34}(4\leftrightarrow 5),\quad N_{55}=N_{44}(4\leftrightarrow 5).$$

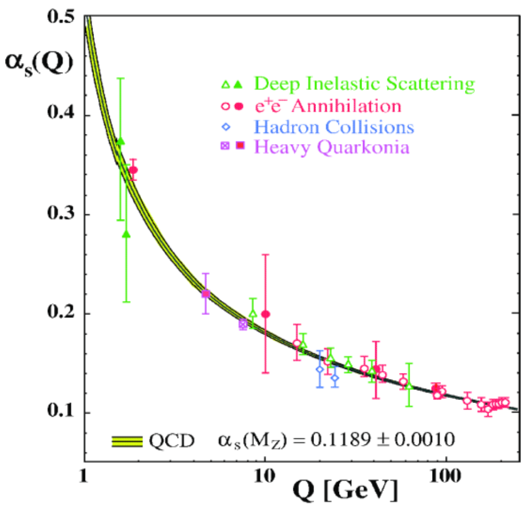


improved Gunion Bertsch (GB) approximation

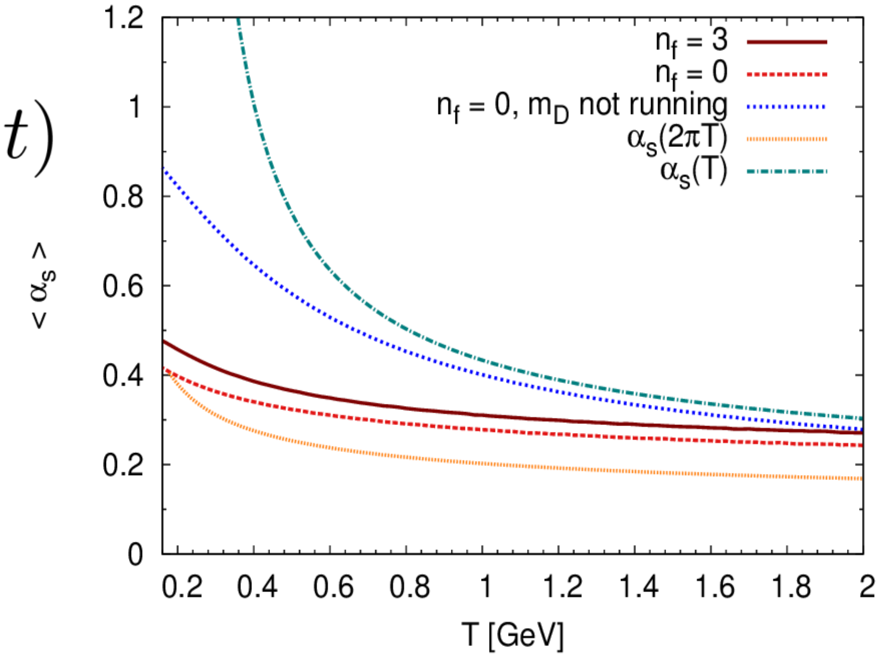
$$\left|\overline{\mathcal{M}}_{qQ\to qQg}\right|^2 = 12g^2(1-x)^2\left|\overline{\mathcal{M}}_0^{qQ}\right|^2\left[\frac{\mathbf{k}_\perp}{k_\perp^2+x^2M^2}+\frac{\mathbf{q}_\perp-\mathbf{k}_\perp}{(\mathbf{q}_\perp-\mathbf{k}_\perp)^2+x^2M^2}\right]^2$$

Fochler, Uphoff, Xu, CG, PRD 88 (2013) 014018

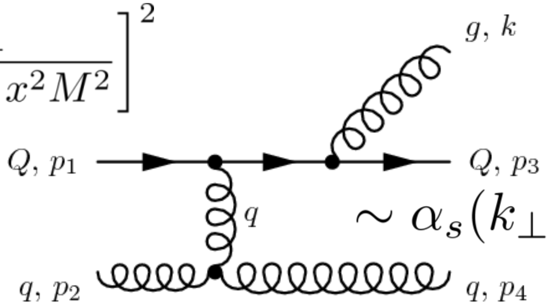
Running coupling



$$\alpha_s = \alpha_s(t)$$



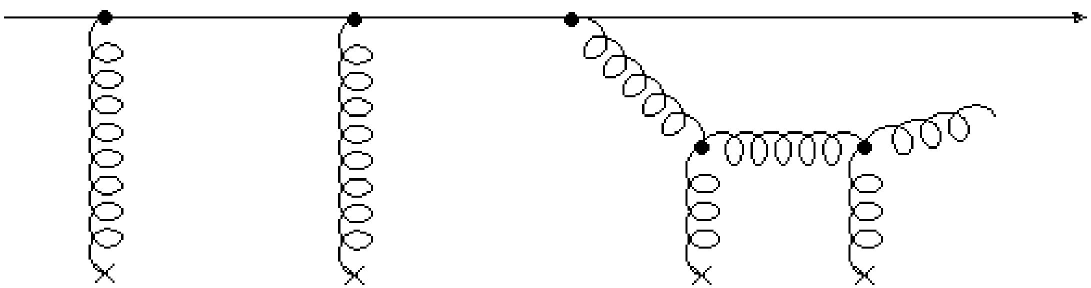
$$\left|\overline{\mathcal{M}}_{qQ\to qQg}\right|^2 = 12g^2(1-\bar{x})^2 \left|\overline{\mathcal{M}}_0^{qQ}\right|^2 \left[\frac{\mathbf{k}_\perp}{k_\perp^2 + x^2M^2} + \frac{\mathbf{q}_\perp - \mathbf{k}_\perp}{(\mathbf{q}_\perp - \mathbf{k}_\perp)^2 + x^2M^2}\right]^2$$



$$\sim \alpha_s(k_\perp)$$

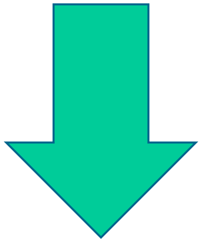
$$\sim \alpha_s^2(t)$$

LPM effect



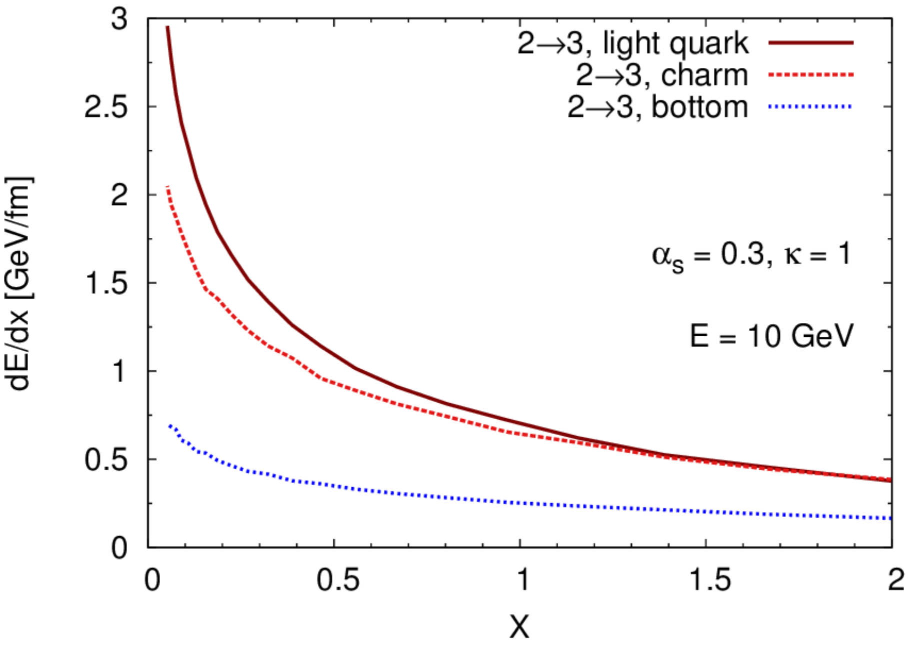
Only completely independent scatterings

$$\lambda > \tau$$

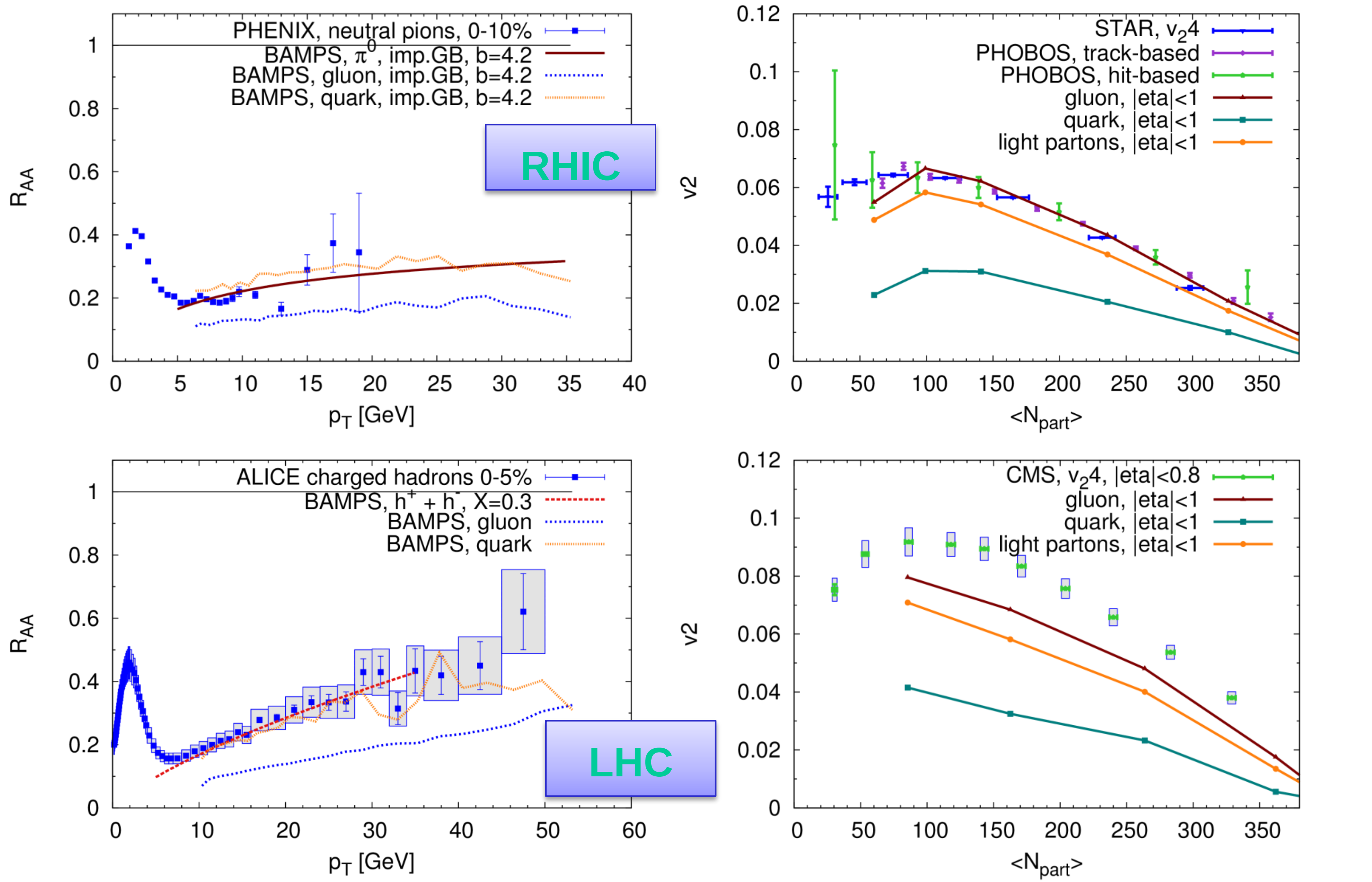


Interference effects: X < 1 expected

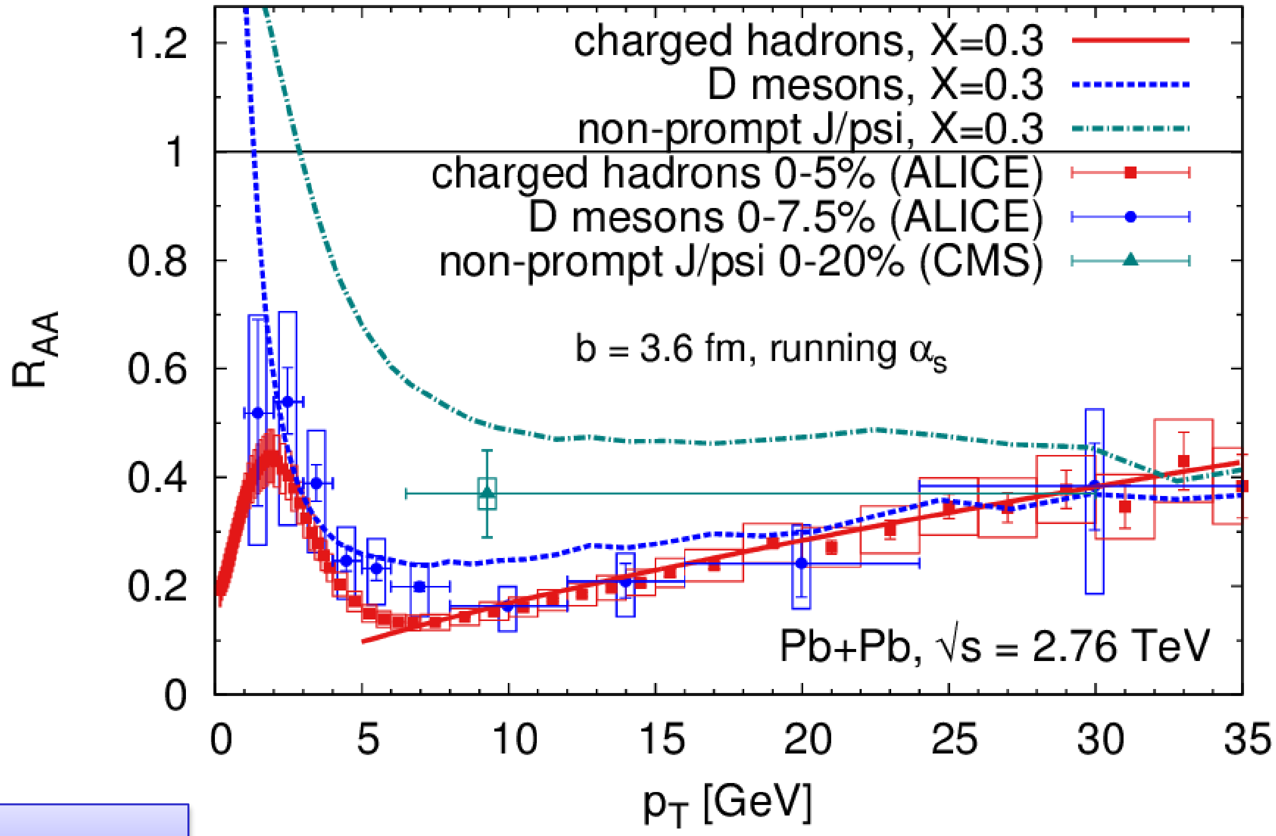
$$\lambda > X\,\tau$$



R_{AA} and v_2 for X=0.3 - preliminary



Heavy flavor and charged hadron R_{AA} at LHC



LHC

preliminary

Summary

Inelastic/radiative pQCD interactions (23 + 32) explain:

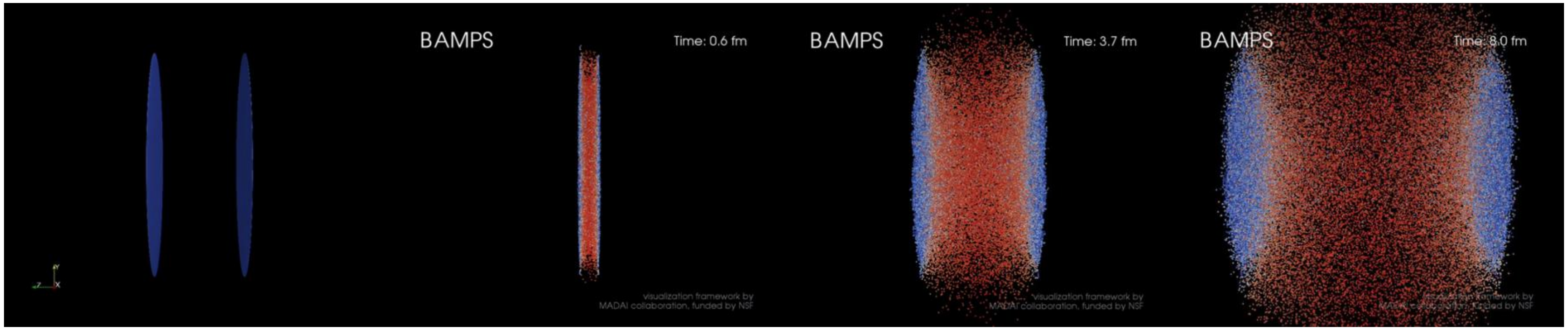
Gunion-Bertsch approximation needs corrections

faster thermalization

larger collective flow, also of heavy quarks

smaller shear viscosity of QCD

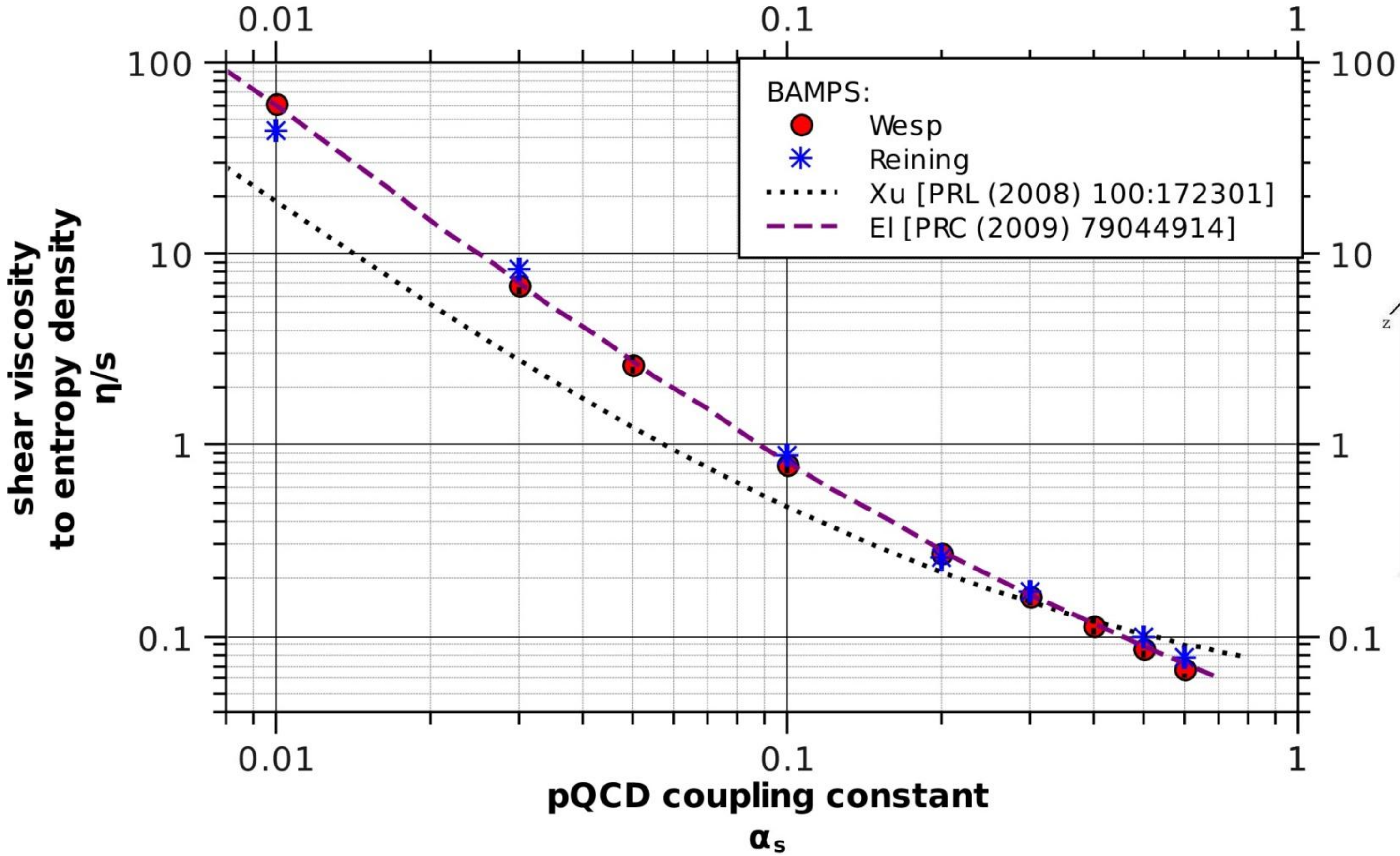
semirealistic jet-quenching , also of heavy quarks



numerical extraction of viscosity

Green-Kubo relation: $$\eta = \frac{1}{10T}\int_0^\infty dt \int_V d^3r \left\langle \pi^{ij}(\mathbf{r},t)\,\pi^{ij}(\mathbf{0},0)\right\rangle$$

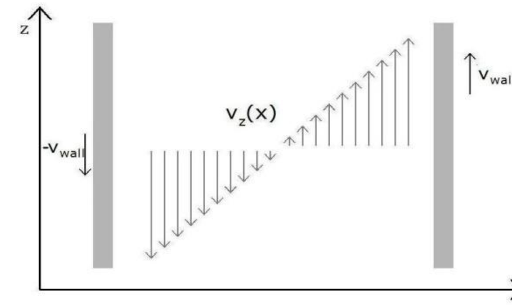
$$\Rightarrow \quad \frac{\eta}{s} = \frac{1}{20\pi}\left(\frac{T\cdot\tau}{0.197\,GeVfm}\right)$$



C. Wesp

Christian Wesp et al, Phys. Rev. C 84(2011)

$$\frac{F_z}{A} = -\eta\,\frac{\partial v_z}{\partial x}$$



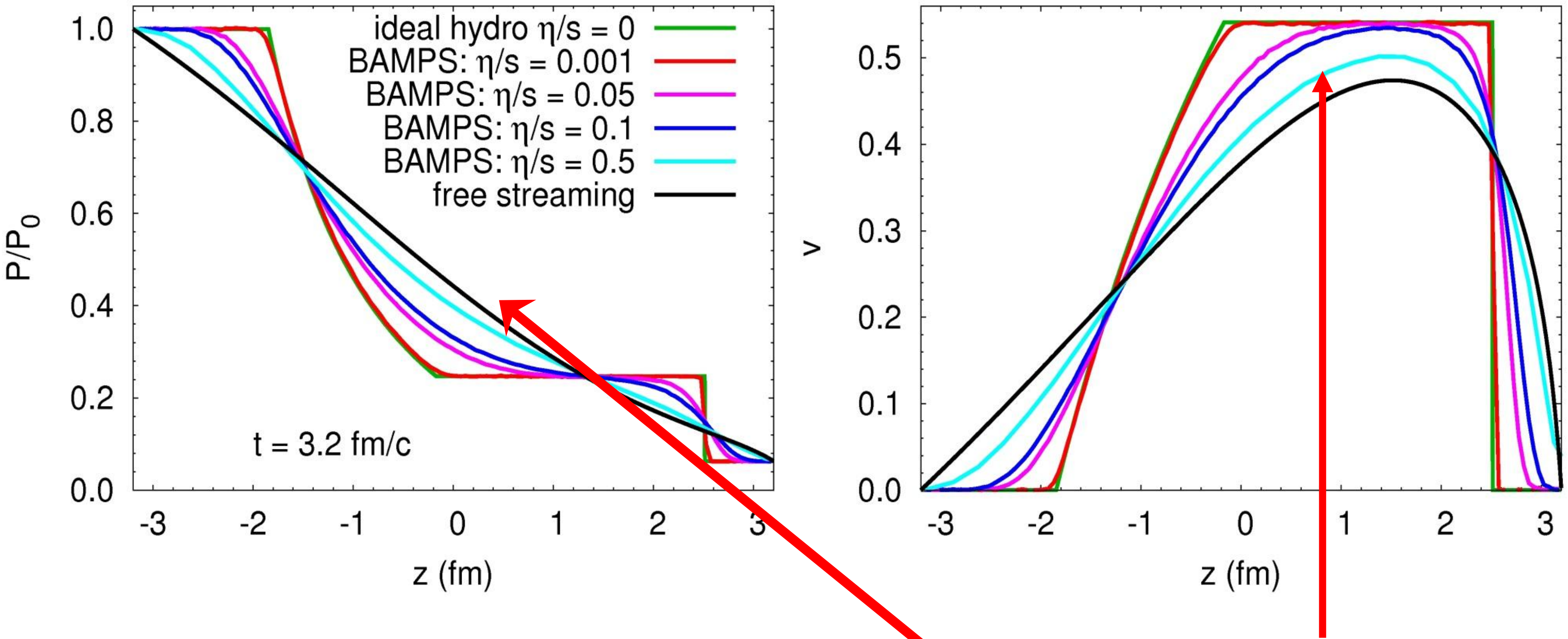
F. Reining

Felix Reining et al, Phys. Rev. E 85 (2012)

Riemann problem at finite viscosity

$$p^\mu \partial_\mu f = C$$

I. Bouras et al, PRL 103:032301 (2009)

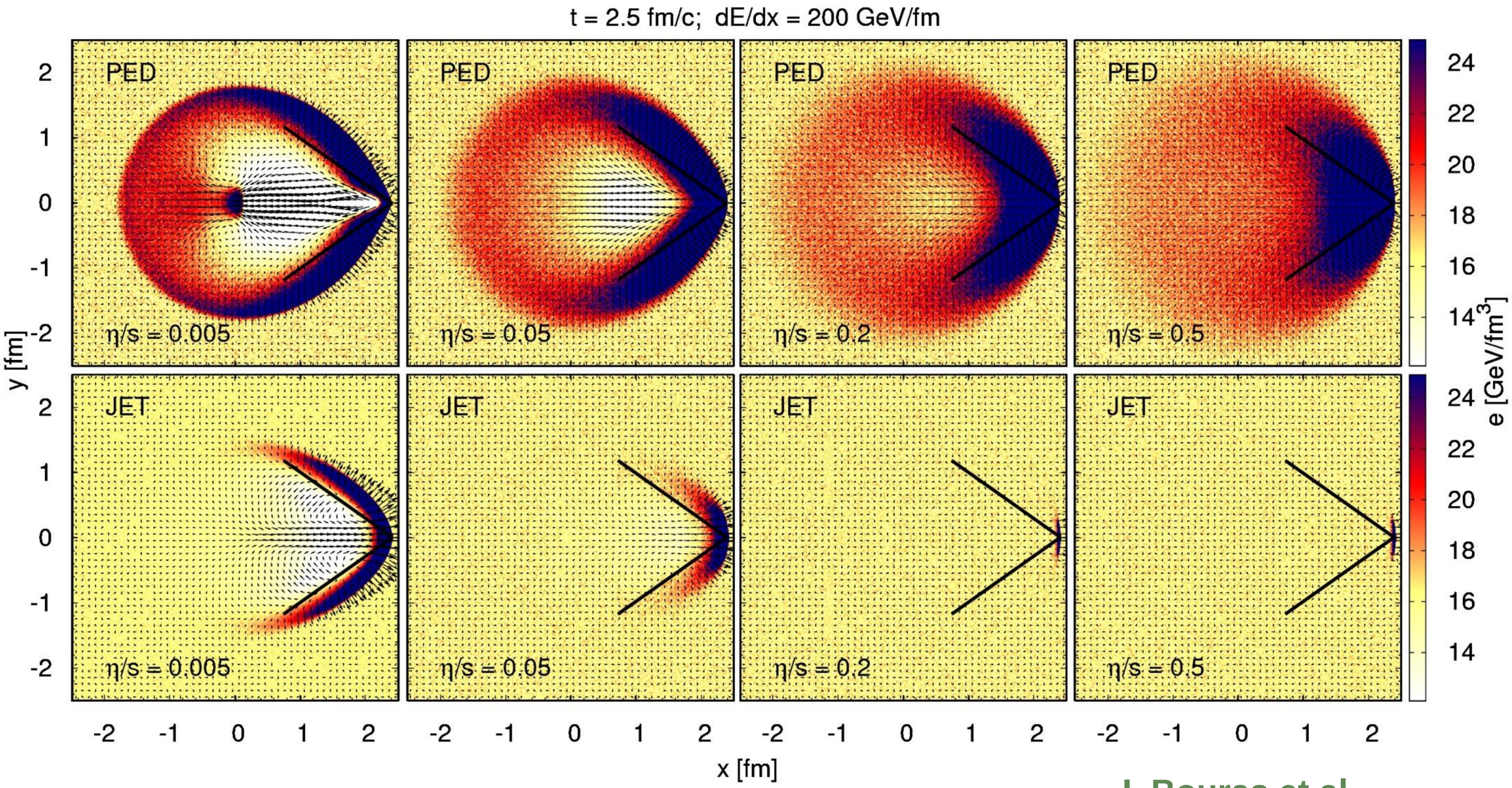


Tleft = 400 MeV
Tright = 200 MeV
t = 1.0 fm/c

Development of a shock plateau

η/s less than 0.1-0.2

VISCOUS Solutions



I. Bouras et al, PLB 710 (2012) 641

… the death of Mach Cones ?