Light flavor at e^+e^-colliders: Impact of hadronic cross sections on $g_\mu - 2$

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On behalf of the *BABAR* Collaboration

FPCP
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Outline

1. Motivation: muon-anomaly $(g-2)_\mu$
2. Initial State Radiation (ISR) analyses at *BABAR*
3. Status of Hadronic Cross Section Measurements
   - $e^+e^- \to p\bar{p}$
   - $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$
   - $e^+e^- \to K^+K^-$
4. Summary

The anomalous magnetic moment of the muon $(g-2)_\mu$

gyromagnetic ratio: g

$$\vec{\mu} = g \frac{e\hbar}{2mc} \cdot \vec{S}$$

spin $\frac{1}{2} \rightarrow$ Dirac theory: $g = 2$

QFT: $g \neq 2$

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muon anomaly: $a_\mu = (g-2)_\mu/2$

$$a_\mu^{\text{theory}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$

BNL E821	11 659 208.9	± 6.4

a_μ units in 10^{-10}

Brookhaven National Laboratory (BNL)
[G.W. Bennett *et al.*, PRD**73**, 072003 (2006)]

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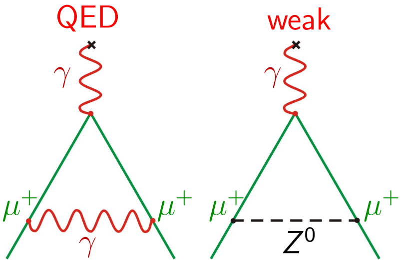

BNL E821	11 659 208.9	±6.4
QED	11 658 471.809	±0.015
weak	15.4	±0.2

[T.Kinoshita *et al.*, PRD**73**, 013003 (2006)]

[A.Czarnecki *et al.*, PRD**67**, 073006 (2003)
Erratum-ibid. D**73**, 119901 (2006)]
[M.Knecht *et al.*, JHEP 0211, 003 (2002)]

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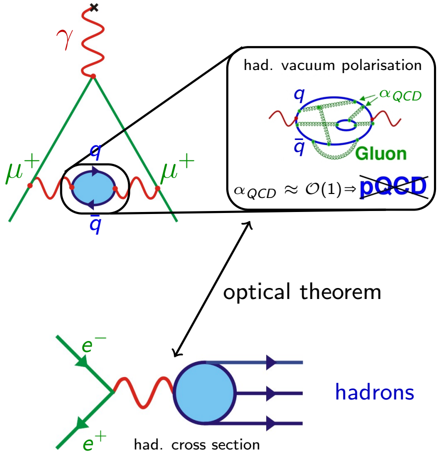
had	693.0	±4.9

a_μ units in 10^{-10}

hadronic



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weak	15.4	± 0.2
had	693.0	± 4.9
BNL−SM	28.7	± 8.0

a_μ units in 10^{-10}

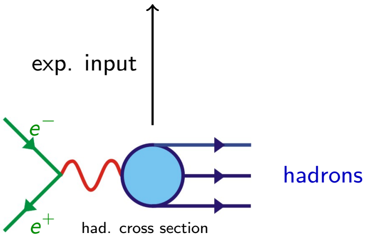
$\Downarrow$ 3.6σ [M. Davier *et al.*, EPJ **C**71, 1515 (2011)]

dispersion relation:

$$a_{\mu,LO}^{\text{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\,\sigma_{\text{had}}(s)$$

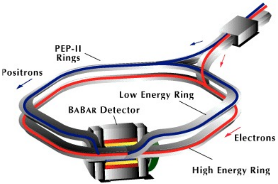
$$K(s) \sim 1/s \quad \& \quad \sigma_{\text{had}}(s) \sim 1/s$$

$\rightarrow \sim 1/s^2$ (low energies important!)

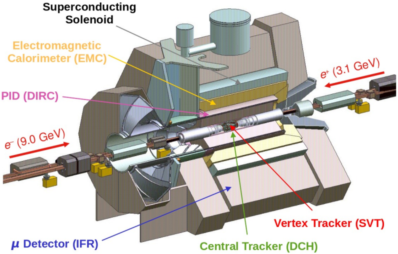


PEP-II and the *BABAR* detector at SLAC

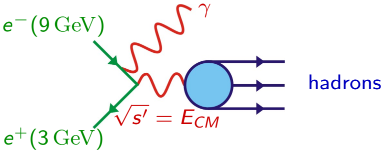
- asymmetric e^+e^--collider: $9\,\mathrm{GeV}$ (e^-) and $3.1\,\mathrm{GeV}$ (e^+)
- $\sqrt{s} = 10.58\,\mathrm{GeV} \Rightarrow \Upsilon(4S)$ $\Rightarrow$ above $B\bar{B}$-threshold

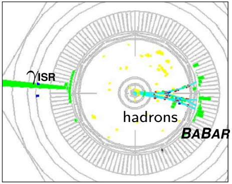


- main purpose: B-physics
- multi purpose detector
- data taken from 1999 – 2008
- integrated luminosity: $531\,\mathrm{fb}^{-1}$ on $\Upsilon(4S)$: $454\,\mathrm{fb}^{-1}$ $\approx 600 \cdot 10^6$ $B\bar{B}$-pairs



Initial State Radiation (ISR) events at *BABAR*





ISR selection

- Detected high energy photon: $E_\gamma > 3\,\mathrm{GeV}$
  $\rightarrow$ defines E_{CM} & provides strong background rejection
- Event topology: γ_{ISR} back-to-back to hadrons
  $\rightarrow$ high acceptance
- Kinematic fit including γ_{ISR}
  $\rightarrow$ very good energy resolution $(4 - 15\,\mathrm{MeV})$
- e^+e^--boost into the laboratory reference frame
  $\rightarrow$ high efficiency at production threshold of hadronic system
- Continuous measurement from threshold to $\sim$4.5 GeV
  $\rightarrow$ provides common, consistent systematic uncertainties

ISR analyses at *BABAR*

published

$e^+e^- \to \pi^+\pi^-$	PRD 86 (2012) 032013, PRL 103 (2009) 231801
$e^+e^- \to \phi f_0(980)$	PRD 74 (2006) 091103, PRD 76 (2007) 012008
$e^+e^- \to \pi^+\pi^-\pi^0$	PRD 70 (2004) 072004
$e^+e^- \to K^+K^-\eta, K^+K^-\pi^0, K_S^0K^\pm\pi^\mp$	PRD 77 (2008) 092002, PRD 71 (2005) 052001
$e^+e^- \to 2(\pi^+\pi^-)$	PRD 85 (2012) 112009, PRD 76 (2007) 012008
$e^+e^- \to K^+K^-\pi^0\pi^0, K^+K^-\pi^+\pi^-, 2(K^+K^-)$	PRD 86 (2012) 012008, PRD 76 (2007) 012008
$e^+e^- \to 2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$	PRD 76 (2007) 092005
$e^+e^- \to 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-)K^+K^-$	PRD 73 (2006) 052003
$e^+e^- \to p\bar{p}$	PRD 87 (2013) 092005, PRD 73 (2006) 012005
$e^+e^- \to \Lambda\bar{\Lambda}, \Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$	PRD 76 (2007) 092006
$e^+e^- \to c\bar{c} \to \ldots$	... ...

about to be submitted for publication to PRD

$e^+e^- \to K^+K^-$

ongoing analyses

$e^+e^- \to K_S^0K_L^0, \pi^+\pi^-\pi^0\pi^0, K_S^0K^\pm\pi^\mp\pi^0/\eta$

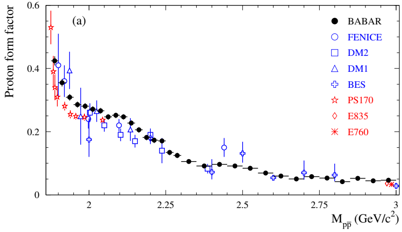
$e^+e^- \to p\bar{p}$

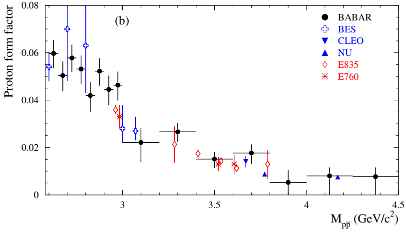
- Based on $469\,\text{fb}^{-1}$: PRD 87 (2013) 092005
- Update of PRD 73 (2006) 012005 based on $232\,\text{fb}^{-1}$
- Efficiency obtained from simulation [Kühn *et al.*, EPJC 18 (2001),497]
- Measure Cross Section σ
- Extract effective form factor:

  $\sigma = \frac{4\pi\alpha^2\beta C}{3m^2_{p\bar{p}}}|FF|^2$, $|FF| = \sqrt{|G_M|^2 + \frac{1}{2\tau}|G_E|^2}$
- Measure the ratio $|G_E/G_M|$ from angular distributions

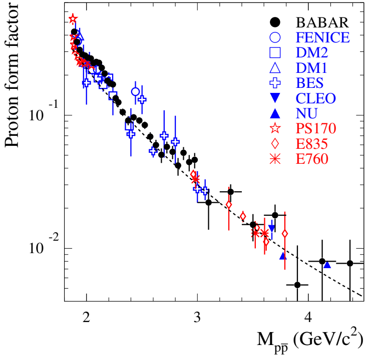
  $\frac{d\sigma}{dcos\theta} \sim (1 + cos^2\theta) + \tau|\frac{G_E}{G_M}|^2 sin^2\theta$

Form factor in comparison to other experiments





- Steep rise at threshold seen by PS170 confirmed → tail of a resonance below threshold?
- FF exhibits sharp drops at $M_{p\bar{p}}$ =2.2 GeV and 3 GeV
- Good fit to pQCD prediction: Brodsky-Lepage [PRL 43 (1979) 545]: $FF \sim \frac{\alpha_S^2(M_{p\bar{p}})}{M_{p\bar{p}}^4}$ $(M_{p\bar{p}} > 3\,\mathrm{GeV}/c^2)$



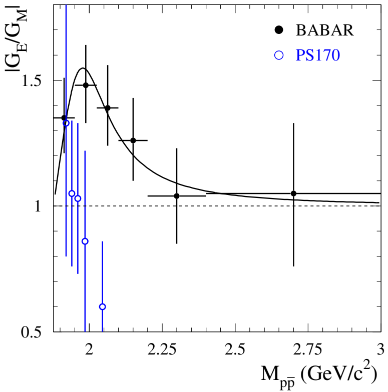
Time-like $|G_E/G_M|$ measurements

BABAR measurement:

- Angular distributions from threshold up to $3\,\mathrm{GeV}$
- Observe maximum at $M_{p\bar{p}} \approx 2\,\mathrm{GeV}/c^2$
- Inconsistent with PS170 measurement at LEAR
- ISR method → weak angular dependence of detection efficiency

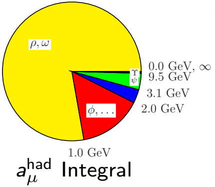
$$\frac{d\sigma(G_M)}{dcos\theta} \sim 1 + cos^2\theta_p$$

$$\frac{d\sigma(G_E)}{dcos\theta} \sim sin^2\theta_p$$



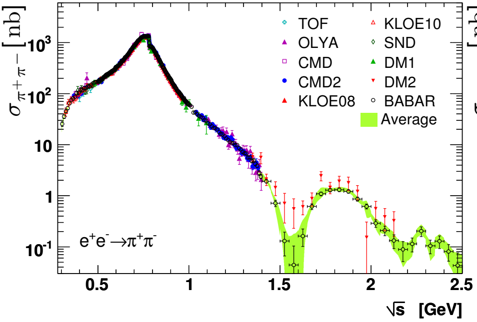
Contributions of Exclusive Final States to $g_\mu - 2$

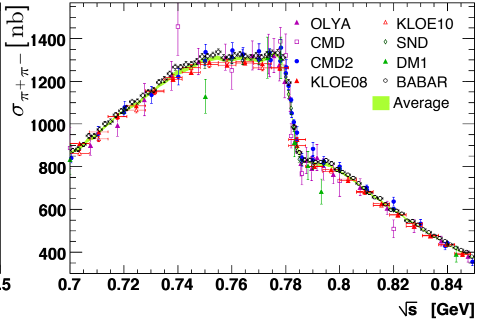
Contributions of different energy regions to the dispersion integral



→ $E < 1\,\mathrm{GeV}$ region dominates
→ $\pi^+\pi^-$ channel needed!

$\pi^+\pi^-$ Cross Section



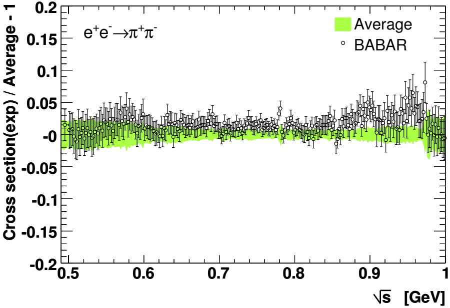


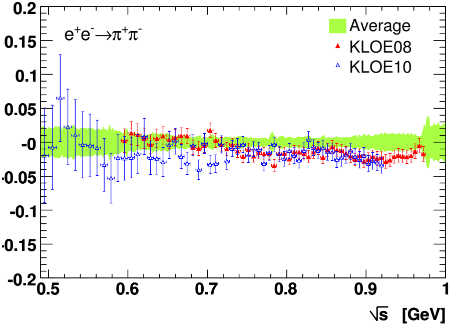
- ρ peak
- $\rho - \omega$ interference
- Dip at 1.6 GeV: excited ρ states
- Dip at 2.2 GeV
- Contribution to a_μ^{had}: 75%!

Systematic Uncertainties

BABAR:	0.5%
CMD-2:	0.8%
SND:	1.5%
KLOE:	0.8%

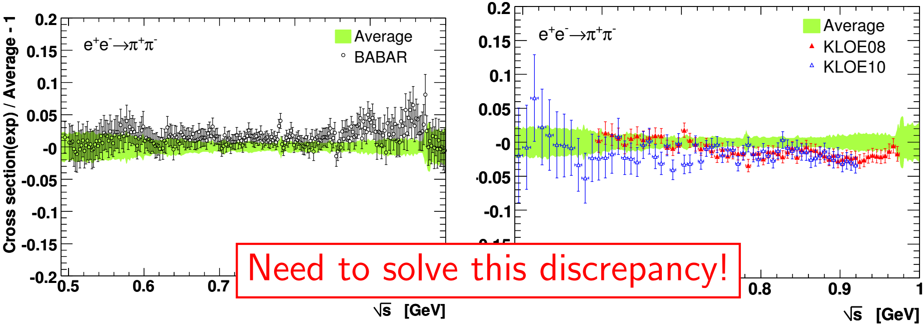
$\pi^+\pi^-$ Cross Section





- KLOE and *BABAR* dominate the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference, especially above ρ peak
- Difference $\rightarrow$ relatively large uncertainty for a_μ^{had}

$\pi^+\pi^-$ Cross Section



Need to solve this discrepancy!

- KLOE and *BABAR* dominate the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference, especially above ρ peak
- Difference $\rightarrow$ relatively large uncertainty for a_μ^{had}

Contributions of Exclusive Final States

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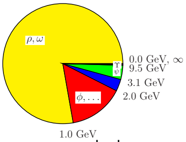
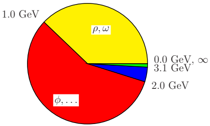
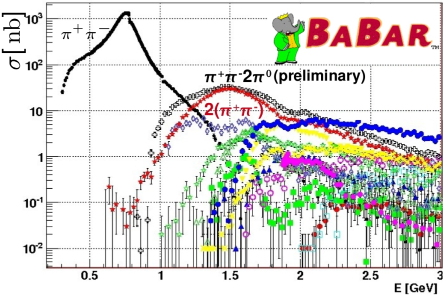


Integral a_μ^{had}



Uncertainty $\delta a_\mu^{\text{had}}$

$\Rightarrow$ Precise measurements $1\,\text{GeV} < E < 2\,\text{GeV}$ needed!



$\Rightarrow$ High multiplicity channels!
Why new analyses?

- Improve Statistics
- Improve Systematics
- Use existing data for bkg subtraction

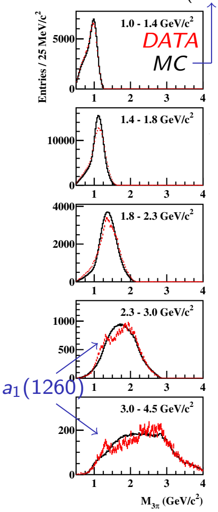
$e^+e^- \to \pi^+\pi^-\pi^+\pi^-$

Phys. Rev. D**85**, 112009 (2012), based on 454 fb^{-1}

supersedes our previous publication,
based on 89 fb^{-1} of the data:
Phys. Rev. D**71**, 052001 (2005).

Internal structure in various E_{CM} energy slices

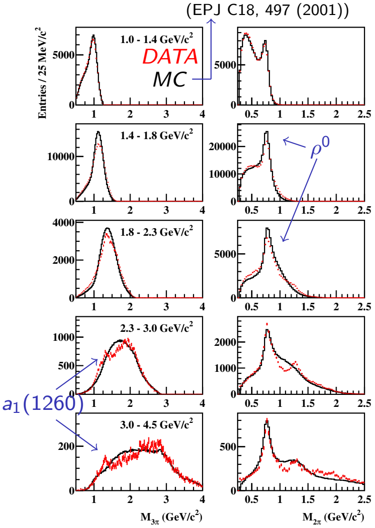
(EPJ C18, 497 (2001))



First column (4 entries/event):

$a_1(1260)$

Internal structure in various E_{CM} energy slices



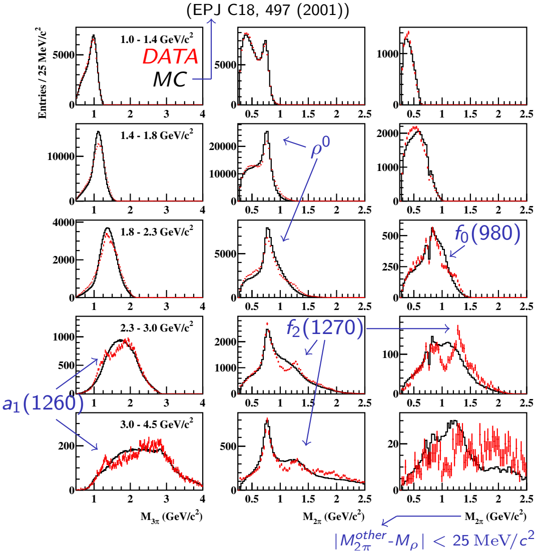
First column (4 entries/event):

$a_1(1260)$

Second column (4 entries/event):

strong ρ^0 contribution
e.g. for $M_{4\pi} > 1.4\,\mathrm{GeV}/c^2$:
$1/4th$ of entries in ρ^0 peak
$\rho^0\rho^0$ is forbidden
$\to \rho^0$ in each event!

Internal structure in various E_{CM} energy slices



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$\rho^0\rho^0$ is forbidden
$\rightarrow \rho^0$ in each event!

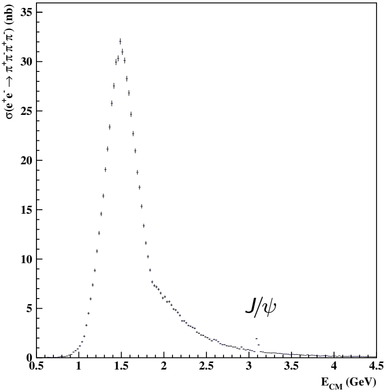
Third column (1 entry/event):

2π lie within ρ^0 mass
$\rightarrow$ other $\pi^+\pi^-$'s mass plotted

$f_2(1270)$, $a_1(1260)$, $f_0(980)\ldots$?

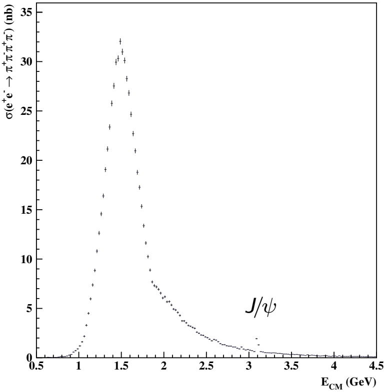
$\rightarrow$ Partial Wave Analysis needed

Cross section for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$



- Systematic uncertainties
  2.4% in peak region (1.1-2.8 GeV)
  11% (0.6-1.1 GeV)
  4% (2.8-4.0 GeV)
- J/ψ visible

Cross section for $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$



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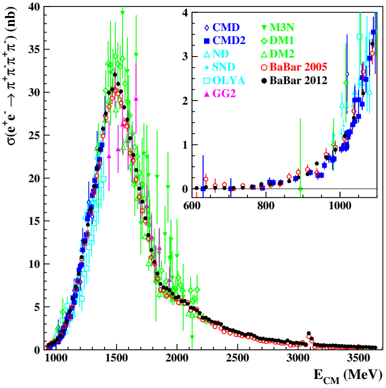
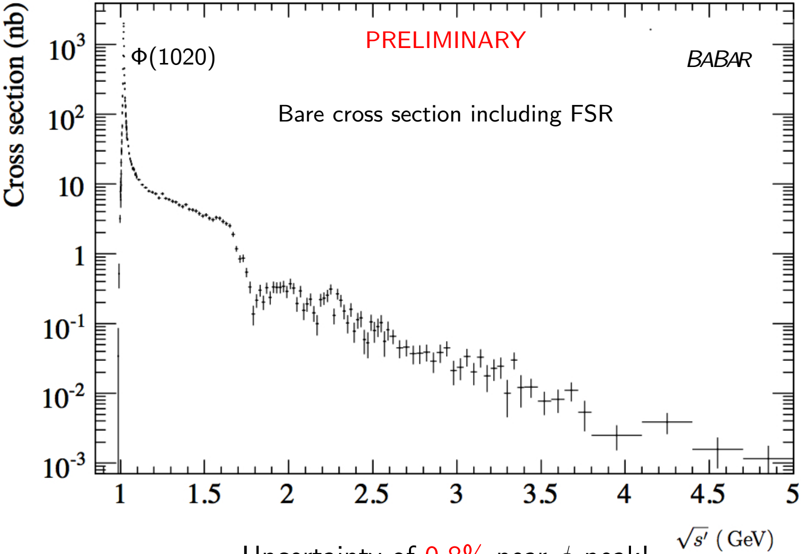


- $< 1.4\,\mathrm{GeV}$: agreement with previous *BABAR* results, SND and CMD-2 data
- $> 1.4\,\mathrm{GeV}$: highest precision (DM2, 20%)

$e^+e^- \to K^+K^-$

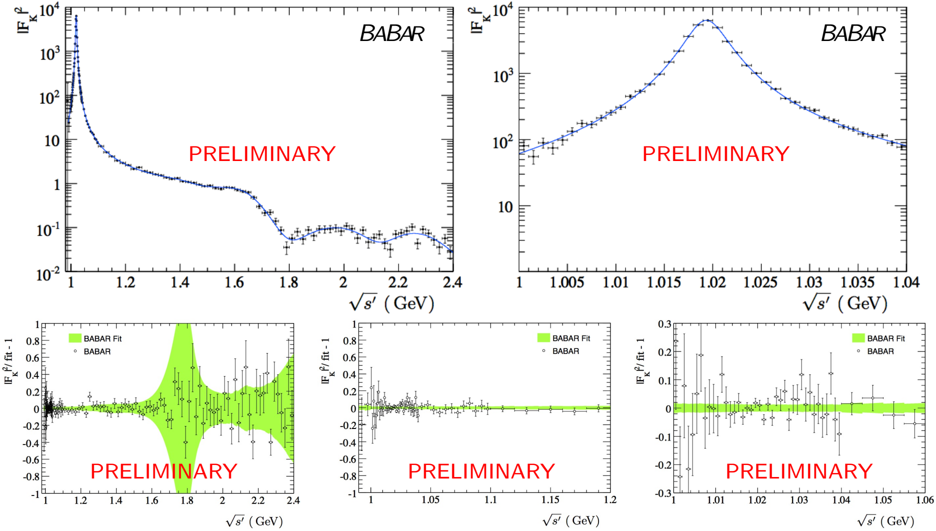
- PRELIMINARY
  $\to$ about to be submitted to PRD, based on $232\,\mathrm{fb}^{-1}$
- Efficiency obtained from simulation [Kühn *et al.*, EPJC 18 (2001),497]
  $\to$ data/MC corrections of utmost importance:
  trigger, tracking, K-ID and mis-ID
- Unfolding bkg-subtracted and data/MC corrected mass spectrum
- PHOKHARA [Czyż *et al.*, EPJC35 (2004) 527; EPJC39 (2005), 411]
  $\to$ Geometrical acceptance
  $\to$ 2^{nd}order ISR corrections
- ISR effective luminosity from $\mu\mu\gamma(\gamma)$: KK/$\mu\mu$ ratio

Cross section $\sigma(e^+e^- \to K^+K^-)$

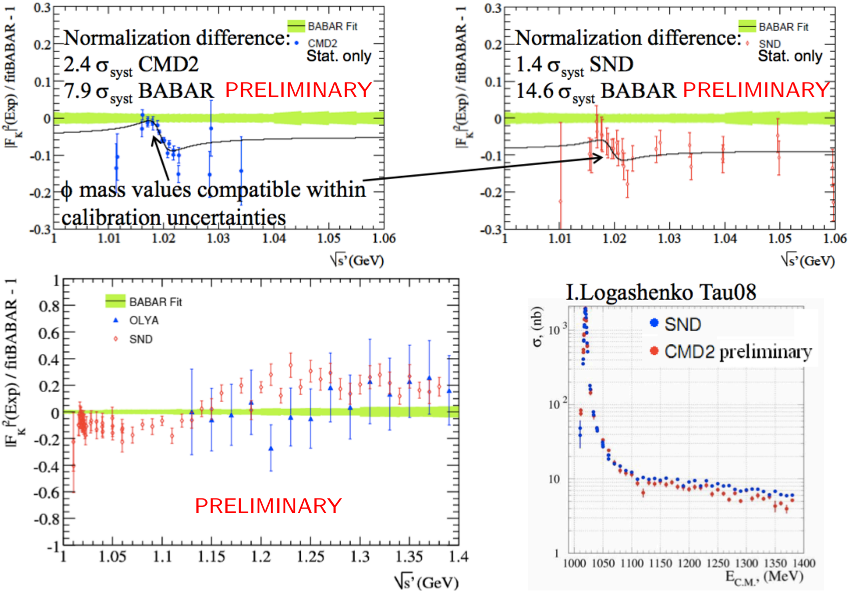


Uncertainty of 0.8% near ϕ peak!

A phenomenological fit to the form factor



Comparison to other experiments



The Φ parameters

m_Φ and Γ_Φ obtained from the fit of the form factor

BABAR

$m_\Phi = 1019.51 \pm 0.02 \pm 0.05_{sys}\,\mathrm{MeV}$
$\Gamma_\Phi = 4.29 \pm 0.04 \pm 0.06_{sys}\,\mathrm{MeV}$

PDG

$m_\Phi = 1019.455 \pm 0.020\,\mathrm{MeV}$
$\Gamma_\Phi = 4.26 \pm 0.04\,\mathrm{MeV}$

$\rightarrow$ good agreement

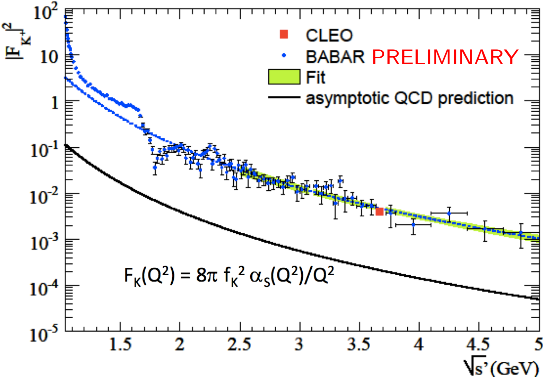
From integrated Φ peak: $\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \to K^+K^-) = \frac{\alpha^2\beta^3(s,m_K)}{324}\frac{m_\Phi^2}{\Gamma_\Phi}a_\Phi^2 C_{FS}$

BABAR:
$\Gamma_\Phi^{ee}\times\mathcal{B}(\Phi \to K^+K^-) = 0.6344\pm0.0059_{exp}\pm0.0028_{fit}\pm0.0015_{cal}\,\mathrm{keV}(1.1\%)$
CMD2:
$\Gamma_\Phi^{ee} \times \mathcal{B}(\Phi \to K^+K^-) = 0.605 \pm 0.002 \pm 0.013\,\mathrm{keV}(2.1\%)$

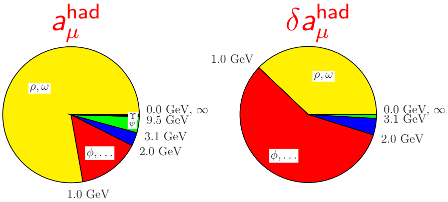
Charged kaon form factor at large Q^2

Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)

- Power law: $F_K \sim \alpha_S(Q^2)Q^{-n}$ with n=2
  $\rightarrow$in good agreement with the data (2.5-5 GeV $n = 2.10 \pm 0.23$)
- HOWEVER: data on $|F_K|^2$ factor $\sim$ 20 above prediction!
- No trend in data up to 25 GeV^2 for approaching the asymp. QCD prediction



Impact on $g_\mu - 2$



[PR 477, 1 (2009).]

$$a_\mu^{had}(K^+K^-) = 216.3 \pm 2.7 \pm 6.8$$
$$\downarrow$$
$$a_\mu^{had}(K^+K^-) = 229.5 \pm 1.4 \pm 2.2$$

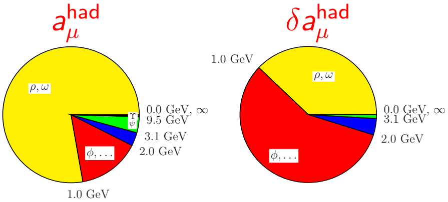
calculation only based on *BABAR* 2013 data!

$$a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-) = 133.5 \pm 1.0 \pm 5.2$$
$$\downarrow$$
$$a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-) = 136.4 \pm 0.3 \pm 3.6$$

calculation only based on *BABAR* 2012 data!

(all a_μ units in 10^{-11})

Impact on $g_\mu - 2$



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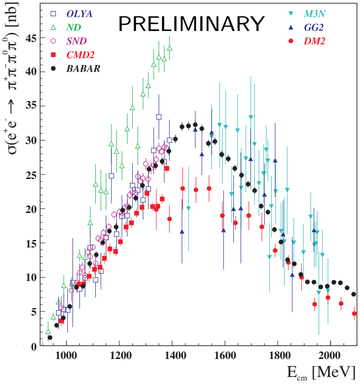
$$a_\mu^{had}(K^+K^-) = 229.5 \pm 1.4 \pm 2.2$$
calculation only based on *BABAR* 2013 data!

$$a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-) = 133.5 \pm 1.0 \pm 5.2$$
$$\downarrow$$
$$a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-) = 136.4 \pm 0.3 \pm 3.6$$
calculation only based on *BABAR* 2012 data!



(all a_μ units in 10^{-11})

dominant contribution to δa_μ^{had}:
$$a_\mu^{had}(\pi^+\pi^-\pi^0\pi^0) = 180.1 \pm 0.3 \pm 12.4$$
$$\Downarrow$$
BABAR analysis in progress

Summary

Measurement of hadronic cross sections via ISR is a very productive field in addition to B-physics at *BABAR*

- Most accurate measurements of $\sigma(e^+e^- \to p\bar{p}/K^+K^-/\pi^+\pi^-\pi^+\pi^-)$
- From threshold of the invariant mass up to $\sim 4.5\,\mathrm{GeV}/c^2$

$e^+e^- \to p\bar{p}$

- Enhancement at threshold of the FF confirmed
- $|G_E/G_M|$ measured via angular distributions for $m_{p\bar{p}} < 3\,\mathrm{GeV}$

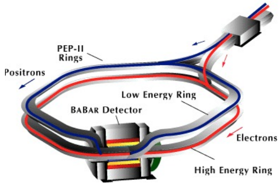
$e^+e^- \to K^+K^-/\pi^+\pi^-\pi^+\pi^-$

- Important for theoretical predictions of $(g_\mu - 2)$
  $\to$ Hint for new physics?
  $\to$ In combination with other measurements: $a_\mu^{\text{exp}} - a_\mu^{\text{theory}} \approx 3 - 4\sigma$

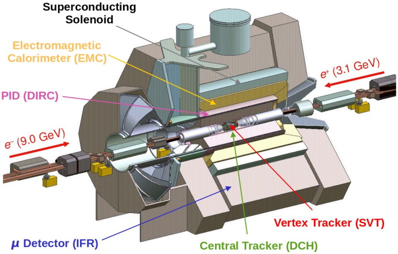
backup slides

PEP-II and the *BABAR* detector at SLAC

- asymmetric e^+e^--collider: $9\,\mathrm{GeV}$ (e^-) and $3.1\,\mathrm{GeV}$ (e^+)
- $\sqrt{s} = 10.58\,\mathrm{GeV} \Rightarrow \Upsilon(4S)$ $\Rightarrow$ above $B\bar{B}$-threshold

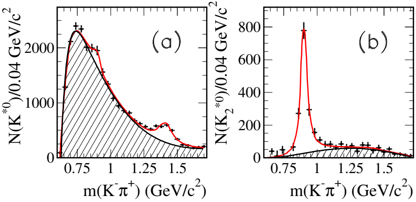


- main purpose: B-physics
- multi purpose detector
- data taken from 1999 – 2008
- integrated luminosity: $531\,\mathrm{fb}^{-1}$ on $\Upsilon(4S)$: $454\,\mathrm{fb}^{-1}$ $\approx 600 \cdot 10^6$ $B\bar{B}$-pairs



Coherent K^*K^* contribution in $e^+e^- \to K^+K^-\pi\pi$

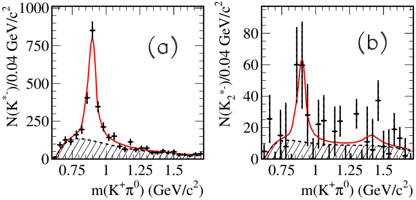
$K^+K^-\pi^+\pi^-$



Extract number of $K^*(892)^0$ and $K_2^*(1430)^0$ by fitting $K^+\pi^-$ mass in every 40 MeV/c^2 bin of $K^-\ \pi^+$ mass

$\to$ less than 1% $K^*(892)^0 K^*(892)^0$

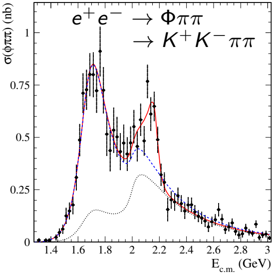
$K^+K^-\pi^0\pi^0$

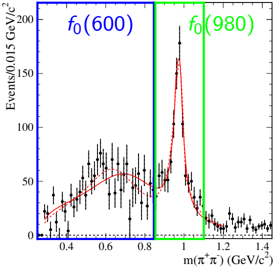


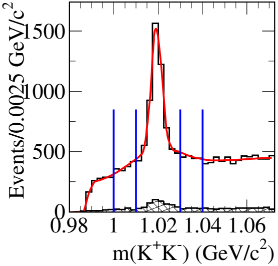
Extract number of $K^*(892)^+$ and $K_2^*(1430)^+$ by fitting 40 MeV/c^2 bins of $K^-\pi^0$ mass

$\to$ 30% $K^*(892)^{\pm} K^*(892)^{\mp}$

$e^+e^- \rightarrow \phi\pi\pi \rightarrow K^+K^-\pi\pi$

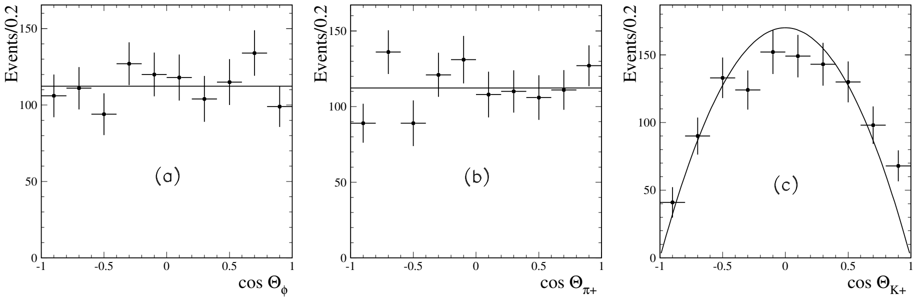


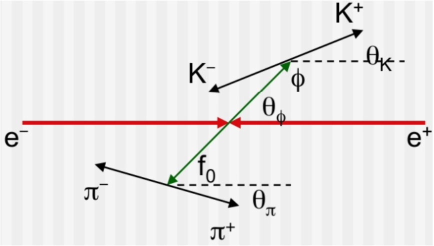




- minimum 2 peaks!
- resonance confirmed: $J^{PC} = 1^{--}$
  M=$2176 \pm 14 \pm 4\,\mathrm{MeV}/c^2$; $\Gamma = 90 \pm 22 \pm 10\,\mathrm{MeV}$

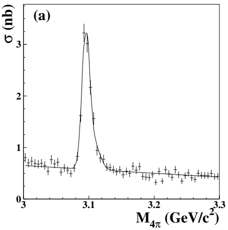
$e^+e^- \to \phi\pi^+\pi^- \to K^+K^-\pi^+\pi^-$





- ϕ and $\pi^+\pi^-$ system are in S-wave
- pions in $\pi^+\pi^-$ system are in S-wave
- kaons from ϕ are in P-wave (as expected)

charmonium branching ratios

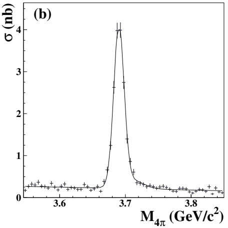


$$\mathcal{B}_{J/\psi \to 2(\pi^+\pi^-)} \cdot \sigma_{int}^{J/\psi} = \frac{N(J/\psi \to 2(\pi^+\pi^-))}{d\mathcal{L}/dE \cdot \epsilon_{MC}} = (48.9 \pm 2.1_{stat} \pm 1.0_{syst})\,\mathrm{MeV}/c^2\,\mathrm{nb}$$

$$\mathcal{B}_{J/\psi \to 2(\pi^+\pi^-)} = (3.67 \pm 0.16_{stat} \pm 0.08_{syst} \pm 0.09_{ext}) \cdot 10^{-3}$$

$$\mathcal{B}^{PDG}_{J/\psi \to 2(\pi^+\pi^-)} = (3.55 \pm 0.23) \cdot 10^{-3}$$

$\to$ agrees with PDG, higher in precision



$$\mathcal{B}_{\psi(2S) \to J/\psi\,\pi^+\pi^-} \cdot \mathcal{B}_{J/\psi \to \mu^+\mu^-} \cdot \sigma_{int}^{\psi(2S)} = \frac{N(\psi(2S) \to \pi^+\pi^-\mu^+\mu^-)}{d\mathcal{L}/dE \cdot \epsilon_{MC}} = (84.7 \pm 2.2_{stat} \pm 1.8_{syst})\,\mathrm{MeV}/c^2\,\mathrm{nb}$$

$$\mathcal{B}_{\psi(2S) \to J/\psi\,\pi^+\pi^-} = 0.354 \pm 0.009_{stat} \pm 0.007_{syst} \pm 0.007_{ext}$$

$$\mathcal{B}^{PDG}_{\psi(2S) \to J/\psi\,\pi^+\pi^-} = 0.336 \pm 0.004$$

$$\mathcal{B}^{CLEO}_{\psi(2S) \to J/\psi\,\pi^+\pi^-} = 0.3504 \pm 0.0007_{syst} \pm 0.0077_{ext}$$

$\to$ agrees with recent CLEO result (PRD 78, 011102 (2008))