Recent developments on CKM angles

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Why are we particularly interested in determining the CKM angles?

- Direct search at hadron collider
  Higgs vs New particles beyond the SM
- Flavor Physics
  Test of SM
  Indirect search of new degree of freedom

CKM unitarity triangle and CPV parameter convention

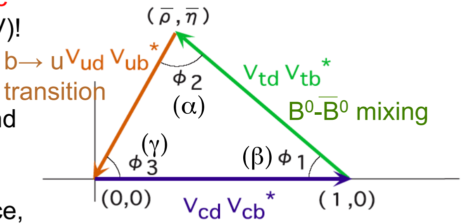
$$V=\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Wolfenstein parametrization

Irreducible complex phase causes CP Violation (CPV)!

Comprehensive test; measure all the angles and sides.

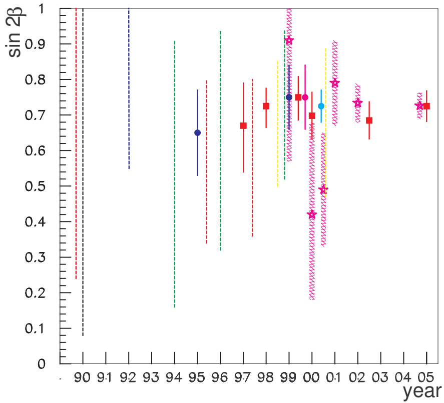
B system : very good place, all the angles are $O(0.1)$!

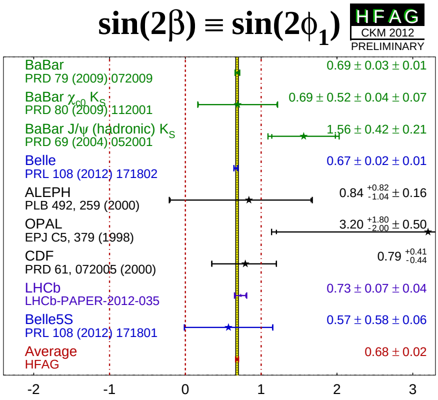


$$V_{td}V_{tb}^* + V_{cd}V_{cb}^* + V_{ud}V_{ub}^* = 0$$

Status

Left: evolution plot (UTFit); Right: HFAG Average of $\sin(2\beta/\phi_1)$ from all experiments.

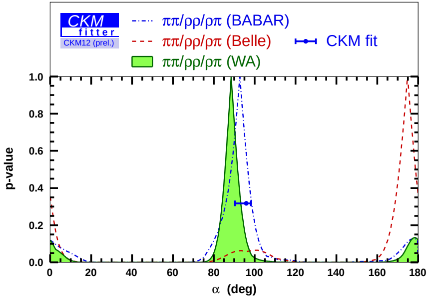


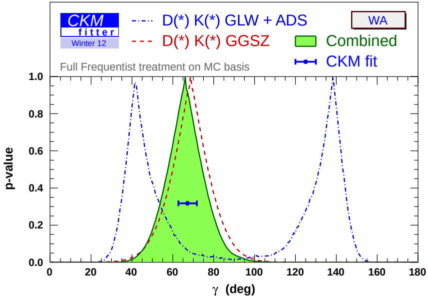


$$\beta = \phi_1 = (21.5^{+0.8}_{-0.7})^\circ.$$

Status

Average taken from CKMFitter





$$\alpha = \phi_2 = (88.5^{+4.7}_{-4.4})^\circ \qquad \gamma = \phi_3 = (66 \pm 12)^\circ.$$

$$\alpha + \beta + \gamma = (176 \pm 13)^\circ.$$

Status

Recent results for γ from $B \to DK$ (from HFAG)

	BaBar	Belle	LHCb
γ	$(69^{+17}_{-16})^\circ$	$(68^{+15}_{-14})^\circ$	$(71.1^{+16.6}_{-15.7})^\circ$
r_B	$0.092^{+0.013}_{-0.012}$	$0.112^{+0.014}_{-0.015}$	$0.0919^{+0.0083}_{-0.0082}$
δ_B	$(105^{+16}_{-17})^\circ$	$(116^{+18}_{-21})^\circ$	$(112.0^{+12.6}_{-15.5})^\circ$
Ref.	arXiv:1301.1029	CKM2012 preliminary	LHCb-CONF-2012-032

- Naive average: $\gamma = (69.3 \pm 9.3)^\circ$.
  - $\alpha + \beta + \gamma = (179.3 \pm 10.4)^\circ$.

Development:
New Measurements
+ New Channels
+ New Effects

New measurements of α/ϕ_2:

cf talk by Pit VANHOEFER in this conference

New measurements of β/ϕ_1:

cf talk by Riccardo DE SANGRO in this conference

New measurements of γ/ϕ_3:

cf talk by Matteo RAMA, Till Moritz KARBACH in this conference

We shall do everything to reduce the errors.

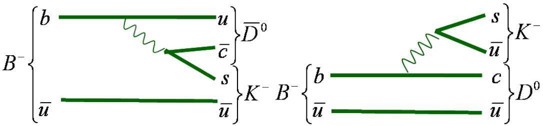
New Channels

$\gamma = \phi_3$ constraint via $B \to DK$

Principle	Methods and Reference
D^0 or $D^{*0} \to$ CP eigenstate	GLW PLB 253, 483 (1991) PLB265,172(1991)
Enhance CP asymmetry by suppressed D decay	ADS, PRL78,3357(1997) PRD63,036005(2001)
Dalitz distribution in three-body D decay ($K_S\pi^+\pi^-$, etc)	GGSZ, PRD68,054018(2003)

Very clean weak phase information

γ from $B \to DK^*_{0,2}$ in GLW method



In $B \to D_{CP}K$, sensitive to a small ratio of interfering amplitudes.

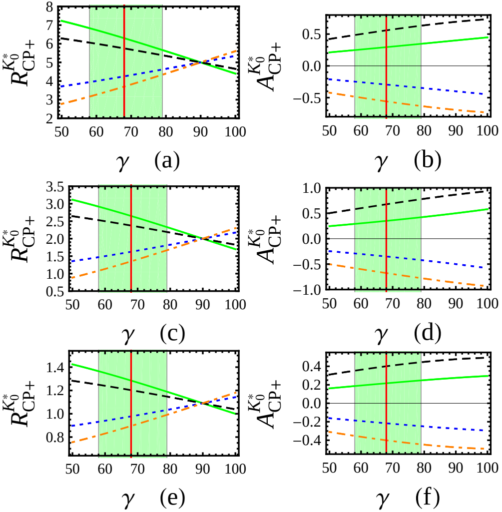
$$\sqrt{2}A(B^+ \to D^0_\pm K^+) = A(B^+ \to D^0 K^+) \pm A(B^+ \to \bar{D}^0 K^+),$$
$$\sqrt{2}A(B^- \to D^0_\pm K^-) = A(B^- \to D^0 K^-) \pm A(B^- \to \bar{D}^0 K^-),$$

Vanishing decay constant for $K^*_{0,2}$:

$$\langle K_0^{*-}(1430)|\bar{s}\gamma^\mu u|0\rangle = f_{K_0^*} p^\mu_{K_0^*} \sim 0, \langle K_2^{*-}(1430)|\bar{s}\gamma^\mu u|0\rangle = 0.$$

Color-allowed and color-suppressed amplitudes are comparable in $B \to DK^*_{0,2}$!

γ from $B \to DK^*_{0,2}$ in GLW method



Large CP asymmetries are expected in $B \to DK_{0,2}$. Even better in $B \to D\pi$ counterpart.

WW, 1110.5194

Determining γ via Three-body $B \to K\bar{K}K$ and $B \to K\pi\pi$:

Solution	Fit 1	Fit 2	Fit 3
I	31^{+3}_{-2}	31 ± 2	31^{+2}_{-3}
II	76^{+3}_{-2}	78^{+2}_{-3}	77 ± 3
III	261^{+2}_{-4}	259 ± 3	258^{+4}_{-3}
IV	314 ± 2	315 ± 2	315^{+3}_{-2}

Bhattacharya, Imbeault, London, 1303.0846

Based on SU(3) symmetry analysis of the Dalitz plot.
cf David LONDON's talk in this conference

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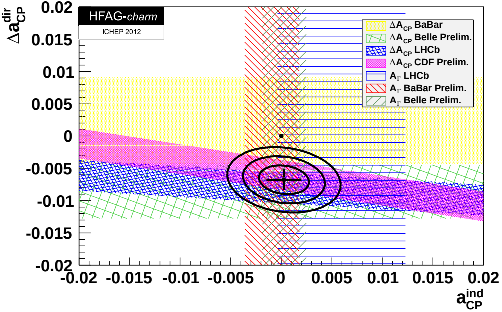
$$\alpha + \beta + \gamma = (187 \pm 5.6)^\circ$$

New Effects

CPA effects on γ

Large CPA in $D^0 \to (K^+K^-, \pi^+\pi^-)$ was observed by LHCb, CDF and Belle (2011, 2012):



$$\Delta a_{CP}^{\rm dir} = a_{CP}^{K^+K^-,{\rm dir}} - a_{CP}^{\pi^+\pi^-,{\rm dir}} = (-0.678 \pm 0.147)\%$$

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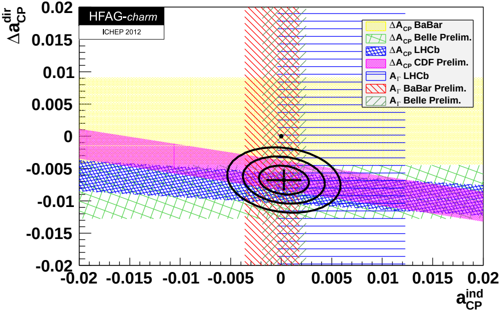

$$\Delta a^{\rm dir}_{CP} = a^{K^+K^-,{\rm dir}}_{CP} - a^{\pi^+\pi^-,{\rm dir}}_{CP} = (-0.678 \pm 0.147)\%$$

Question: How large is the impact on the extraction of γ if $A_{CP} \sim \mathcal{O}(0.1 - 1\%)$?

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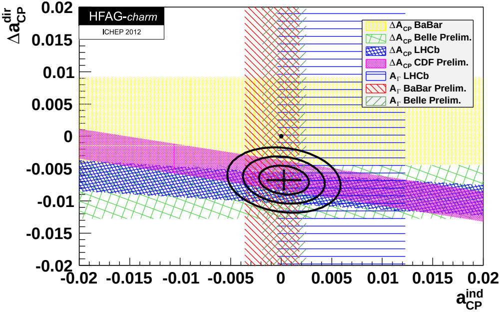

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Warning: Large CPA is not confirmed by the recent LHCb measurement: 1303.2614; LHCb-CONF-2013-003

CPA effects on γ in GLW method

$$A(D^0 \to f) = T_D^f (1 + r_D^f e^{-i\gamma + i\delta_D^f}),$$
$$A(\bar{D}^0 \to f) = T_D^f (1 + r_D^f e^{i\gamma + i\delta_D^f}),$$

$$\begin{aligned} A_+^K &= \frac{\mathcal{B}(B^- \to D_+^0 K^-) - \mathcal{B}(B^+ \to D_+^0 K^+)}{\mathcal{B}(B^- \to D_+^0 K^-) + \mathcal{B}(B^+ \to D_+^0 K^+)} \\ &= \frac{1}{R_+^K}\left[(1 - (r_B^K)^2) A_{CP}^{dir}(D^0 \to f) + \frac{2r_B^K (1 + (r_D^f)^2) \sin\delta_B^K \sin\gamma}{1 + (r_D^f)^2 + 2r_D^f \cos\delta_D^f \cos\gamma}\right] \\ &\equiv 2r_B^K \sin\delta_B^K \sin\gamma_{eff} / R_+^K. \end{aligned} \quad (1)$$

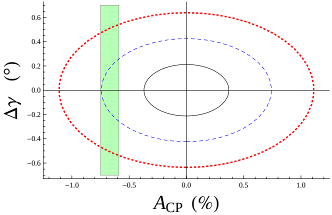
$$\Delta\gamma \equiv \gamma_{eff} - \gamma = \mathcal{O}(A_{CP}^{\rm dir}/r_B^K) \sim \text{a few degrees!}$$

WW, 1211.4539; Martone, Zupan, 1212.0165; Bhattacharya, London, Gronau, Rosner, 1301.5631

CPA effects on γ

The direct CP asymmetry effects in R_+:

$$R_+^K = 2\frac{\mathcal{B}(B^- \to D_+^0 K^-) + \mathcal{B}(B^+ \to D_+^0 K^+)}{\mathcal{B}(B^- \to D^0 K^-) + \mathcal{B}(B^+ \to \bar{D}^0 K^+)}$$
$$= 1 + (r_B^K)^2 + \frac{2r_B^K \cos\delta_B[(1+(r_D^f)^2)\cos\gamma + 2r_D^f \cos\delta_D^f]}{1 + (r_D^f)^2 + 2r_D^f \cos\gamma \cos\delta_D^f}$$
$$\equiv 1 + (r_B^K)^2 + 2r_B^K \cos\delta_B^K \cos\gamma_{eff} \quad (2)$$



Depending on the strong phase difference in $D \to K^+K^-/\pi^+\pi^-$ decays, the shift of γ can reach $0.5°$!

CPA effects on γ in $B \to DK$ ($D \to K_S\pi^+\pi^-$)

Dalitz plot analysis

Resonance	Contribution to γ bias ($^\circ$)	
	Amplitude	Phase
CF $K^*(892)$	$+0.09 \pm 0.27$	-0.87 ± 2.09
CF $K_0^*(1430)$	-0.05 ± 0.05	-0.23 ± 0.35
CF $K_2^*(1430)$	$+0.07 \pm 0.12$	-0.04 ± 0.07
CF $K^*(1410)$	$+0.01 \pm 0.02$	-0.21 ± 0.37
$\rho(770)$	$+0.27 \pm 0.89$	-0.24 ± 0.97
ω	-0.32 ± 0.21	-0.25 ± 0.36
$f_0(980)$	-0.02 ± 0.13	-0.02 ± 0.38
$f_2(1270)$	-0.09 ± 0.10	-0.06 ± 0.09
$f_0(1370)$	-0.09 ± 1.06	$+0.01 \pm 0.26$
$\rho(1450)$	-0.02 ± 0.19	-0.09 ± 0.22
σ_1	-0.31 ± 0.78	-0.09 ± 0.62
σ_2	-0.07 ± 0.08	-0.04 ± 0.56
DCS $K^*(892)$	-0.04 ± 0.24	$+0.22 \pm 0.15$
DCS $K_0^*(1430)$	$+0.23 \pm 0.44$	-0.12 ± 0.21
DCS $K_2^*(1430)$	-0.30 ± 0.56	$+0.03 \pm 0.04$
Total	-2.65 ± 3.17	

Bondar, Dolgov, Poluektov, Vorobiev, arXiv:1303.6305
CP violating contributions in $D \to K_S\pi^+\pi^-$ decay to the γ measurement bias

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ϕ_s

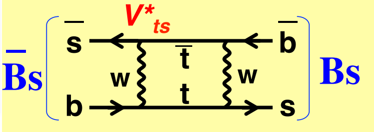
CKM ansatz: CPV is due to a complex phase in the quark mixing matrix

$$V_{n=3} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\mathcal{O}(\lambda^6)$

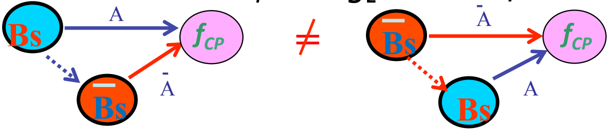
$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda\left[1+\frac{1}{2}A^2\lambda^4(2\rho-1)+iA^2\lambda^4\eta\right] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\left(4A^2+1\right)\lambda^4 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2\left[1+\frac{1}{2}\lambda^2(2\rho-1)+i\lambda^2\eta\right] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

Bs-$\bar{\text{B}}$s mixing



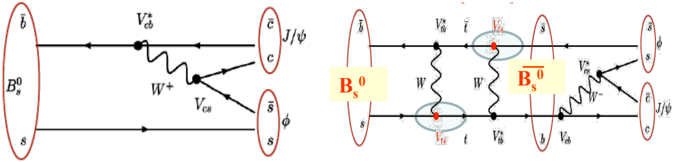
mixing induced CP violation

β_s=arg[−VtsVtb*/VcsVcb*]



ϕ_s from $B_s \to J/\psi\phi$

$B_s(\bar{B}_s) \to J/\psi(\mu^+\mu^-)\ \phi(K^+K^-)$ can proceed directly or through mixing



Angular analysis to disentangle different CP-eigenstates

$$\frac{d^4\Gamma(B_s^0 \to J/\psi\,\phi)}{dt\ d\Omega} \propto \sum_{k=1}^{10} h_k(t)\, f_k(\Omega)\,.$$

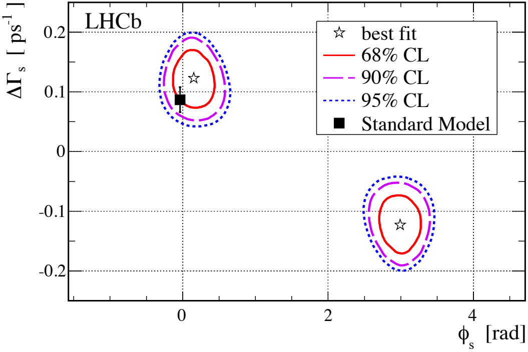
The time-dependent functions $h_k(t)$ can be written

$$h_k(t) = N_k e^{-\Gamma_s t}\left[c_k \cos(\Delta m_s t) + d_k \sin(\Delta m_s t) + a_k \cosh\left(\tfrac{1}{2}\Delta\Gamma_s t\right) + b_k \sinh\left(\tfrac{1}{2}\Delta\Gamma_s t\right)\right].$$

ϕ_s from $B_s \to J/\psi\phi$

$$SM: \phi_s = -2\beta_s = -0.04$$

$$LHCb: \phi_s = 0.15 \pm 0.18 \pm 0.06$$



LHCb

arXiv:1112.3183

Penguin pollutions?

ϕ_s from $B_s \to J/\psi\phi$

$$h_k(t) = N_k e^{-\Gamma_s t}\left[c_k \cos(\Delta m_s t) + d_k \sin(\Delta m_s t) + a_k \cosh\left(\tfrac{1}{2}\Delta\Gamma_s t\right) + b_k \sinh\left(\tfrac{1}{2}\Delta\Gamma_s t\right)\right].$$

c_k
0
0
0
$\sin(\delta_\perp - \delta_\parallel)$
0
$\sin(\delta_\perp - \delta_0)$
0
$\cos(\delta_\parallel - \delta_S)$
0
$\cos(\delta_0 - \delta_S)$

$4\phi_s$!

$$\lambda_i^j = \eta_i^j \frac{q}{p}\frac{\overline{A_i^j}}{A_i^j} \equiv |\lambda_i^j| e^{-i\phi_i^j}$$

c_k
$\frac{1-\lvert\lambda_0^1\rvert^2}{1+\lvert\lambda_0^1\rvert^2}$
$\frac{1-\lvert\lambda_\parallel^1\rvert^2}{1+\lvert\lambda_\parallel^1\rvert^2}$
$\frac{1-\lvert\lambda_\perp^1\rvert^2}{1+\lvert\lambda_\perp^1\rvert^2}$
$\frac{1}{2}\left[\sin(\delta_\perp^1 - \delta_\parallel^1) + \lvert\lambda_\perp^1\rvert\lvert\lambda_\parallel^1\rvert \sin(\delta_\perp^1 - \delta_\parallel^1 - \phi_\perp^1 + \phi_\parallel^1)\right]$
$\frac{1}{2}\left[\cos(\delta_0^1 - \delta_\parallel^1) - \lvert\lambda_0^1\rvert\lvert\lambda_\parallel^1\rvert \cos(\delta_0^1 - \delta_\parallel^1 - \phi_0^1 + \phi_\parallel^1)\right]$
$\frac{1}{2}\left[\sin(\delta_0^1 - \delta_\perp^1) + \lvert\lambda_0^1\rvert\lvert\lambda_\perp^1\rvert \sin(\delta_0^1 - \delta_\perp^1 - \phi_0^1 + \phi_\perp^1)\right]$
$\frac{1-\lvert\lambda_0^0\rvert^2}{1+\lvert\lambda_0^0\rvert^2}$
$\frac{1}{2}\left[\cos(\delta_0^0 - \delta_\parallel^1) + \lvert\lambda_0^0\rvert\lvert\lambda_\parallel^1\rvert \cos(\delta_0^0 - \delta_\parallel^1 - \phi_0^0 + \phi_\parallel^1)\right]$
$\frac{1}{2}\left[\sin(\delta_0^0 - \delta_\perp^1) - \lvert\lambda_0^0\rvert\lvert\lambda_\perp^1\rvert \sin(\delta_0^0 - \delta_\perp^1 - \phi_0^0 + \phi_\perp^1)\right]$
$\frac{1}{2}\left[\cos(\delta_0^0 - \delta_0^1) + \lvert\lambda_0^0\rvert\lvert\lambda_0^1\rvert \cos(\delta_0^0 - \delta_0^1 - \phi_0^0 + \phi_0^1)\right]$

X.Liu, WW, Y.H. Xie,

in preparation.

Bhattacharya, A. Datta,
D. London 1209.1413

Conclusions

- New measurements are reducing the errors and soon we will be able to test the unitarity of CKM:

$$\alpha + \beta + \gamma = 180°? :: (179.3 \pm 10.4)°.$$

- New Channels can provide complementary power and useful to increase the statistical significance.
- There are always something unexpected!

Thank you for your attention!

Backup: STOP

ϕ_s from $B_s \to J/\psi\phi$

TABLE I. The angular and time-dependent functions used in Eqs. (7) and (8) , as discussed in the text. In the amplitude A_i^J, the superscript J denotes the spin of the K^+K^- state, while the subscript $i = 0, ||, \perp$ corresponds to the three polarization configurations. A_0^0 is also usually referred to as A_S in the literature. Some abbreviations have been used for cosine and sine functions: $c_K = \cos\theta_K$, $s_K = \sin\theta_K$.

k	f_k	N_k	a_k	b_k	c_k	d_k
1	$c_K^2 s_l^2$	$\frac{\vert A_0^1\vert^2+\vert\bar{A}_0^1\vert^2}{2}$	1	$-\frac{2\vert\lambda_0^1\vert}{1+\vert\lambda_0^1\vert^2}\cos(\phi_0^1)$	$\frac{1-\vert\lambda_0^1\vert^2}{1+\vert\lambda_0^1\vert^2}$	$\frac{2\vert\lambda_0^1\vert}{1+\vert\lambda_0^1\vert^2}\sin(\phi_0^1)$
2	$\frac{s_K^2(1-c_\phi^2 c_l^2)}{2}$	$\frac{\vert A_{\vert\vert}^1\vert^2+\vert\bar{A}_{\vert\vert}^1\vert^2}{2}$	1	$-\frac{2\vert\lambda_{\vert\vert}^1\vert}{1+\vert\lambda_{\vert\vert}^1\vert^2}\cos(\phi_{\vert\vert}^1)$	$\frac{1-\vert\lambda_{\vert\vert}^1\vert^2}{1+\vert\lambda_{\vert\vert}^1\vert^2}$	$\frac{2\vert\lambda_{\vert\vert}^1\vert}{1+\vert\lambda_{\vert\vert}^1\vert^2}\sin(\phi_{\vert\vert}^1)$
3	$\frac{s_K^2(1-s_\phi^2 c_l^2)}{2}$	$\frac{\vert A_\perp^1\vert^2+\vert\bar{A}_\perp^1\vert^2}{2}$	1	$\frac{2\vert\lambda_\perp^1\vert}{1+\vert\lambda_\perp^1\vert^2}\cos(\phi_\perp^1)$	$\frac{1-\vert\lambda_\perp^1\vert^2}{1+\vert\lambda_\perp^1\vert^2}$	$\frac{2\vert\lambda_\perp^1\vert}{1+\vert\lambda_\perp^1\vert^2}\sin(\phi_\perp^1)$
4	$s_K^2 s_l^2 s_\phi c_\phi$	$\vert A_\perp^1\vert\vert A_{\vert\vert}^1\vert$	$\frac{1}{2}\Big[\sin(\delta_\perp^1-\delta_{\vert\vert}^1)-\vert\lambda_\perp^1\vert\vert\lambda_{\vert\vert}^1\vert \sin(\delta_\perp^1-\delta_{\vert\vert}^1-\phi_\perp^1+\phi_{\vert\vert}^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_\perp^1\vert\sin(\delta_\perp^1-\delta_{\vert\vert}^1-\phi_\perp^1) +\vert\lambda_{\vert\vert}^1\vert\sin(\delta_{\vert\vert}^1-\delta_\perp^1-\phi_{\vert\vert}^1)\Big]$	$\frac{1}{2}\Big[\sin(\delta_\perp^1-\delta_{\vert\vert}^1)+\vert\lambda_\perp^1\vert\vert\lambda_{\vert\vert}^1\vert \sin(\delta_\perp^1-\delta_{\vert\vert}^1-\phi_\perp^1+\phi_{\vert\vert}^1)\Big]$	$-\frac{1}{2}\Big[\vert\lambda_\perp^1\vert\cos(\delta_\perp^1-\delta_{\vert\vert}^1-\phi_\perp^1) +\vert\lambda_{\vert\vert}^1\vert\cos(\delta_{\vert\vert}^1-\delta_\perp^1-\phi_{\vert\vert}^1)\Big]$
5	$\sqrt{2}s_K c_K s_l c_l c_\phi$	$\vert A_0^1 A_{\vert\vert}^1\vert$	$\frac{1}{2}\Big[\cos(\delta_0^1-\delta_{\vert\vert}^1)+\vert\lambda_0^1\vert\vert\lambda_{\vert\vert}^1\vert \cos(\delta_0^1-\delta_{\vert\vert}^1-\phi_0^1+\phi_{\vert\vert}^1)\Big]$	$-\frac{1}{2}\Big[\vert\lambda_0^1\vert\cos(\delta_0^1-\delta_{\vert\vert}^1-\phi_0^1) +\vert\lambda_{\vert\vert}^1\vert\cos(\delta_{\vert\vert}^1-\delta_0^1-\phi_{\vert\vert}^1)\Big]$	$\frac{1}{2}\Big[\cos(\delta_0^1-\delta_{\vert\vert}^1)-\vert\lambda_0^1\vert\vert\lambda_{\vert\vert}^1\vert \cos(\delta_0^1-\delta_{\vert\vert}^1-\phi_0^1+\phi_{\vert\vert}^1)\Big]$	$-\frac{1}{2}\Big[\vert\lambda_0^1\vert\sin(\delta_0^1-\delta_{\vert\vert}^1-\phi_0^1) +\vert\lambda_{\vert\vert}^1\vert\sin(\delta_{\vert\vert}^1-\delta_0^1-\phi_{\vert\vert}^1)\Big]$
6	$\sqrt{2}s_K c_K s_l c_l s_\phi$	$\vert A_0^1 A_\perp^1\vert$	$\frac{1}{2}\Big[\sin(\delta_0^1-\delta_\perp^1)-\vert\lambda_0^1\vert\vert\lambda_\perp^1\vert \sin(\delta_0^1-\delta_\perp^1-\phi_0^1+\phi_\perp^1)\Big]$	$-\frac{1}{2}\Big[\vert\lambda_0^1\vert\sin(\delta_0^1-\delta_\perp^1-\phi_0^1) +\vert\lambda_\perp^1\vert\sin(\delta_\perp^1-\delta_0^1-\phi_\perp^1)\Big]$	$\frac{1}{2}\Big[\sin(\delta_0^1-\delta_\perp^1)+\vert\lambda_0^1\vert\vert\lambda_\perp^1\vert \sin(\delta_0^1-\delta_\perp^1-\phi_0^1+\phi_\perp^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_0^1\vert\cos(\delta_0^1-\delta_\perp^1-\phi_0^1) +\vert\lambda_\perp^1\vert\cos(\delta_\perp^1-\delta_0^1-\phi_\perp^1)\Big]$
7	$\frac{1}{3}s_l^2$	$\frac{\vert A_0^0\vert^2+\vert\bar{A}_0^0\vert^2}{2}$	1	$\frac{2\vert\lambda_0^0\vert}{1+\vert\lambda_0^0\vert^2}\cos(\phi_0^0)$	$\frac{1-\vert\lambda_0^0\vert^2}{1+\vert\lambda_0^0\vert^2}$	$\frac{2\vert\lambda_0^0\vert}{1+\vert\lambda_0^0\vert^2}\sin(\phi_0^0)$
8	$\frac{2s_K s_l c_l c_\phi}{\sqrt{6}}$	$\vert A_0^0 A_{\vert\vert}^1\vert$	$\frac{1}{2}\Big[\cos(\delta_0^0-\delta_{\vert\vert}^1)-\vert\lambda_0^0\vert\vert\lambda_{\vert\vert}^1\vert \cos(\delta_0^0-\delta_{\vert\vert}^1-\phi_0^0+\phi_{\vert\vert}^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_0^0\vert\cos(\delta_0^0-\delta_{\vert\vert}^1-\phi_0^0) -\vert\lambda_{\vert\vert}^1\vert\cos(\delta_{\vert\vert}^1-\delta_0^0-\phi_{\vert\vert}^1)\Big]$	$\frac{1}{2}\Big[\cos(\delta_0^0-\delta_{\vert\vert}^1)+\vert\lambda_0^0\vert\vert\lambda_{\vert\vert}^1\vert \cos(\delta_0^0-\delta_{\vert\vert}^1-\phi_0^0+\phi_{\vert\vert}^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_0^0\vert\sin(\delta_0^0-\delta_{\vert\vert}^1-\phi_0^0) -\vert\lambda_{\vert\vert}^1\vert\sin(\delta_{\vert\vert}^1-\delta_0^0-\phi_{\vert\vert}^1)\Big]$
9	$\frac{2s_K s_l c_l s_\phi}{\sqrt{6}}$	$A_0^0 A_\perp^{1*}\vert$	$\frac{1}{2}\Big[\sin(\delta_0^0-\delta_\perp^1)+\vert\lambda_0^0\vert\vert\lambda_\perp^1\vert \sin(\delta_0^0-\delta_\perp^1-\phi_0^0+\phi_\perp^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_0^0\vert\sin(\delta_0^0-\delta_\perp^1-\phi_0^0) -\vert\lambda_\perp^1\vert\sin(\delta_\perp^1-\delta_0^0-\phi_\perp^1)\Big]$	$\frac{1}{2}\Big[\sin(\delta_0^0-\delta_\perp^1)-\vert\lambda_0^0\vert\vert\lambda_\perp^1\vert \sin(\delta_0^0-\delta_\perp^1-\phi_0^0+\phi_\perp^1)\Big]$	$\frac{1}{2}\Big[-\vert\lambda_0^0\vert\cos(\delta_0^0-\delta_\perp^1-\phi_0^0) +\vert\lambda_\perp^1\vert\cos(\delta_\perp^1-\delta_0^0-\phi_\perp^1)\Big]$
10	$\frac{2c_K s_l^2}{\sqrt{3}}$	$\vert A_0^0 A_0^1\vert$	$\frac{1}{2}\Big[\cos(\delta_0^0-\delta_0^1)-\vert\lambda_0^0\vert\vert\lambda_0^1\vert \cos(\delta_0^0-\delta_0^1-\phi_0^0+\phi_0^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_0^0\vert\cos(\delta_0^0-\delta_0^1-\phi_0^0) -\vert\lambda_0^1\vert\cos(\delta_0^1-\delta_0^0-\phi_0^1)\Big]$	$\frac{1}{2}\Big[\cos(\delta_0^0-\delta_0^1)+\vert\lambda_0^0\vert\vert\lambda_0^1\vert \cos(\delta_0^0-\delta_0^1-\phi_0^0+\phi_0^1)\Big]$	$\frac{1}{2}\Big[\vert\lambda_0^0\vert\sin(\delta_0^0-\delta_0^1-\phi_0^0) -\vert\lambda_0^1\vert\sin(\delta_0^1-\delta_0^0-\phi_0^1)\Big]$