Measurement of γ from $B \to DK$ and related modes at LHCb

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Outline

I. LHCb measurements

- two-body GLW/ADS
- four-body ADS
- GGSZ

} 22 observables

$B \to Dh$, followed by:

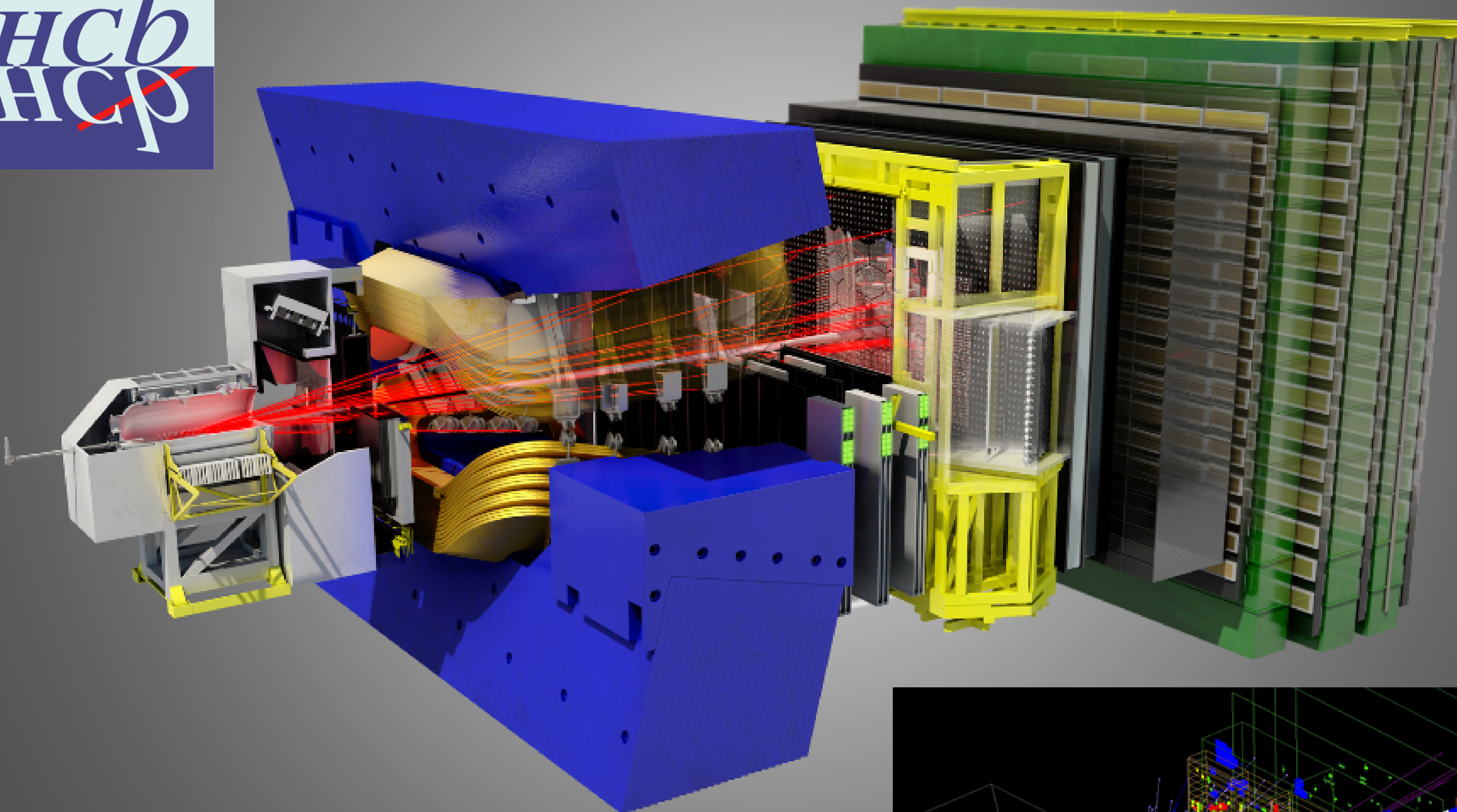
GLW: $D \to$ CP final states
ADS: $D \to$ flavor final states
GGSZ: $D \to$ 3-body self. conj.

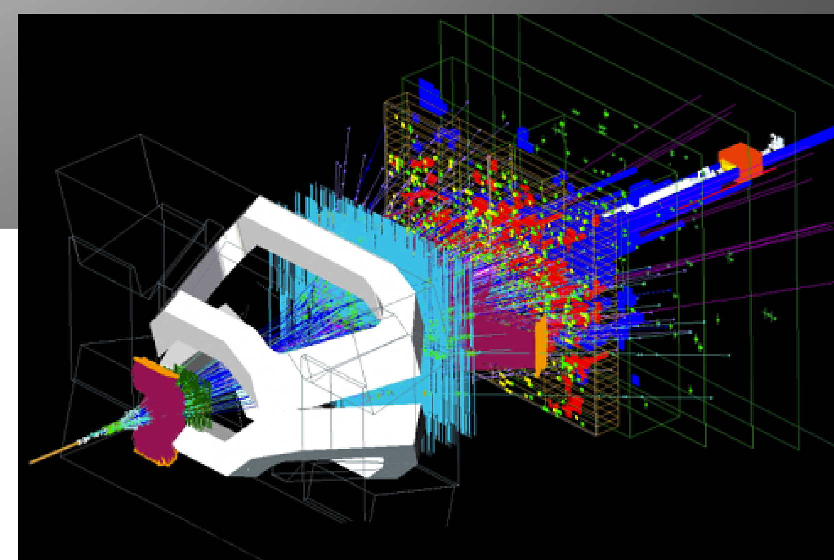
II. Combination

- $B \to DK$
- $B \to D\pi$
- full $B \to DK$ and $B \to D\pi$

III. A new GGSZ result using additional $2fb^{-1}$

→ see also talk by Matteo Rama!



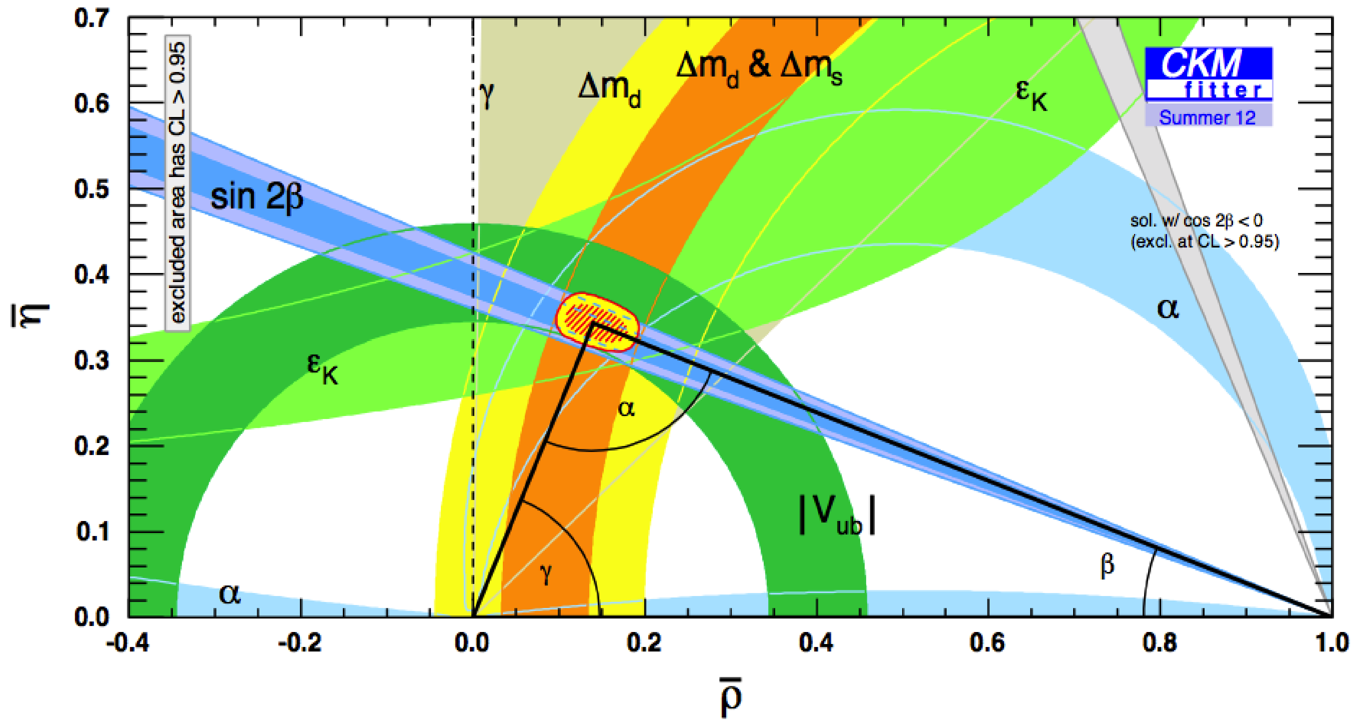


- LHCb is a forward spectrometer operated in collider mode.
- Focus on precision measurements of b and c decays.
- CP violation, rare decays

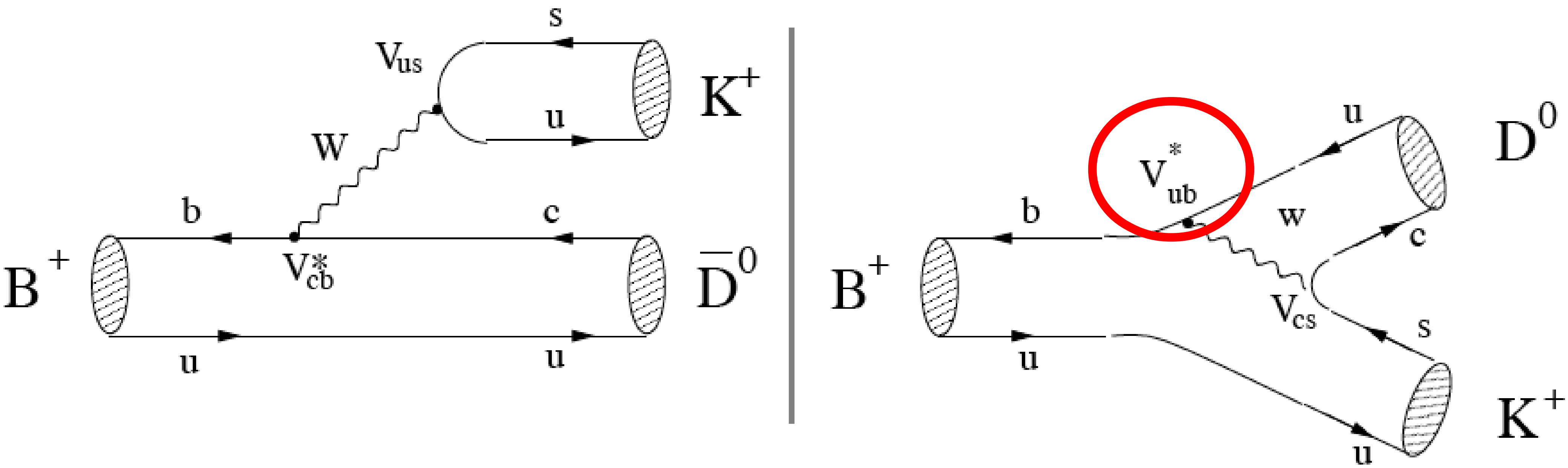
CKM angle γ

γ is the least well known angle of the unitarity triangle.

"combined γ measurements"		**"γ meas. not in triangle fit"**
$\gamma = (66^{+12}_{-12})^\circ$	CKMfitter ICHEP 2012	$\gamma = (68.0^{+4.1}_{-4.6})^\circ$
$\gamma = (70.8 \pm 7.8)^\circ$	UTfit pre-Moriond 2013	$\gamma = (68.6 \pm 3.6)^\circ$



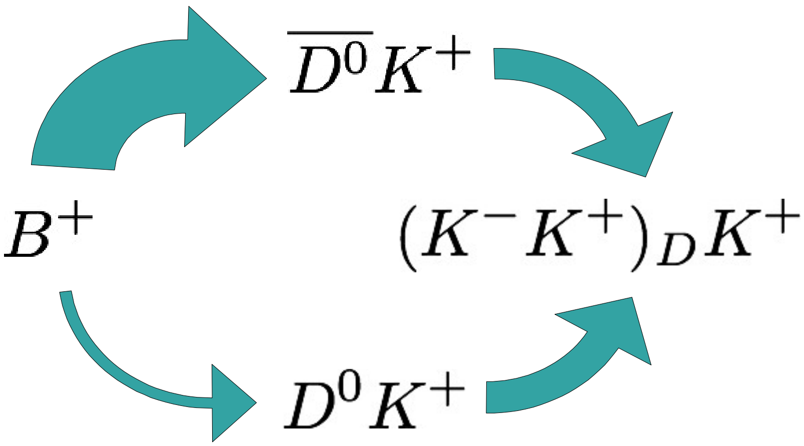
$B \to DK$



- This was, and still is, the most important channel to measure γ.
- We need to reconstruct the $D/\overline{D}$ meson in a final state accessible to both to achieve interference.
- Choice of final state labels the "method": GLW, ADS, GGSZ
- **Also possible: B → Dπ**! But little sensitivity.

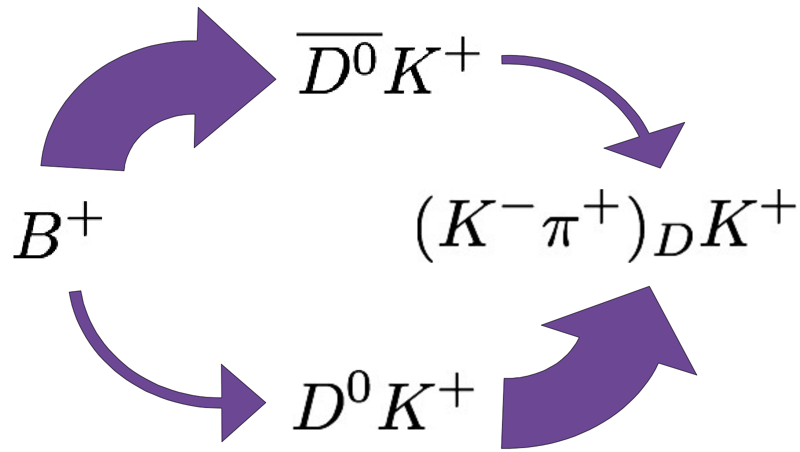
$B \to DK$

“GLW”



Phys.Lett. B253 (1991) 483
Phys.Lett. B265 (1991) 172
Gronau, London, Wyler

“ADS”, “suppressed”



Phys.Rev.Lett 78 (1997) 3257
Phys.Rev. D63 (2001) 036005
Atwood, Dunietz, Soni

“GGSZ”, “Dalitz”

- Use 3-body self-conjugate modes such as $\mathbf{D \to K_S \pi^+ \pi^-}$
- hadronic D parameters vary across Dalitz plot
- Giri, Grossman, Soffer, Zupan, hep-ph/0303187

B → Dh: GLW/ADS observables

- Define observables as **yield ratios** (many systematics cancel).
- Charge **asymmetries**:

$$A_h^f = \frac{\Gamma(B^- \to [f]_D h^-) - \Gamma(B^+ \to [f]_D h^+)}{\Gamma(B^- \to [f]_D h^-) + \Gamma(B^+ \to [f]_D h^+)}$$

- **Kaon/pion** ratio:

$$R_{K/\pi}^f = \frac{\Gamma(B^\pm \to [f]_D K^\pm)}{\Gamma(B^\pm \to [f]_D \pi^\pm)}$$

Form a system of equations. Need more observables than parameters!
→ many different decays

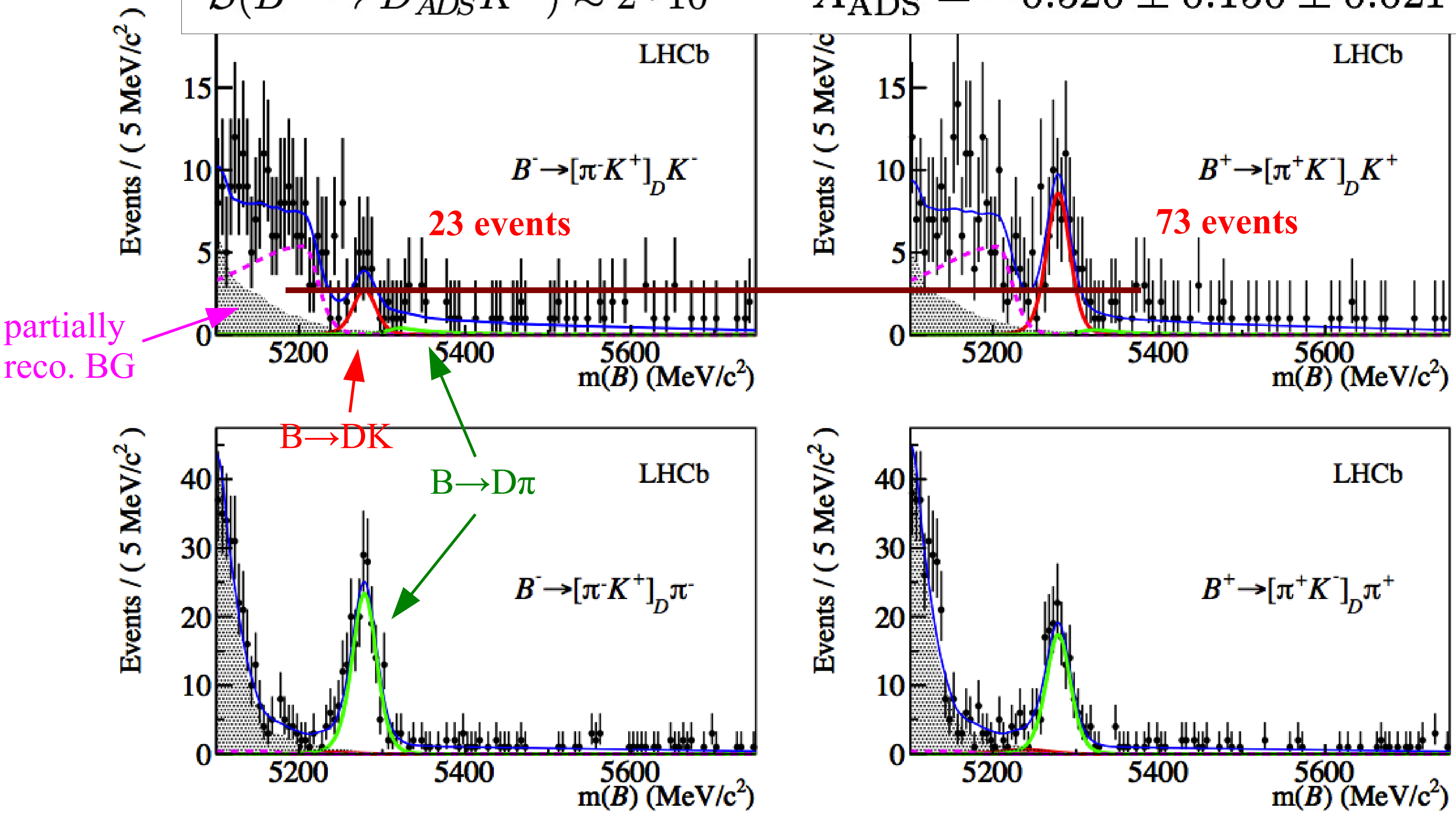
- **Suppressed/favored** decay ratio (2-body example):

$$R_h^\pm = \frac{\Gamma(B^\pm \to [\pi^\pm K^\mp]_D h^\pm)}{\Gamma(B^\pm \to [K^\pm \pi^\mp]_D h^\pm)}$$
$$= r_B^2 + r_D^2 + 2 r_B r_D \cos(\pm\gamma + \delta_B + \delta_D)$$

strong phase difference: different for each decay mode!

$B \to D(\pi K)h$: suppressed ADS mode

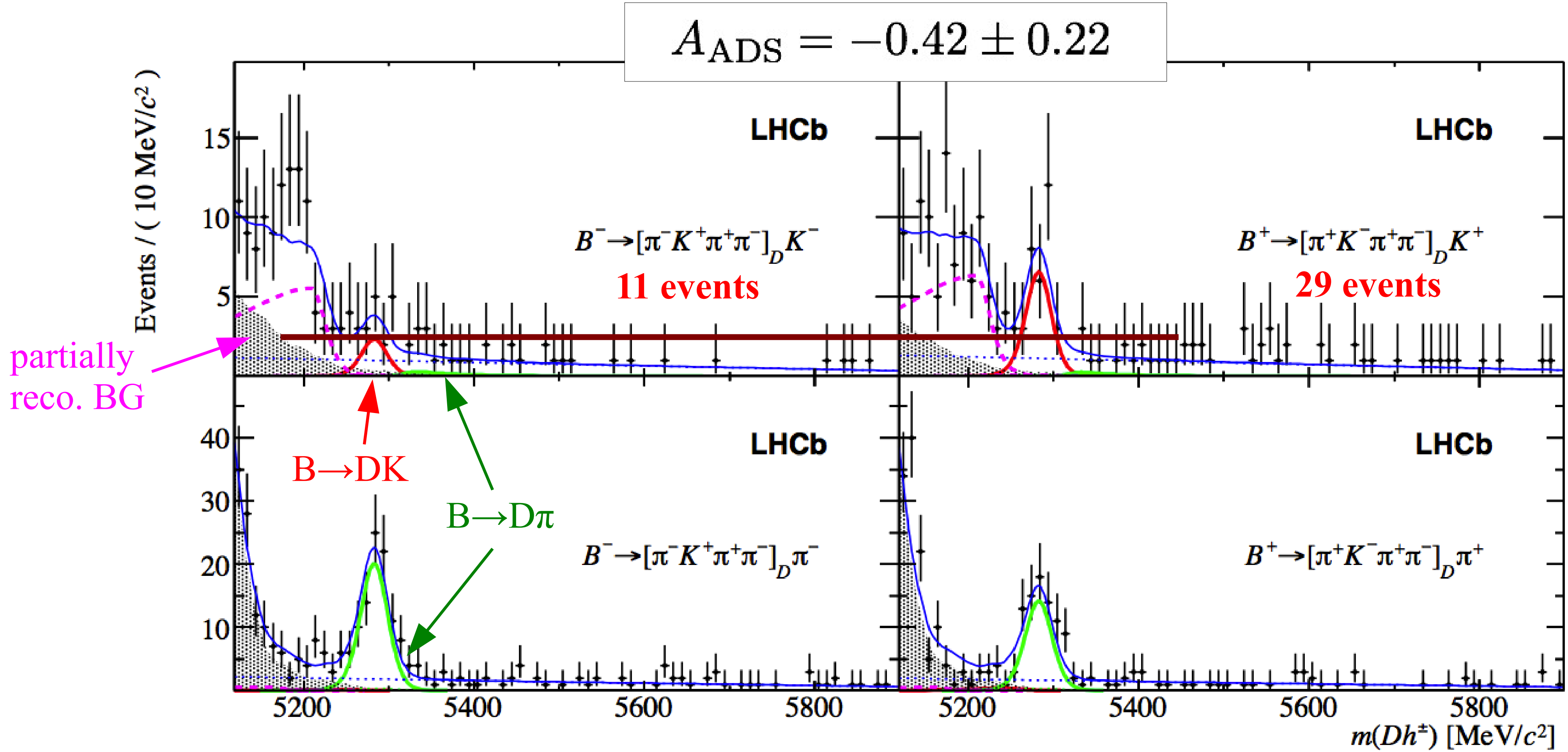
$\mathcal{B}(B^{\pm} \to D_{ADS} K^{\pm}) \approx 2 \cdot 10^{-7}$ $A_{\mathrm{ADS}} = -0.520 \pm 0.150 \pm 0.021$



13 observables in $B \to Dh$, $D \to hh$

arXiv:1203.3662, PLB 712:203–212, 2012.

$B \to D(\pi K \pi\pi)h$: suppressed ADS mode



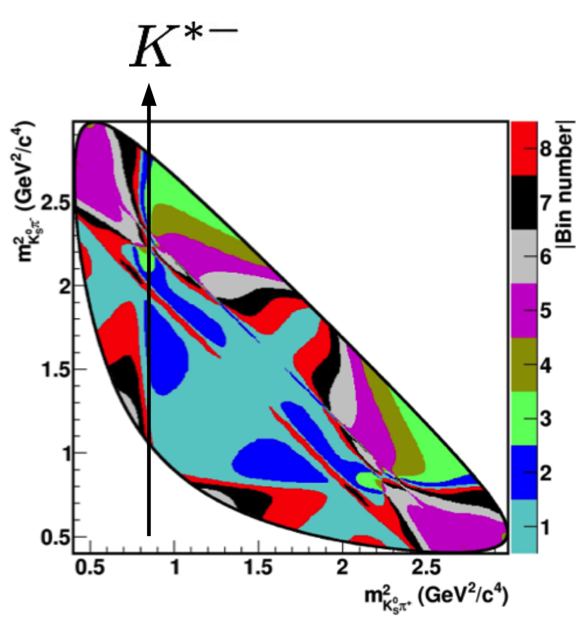
5 observables in $B \to Dh$, $D \to K3\pi$

arXiv:1303.4646, to appear in PLB

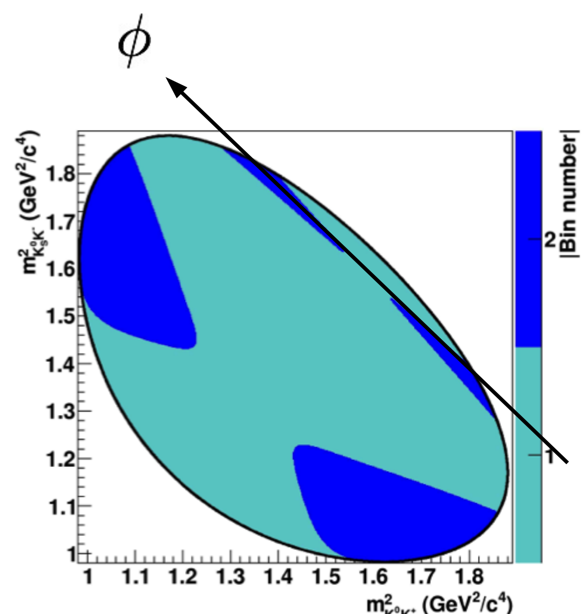
model independent GGSZ

- In the GGSZ method, one considers self-conjugate 3-body final states of the D meson: $D \to K_S^0 \pi^+ \pi^-$ $D \to K_S^0 K^+ K^-$
- A range of resonances introduces strong phase variations – no need for system of equations.
- Phase variation measured by CLEO. Used as input in binned analysis of the D Dalitz plot.

- Only $B^\pm \to DK^\pm$
- Control efficiency variation using $B^\pm \to D\pi^\pm$



$D \to K_S^0 \pi^+ \pi^-$

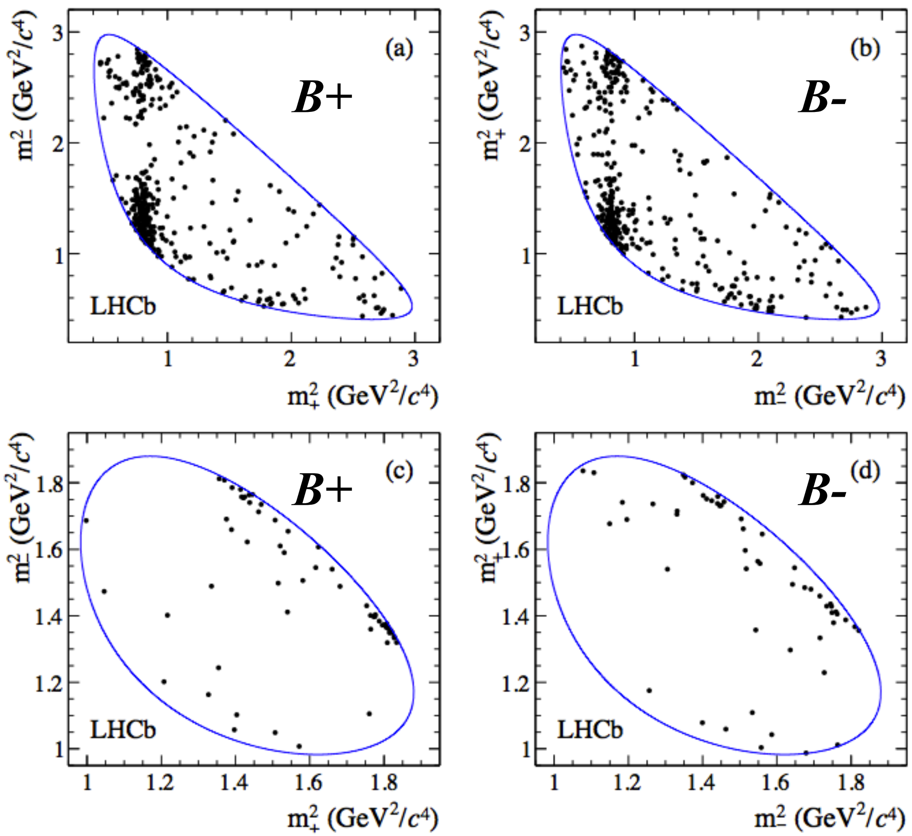


$D \to K_S^0 K^+ K^-$

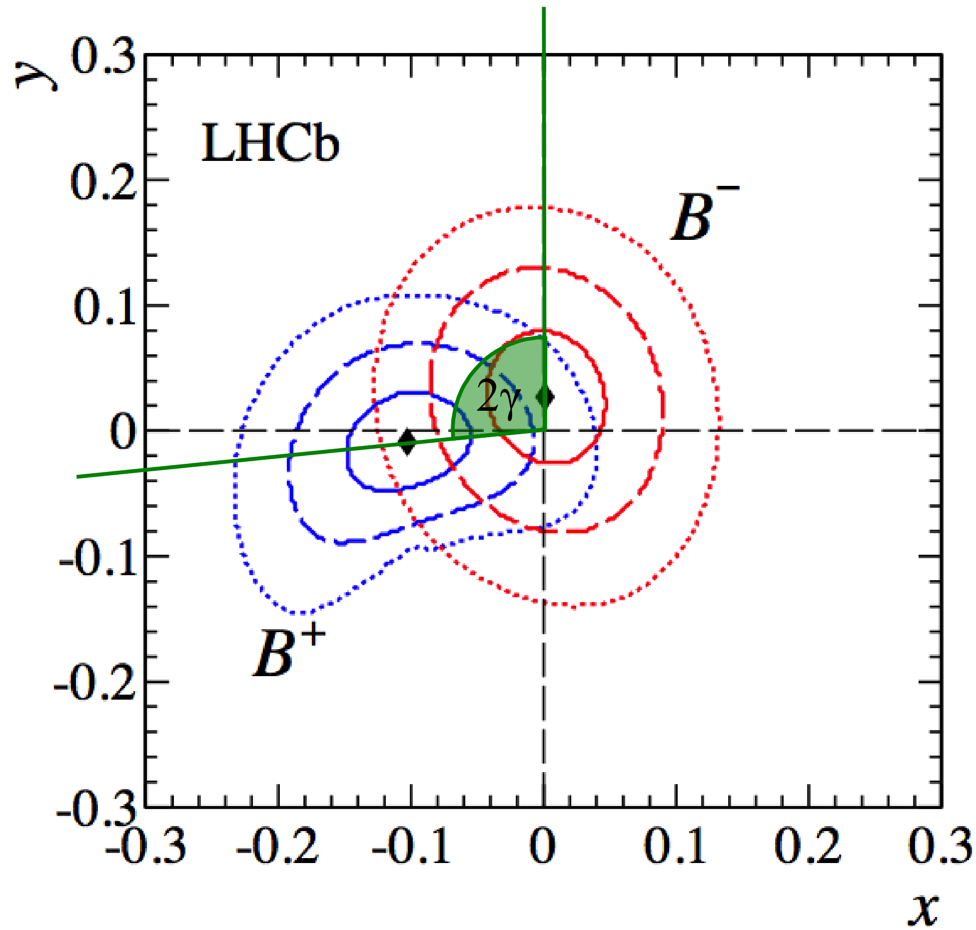
CLEO, Phys. Rev. D 82 112006.

model independent GGSZ

$D^0 \to K^0_S \pi^- \pi^+$



$D^0 \to K^0_S K^- K^+$

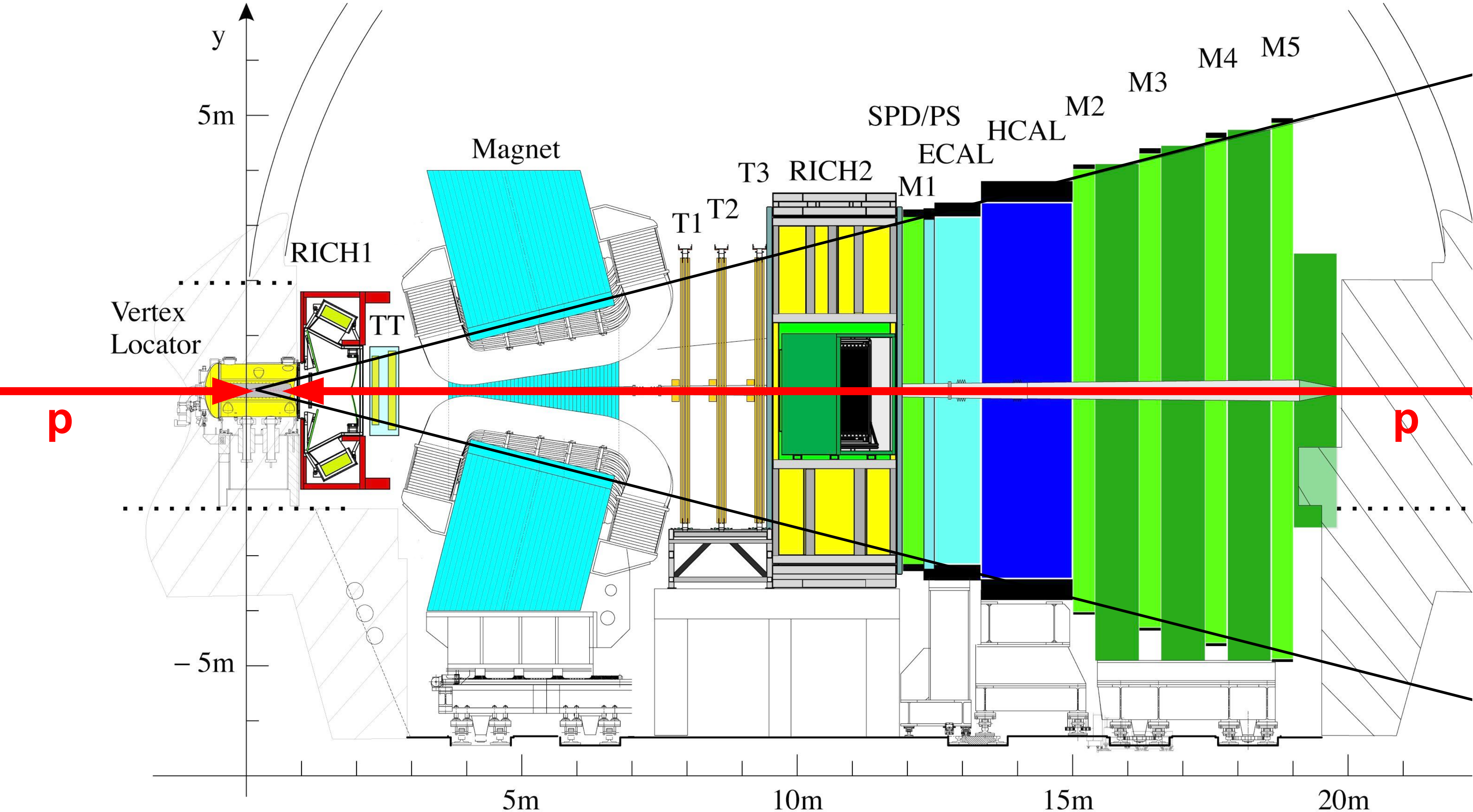


At the B-factories, this method is the best way to measure γ!

4 observables: "cartesian coordinates"

$$x_\pm = r_B \cos(\delta_B \pm \gamma)$$
$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$

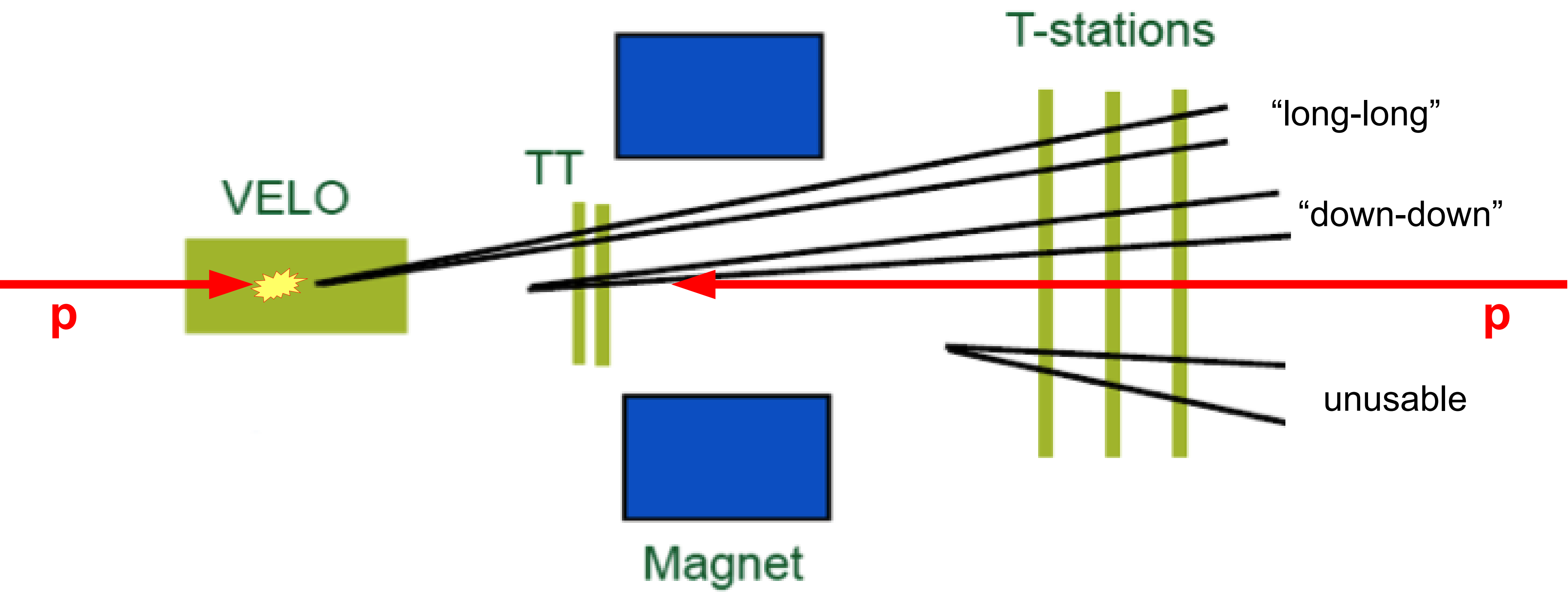
LHCb



y
5m
− 5m
Magnet
RICH1
Vertex
Locator
TT
T1
T2
T3
RICH2
M1
SPD/PS
ECAL
HCAL
M2
M3
M4
M5
p
p
5m
10m
15m
20m

K_S reconstruction

- At LHCb, about 70% of the reconstructible KS decays are "down-down".
- Decays behind first tracker are unusable!



Combination

- We now have measured **22 γ-related observables**. What does it mean for γ?

- **Combine the inputs!**
  - frequentist procedure
  - assume (mostly) Gaussian observables
  - assume Gaussian systematics
  - correct for undercoverage and some neglected systematic correlations

- **Strategy:**
  - for the first time include the B → Dπ system
  - consider CP violation in charm decays
  - partially consider charm mixing

exp. covariance

"truth" relations

$$\mathcal{L}(\vec{y}) = \frac{1}{N}\exp\left(-\frac{1}{2}(\vec{y}-\vec{y}_{\rm t})^T V_{\rm cov}^{-1}(\vec{y}-\vec{y}_{\rm t})\right)$$

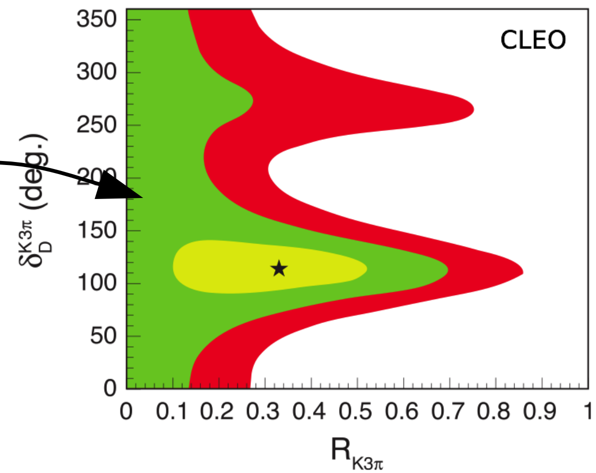
$$\chi^2(\vec{y}) = -2\ln\mathcal{L}(\vec{y})\ .$$

observables

Combination

- **Three LHCb input measurements:**
  - B → Dh, D → hh (two-body GLW/ADS)
  - B → Dh, D → Kπππ (four-body ADS)
  - B → DK, D → Kshh (GGSZ)

CLEO: arXiv:0903.4853



- **Other inputs:**
  - CLEO measurement of D → hh, Kπππ systems
  - Heavy Fl. Avg. Group averages for CPV in charm
  - (as crosscheck:) LHCb charm mixing result (arXiv:1211.1230 / PRL)

- Results are presented for **three combinations**:
  - "DK only" (in-line with previous experiments)
  - "Dπ only"
  - "DK & Dπ"

statistical treatment

- The combined likelihood has a very rich structure:
  - many **nuisance parameters**
  - many trigonometrical functions, thus **many local minima**
  - **varying dimensionality** of the likelihood, depending on the value of the nuisance parameters

direct product of r_B and angular terms:
$x_\pm = r_B \cos(\delta_B \pm \gamma)$

- Use a Feldman-Cousins based frequentist method.
- Compute the actual distribution of the test statistic ($\Delta\chi^2$) using toy Monte Carlo.
- Nuisances assume their profiled best-fit values.

"plug-in" method

CP violation in D^0 decays / D^0 mixing

- Any **CP violation** in the decays D → KK or D → ππ will affect the GLW method.

$$\left.\begin{aligned} A_{CP}^{\text{dir}}(KK) &= (-0.31 \pm 0.24) \times 10^{-2} \\ A_{CP}^{\text{dir}}(\pi\pi) &= (+0.36 \pm 0.25) \times 10^{-2} \end{aligned}\right\}$$

measurements combined by the Heavy Fl. Avg. Group

- We take this into account by modifying the GLW asymmetries, but leaving the ratios unchanged:

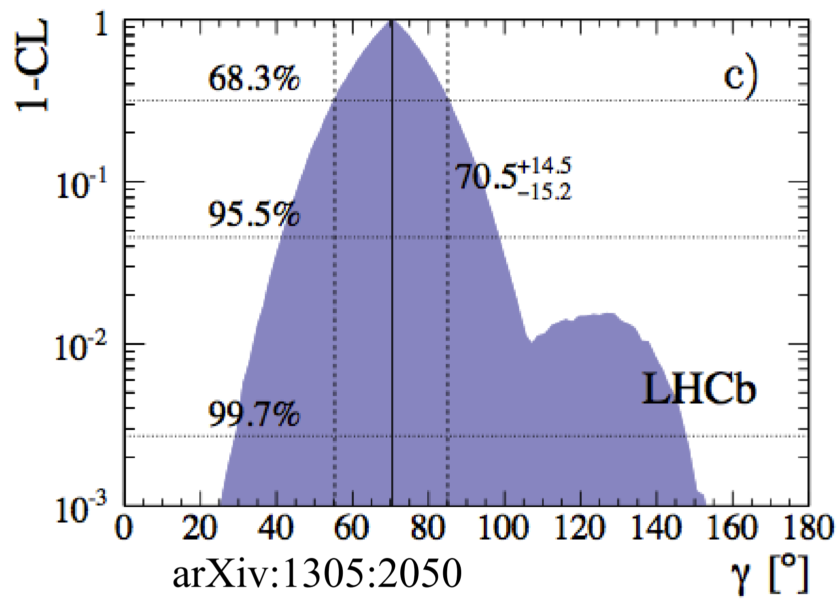
$$A_{\pi}^{KK} = \frac{2r_B^{\pi}\sin\delta_B^{\pi}\sin\gamma}{1+(r_B^{\pi})^2+2r_B^{\pi}\cos\delta_B^{\pi}\cos\gamma} + A_{CP}^{\text{dir}}(KK)$$

- This is valid up to a small weak phase in the D decay (London et al., arXiv:1301.5631).

- **D^0 mixing:** considered in description of D decay (constrained through CLEO measurement), but ignored in B decay: possible γ shift of $\mathcal{O}(x_D, y_D)$
  → will have to be fixed!

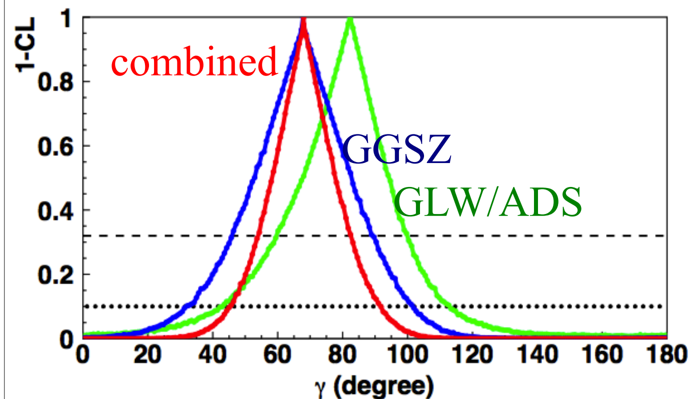
$B \to DK$

LHCb: $\gamma = (70.5^{+14.9}_{-15.6})^\circ$ (corrected)



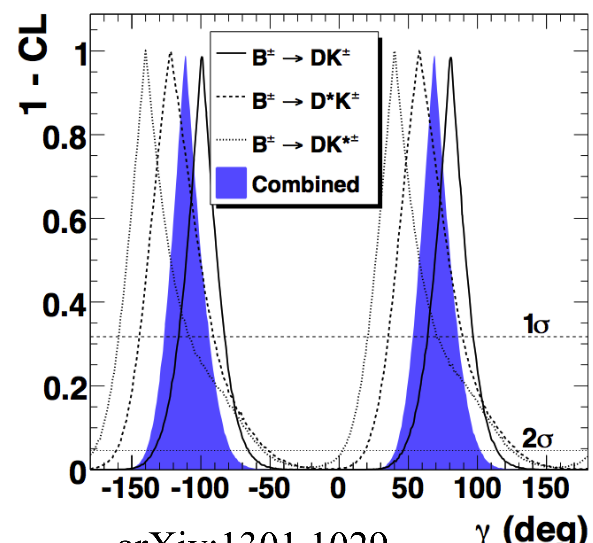
arXiv:1305:2050

Belle: $\gamma = (68^{+15}_{-14})^\circ$



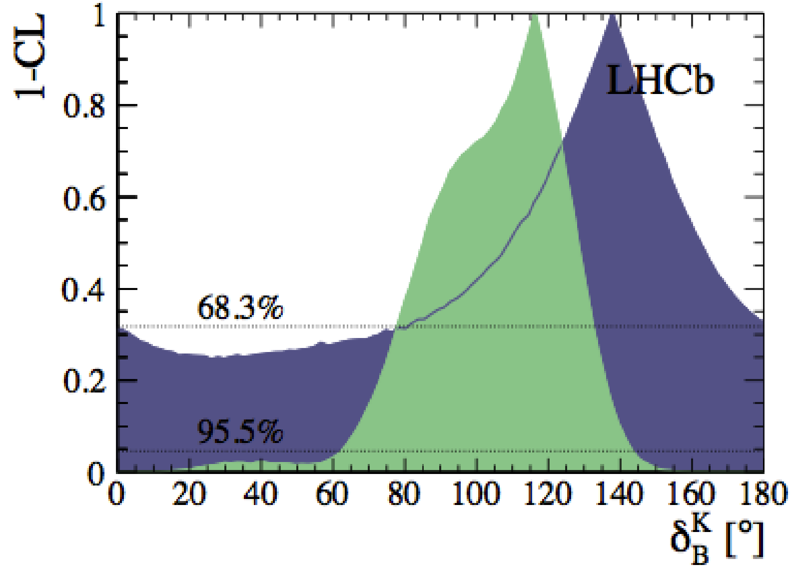
arXiv:1301.2033

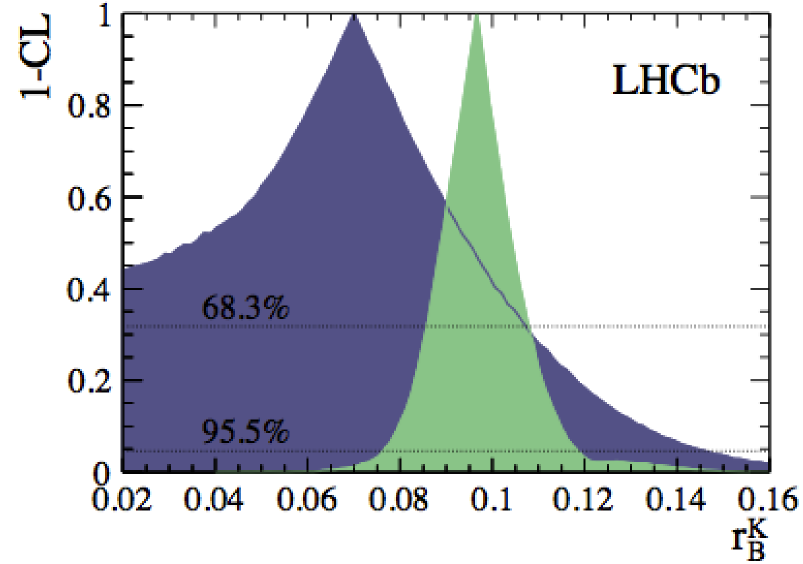
BaBar: $\gamma = (69^{+17}_{-16})^\circ$

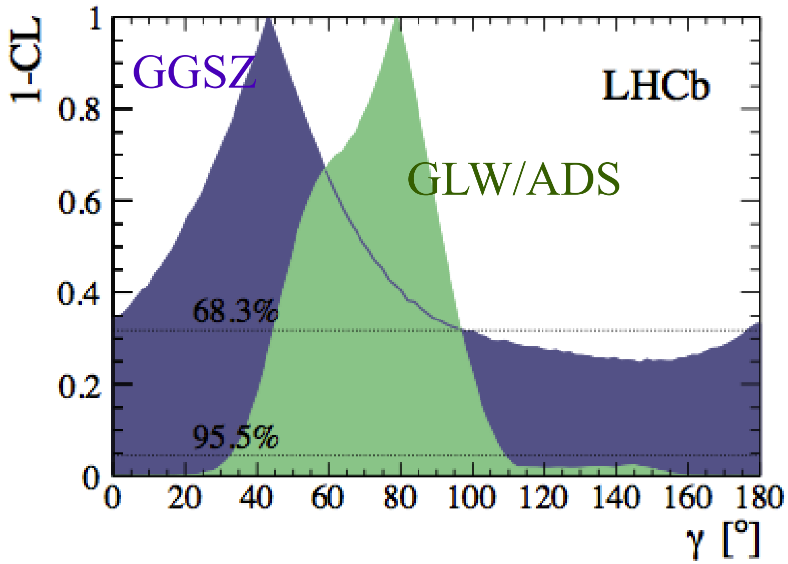


arXiv:1301.1029

$B \to DK$





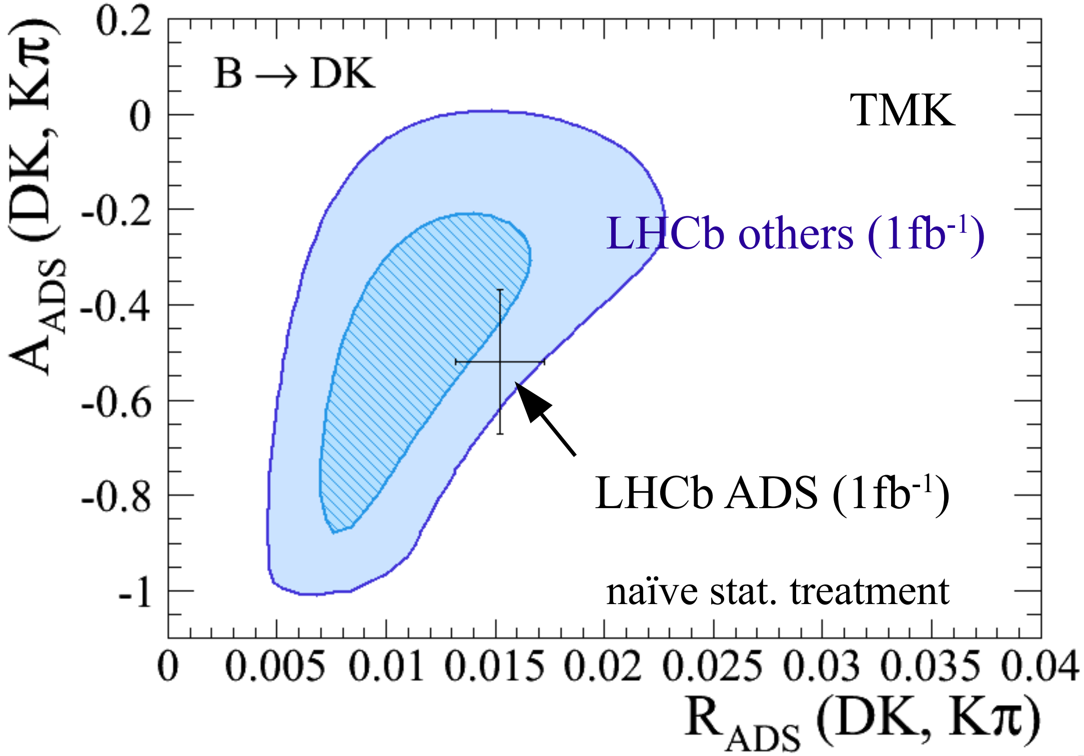


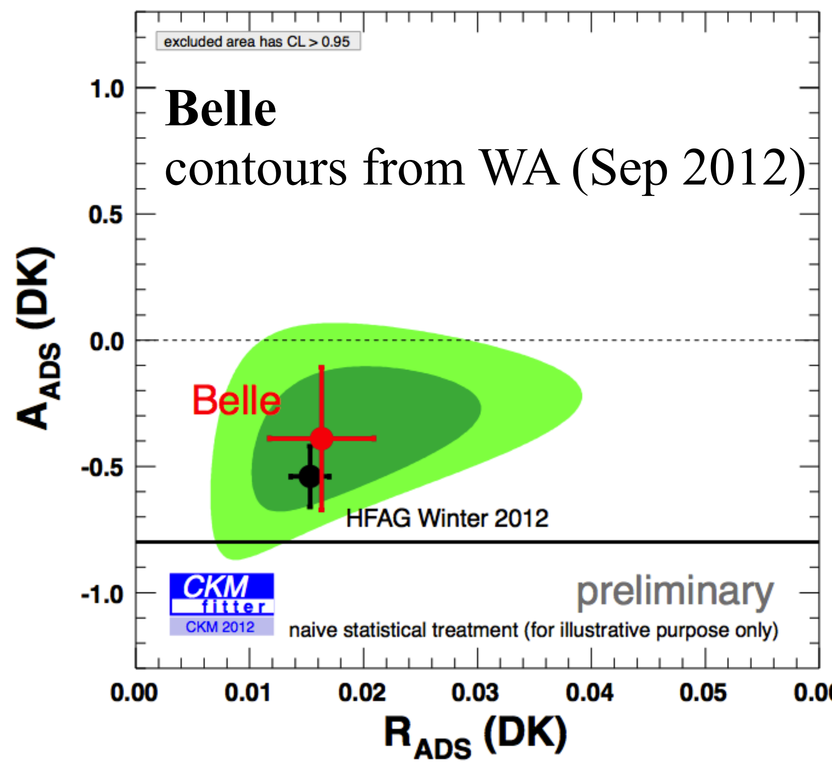
Comparing:
$1 fb^{-1}$ GLW/ADS and
$1 fb^{-1}$ GGSZ

Agreement of inputs

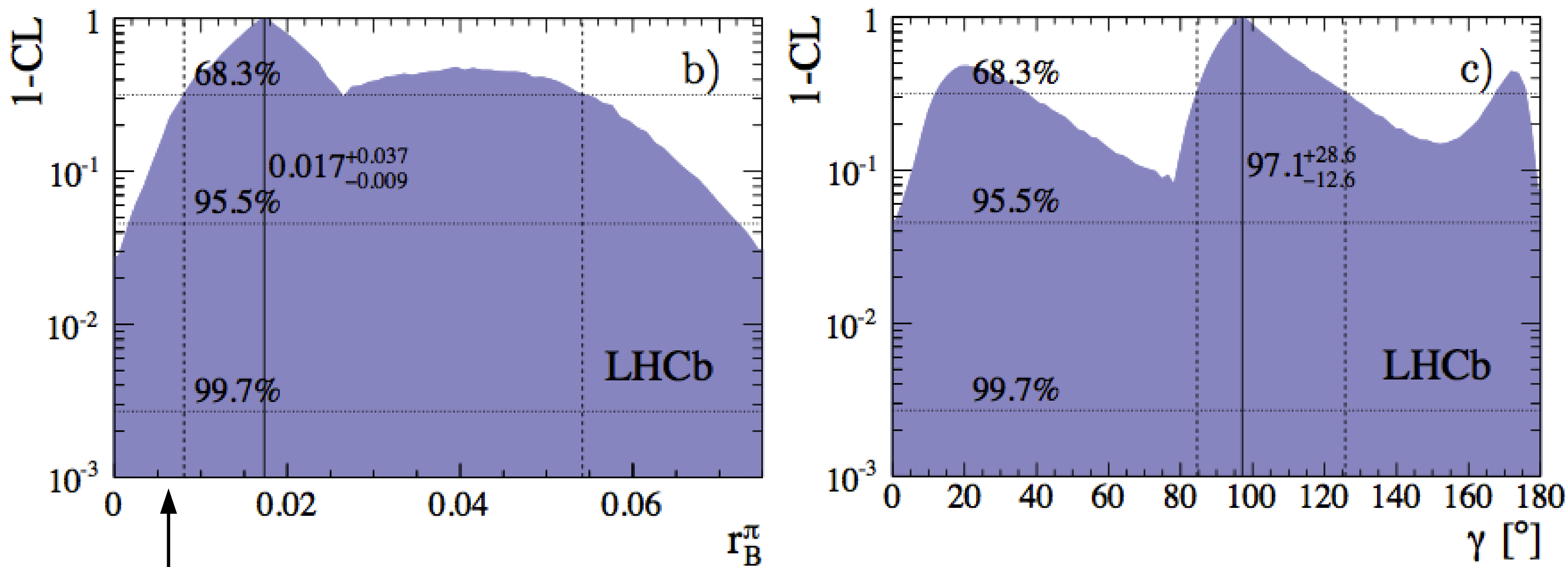
Make a test:

- predict the traditional ADS observables, R_{ADS}, A_{ADS}, in $B \to DK$, $D \to K\pi$, using all other LHCb 1fb^{-1} inputs
- (the combination uses R_+, R_- instead)
- the agreement is impressive





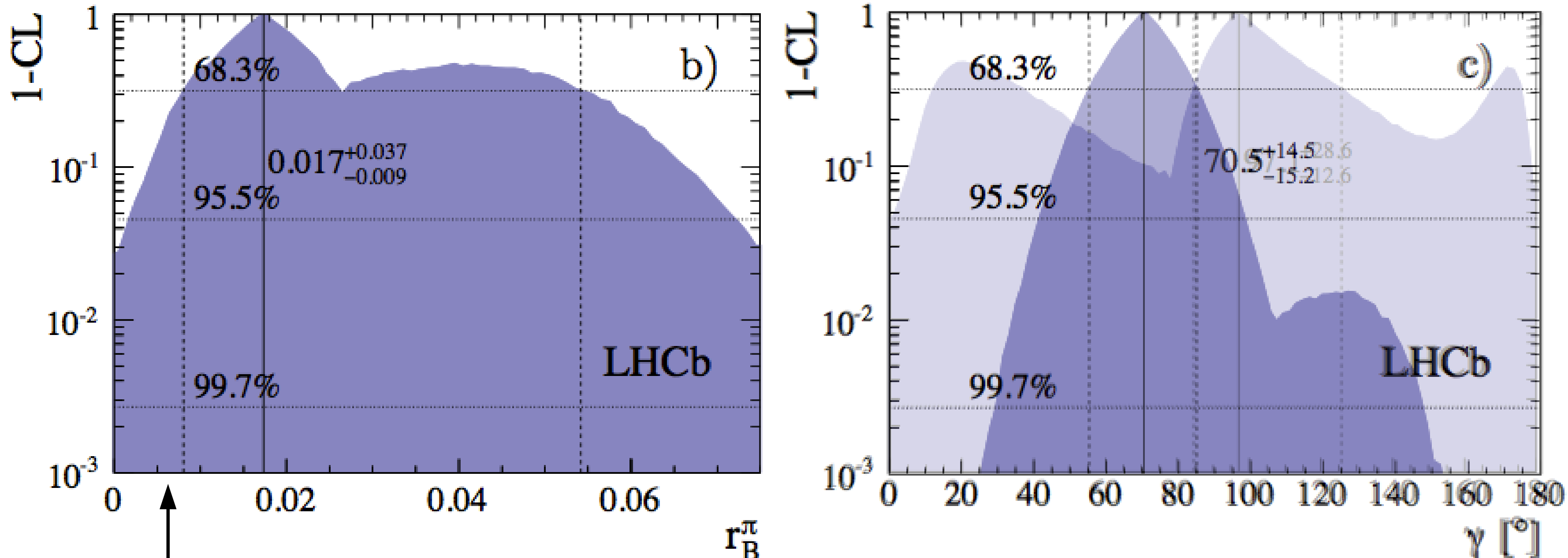
B → Dπ



$$r_B^\pi \approx |(V_{ub}^* V_{cd})/(V_{cb}^* V_{ud})| \times \underbrace{|C|/|T+C|}_{\text{color suppression}} \approx 0.006$$

- For the first time, we include B → Dπ into a γ measurement.
- Data are compatible with rather high values of r_B^π
- Sensitivity scales roughly like $1/r_B^\pi$

B → Dπ



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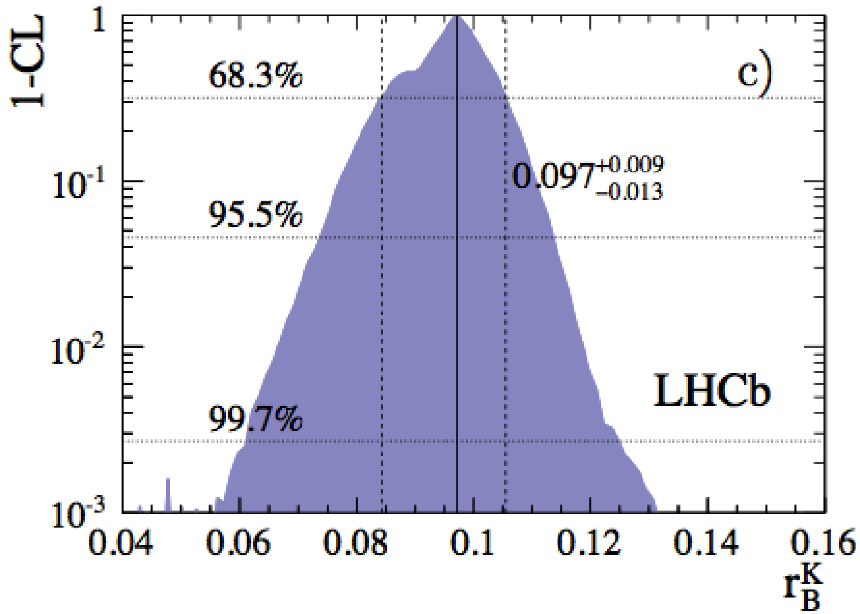
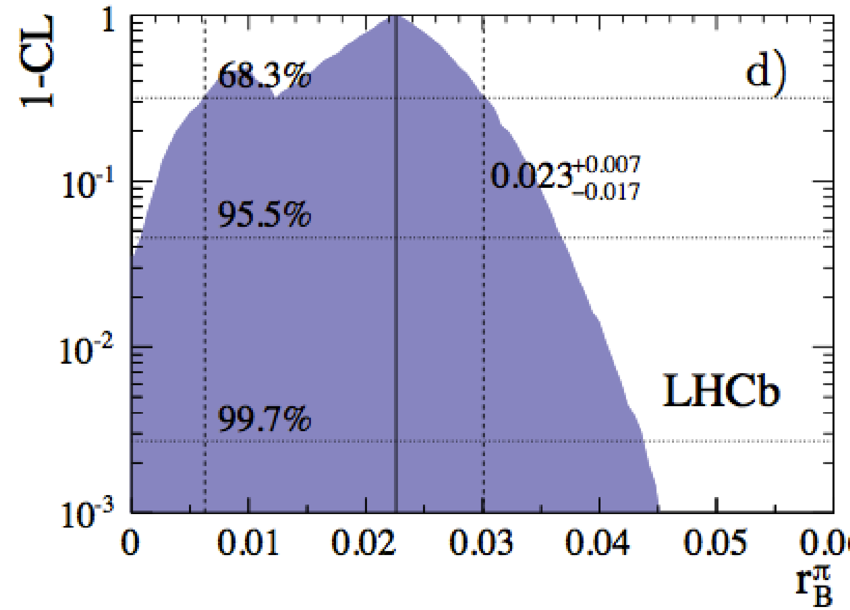
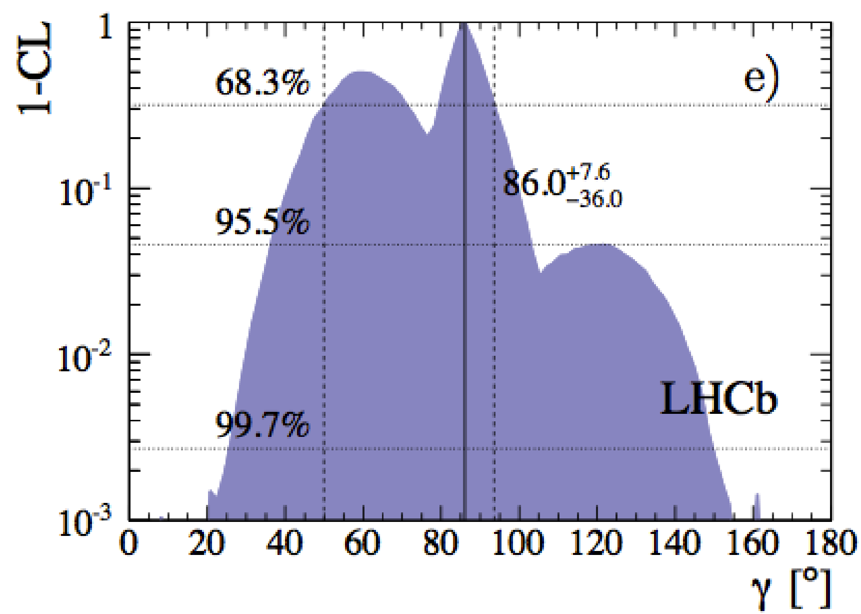
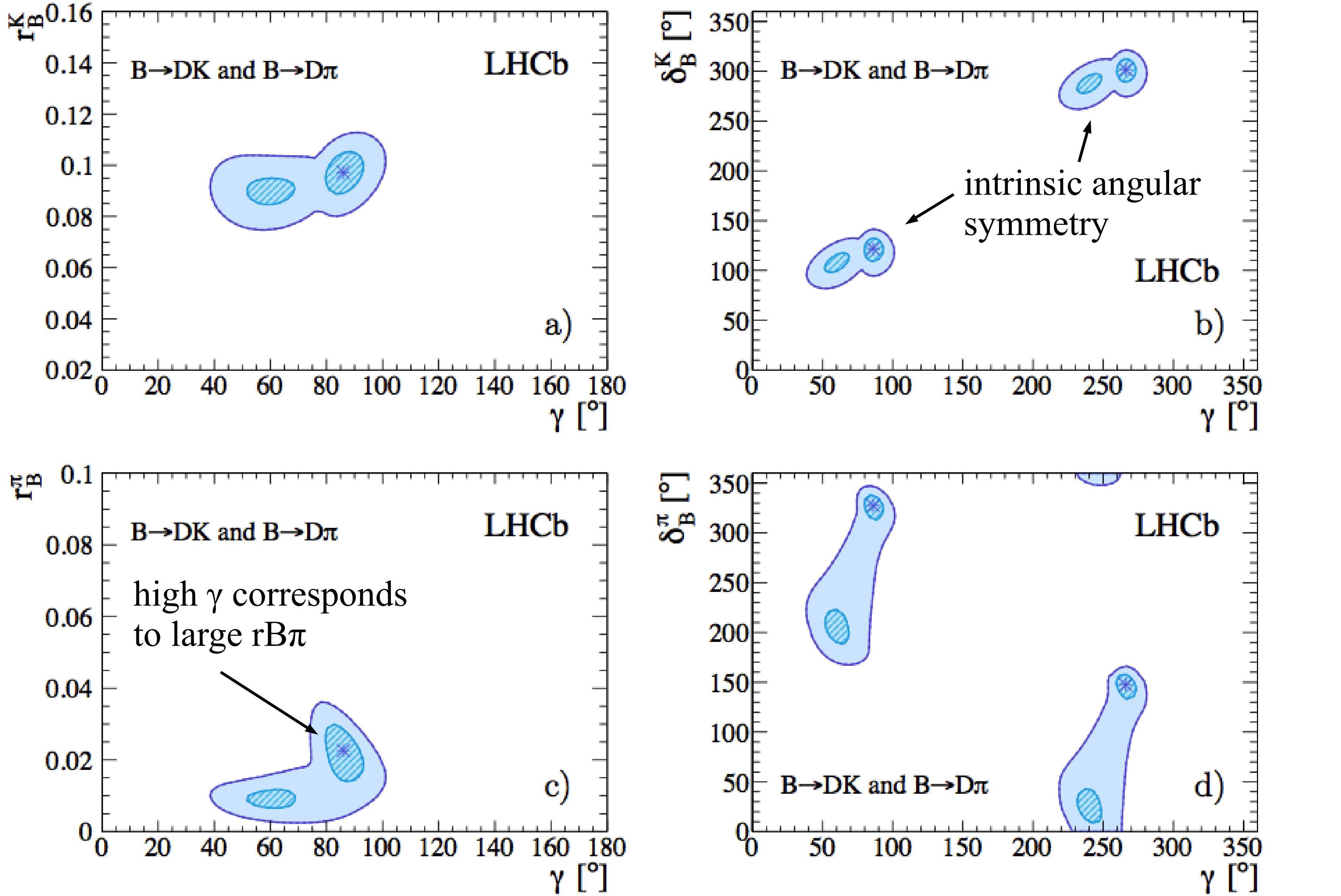
B → DK and B → Dπ


1-CL
c)
68.3%
95.5%
99.7%
$0.097^{+0.009}_{-0.013}$
LHCb
1
10^{-1}
10^{-2}
10^{-3}
0.04 0.06 0.08 0.1 0.12 0.14 0.16
r_B^K


1-CL
d)
68.3%
95.5%
99.7%
$0.023^{+0.007}_{-0.017}$
LHCb
1
10^{-1}
10^{-2}
10^{-3}
0 0.01 0.02 0.03 0.04 0.05 0.06
r_B^π


1-CL
e)
68.3%
95.5%
99.7%
$86.0^{+7.6}_{-36.0}$
LHCb
1
10^{-1}
10^{-2}
10^{-3}
0 20 40 60 80 100 120 140 160 180
γ [°]

B → DK and B → Dπ



naïve statistical treatment

Validation

- Goodness-of-fit probability:

Combination	n_{obs}	n_{fit}	χ^2_{min}	$P[\%]$ (pseudoexperiments)
$DK^{\pm}$	26	15	7.47	83.2 ± 0.9
$D\pi^{\pm}$	19	14	3.00	86.3 ± 0.8
full	35	17	12.24	88.2 ± 0.7

- Coverage test. Intervals for γ are **corrected for undercoverage**.

Combination	η	α (plug-in)	α (profile likelihood)	
$DK^{\pm}$	0.6827 (1σ)	0.6646 ± 0.0067	0.6299 ± 0.0069	!
	0.9545 (2σ)	0.9453 ± 0.0032	0.9318 ± 0.0036	
$D\pi^{\pm}$	0.6827 (1σ)	0.6532 ± 0.0048	0.6019 ± 0.0050	
	0.9545 (2σ)	0.9492 ± 0.0022	0.9389 ± 0.0024	
$DK^{\pm}$ and $D\pi^{\pm}$	0.6827 (1σ)	0.6616 ± 0.0067	0.6156 ± 0.0069	
	0.9545 (2σ)	0.9586 ± 0.0028	0.9352 ± 0.0035	

- Berger-Boos-like method: confirms intervals.
- Bayesian approach: confirms intervals.
- Assign systematic error due to some neglected syst. correlations.

arXiv:1305:2050, submitted to PLB

corrected results

The results, corrected for undercoverage and neglected systematic correlations, are:

$$B^{\pm} \to DK^{\pm}$$

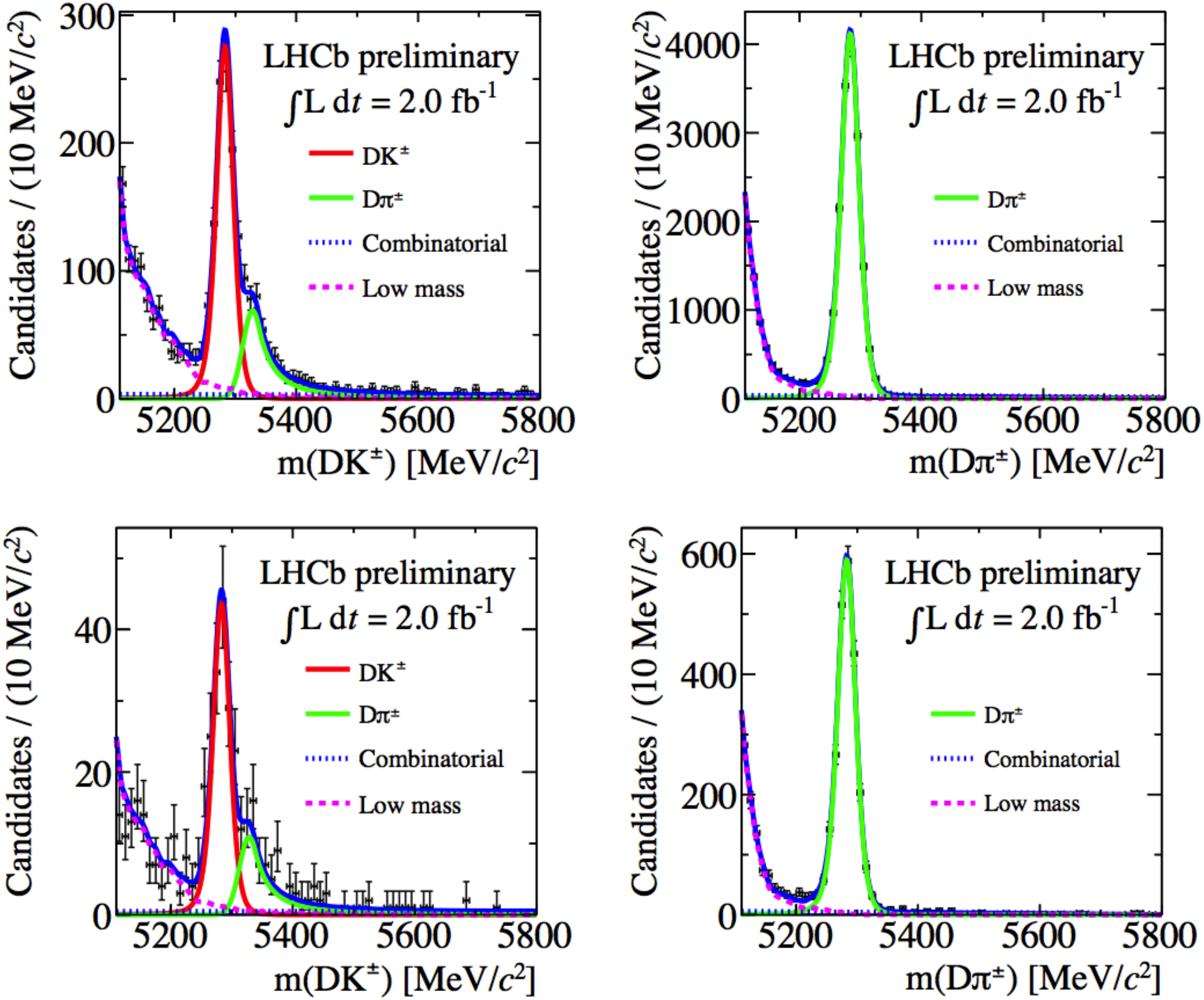
$$\gamma = (70.5^{+14.9}_{-15.6})^{\circ} \text{ at } 68\% \text{ CL}$$

$$B^{\pm} \to DK^{\pm} \text{ and } B^{\pm} \to D\pi^{\pm}$$

$$\gamma = 86.0^{\circ}$$

$$\gamma \in [49.0, 72.5]^{\circ} \quad \cup \quad [79.1, 94.2]^{\circ} \quad \text{at } 68\% \text{ CL}$$

A new GGSZ result



$B^{\pm} \to DK^{\pm}$,

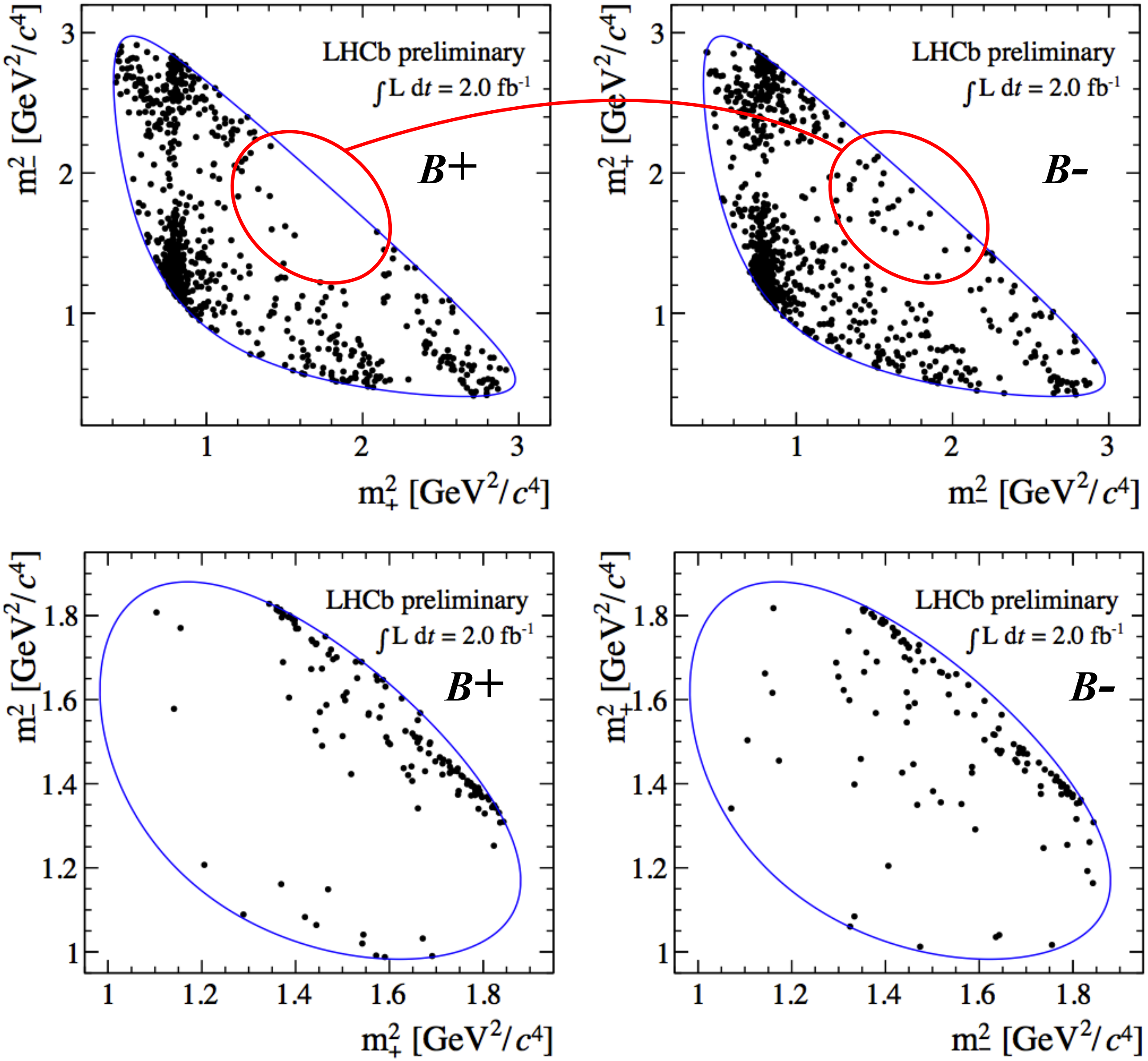
$D \to K^0_S \pi^+ \pi^-$

new!

$D \to K^0_S K^+ K^-$

plots show "down-down" K_S reconstruction only

A new GGSZ result



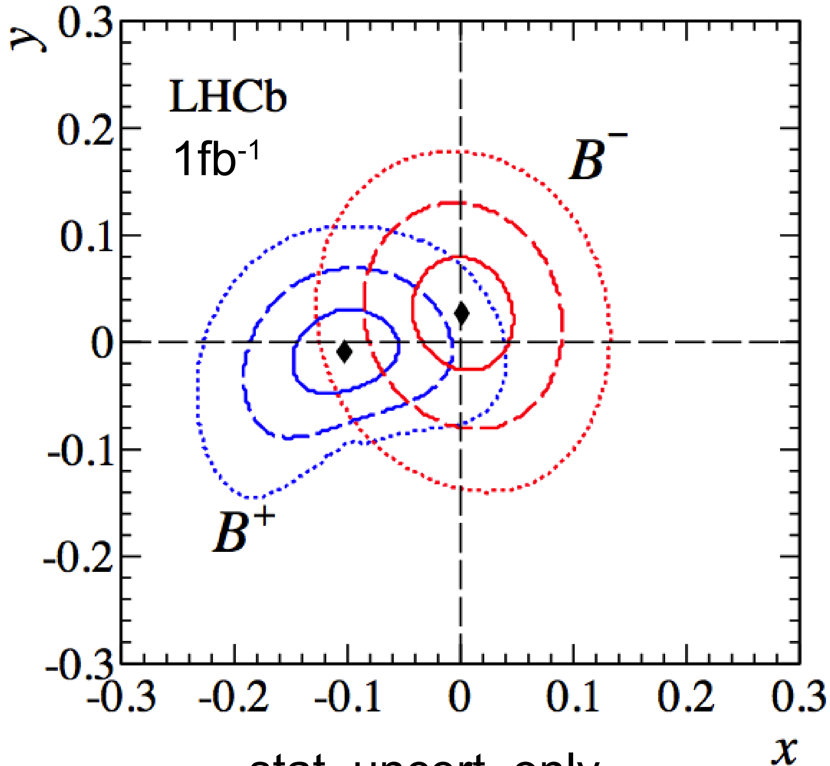
$B^{\pm} \to DK^{\pm}$,

$D \to K^0_S \pi^+ \pi^-$

$D \to K^0_S K^+ K^-$

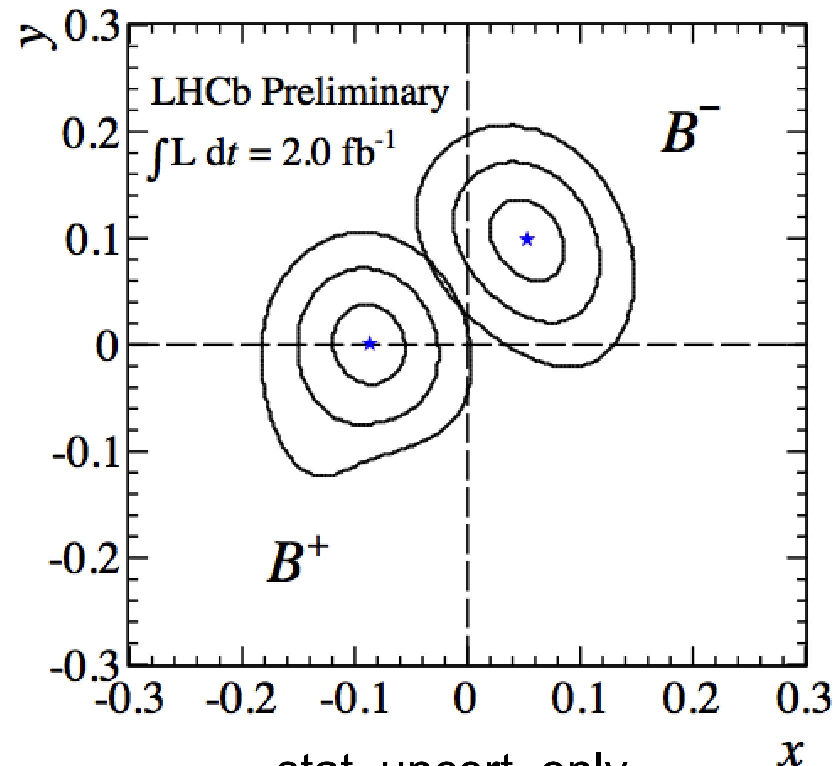
A new GGSZ result

7 TeV 2011



stat. uncert. only

8 TeV 2012



stat. uncert. only

new!

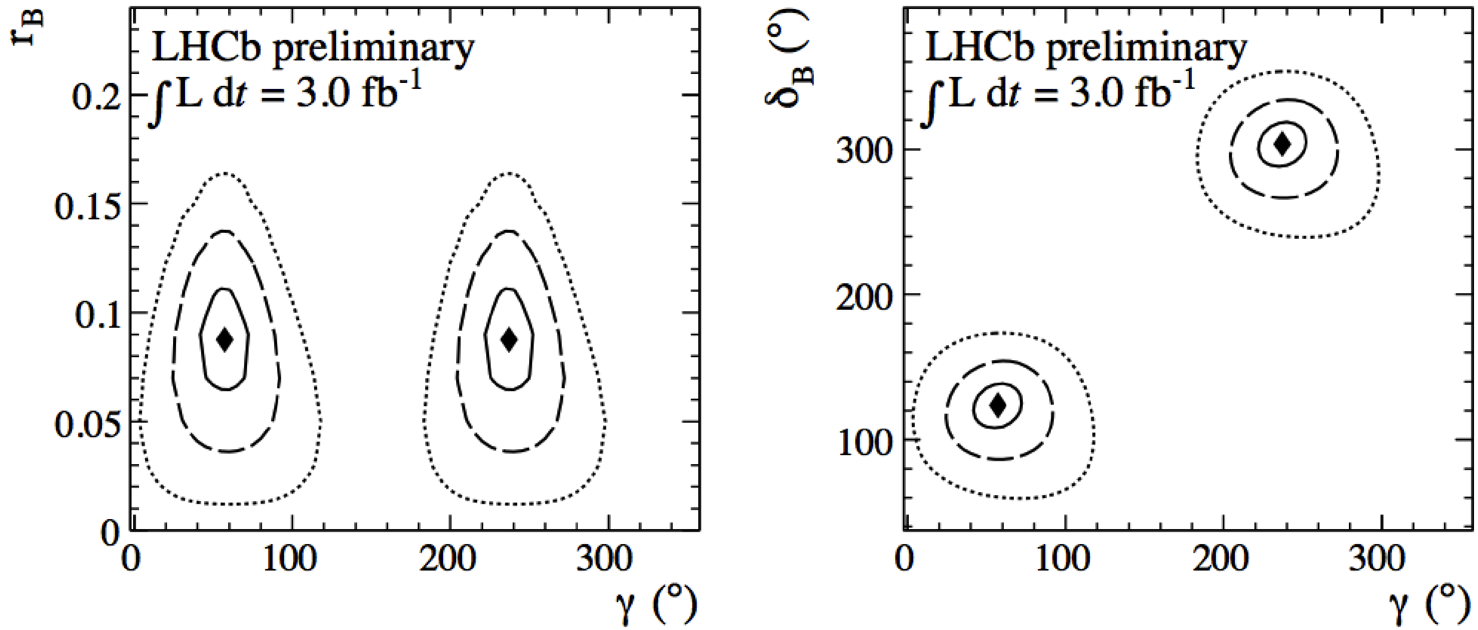
2012 result

dominant internal systematics:
assumption of no CPV in B → Dπ
second leading: fit shape

$$\begin{cases} x_+ = (-8.7 \pm 3.1 \pm 1.6 \pm 0.6) \times 10^{-2} \\ x_- = (5.3 \pm 3.2 \pm 0.9 \pm 0.9) \times 10^{-2} \\ y_+ = (0.1 \pm 3.6 \pm 1.4 \pm 1.9) \times 10^{-2} \\ y_- = (9.9 \pm 3.6 \pm 2.2 \pm 1.6) \times 10^{-2} \end{cases}$$

↑ CLEO

combined 1fb^{-1}+2fb^{-1} GGSZ result



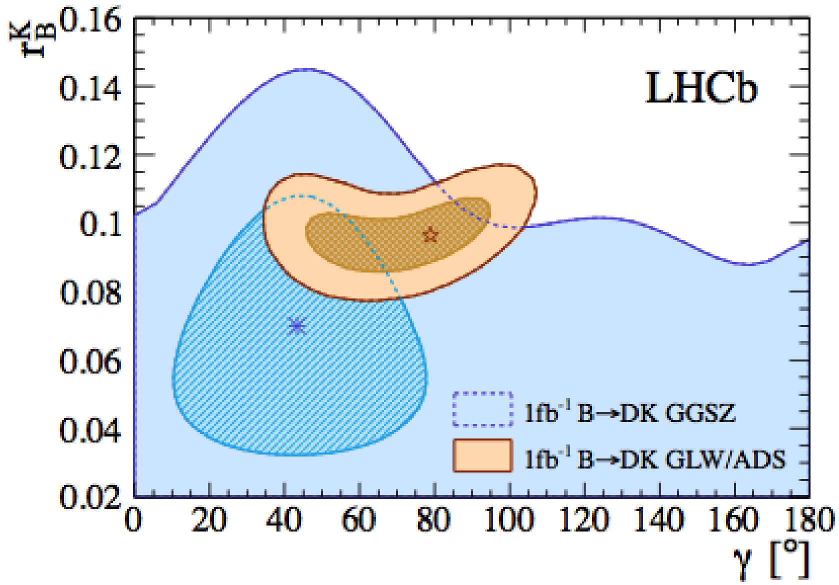
$$\langle x_+\rangle = (-8.9 \pm 3.1) \times 10^{-2}$$
$$\langle y_+\rangle = (-0.1 \pm 3.7) \times 10^{-2}$$
$$\langle x_-\rangle = (3.5 \pm 2.9) \times 10^{-2}$$
$$\langle y_-\rangle = (7.9 \pm 3.8) \times 10^{-2}$$

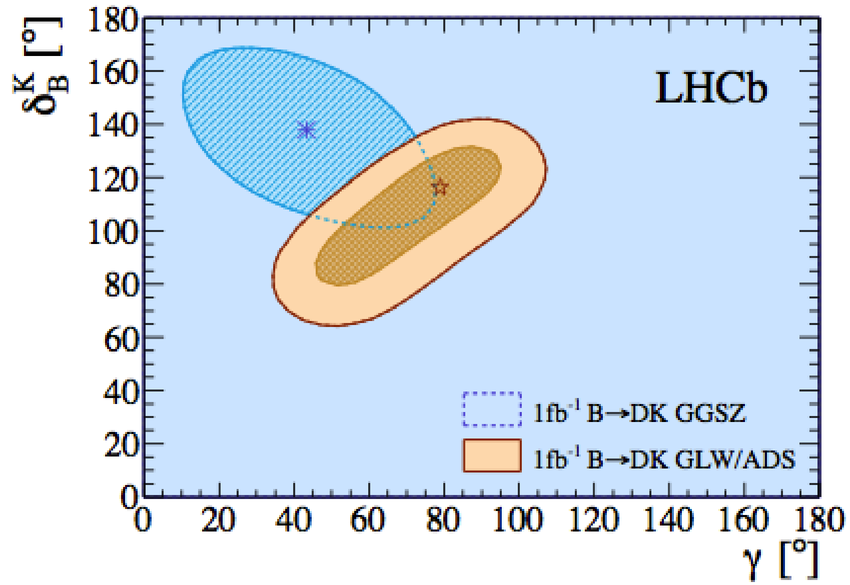
combined taking into account systematic correlations (CLEO phase information)

3-dimensional Feldman-Cousins, projecting the 20% CL shape

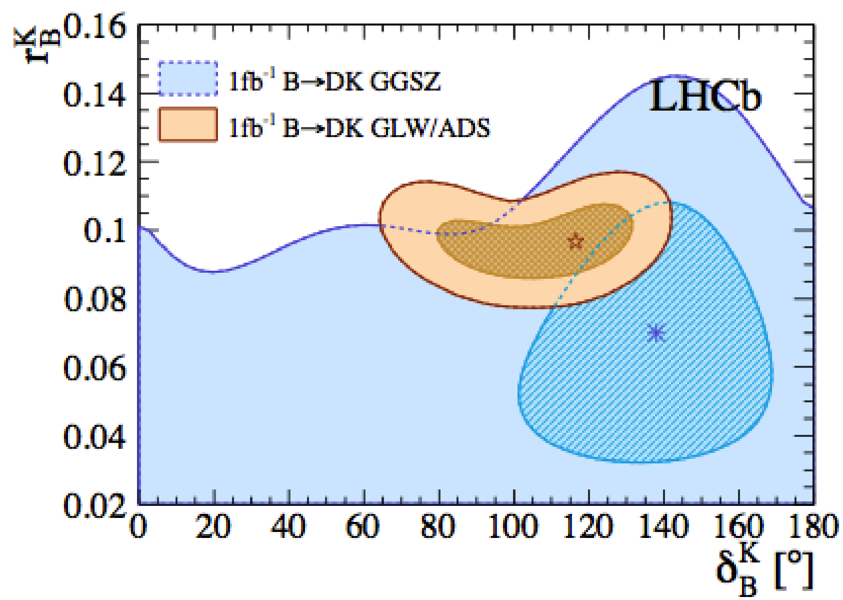
$$\gamma = (57 \pm 16)^\circ$$
$$\delta_B^K = (124^{+15}_{-17})^\circ$$
$$r_B^K = (8.8^{+2.3}_{-2.4}) \times 10^{-2}$$

impact on LHCb γ (B → DK)



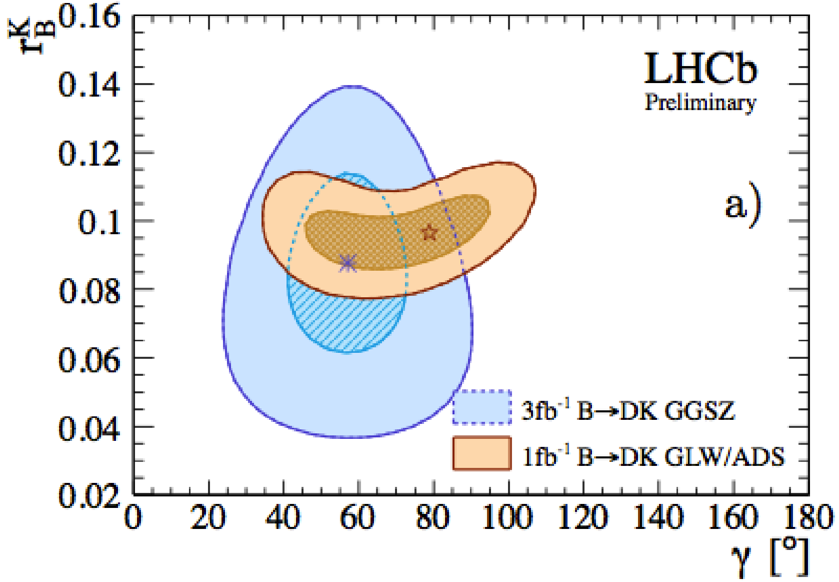


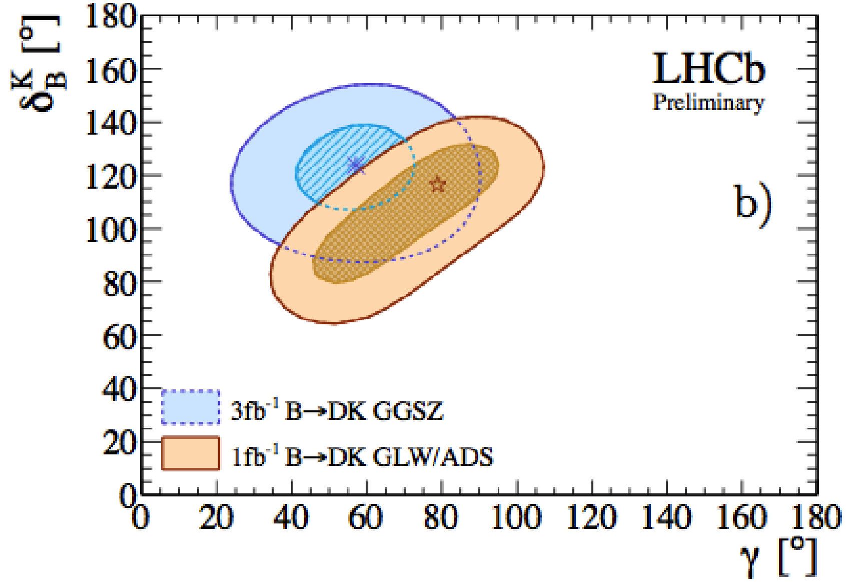
Comparing:
1fb^{-1} GLW/ADS and
1fb^{-1} GGSZ



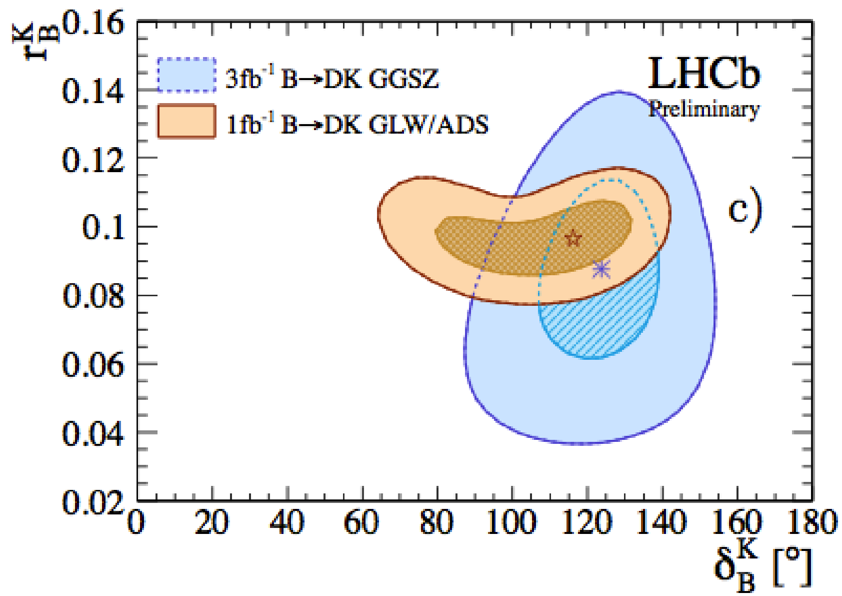
naïve statistical
treatment

impact on LHCb γ (B → DK)





Comparing:
1fb^{-1} GLW/ADS and
3fb^{-1} GGSZ



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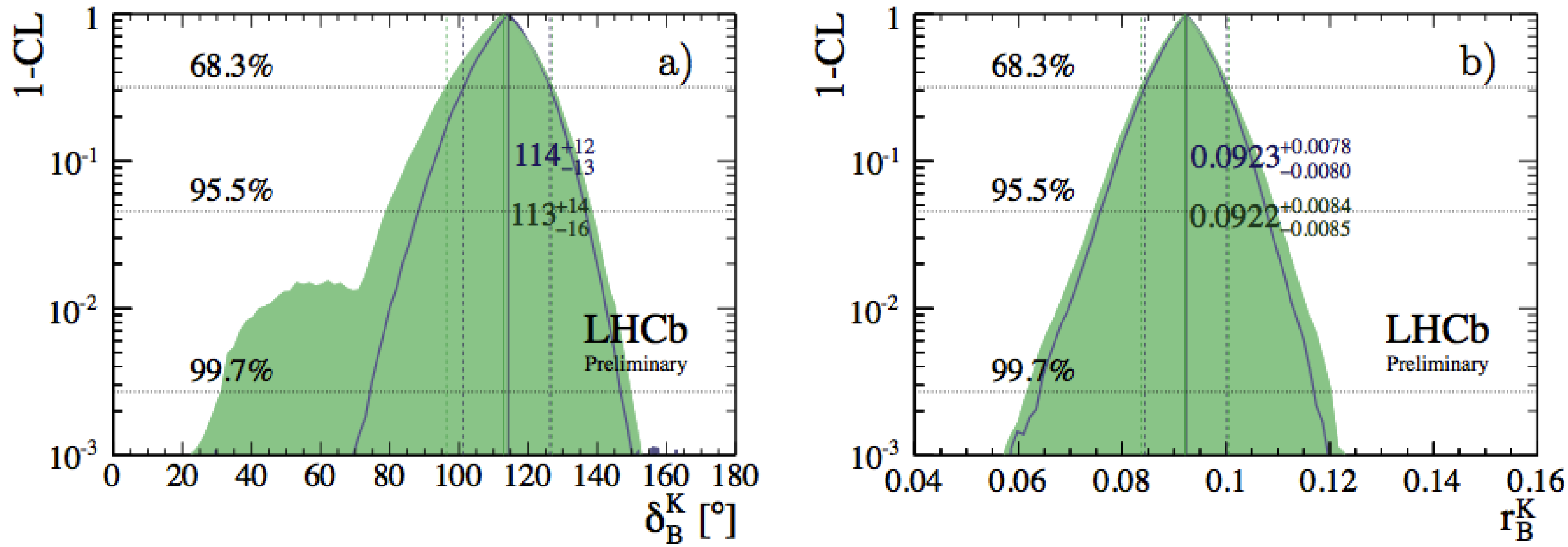
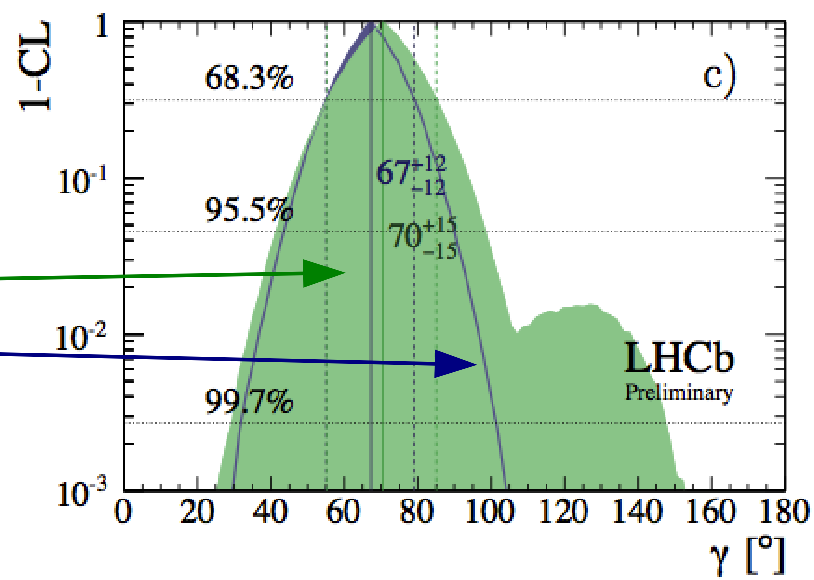




Comparing:

1fb^{-1} B → DK

3fb^{-1} B → DK

full statistical treatment

$$\gamma = (67 \pm 12)^\circ \text{ at } 68\% \text{ CL}$$

preliminary

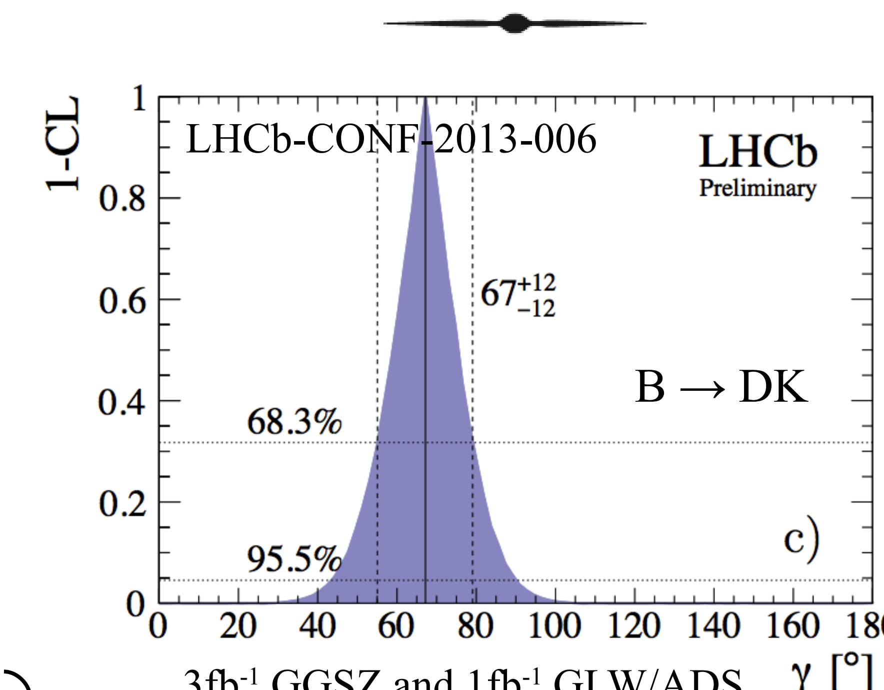
Conclusion

- LHCb has a complete set of 1fb^{-1} results: GLW, ADS, GGSZ
- New results using 3fb^{-1} start to appear.
- The "factory approach" by LHCb starts going beyond the traditional methods.
- B → Dπ modes used to measure γ.
- As the precision increases, we will soon have to be more accurate with D mixing.
- The overall consistency is impressive: goodness-of-fit, predictions of observables, agreement with BaBar and Belle, ...

We understand what we're doing!

$$\gamma = (70.5^{+14.9}_{-15.6})^\circ \text{ at } 68\% \text{ CL}$$

1fb^{-1} LHCb measurements



3fb^{-1} GGSZ and 1fb^{-1} GLW/ADS

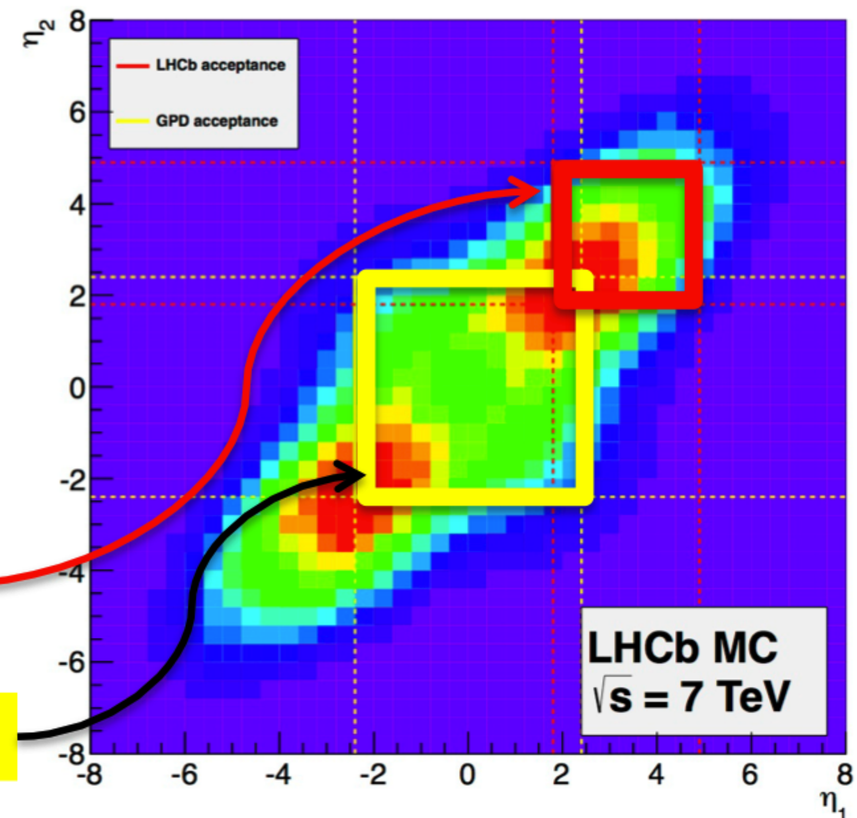
Backup

Outlook

- model dependent GGSZ
- model independent GGSZ: $B \to D\pi$
- $B \to DK$, $D \to K_S K\pi$ (ADS)
- time dependent $B_s \to D_s K$
- time dependent $B^0 \to D\pi$
- Bayesian combination
- …

LHCb

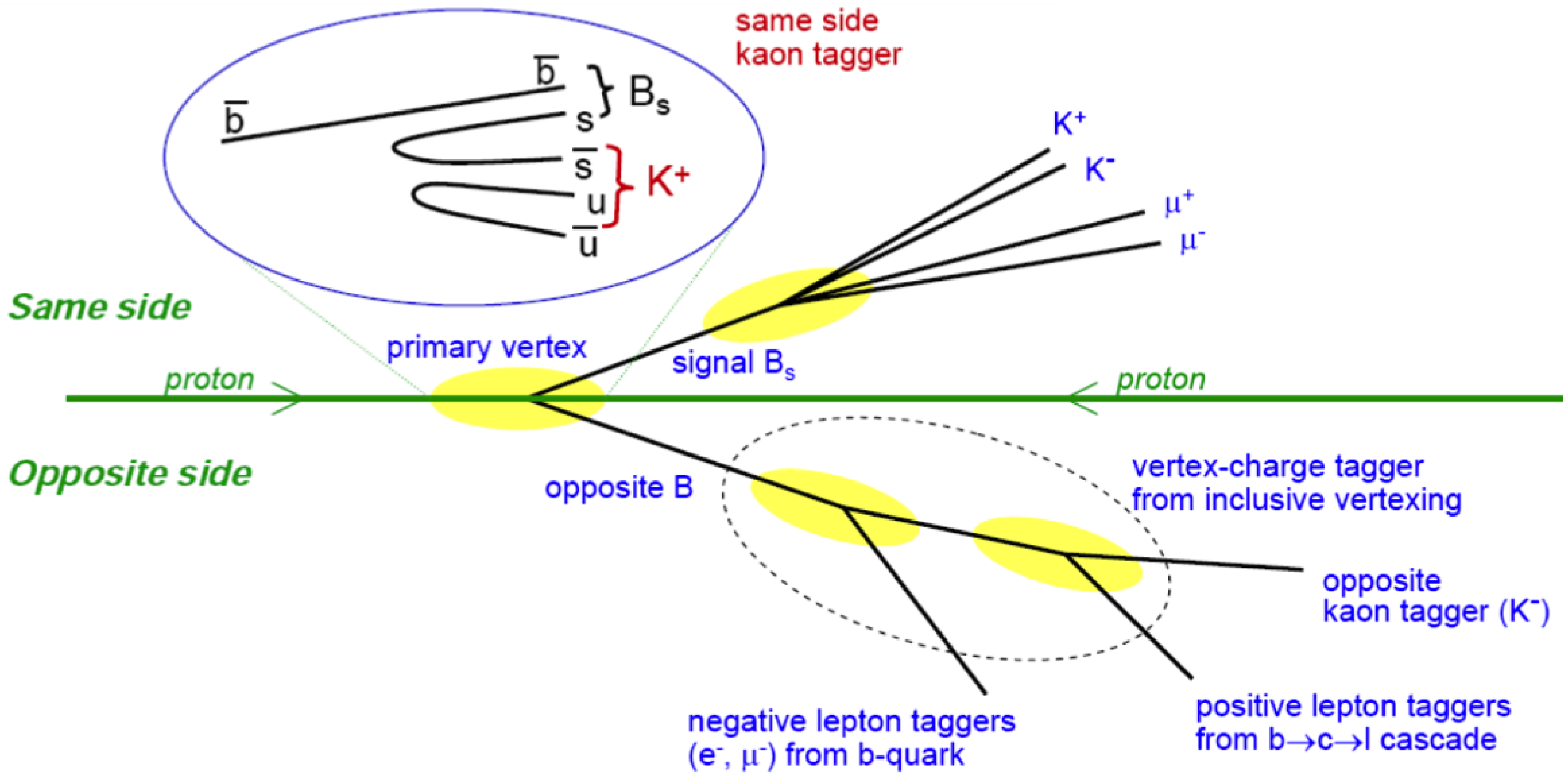
- $b\bar{b}$ pair production angles strongly correlated
- covers **1.9 < η < 4.9**
- 100'000 bb pairs produced per second (10^4 x B factories)



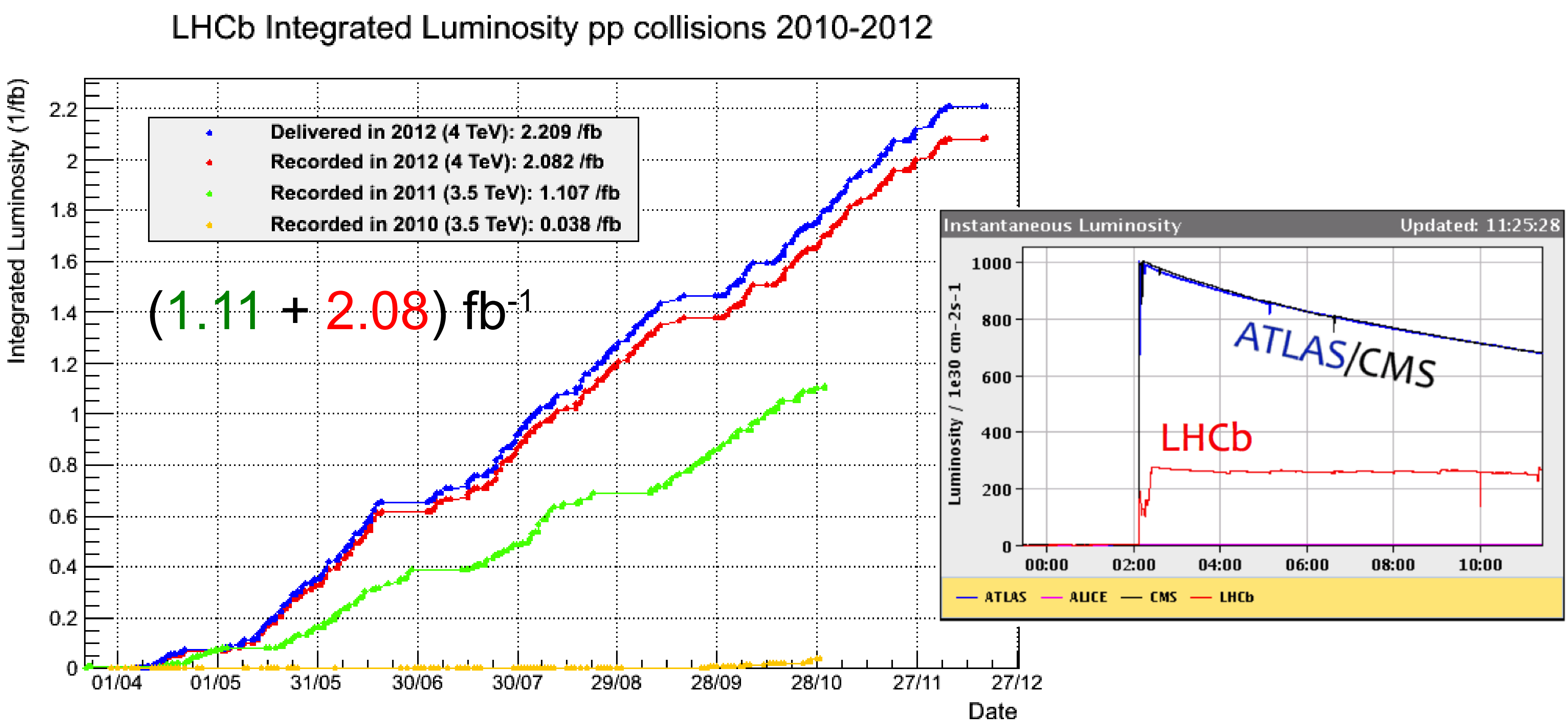
$\sigma(b\bar{b}) = 284 \pm 53 \mu\mathrm{b}$ [PLB 694 (2010) 209]

$\sigma(c\bar{c}) \approx 20 \times \sigma(b\bar{b})$ [LHCb-CONF−2010−013]

flavor tagging



Luminosity

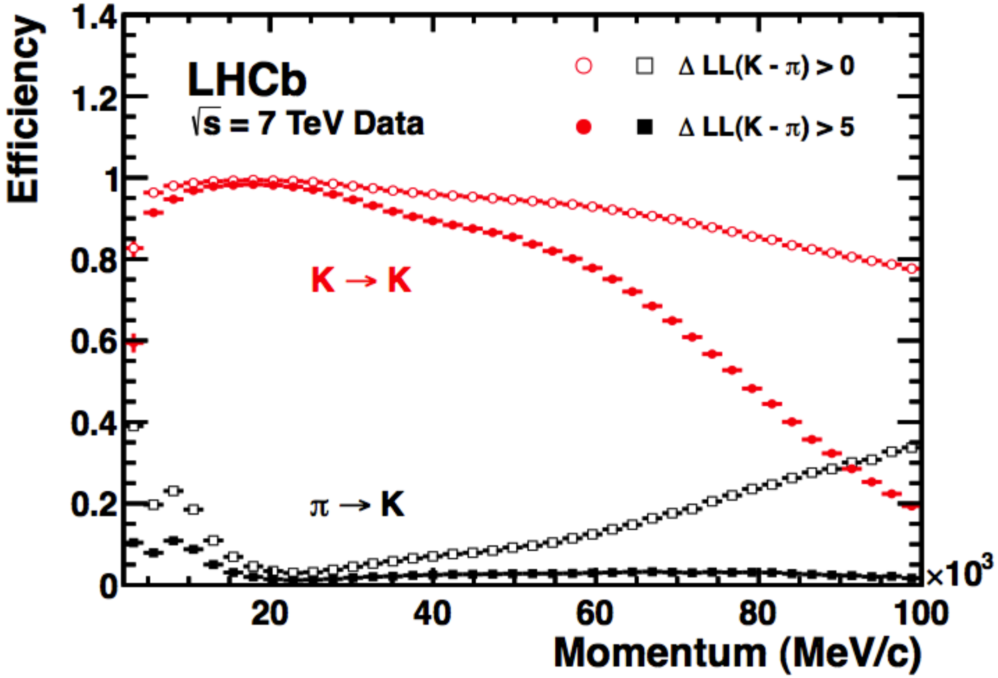


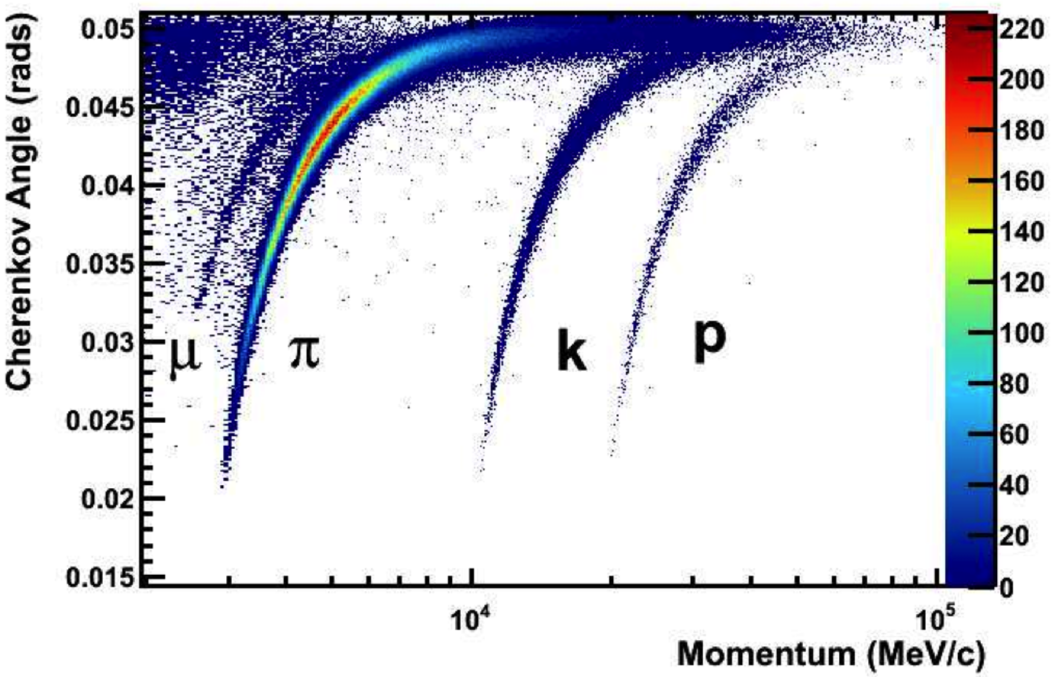
LHCb Integrated Luminosity pp collisions 2010-2012
Delivered in 2012 (4 TeV): 2.209 /fb
Recorded in 2012 (4 TeV): 2.082 /fb
Recorded in 2011 (3.5 TeV): 1.107 /fb
Recorded in 2010 (3.5 TeV): 0.038 /fb
(1.11 + 2.08) fb-1
Integrated Luminosity (1/fb)
Date
01/04
01/05
31/05
30/06
30/07
29/08
28/09
28/10
27/11
27/12
Instantaneous Luminosity
Updated: 11:25:28
Luminosity / 1e30 cm-2s-1
ATLAS/CMS
LHCb
00:00
02:00
04:00
06:00
08:00
10:00
ATLAS
ALICE
CMS
LHCb

LHCb – Kaon/pion separation

- Ring Imaging Cherenkov Detectors
- 3 radiators covering wide momentum range

$$\cos\theta = \frac{1}{\beta n}$$





B → D(hh)K: Results

$$
\begin{aligned}
R^{K\pi}_{K/\pi} &= 0.0774 \pm 0.0012 \pm 0.0018 \\
R^{KK}_{K/\pi} &= 0.0773 \pm 0.0030 \pm 0.0018 \\
R^{\pi\pi}_{K/\pi} &= 0.0803 \pm 0.0056 \pm 0.0017 \\
A^{K\pi}_{\pi} &= -0.0001 \pm 0.0036 \pm 0.0095 \\
A^{K\pi}_{K} &= 0.0044 \pm 0.0144 \pm 0.0174 \\
A^{KK}_{K} &= 0.1480 \pm 0.0369 \pm 0.0097 \\
A^{\pi\pi}_{K} &= 0.1351 \pm 0.0661 \pm 0.0095 \\
A^{KK}_{\pi} &= -0.0199 \pm 0.0091 \pm 0.0116 \\
A^{\pi\pi}_{\pi} &= -0.0009 \pm 0.0165 \pm 0.0099 \\
R^{-}_{K} &= 0.0073 \pm 0.0023 \pm 0.0004 \\
R^{+}_{K} &= 0.0232 \pm 0.0034 \pm 0.0007 \\
R^{-}_{\pi} &= 0.00469 \pm 0.00038 \pm 0.00008 \\
R^{+}_{\pi} &= 0.00352 \pm 0.00033 \pm 0.00007
\end{aligned}
$$

multi-body D decays

- Interference can only occur at same points in phase space, i.e. the requirement "same final state" is not enough.
- The magnitudes of the D decay amplitudes and the strong phase difference become **functions of the phase space**.
- Introduce effective quantities averaged over phase space!

$$r_{K3\pi}^2 = \frac{\int \bar{A}_D(\vec{m})^2 d\vec{m}}{\int A_D(\vec{m})^2 d\vec{m}}$$

phase space point

$$\kappa_{K3\pi} e^{i\delta_{K3\pi}} = \frac{\int A_D(\vec{m})\bar{A}_D(\vec{m})e^{i\delta(\vec{m})}d\vec{m}}{\sqrt{\int \bar{A}_D(\vec{m})^2 d\vec{m} \times \int A_D(\vec{m})^2 d\vec{m}}}$$

$$R_\pm = r_B^2 + r_{K3\pi}^2 + 2\underbrace{\kappa_{K3\pi}}\, r_B r_{K3\pi} \cos(\pm\gamma + \delta_B + \underbrace{\delta_{K3\pi}})$$

the "coherence factor", external input

a new (eff.) strong phase diff.

four-body ADS

"LHCb-style" observables:

$$R_{K/\pi}^{K3\pi} = R_{\rm cab} \frac{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\delta_B - \delta_{K3\pi}) \cos\gamma}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \cos(\delta_B^\pi - \delta_{K3\pi}) \cos\gamma},$$

$$A_\pi^{K3\pi} = \frac{2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \sin(\delta_B^\pi - \delta_{K3\pi}) \sin(\gamma)}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \cos(\delta_B^\pi - \delta_{K3\pi}) \cos\gamma},$$

$$A_K^{K3\pi} = \frac{2\kappa_{K3\pi} r_B r_{K3\pi} \sin(\delta_B - \delta_{K3\pi}) \sin\gamma}{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\delta_B - \delta_{K3\pi}) \cos\gamma},$$

$$R_{\pi-}^{K3\pi} = \frac{r_B^{\pi 2} + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \cos(\delta_B^\pi + \delta_{K3\pi} - \gamma)}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \cos(\delta_B^\pi - \delta_{K3\pi} - \gamma)},$$

$$R_{\pi+}^{K3\pi} = \frac{r_B^{\pi 2} + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \cos(\delta_B^\pi + \delta_{K3\pi} + \gamma)}{1 + r_B^{\pi 2} r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B^\pi r_{K3\pi} \cos(\delta_B^\pi - \delta_{K3\pi} + \gamma)},$$

$$R_{K-}^{K3\pi} = \frac{r_B^2 + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\delta_B + \delta_{K3\pi} - \gamma)}{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\delta_B - \delta_{K3\pi} - \gamma)},$$

$$R_{K+}^{K3\pi} = \frac{r_B^2 + r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\delta_B + \delta_{K3\pi} + \gamma)}{1 + r_B^2 r_{K3\pi}^2 + 2\kappa_{K3\pi} r_B r_{K3\pi} \cos(\delta_B - \delta_{K3\pi} + \gamma)}.$$

$$D^0 \to K\pi\pi\pi$$

four-body ADS

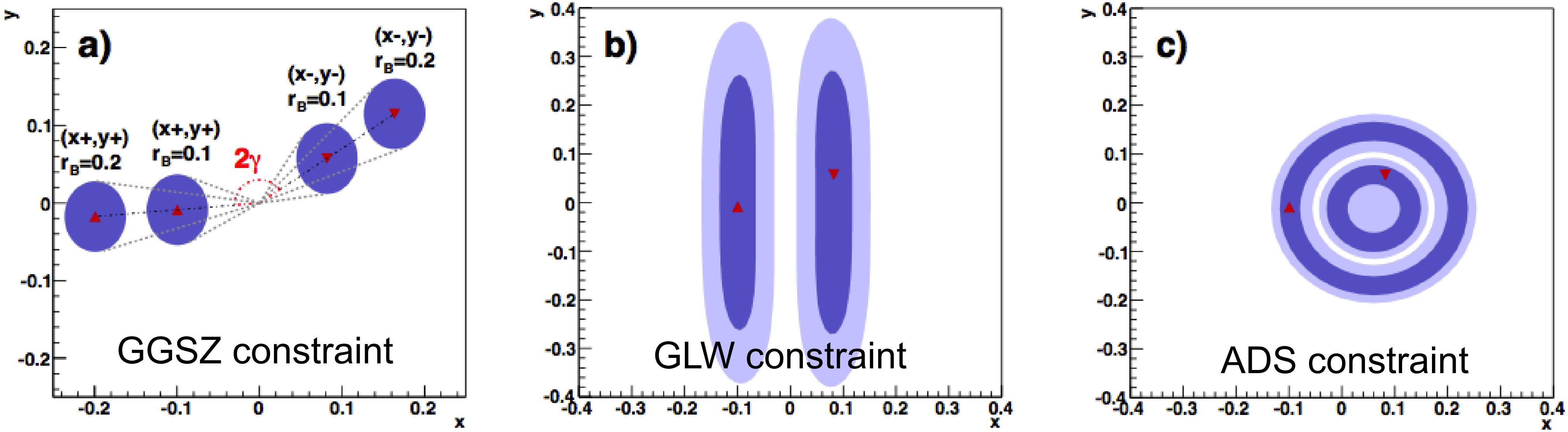
$$R_{K/\pi}^{K3\pi} \equiv \frac{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^+]_D K^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D K^+)}{\Gamma(B^- \to [K^-\pi^+\pi^-\pi^+]_D \pi^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D \pi^+)},$$

$$A_h^{K3\pi} \equiv \frac{\Gamma(B^- \to [K^-\pi^+\pi^+\pi^-]_D h^-) - \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D h^+)}{\Gamma(B^- \to [K^-\pi^+\pi^+\pi^-]_D h^-) + \Gamma(B^+ \to [K^+\pi^-\pi^+\pi^-]_D h^+)},$$

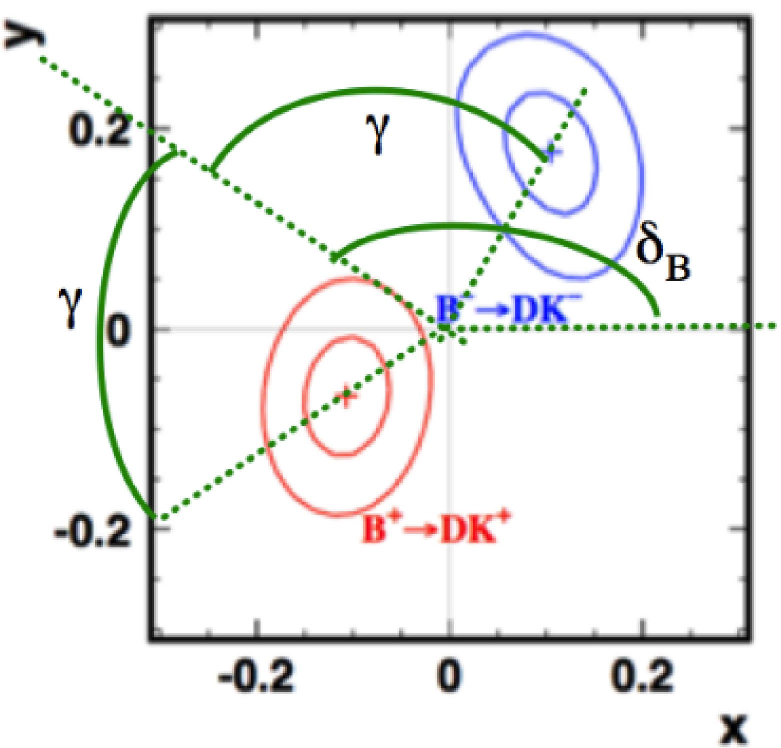
$$R_h^{K3\pi,\pm} \equiv \frac{\Gamma(B^\pm \to [\pi^\pm K^\mp \pi^+\pi^-]_D h^\pm)}{\Gamma(B^\pm \to [K^\pm \pi^\mp \pi^+\pi^-]_D h^\pm)}.$$

$$\begin{aligned}
R_{K/\pi}^{K3\pi} &= 0.0771 \pm 0.0017 \pm 0.0026 \\
A_K^{K3\pi} &= -0.029 \pm 0.020 \pm 0.018 \\
A_\pi^{K3\pi} &= -0.006 \pm 0.005 \pm 0.010 \\
R_K^{K3\pi,-} &= 0.0072 \, ^{+0.0036}_{-0.0032} \pm 0.0008 \\
R_K^{K3\pi,+} &= 0.0175 \, ^{+0.0043}_{-0.0039} \pm 0.0010 \\
R_\pi^{K3\pi,-} &= 0.00417 \, ^{+0.00054}_{-0.00050} \pm 0.00011 \\
R_\pi^{K3\pi,+} &= 0.00321 \, ^{+0.00048}_{-0.00045} \pm 0.00011
\end{aligned}$$

GGSZ Cartesian Coordinates



Matteo Rama at FPCP2009



Express GLW observables in terms of cart. coordinates:

$$x_\pm = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{4}$$

$$r^2 = x_\pm^2 + y_\pm^2 = \frac{R_{CP+} + R_{CP-} - 2}{2} .$$

Plugin method

Scan for one specific physics parameter, x:

1. Find global minimum $\chi^2_{\rm min}$ and the most probable values for $\vec{x}$.
2. Fix x to x_0 and minimize with respect to the non-fixed parameters, i.e. obtain $\vec{x}'$, and $(\chi^2_{\rm min})'$. Calculate $\Delta\chi^2 = \chi^2_{\rm min} - (\chi^2_{\rm min})'$.
3. Generate a Toy MC result for $\vec{y}$, $\vec{y}_{\rm toy}$, by interpreting the likelihood as a PDF of $\vec{y}$.
4. Repeat the first two steps on the toy result, i.e. calculate $\Delta\chi^2_{\rm toy}$.
5. Calculate $(1 - \mathrm{CL})$ as the fraction

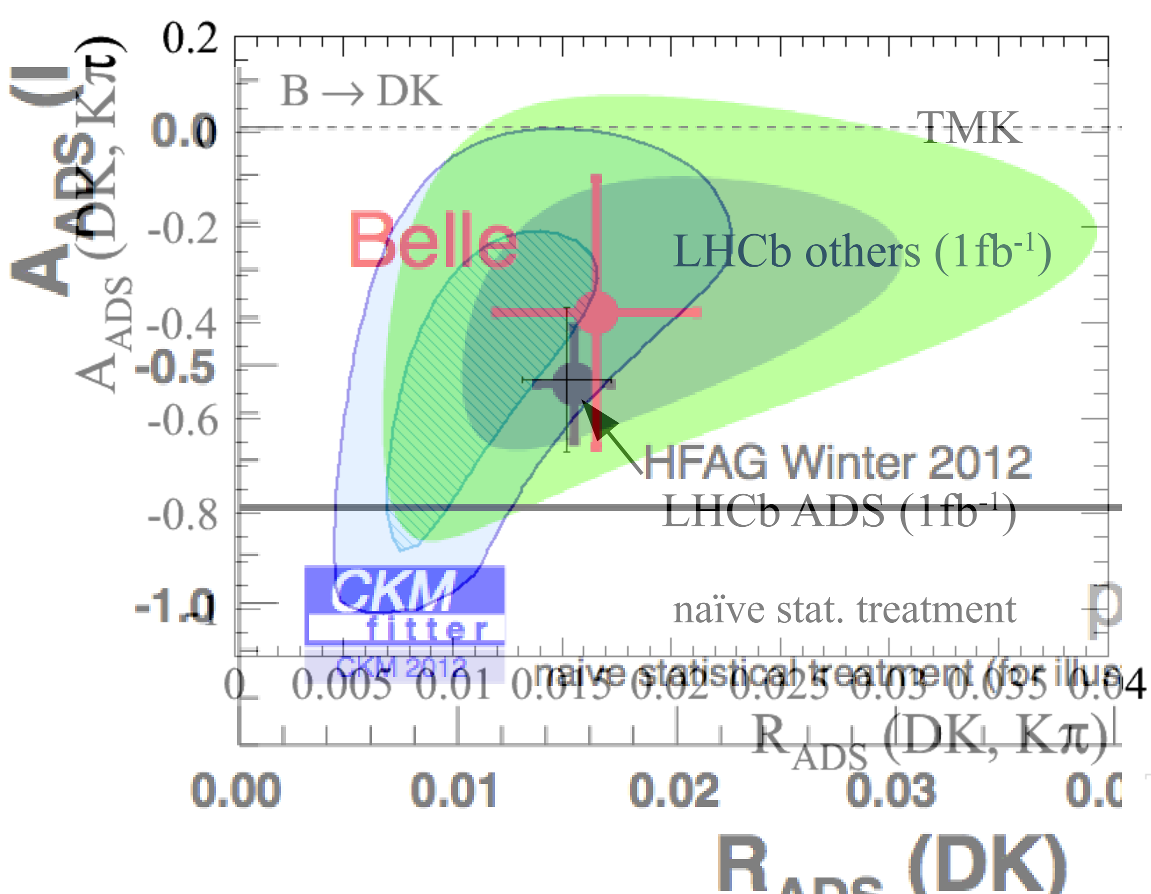
$$1 - \mathrm{CL} = \frac{N(\Delta\chi^2_{\rm toy} > \Delta\chi^2)}{N_{\rm toy}}. \qquad (5)$$

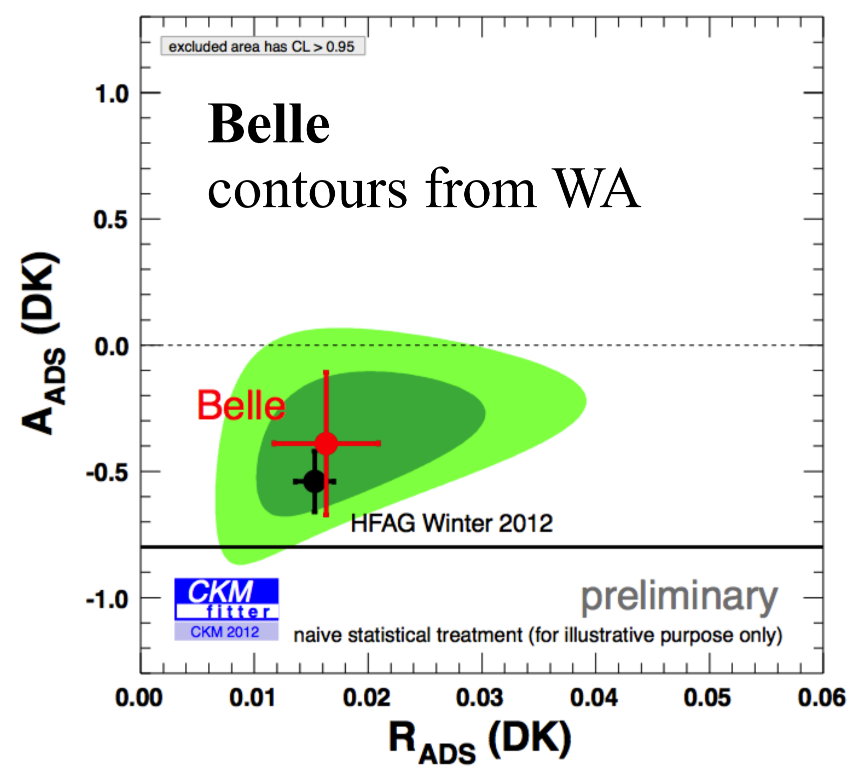
Use the best fit-values values for the parameters.

Doesn't guarantee coverage (but tends to be close).

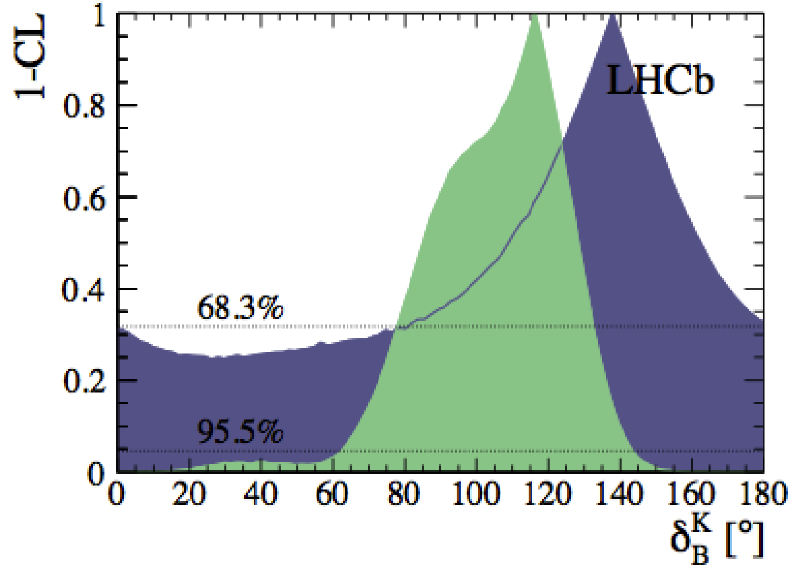
Agreement of inputs

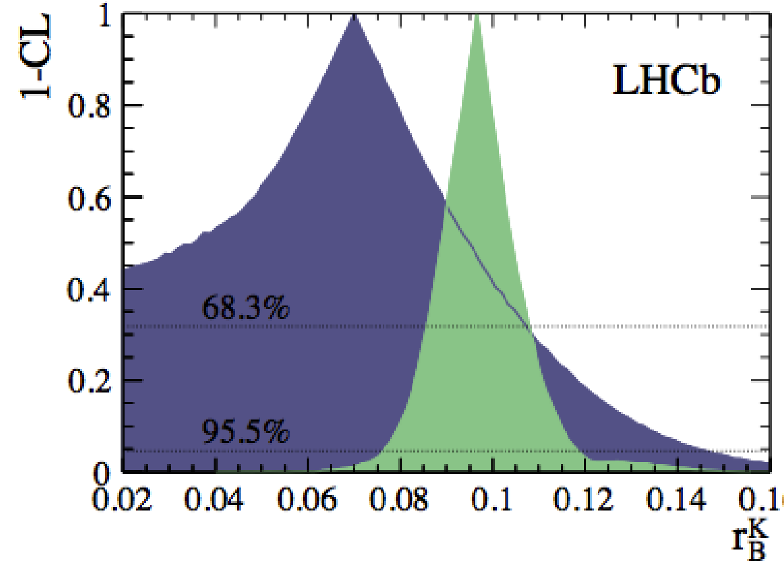
- Make a test:
  - predict the traditional ADS observables, R_{ADS}, A_{ADS}, in $B \rightarrow DK$, $D \rightarrow K\pi$, using all other LHCb $1fb^{-1}$ inputs
  - the agreement is impressive

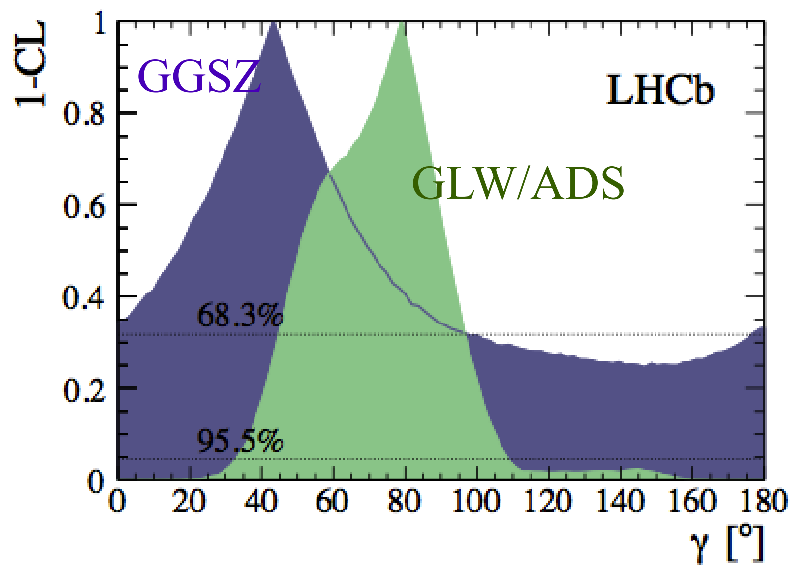




impact on LHCb γ (B → DK)



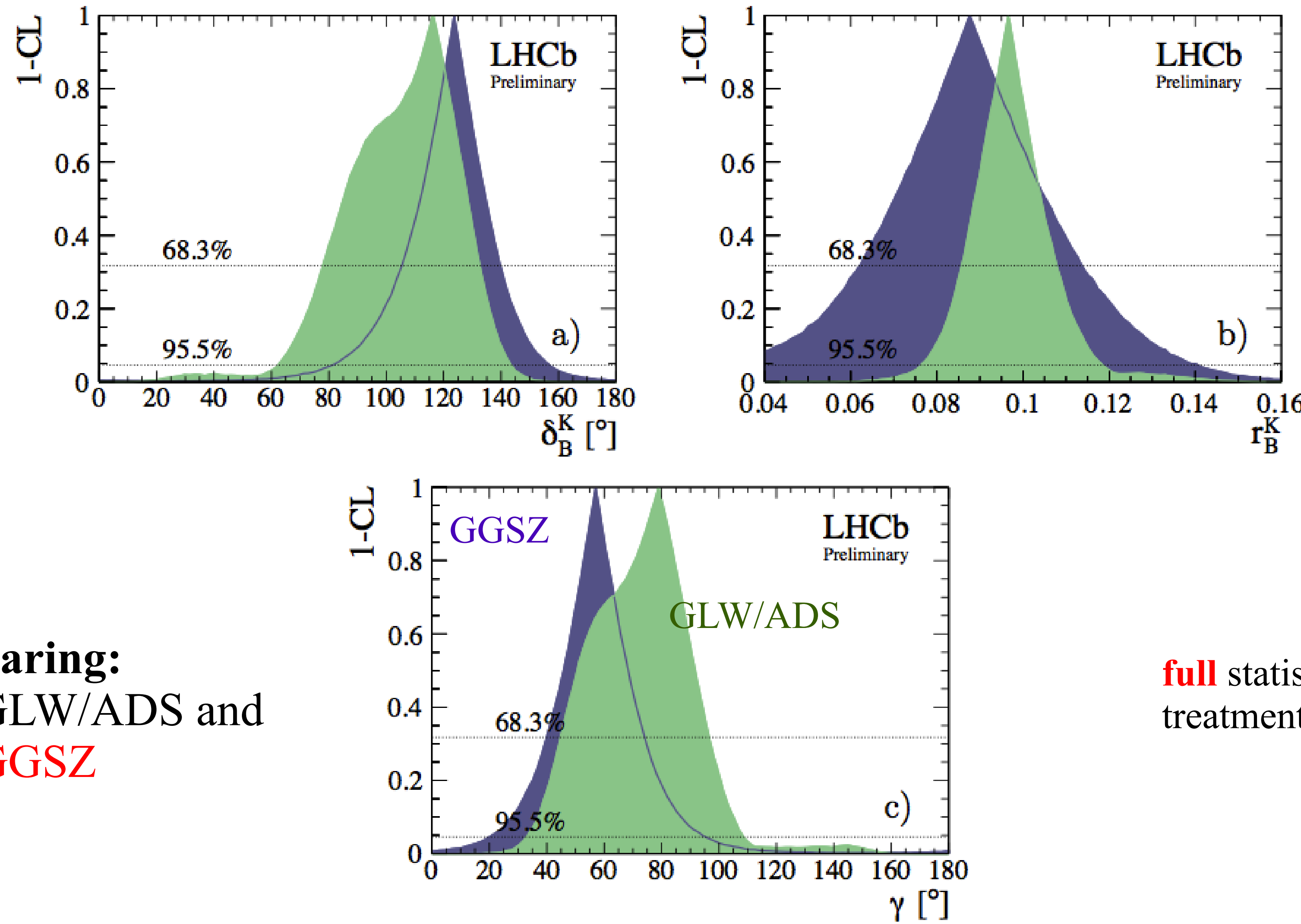




Comparing:
$1 fb^{-1}$ GLW/ADS and
$1 fb^{-1}$ GGSZ

full statistical treatment

impact on LHCb γ (B → DK)



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