Measurement of γ using $B \to K\pi\pi$ and $B \to KK\bar{K}$ decays

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work done in collaboration with B. Bhattacharya and M. Imbeault.

The standard way to obtain clean information about CKM phases is through the measurement of indirect CPV in $B/\bar{B} \to f$. Conventional wisdom: one cannot obtain such clean information from 3-body decays.

Two reasons: (i) f must be a CP eigenstate, but 3-body final states are, in general, not CP eigenstates. E.g., $K_S\pi^+\pi^-$: the value of its CP depends on whether the relative $\pi^+\pi^-$ angular momentum is even (CP $+$) or odd (CP $-$).

(ii) Can only get clean weak-phase information from indirect CP asymmetries if decay is dominated by amplitudes with a single weak phase. But 3-body decays generally receive significant contributions from amplitudes with different weak phases. Even if final-state CP could be fixed, need a way of dealing with this “pollution.”

Recently it was shown that all of these difficulties can be overcome.

M. Imbeault, N. Rey-Le Lorier, D. L., Phys. Rev. D **84**, 034040 (2011), 034041 (2011);

N. Rey-Le Lorier, D. L., Phys. Rev. D **85**, 016010 (2012).

Fundamental idea: it is common to combine observables from different 2-body B decays in order to extract weak-phase information. E.g., $B \to \pi\pi$ (α), $B \to DK$ (γ), $B \to \pi K$ (the $B \to \pi K$ puzzle).

In 3-body B decays, the idea is the same, **except** that the analysis applies to each point in the Dalitz plot. (That is, the analysis is momentum dependent.)

Disadvantage: analysis is more complicated. Big advantage: since it holds at each point in the Dalitz plot, analysis really represents many independent determinations of the weak-phase information. These can be combined, considerably reducing the error.

$\exists$ 3 ingredients in the analysis.

Dalitz Plots

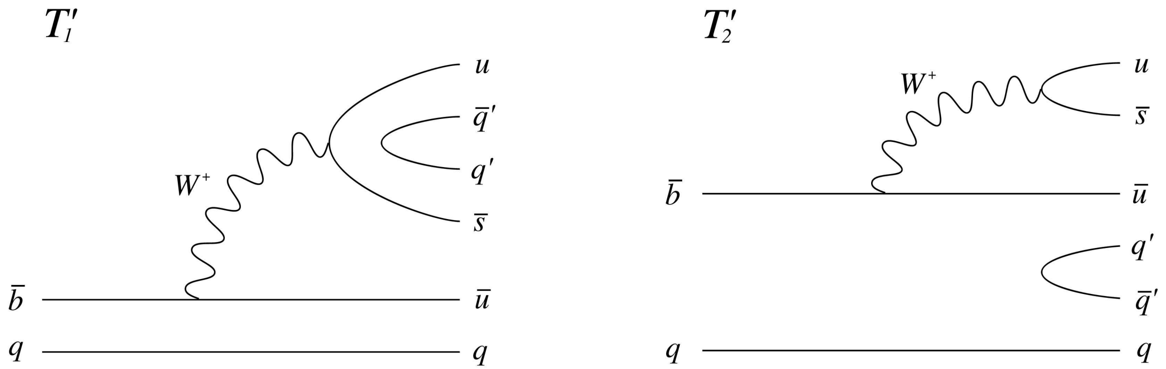
In the decay $B \to P_1 P_2 P_3$, one defines the three Mandelstam variables $s_{ij} \equiv (p_i + p_j)^2$, where p_i is the momentum of P_i. (The three s_{ij} are not independent, but obey $s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2$.) The Dalitz plot is given in terms of two Mandelstam variables, say s_{12} and s_{13}. Key point: can reconstruct the full decay amplitude $\mathcal{M}(B \to P_1 P_2 P_3)(s_{12}, s_{13})$.

The amplitude for a state with a given symmetry is then found by applying this symmetry to $\mathcal{M}(s_{12}, s_{13})$. E.g., the amplitude for the final state $K_S \pi^+ \pi^-$ with CP $+$ is symmetric in $2 \leftrightarrow 3$. This is given by $[\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12})]/\sqrt{2}$.

This amplitude is then used to compute all the observables for the decay. Note: all observables are momentum dependent – they take different values at each point in the Dalitz plot.

Diagrams

In order to remove the pollution due to additional decay amplitudes, one first expresses the full amplitude in terms of diagrams. These are similar to those of two-body B decays (T, C, etc.), but here one has to "pop" a quark pair from the vacuum. We add the subscript "1" ("2") if the popped quark pair is between two non-spectator final-state quarks (two final-state quarks including the spectator).



The above figure shows the T_1' and T_2' diagrams contributing to $B \to K\pi\pi$ (as this is a $\bar{b} \to \bar{s}$ transition, the diagrams are written with primes).

Note: unlike the 2-body diagrams, the 3-body diagrams are momentum dependent. This must be taken into account whenever the diagrams are used.

EWP-Tree Relations

As is the case in two-body decays, under flavor SU(3) there are relations between the EWP and tree diagrams for $\bar{b} \to \bar{s}$ transitions. Taking $c_1/c_2 = c_9/c_{10}$ (which holds to about 5%), these take the simple form

$$P'_{EWi} = \kappa T'_i \ , \quad P'^C_{EWi} = \kappa C'_i \quad (i = 1, 2) \ ,$$

where

$$\kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2} \ ,$$

with $\lambda_p^{(s)} = V^*_{pb} V_{ps}$.

$\exists$ important caveat. Under SU(3), the final state in $B \to K\pi\pi$ involves three identical particles, so that the six permutations of these particles must be taken into account. But the EWP-tree relations hold only for the totally symmetric state. This state, $\mathcal{M}_{\rm fs}$ ('fs' = 'fully symmetric'), is found by symmetrizing $\mathcal{M}(s_{12}, s_{13})$ under all permutations of 1,2,3. The analysis must therefore be carried out for this state.

$B \to K\pi\pi$ and $B \to KK\bar{K}$

We consider the 5 decays $B_d^0 \to K^+\pi^0\pi^-$, $B_d^0 \to K^0\pi^+\pi^-$, $B^+ \to K^+\pi^+\pi^-$, $B_d^0 \to K^+K^0K^-$, and $B_d^0 \to K^0K^0\bar{K}^0$. The $B \to K\pi\pi$ amplitudes are written in terms of diagrams with a popped $u\bar{u}$ or $d\bar{d}$ quark pair (these are equal under isospin); the diagrams of the $B \to KK\bar{K}$ amplitudes have a popped $s\bar{s}$ pair. But flavor-SU(3) symmetry (needed for EWP-relations) implies that all diagrams are equal, so that the 5 amplitudes are written in terms of the same diagrams.

Note, however, that flavor-SU(3) symmetry is not exact. It is therefore important to keep track of a possible difference between $B \to K\pi\pi$ and $B \to KK\bar{K}$ decays.

Can combine the diagrams into "effective diagrams:"

$$a \equiv -\tilde{P}'_{tc} + \kappa \left(\frac{2}{3} T'_1 + \frac{1}{3} C'_1 + \frac{1}{3} C'_2 \right) ,$$

$$b \equiv T'_1 + C'_2 , \quad c \equiv T'_2 + C'_1 , \quad d \equiv T'_1 + C'_1 .$$

The decay amplitudes can now be written in terms of five diagrams, a-d and $\tilde{P}'_{uc}$:

$$\begin{aligned}
2A(B_d^0 \to K^+ \pi^0 \pi^-)_{\rm fs} &= b e^{i\gamma} - \kappa c , \\
\sqrt{2} A(B_d^0 \to K^0 \pi^+ \pi^-)_{\rm fs} &= -d e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} - a + \kappa d , \\
\sqrt{2} A(B^+ \to K^+ \pi^+ \pi^-)_{\rm fs} &= -c e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} - a + \kappa b , \\
\sqrt{2} A(B_d^0 \to K^+ K^0 K^-)_{\rm fs} &= \alpha_{SU(3)} (-c e^{i\gamma} - \tilde{P}'_{uc} e^{i\gamma} - a + \kappa b) , \\
A(B_d^0 \to K^0 K^0 \bar{K}^0)_{\rm fs} &= \alpha_{SU(3)} (\tilde{P}'_{uc} e^{i\gamma} + a) ,
\end{aligned}$$

where $\alpha_{SU(3)}$ measures the amount of flavor-SU(3) breaking.

Now, we have $A(B^+ \to K^+\pi^+\pi^-)_{\rm fs} = A(B_d^0 \to K^+K^0K^-)_{\rm fs}$ in the flavor-SU(3) limit ($|\alpha_{SU(3)}| = 1$) $\Longrightarrow$ the B^+ decay does not furnish any new information. The remaining four amplitudes depend on 10 theoretical parameters: 5 magnitudes of diagrams, 4 relative phases, and γ. But $\exists$ 11 experimental observables: the decay rates and direct asymmetries of each of the 4 processes, and the indirect asymmetries of $B_d^0 \to K^0\pi^+\pi^-$, $B_d^0 \to K^+K^0K^-$ and $B_d^0 \to K^0K^0\bar{K}^0$. With more observables than theoretical parameters, γ can be extracted from a fit.

If one allows for SU(3) breaking ($|\alpha_{SU(3)}| \neq 1$), we can add two more observables: the decay rate and direct CP asymmetry for the B^+ decay. In this case it is possible to extract γ even with the inclusion of $|\alpha_{SU(3)}|$ as a fit parameter.

Note: diagrams and observables are both momentum dependent $\Longrightarrow$ above method for extracting γ in fact applies to each point in the Dalitz plot. Since the value of γ is independent of momentum, the method really represents *many* independent measurements of γ. These can be combined, reducing the error on γ.

Isobar Analysis

How to obtain the observables? The $B \to P_1 P_2 P_3$ amplitude is written as

$$\mathcal{M}(s_{12}, s_{13}) = \mathcal{N}_{\rm DP} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}) ~,$$

where the index j runs over all resonant and non-resonant contributions. Each contribution is expressed in terms of isobar coefficients c_j (amplitude) and θ_j (phase), and a dynamical wave function F_j. The F_j take different forms depending on the contribution. The c_j and θ_j are extracted from a fit to the Dalitz-plot event distribution.

BaBar has performed such fits for each of the five decays of interest. $B_d^0 \to K^+\pi^0\pi^-$: J. P. Lees *et al.*, Phys. Rev. D **83**, 112010 (2011); $B_d^0 \to K^0\pi^+\pi^-$: B. Aubert *et al.*, Phys. Rev. D **80**, 112001 (2009); $B^+ \to K^+\pi^+\pi^-$: B. Aubert *et al.*, Phys. Rev. D **78**, 012004 (2008); $B_d^0 \to K^+K^0K^-$: J. P. Lees *et al.*, Phys. Rev. D **85**, 112010 (2012); $B_d^0 \to K^0K^0\bar{K}^0$: J. P. Lees *et al.* Phys. Rev. D **85**, 054023 (2012). Given the c_j, θ_j and F_j, we reconstruct the amplitude for each decay as a function of s_{12} and s_{13}. We then construct $\mathcal{M}_{\rm fs}$ by symmetrizing under all permutations of 1,2,3. This process is repeated for the CP-conjugate process, where we construct $\overline{\mathcal{M}}_{\rm fs}$.

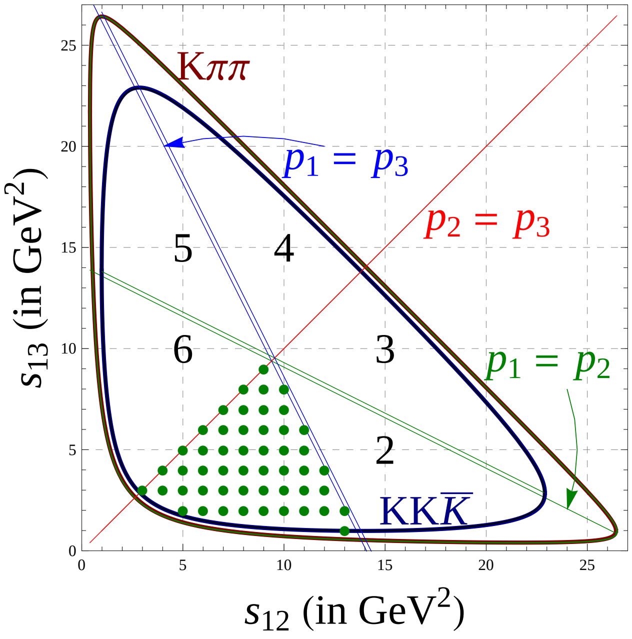
The experimental observables are then obtained as follows:

$$\begin{aligned} X(s_{12},s_{13}) &= |\mathcal{M}_{\rm fs}(s_{12},s_{13})|^2 + |\overline{\mathcal{M}}_{\rm fs}(s_{12},s_{13})|^2 ~, \\ Y(s_{12},s_{13}) &= |\mathcal{M}_{\rm fs}(s_{12},s_{13})|^2 - |\overline{\mathcal{M}}_{\rm fs}(s_{12},s_{13})|^2 ~, \\ Z(s_{12},s_{13}) &= {\rm Im}\left[\mathcal{M}^*_{\rm fs}(s_{12},s_{13})\,\overline{\mathcal{M}}_{\rm fs}(s_{12},s_{13})\right] ~. \end{aligned}$$

The experimental error bars on these quantities are found by varying the input isobar coefficients over their 1σ-allowed ranges. The effective CP-averaged branching ratio (X), direct CP asymmetry (Y), and indirect CP asymmetry (Z) may be constructed for every point on any Dalitz plot. However, Z can be measured only for B_d^0 decays to a CP eigenstate.

One technical point: in its $K_S K_S K_S$ analysis, BaBar takes $A(B_d^0 \to K_S K_S K_S) = A(\bar{B}_d^0 \to K_S K_S K_S)$. This implies that (i) Y and Z vanish for every point of the Dalitz plot, and (ii) the (small) unknown $\tilde{P}'_{uc}$ must be set to zero. The removal of an equal number of unknown parameters (amplitude and phase of $\tilde{P}'_{uc}$) and observables does not affect the viability of the method.

Since the amplitudes used to construct the observables are fully symmetric under the interchange of the three Mandelstam variables, only one sixth of the Dalitz plot provides independent information. In order to avoid multiple counting, we divide each Dalitz plot into six zones by its three axes of symmetry, and use information only from one zone:



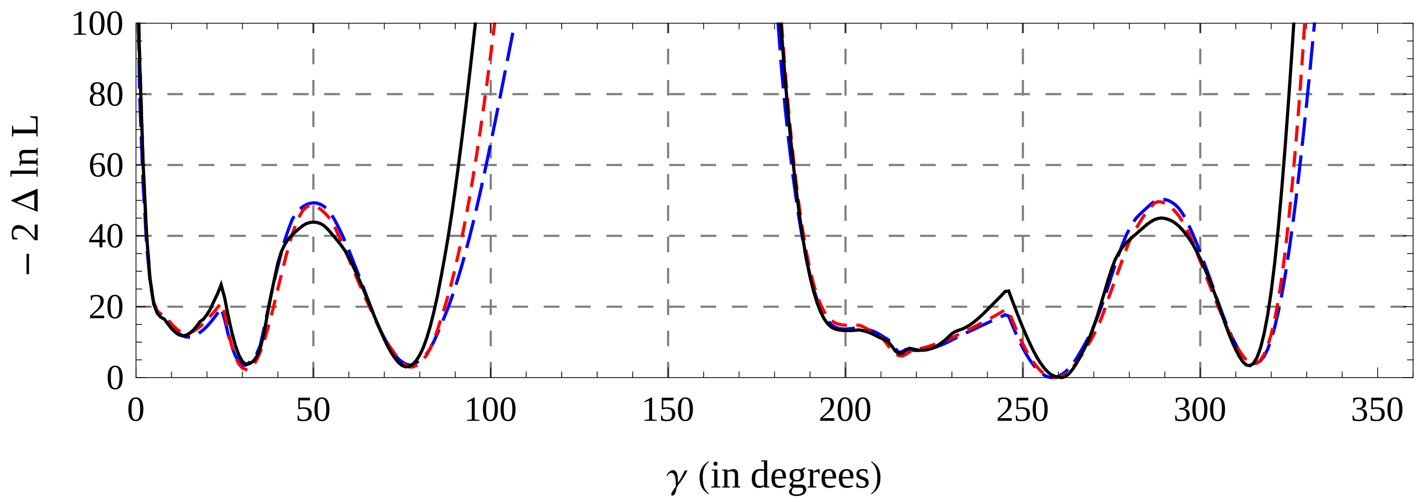
Kinematic boundaries and symmetry axes of the $B \to K\pi\pi$ and $B \to KK\bar{K}$ Dalitz plots. The symmetry axes divide each plot into six zones, five of which are marked 2-6. The fifty points in the region of overlap of the first of six zones from all Dalitz plots are used for the γ measurement.

Maximum Likelihood Fit

For each of the fifty points in the first Dalitz-plot zone, we construct the χ^2 function, which we then minimize over all the hadronic parameters for that point. The sum of such functions over all fifty points gives us a joint likelihood distribution. The local minima of this function are then identified as the most-likely values of γ. The 1σ error bars on γ are given by the condition that $\Delta\chi^2 = 1$.

We perform 3 types of fit:

1. Flavor SU(3) is a good symmetry $\Longrightarrow |\alpha_{SU(3)}| = 1$. The fit involves only the four B^0 decay channels.
2. SU(3) breaking is allowed and treated as follows. The ratio of X's is constructed point by point from the Dalitz plots for $B^+ \to K^+\pi^+\pi^-$ and $B^0 \to K^+K^0K^-$, giving $|\alpha_{SU(3)}|^2(s_{12}, s_{13})$. We use $|\alpha_{SU(3)}|$ found in this way to correct the observables from the $B \to KK\bar{K}$ Dalitz plots and use the corrected numbers in a new maximum-likelihood analysis for finding γ.
3. We consider observables from all five Dalitz plots but now include $|\alpha_{SU(3)}|$ as an additional unknown hadronic parameter.



Results of maximum-likelihood fits. The solid (black) curve represents the fit assuming flavor-SU(3) symmetry. The short dashes (red) represent the fit where flavor-SU(3) breaking is fixed by a point-by-point comparison of Dalitz plots for $B^+ \to K^+\pi^+\pi^-$ and $B^0 \to K^+K^0K^-$. The long dashes (blue) represent the fit with inputs from five Dalitz plots and an extra hadronic fit parameter $|\alpha_{SU(3)}|$.

Very little difference among 3 fits. Consistent with result from fit 2: averaged over the fifty points, we find $|\alpha_{SU(3)}| = 0.97 \pm 0.05$. This shows that, on average, SU(3) breaking is small.

γ and Errors

There are four preferred values for γ:

$$(31^{+2}_{-3})^\circ \ , \quad (77 \pm 3)^\circ \ , \quad (258^{+4}_{-3})^\circ \ , \quad (315^{+3}_{-2})^\circ \ .$$

Three of these indicate new physics (is this a "$K\pi\pi$-$KK\bar{K}$ puzzle"?), but one solution – $(77 \pm 3)^\circ$ – is consistent with the standard model.

In all cases, the error is small, 2-4°. How to understand this? The key point is that this method really involves 50 independent measurements of γ. Roughly speaking, if each measurement has an error of $\pm 20^\circ$, which is somewhat larger than other methods, then when we take a naive average, we divide the error by $\sqrt{50}$, giving a final error of $\sim 3^\circ$.

One potential source of error that has not been included in our method is higher-order flavor-SU(3) breaking. Such breaking may arise due to the nonzero mass difference between pions and kaons, and between intermediate resonances. This said, the error due to leading-order SU(3) breaking is small, and so it is unlikely that the error due to higher-order SU(3) breaking is larger.

CAVEAT: there is one very important error that has not been included, and that can significantly affect our result. All errors considered so far have been entirely statistical (even SU(3) breaking). But there is also the systematic, model-dependent error associated with the isobar analysis. This cannot be treated statistically, i.e., reduced by averaging. This error was not given in the BaBar papers and so we could not include it. Hopefully, the experimentalists themselves will redo this analysis, including all errors.

Recall: the standard way to directly probe γ is via $B^\pm \to DK^\pm$ decays. Although the two-body method is expected to be theoretically clean, it is difficult experimentally, so that the present direct measurement has a large error: $\gamma = (66 \pm 12)^\circ$. The statistical error of 2-4° in the three-body method is far smaller than the two-body error. If the systematic error is not too large, the three-body method could well be the best way to measure γ.

Conclusions

About 2-3 years ago, it was shown that, theoretically, it is possible to cleanly extract weak-phase information from 3-body B decays. In the present study, we demonstrate that this is, in fact, true. Using real data from BaBar, we extract the phase γ from $B \to K\pi\pi$ and $B \to KK\bar{K}$ decays. We find that there is a fourfold discrete ambiguity for the preferred value: $\gamma = 31^\circ$, 77°, 258° or 315°. However, in all cases, the error is small, 2-4°, and it includes leading-order SU(3) breaking. This is due to the fact that, in this method, there are actually 50 independent measurements of γ. When these are combined, the error is considerably reduced.

The one thing that is missing is the systematic, model-dependent error related to the isobar Dalitz-plot analysis. It is only the experimentalists themselves who can properly include it. If the systematic error is not too large, then the 3-body method will likely be the best one for measuring γ. Furthermore, there are undoubtedly other applications which can be done at LHCb or future B factories. Hopefully, the experimentalists will begin to perform this type of analysis and answer the outstanding question regarding the systematic error.