three-body HEAVY meson decays: final state interactions

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three-body HEAVY meson decays:

final state interactions

final state interactions

THEORY

weak vertex

final state interactions

Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

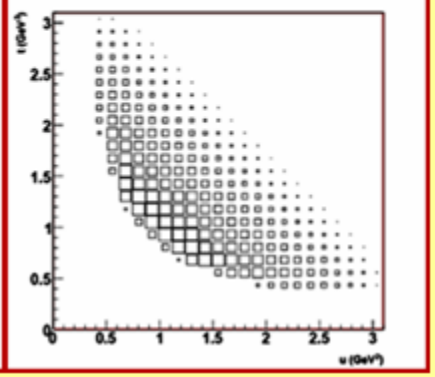
research programme

recent progress

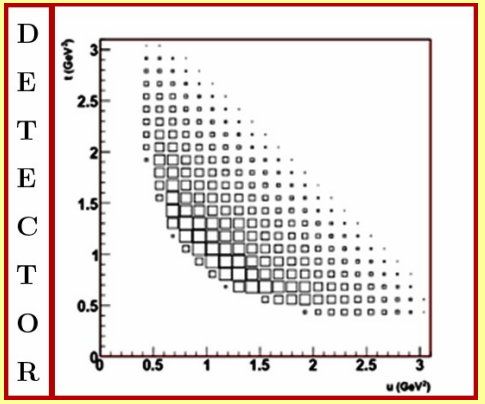
final state interactions

HM

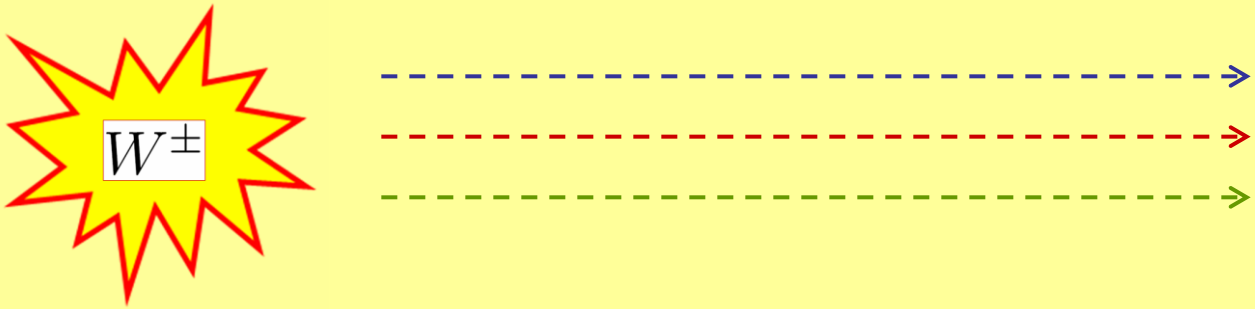
DETECTOR

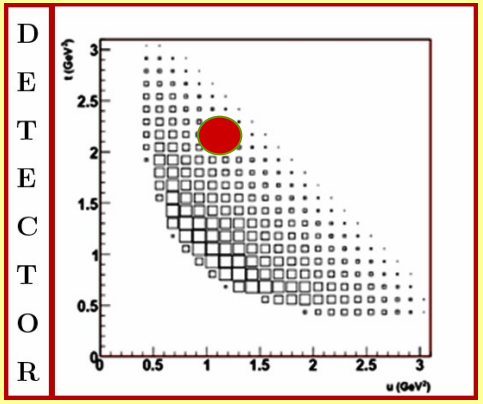


final state interactions

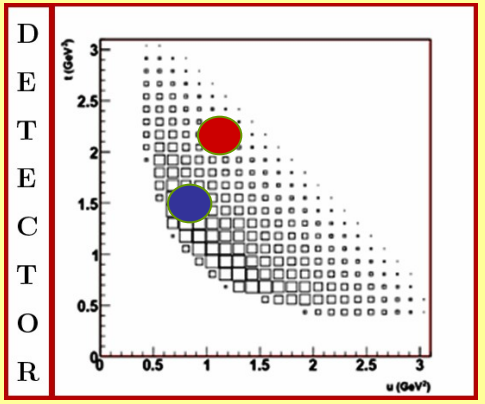


final state interactions





final state interactions



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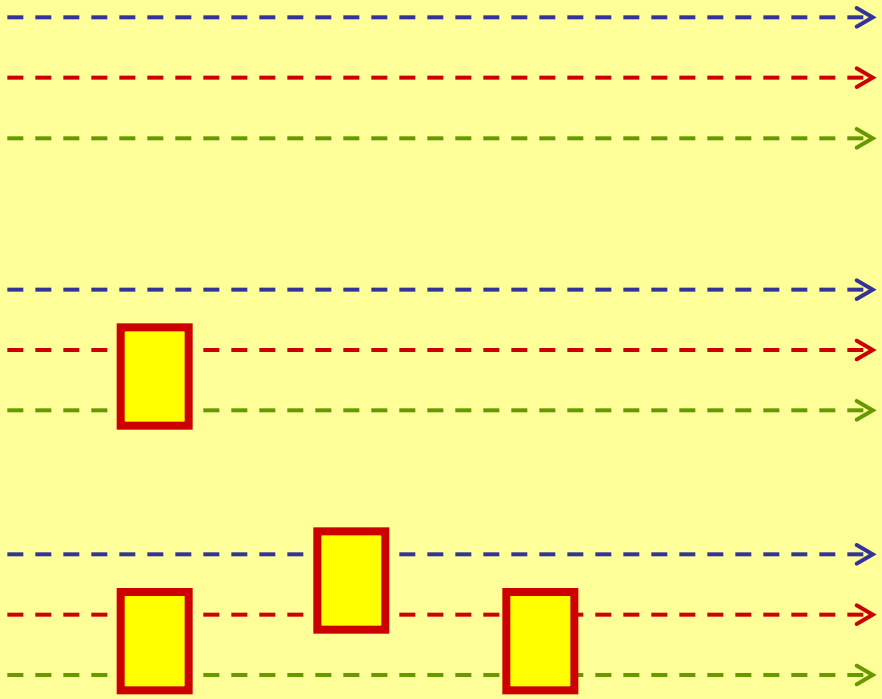
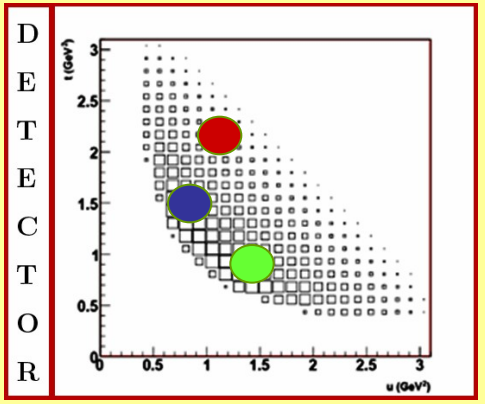
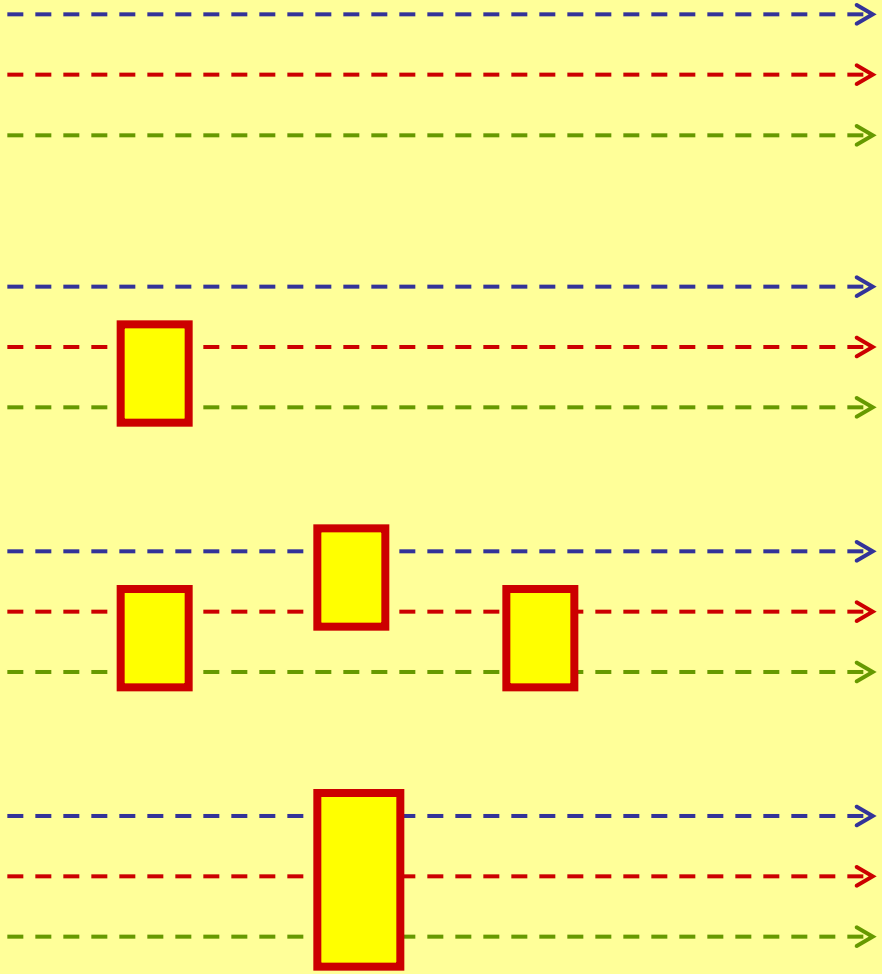
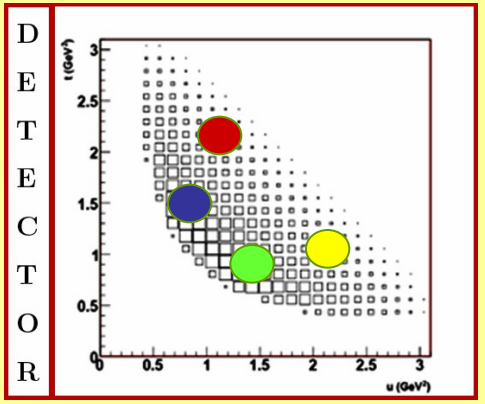




final state interactions





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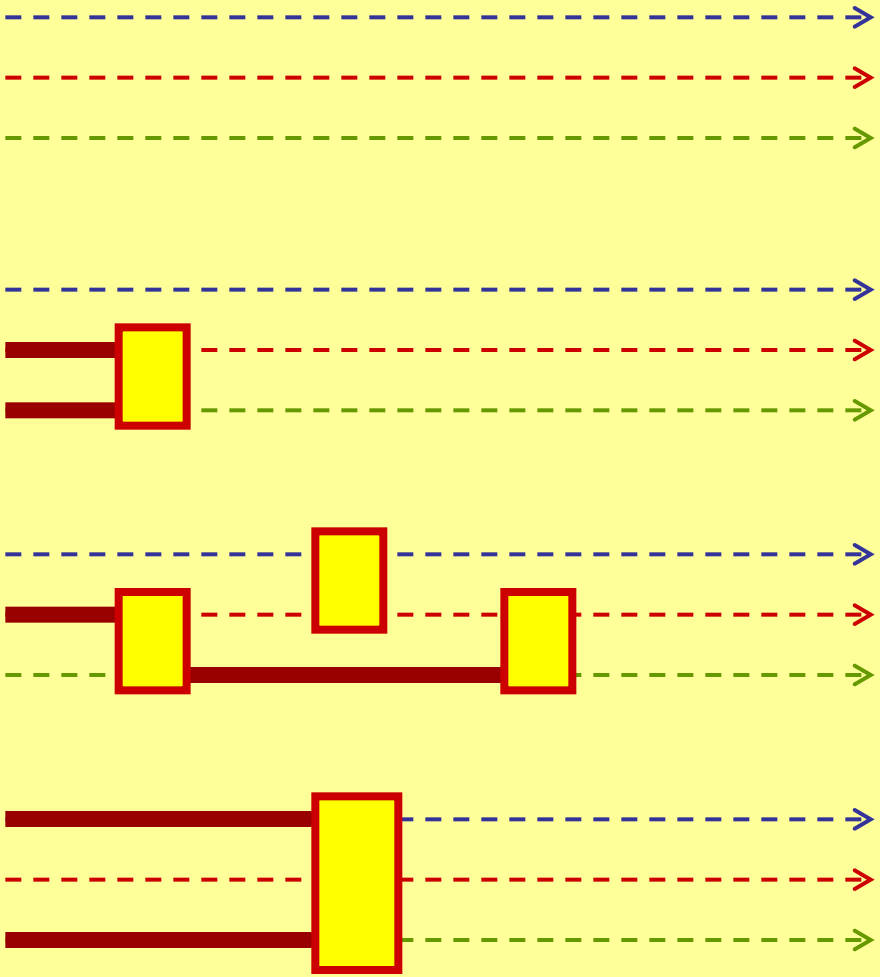

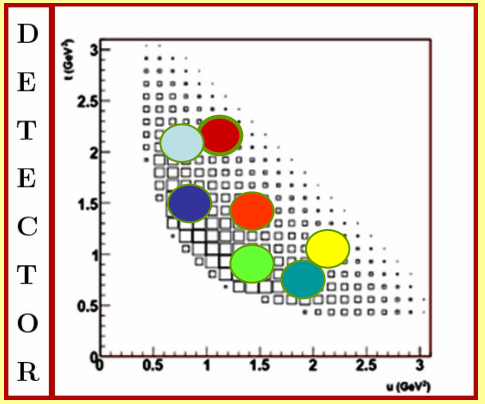

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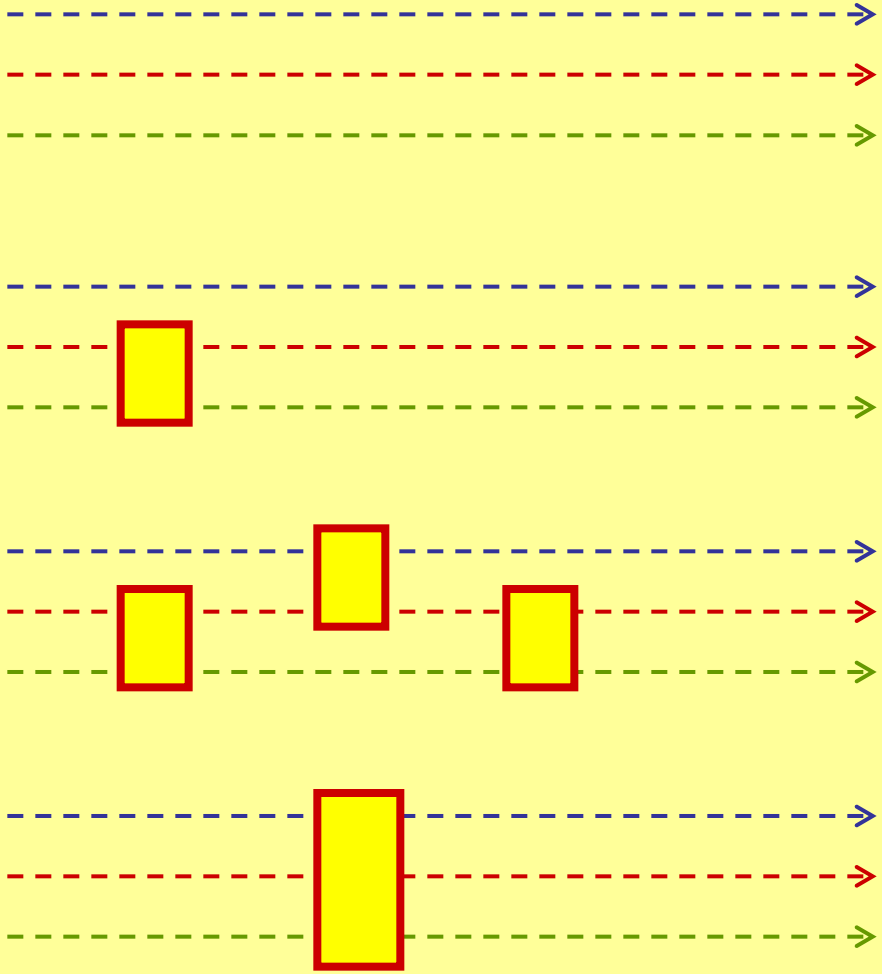
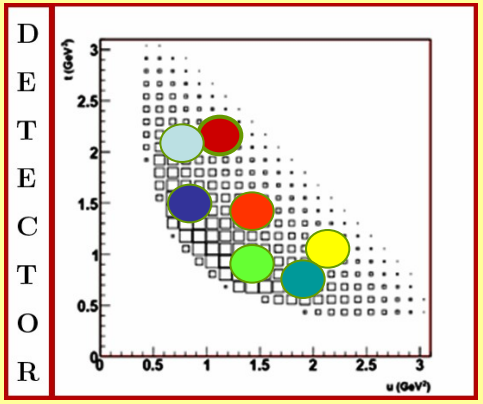
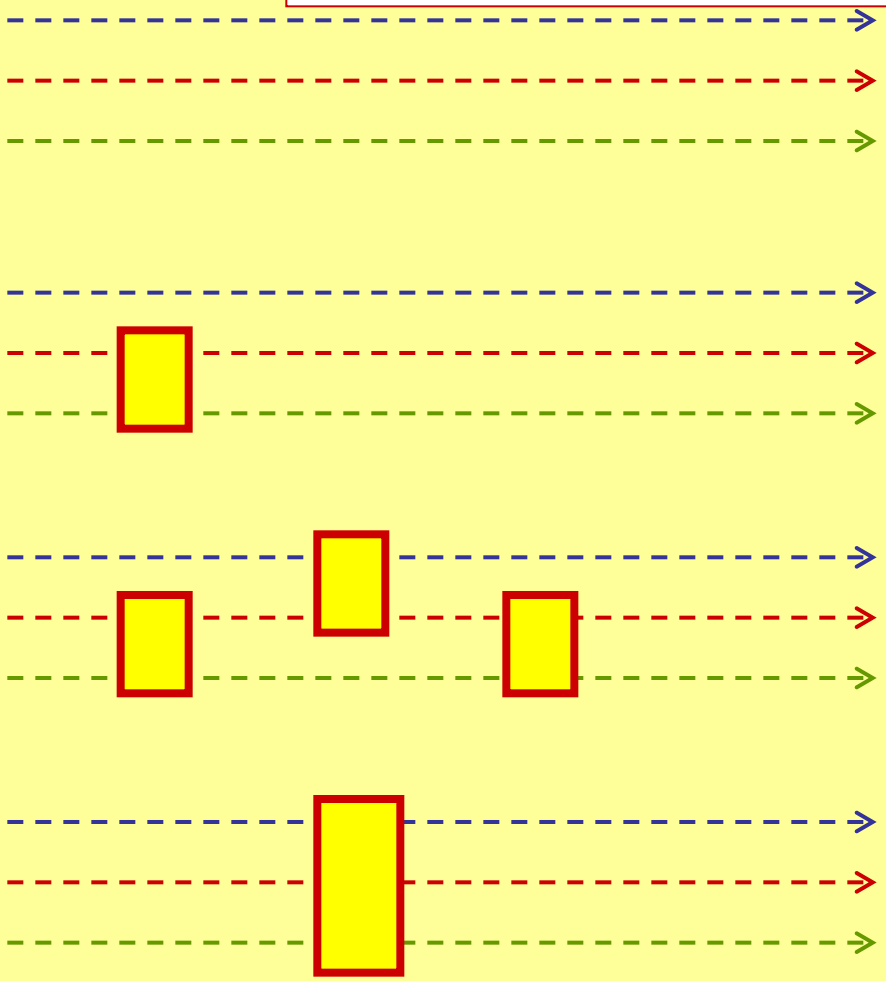




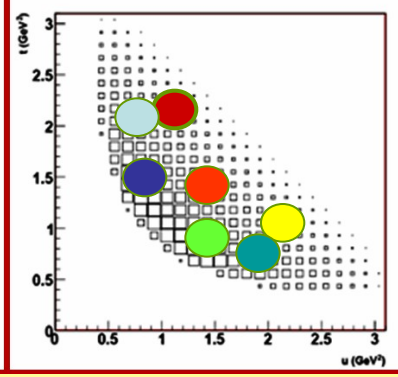
GUESS this MESS ?

THEORY

general guides ?



DETECTOR



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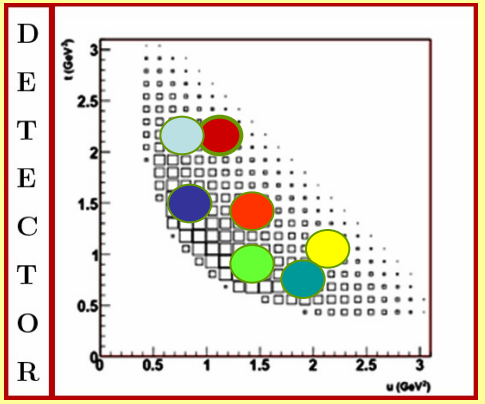
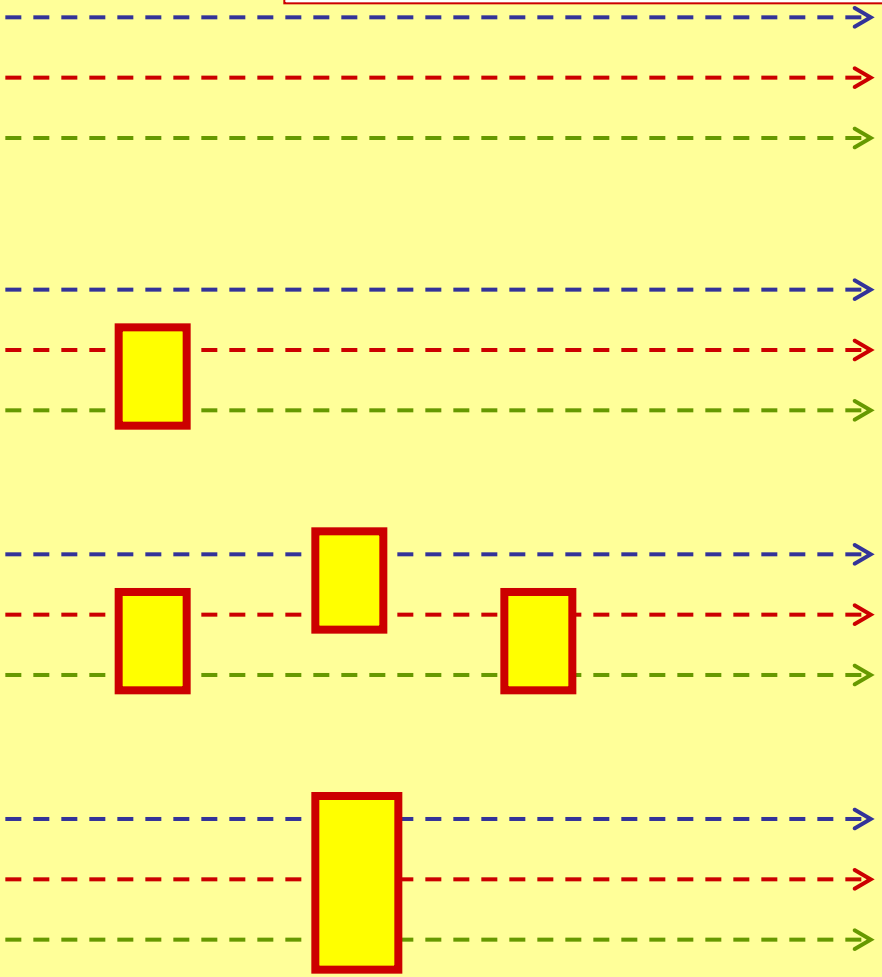
THEORY

general guides ?





atomic
nuclear
physics

methods

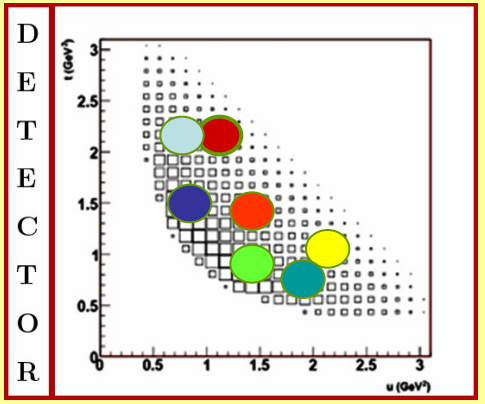
Fadeev
series

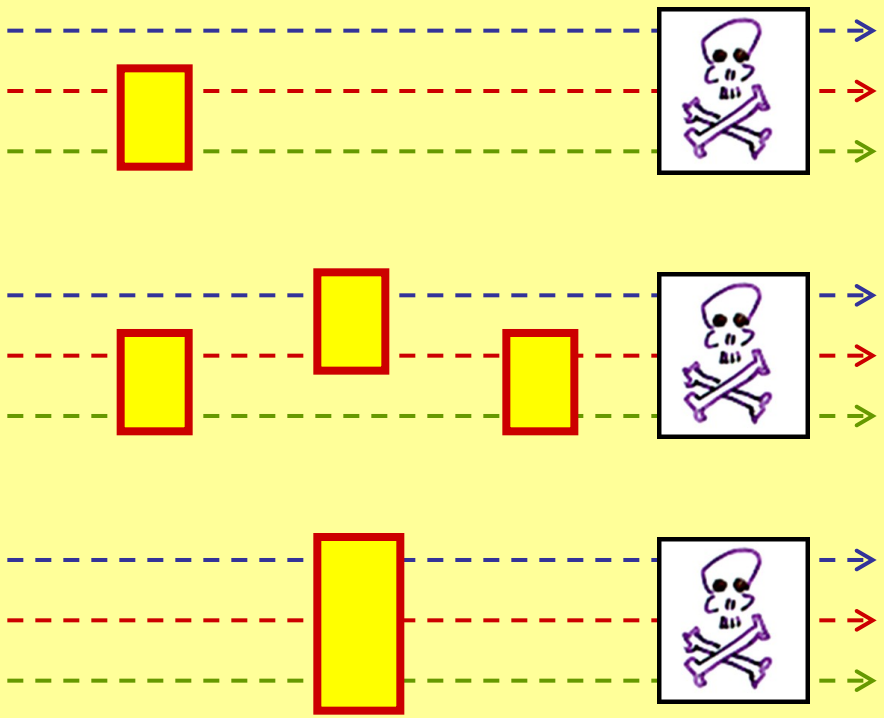
energy

THEORY

general guides ?

$W^{\pm}$





atomic
nuclear
physics

Watson's
theorem

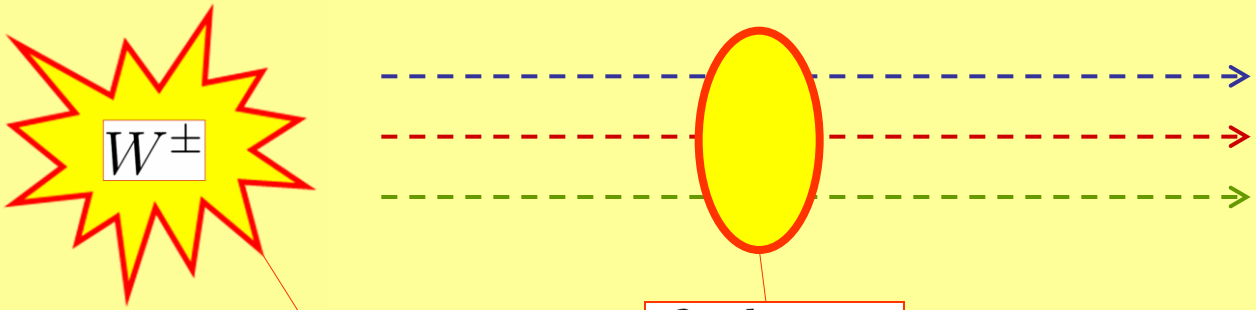
methods

Fadeev
series

energy

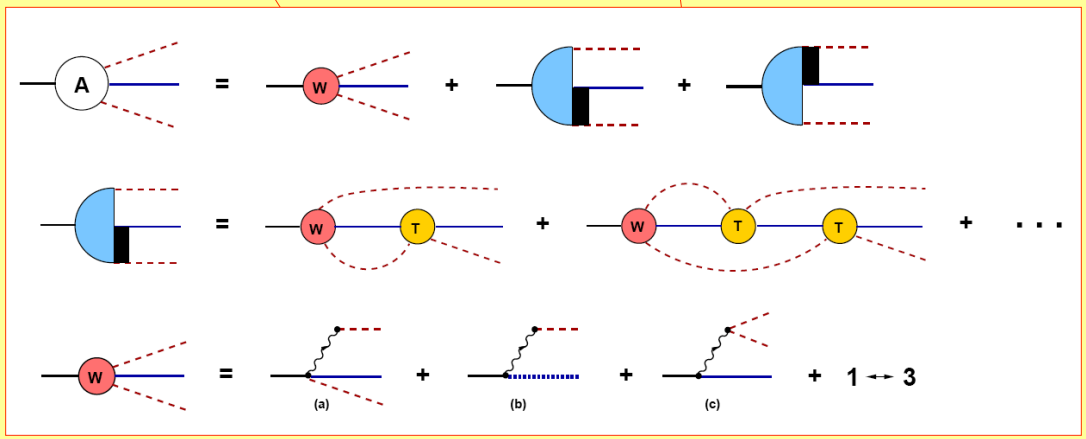
THEORY

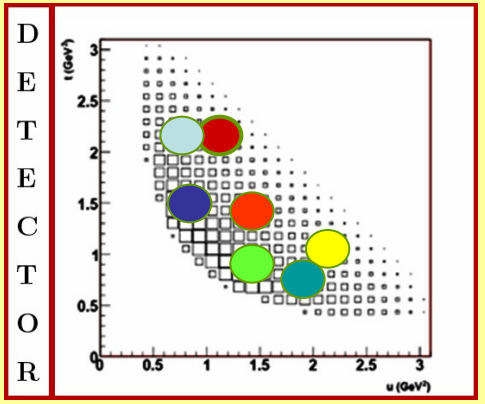
two problems



weak vertex

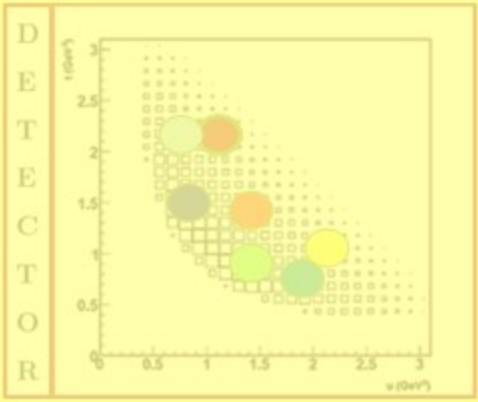
final state interactions

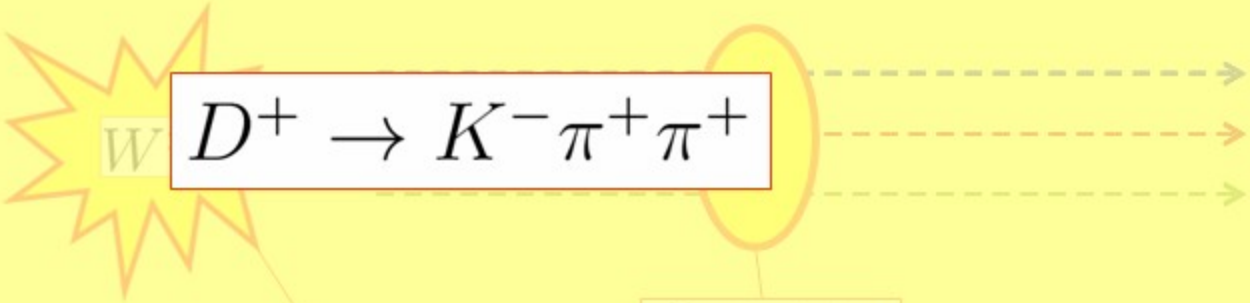




THEORY

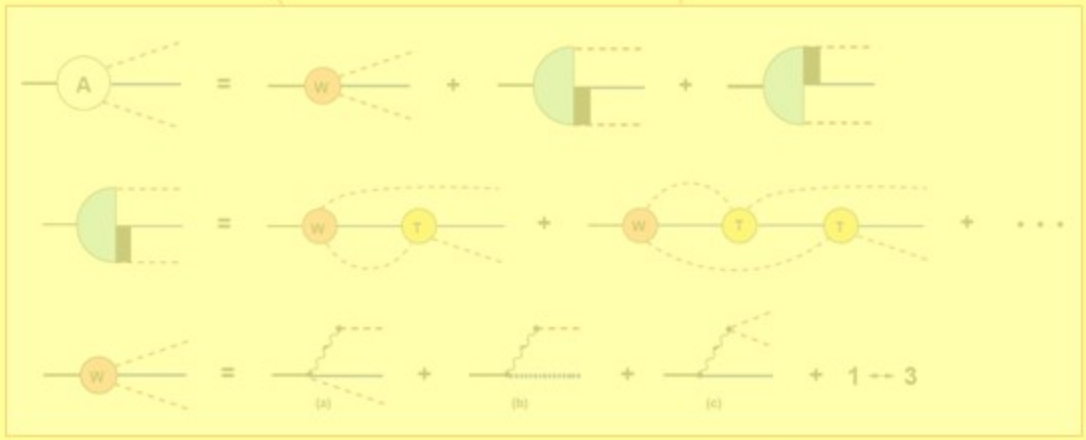
two problems





weak vertex

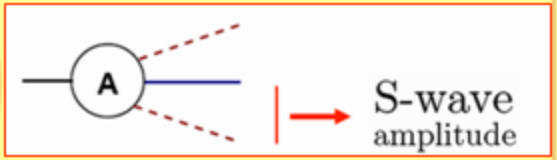
final state interactions

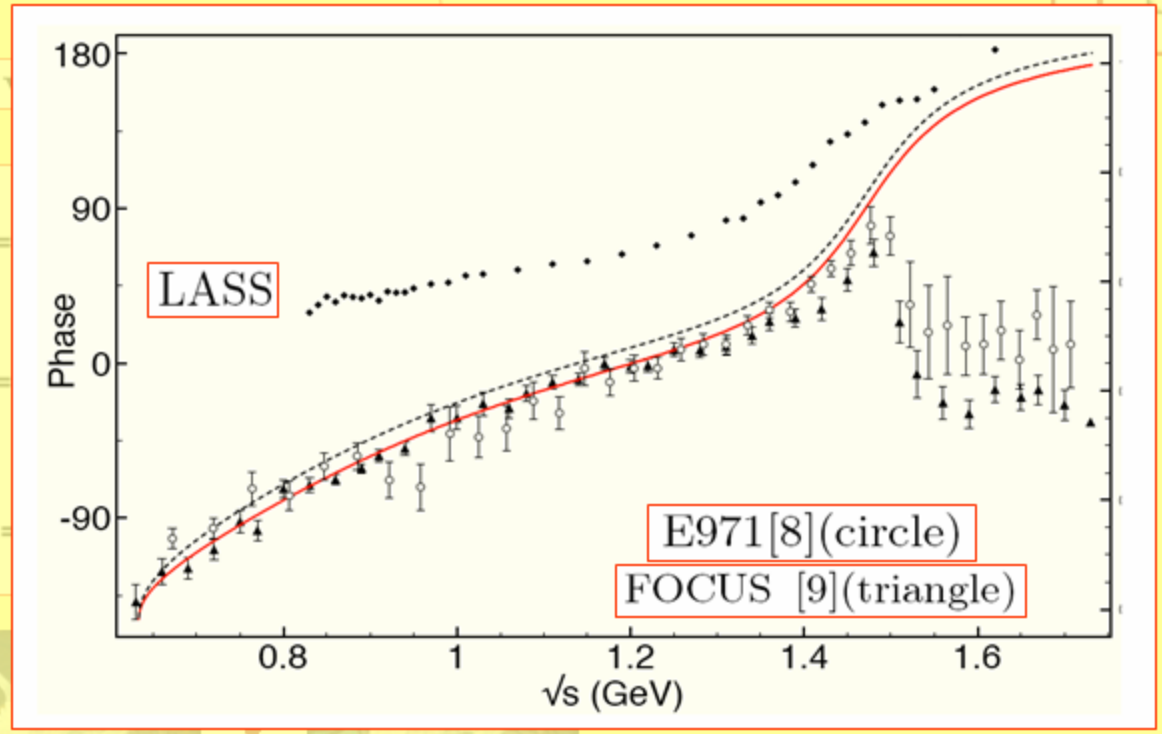


THEORY

two problems

$$D^+ \to K^-\pi^+\pi^+$$

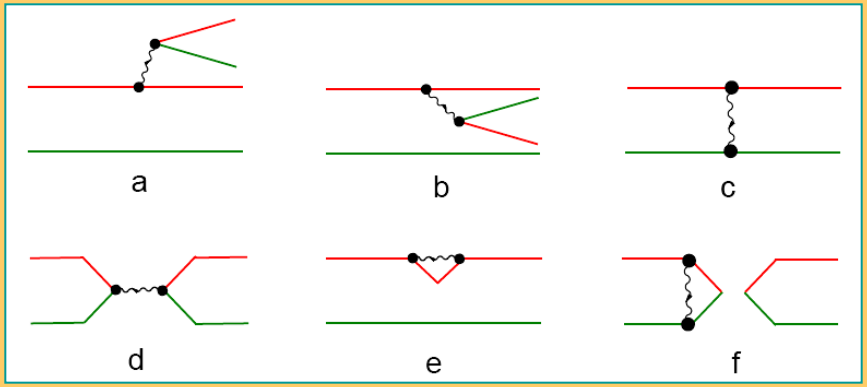




THEORY

weak vertex

$$D^+ \to K^-\pi^+\pi^+$$



THEORY

weak vertex

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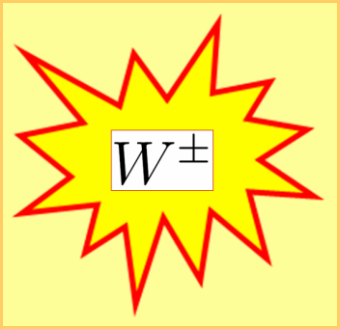
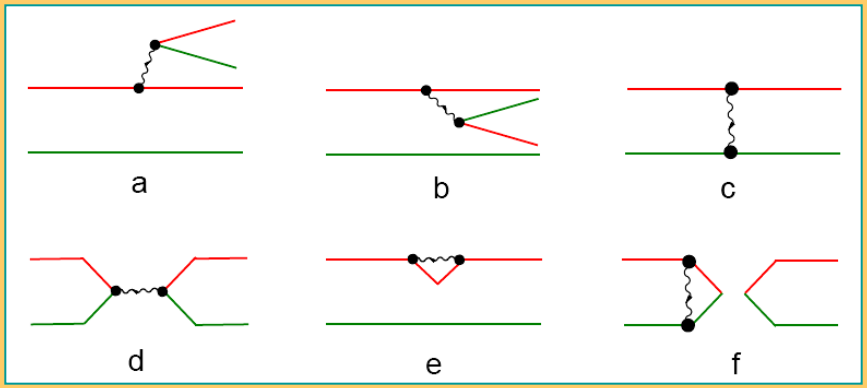


quarks $\to$ hadrons



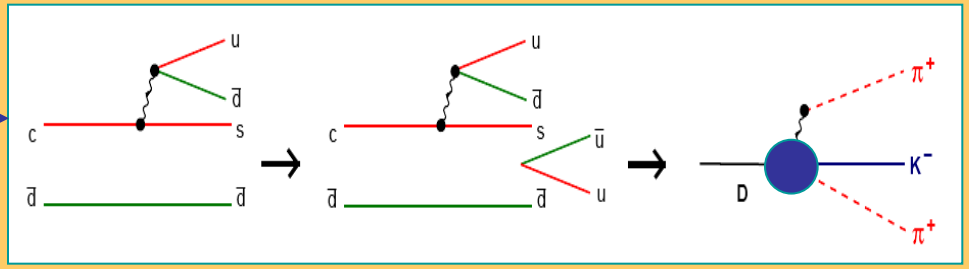
THEORY

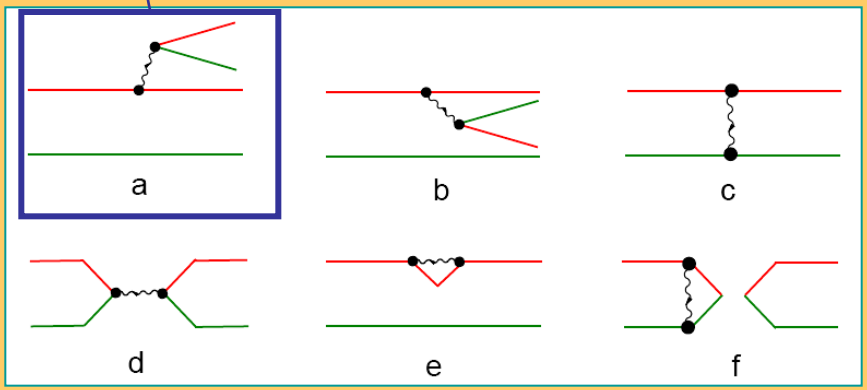
weak vertex

$W^{\pm}$

$D^+ \to K^-\pi^+\pi^+$

quarks $\to$ hadrons





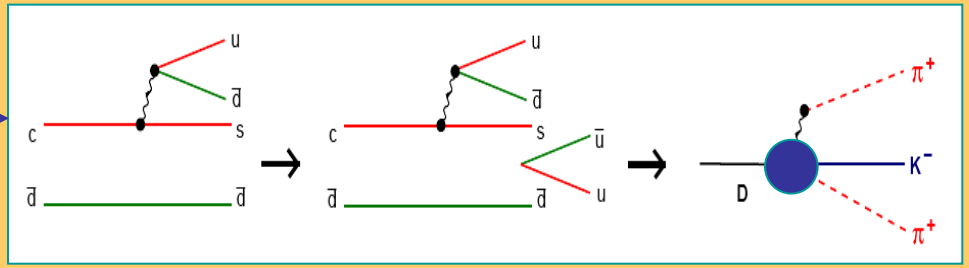
THEORY

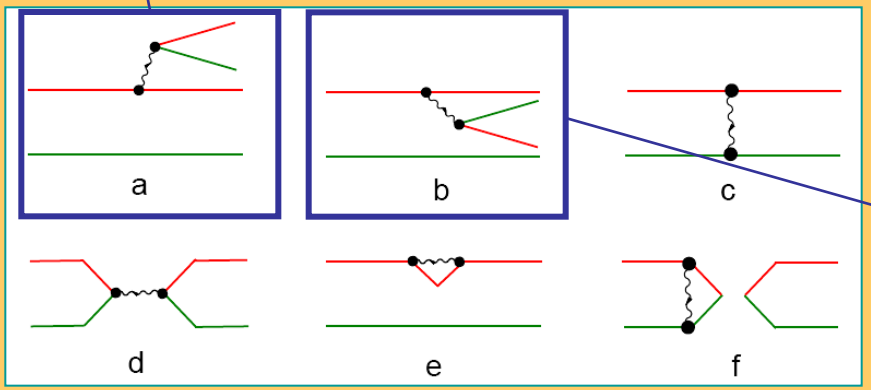
weak vertex

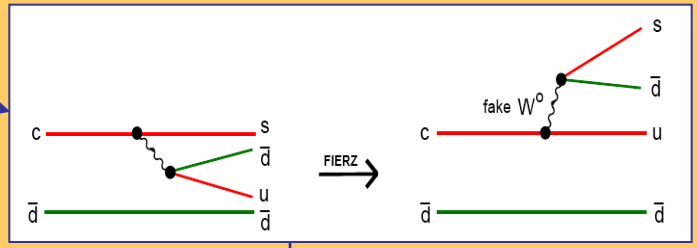
$$D^+ \to K^-\pi^+\pi^+$$

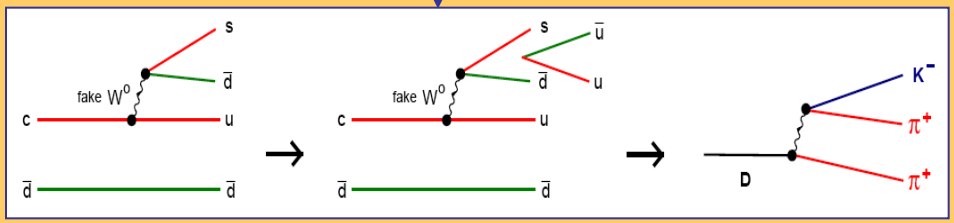
$W^{\pm}$

quarks $\longrightarrow$ hadrons

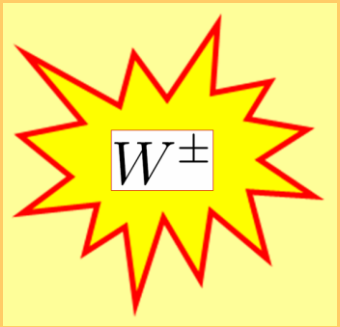








THEORY



$D^+ \to K^- \pi^+ \pi^+$

quarks $\to$ hadrons

phases in QCD ?

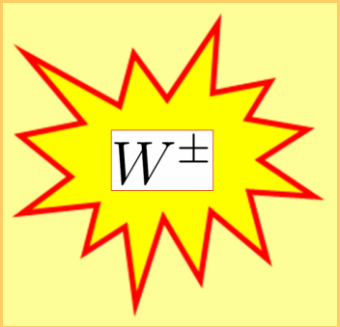
$\to$ unitarity

$\to$ loops

field theory &

dispersion relations

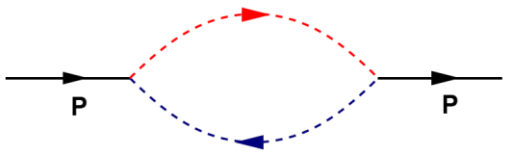
THEORY



$D^+ \to K^-\pi^+\pi^+$

quarks $\to$ hadrons

effective lagrangians



phases in QCD ?

$\to$ unitarity

$\to$ loops

field theory &

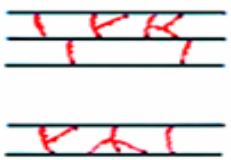
dispersion relations

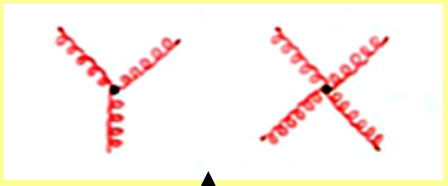
phases from first principles

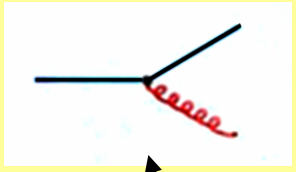
THEORY

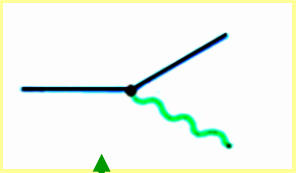
QCD

HADRONS









$$\mathcal{L} = \frac{1}{4}\,\mathcal{E}\,F_{\bullet}^{\mu\nu}F_{\mu\nu}^{\bullet} + \mathcal{E}\,\mathcal{E}\;\bar{q}_{\bullet}^{\bullet}\left[i\gamma^{\mu}\left(\partial_{\mu} - i g \frac{\lambda}{2} A_{\mu}\right) - m_{\bullet}\right] q_{\bullet}^{\bullet}$$

$$F_{\mu\nu}^{\bullet} = \partial_{\mu} A_{\nu}^{\bullet} - \partial_{\nu} A_{\mu}^{\bullet} + g\, f^{\bullet\bullet\bullet} A_{\mu}^{\bullet} A_{\nu}^{\bullet}$$

electroweak probes

external parameter

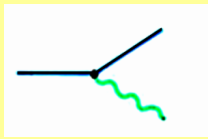
external quantum number

THEORY

QCD

effective lagrangians

$$\mathcal{L} = \text{[blob]} + \sum \bar{q}_i \left[i\gamma^\mu \partial_\mu \text{[blob]} - m_i \right] q_i$$



simplified version

$$\mathcal{L} = \frac{1}{4} \sum F_i^{\mu\nu} F_{\mu\nu}^i + \sum \sum \bar{q}_i^j \left[i\gamma^\mu \left(\partial_\mu - i g \frac{\lambda^i}{2} A_\mu^i \right) - m_i \right] q_j^i$$

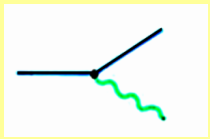
$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g f^{ijk} A_\mu^j A_\nu^k$$

THEORY

QCD

effective lagrangians

$$\mathcal{L} = \text{[blob]} + \sum \bar{q}_i \left[i\gamma^\mu \partial_\mu \text{[blob]} - m_i \right] q_i$$



symmetries → hadrons

- baryon number
- isospin → all quarks have the same m_i
- chiral symmetry → all quarks have $m_i = 0$

THEORY

QCD

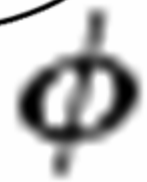
effective lagrangians

$$\mathcal{L} = \text{[blob]} + \sum \bar{q}_i \left[i\gamma^\mu \partial_\mu \text{[blob]} - m_i \right] q_i$$

2 or 3 light quarks

$$\begin{aligned}
\mathcal{L}_M^{(2)} &= 3\,F^2\,B\,s_0 \\
&+ \frac{1}{2}\,\partial_\mu\phi_i\,\partial_\mu\phi_i - B\,(s_0\,\delta_{ij} + s_a\,d_{aij})\,\phi_i\,\phi_j \\
&- \frac{1}{6F^2}\,f_{ijs}\,f_{kls}\,\phi_i\,\partial_\mu\phi_j\,\phi_k\,\partial_\mu\phi_l + \frac{B}{24F^2}\left[s_0\left(\frac{4}{3}\,\delta_{ij}\,\delta_{kl} + 2\,d_{ijs}\,d_{kls}\right)\right. \\
&\left. + s_a\left(\frac{4}{3}\,\delta_{ij}\,d_{kla} + \frac{4}{3}\,d_{ija}\,\delta_{kl} + 2\,d_{ijm}\,d_{kln}\,d_{amn}\right)\right]\phi_i\,\phi_j\,\phi_k\,\phi_l + \cdots
\end{aligned}$$

hadrons



pion field → $|\phi| = F_\pi \sin\theta$ → pion
Goldstone boson

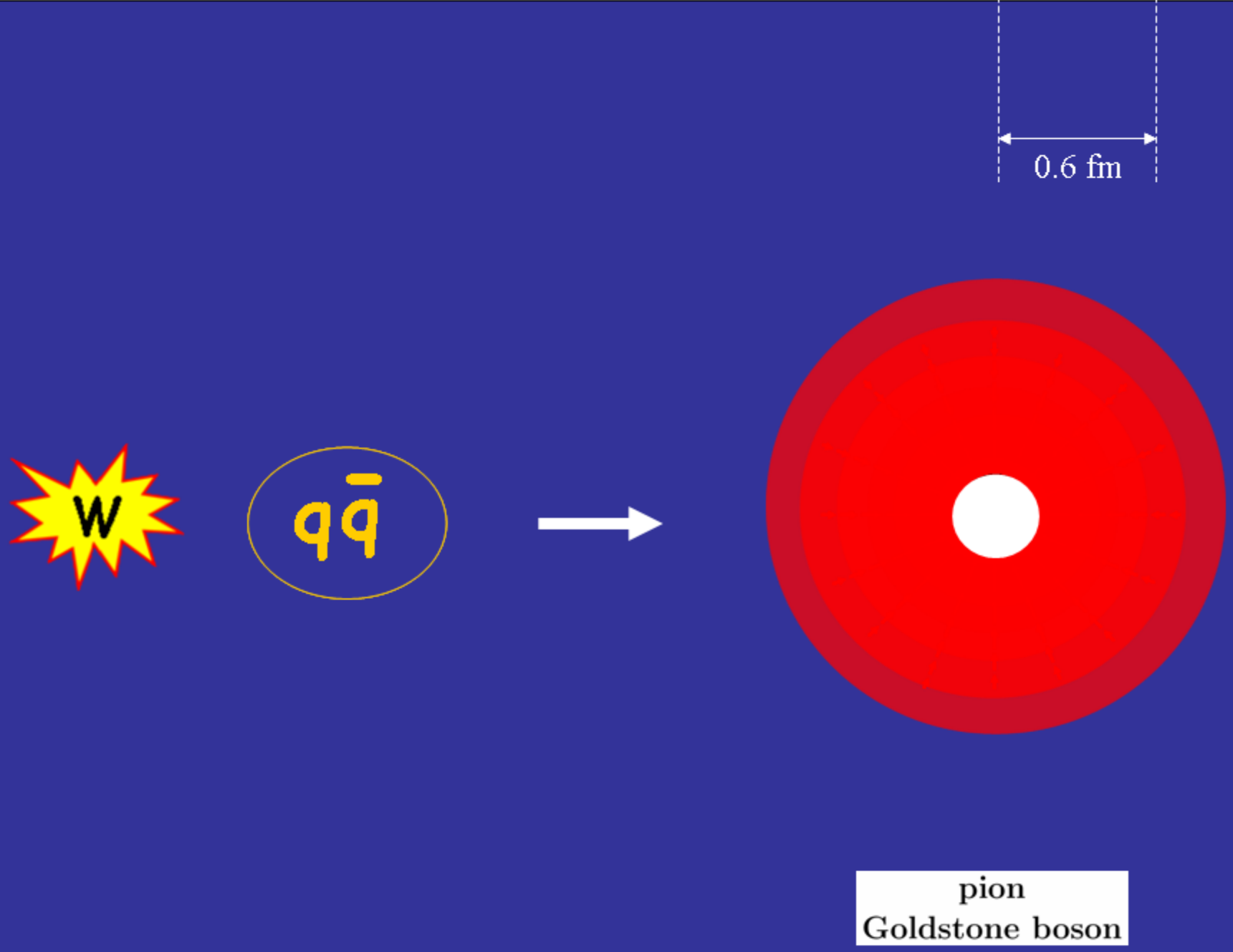
Wigner
Weyl

Nambu
Goldstone

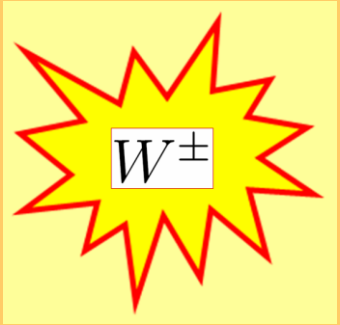
THEORY

quark condensates

pion
Goldstone boson


0.6 fm
W
$q\bar{q}$
pion
Goldstone boson

THEORY



$D^+ \to K^- \pi^+ \pi^+$

quarks → hadrons

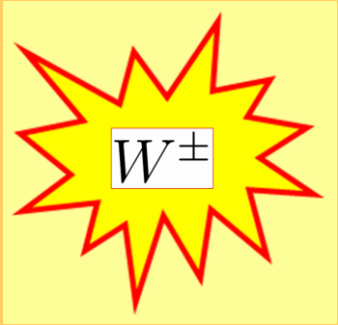
effective lagrangians

light mesons → chiral symmetry → $SU(3) \times SU(3)$

quark condensates

weak interactions → gauge external couplings

THEORY



$D^+ \to K^-\pi^+\pi^+$

quarks → hadrons

effective lagrangians

light mesons → chiral symmetry → $SU(3) \times SU(3)$

quark condensates

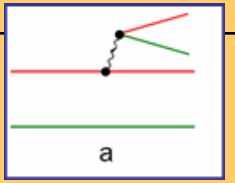
weak interactions → gauge external couplings

heavy mesons → heavy mass symmetry

THEORY

$W^{\pm}$

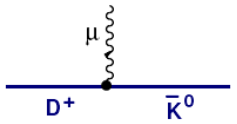
$D^+ \to K^-\pi^+\pi^+$



quarks $\longrightarrow$ hadrons

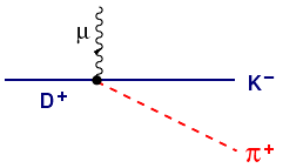
$$\bullet\bullet\bullet\ \mathcal{L}^{(1)}_{wD0} = -\,G_{wD0}\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right] W^-_\mu \left\{\frac{i}{F}\left[\sqrt{2}\,D^0\,\partial^\mu K^+ + \sqrt{2}\,D^+\,\partial^\mu K^0 + \cdots\right] - \frac{1}{2F^2}\left[\sqrt{2}\,D^+\left(\sqrt{2}\,\pi^-\,\partial^\mu K^+ - \pi^0\,\partial^\mu K^0 + \cdots\right)\right]\right\}$$

- $D^+ \to W^+ \bar{K}^0$ [FR−09|2/1]



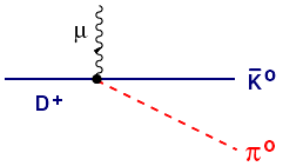
$$\to\ i\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{\sqrt{2}\,G_{wD0}}{F}\right] p^\mu_K$$

- $D^+ \to W^+ \pi^+ K^-$ [FR−09|2/1]



$$\to\ -\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{G_{wD0}}{F^2}\right] p^\mu_K$$

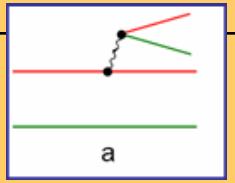
- $D^+ \to W^+ \pi^0 \bar{K}^0$ [FR−09|2/1]

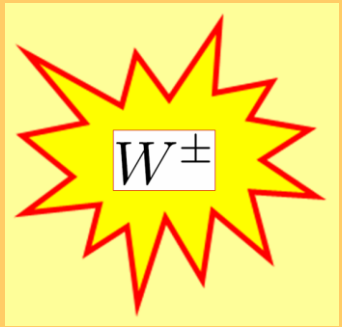


$$\to\ \frac{1}{\sqrt{2}}\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{G_{wD0}}{F^2}\right] p^\mu_K$$

THEORY

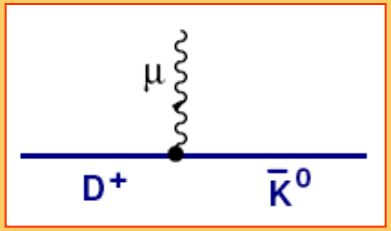
$D^+ \to K^-\pi^+\pi^+$



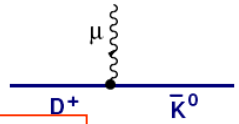


quarks → hadrons

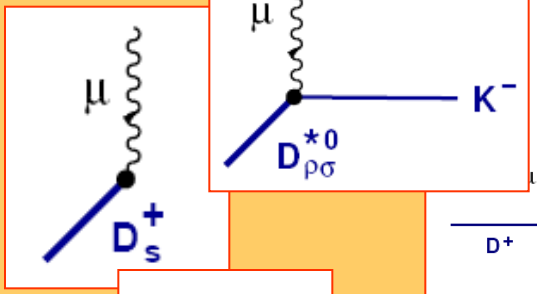
$$\bullet\bullet\bullet\ \mathcal{L}^{(1)}_{wD0} = -\,G_{wD0}\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right] W^-_\mu \left\{\frac{i}{F}\left[\sqrt{2}\,D^0\,\partial^\mu K^+ + \sqrt{2}\,D^+\,\partial^\mu K^0 + \cdots\right] - \frac{1}{2F^2}\left[\sqrt{2}\,D^+\left(\sqrt{2}\,\pi^-\,\partial^\mu K^+ - \pi^0\,\partial^\mu K^0 + \cdots\right)\right]\right\}$$



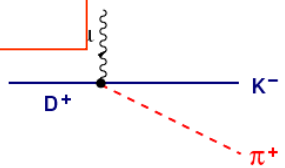
- $D^+ \to W^+ \bar{K}^0$ [FR−09|2/1]



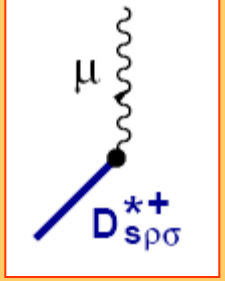
$$\to\ i\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{\sqrt{2}\,G_{wD0}}{F}\right] p^\mu_K$$



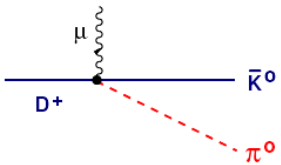
- $W^+ \pi^+ K^-$ [FR−09|2/1]



$$\to\ -\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{G_{wD0}}{F^2}\right] p^\mu_K$$



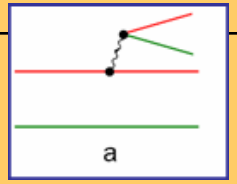
- $D^+ \to W^+ \pi^0 \bar{K}^0$ [FR−09|2/1]

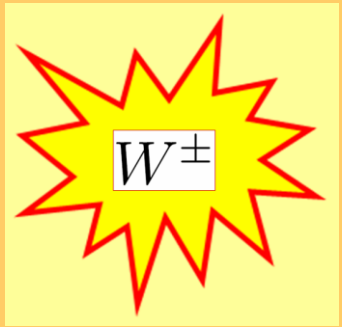


$$\to\ \frac{1}{\sqrt{2}}\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{G_{wD0}}{F^2}\right] p^\mu_K$$

THEORY

$D^+ \to K^-\pi^+\pi^+$

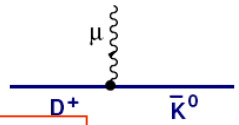




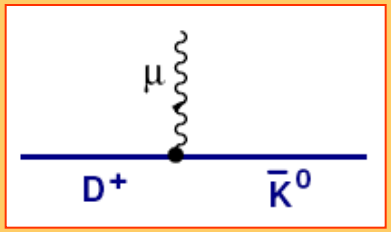
quarks → hadrons

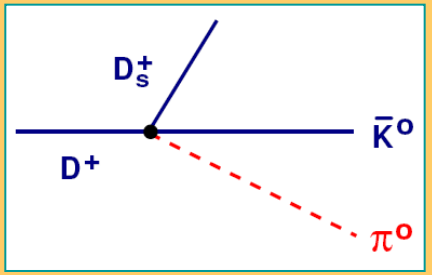
$$\bullet\bullet\bullet\ \mathcal{L}^{(1)}_{wD0} = -\,G_{wD0}\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right] W^-_\mu \left\{\frac{i}{F}\left[\sqrt{2}\,D^0\,\partial^\mu K^+ + \sqrt{2}\,D^+\,\partial^\mu K^0 + \cdots\right] - \frac{1}{2F^2}\left[\sqrt{2}\,D^+\left(\sqrt{2}\,\pi^-\,\partial^\mu K^+ - \pi^0\,\partial^\mu K^0 + \cdots\right)\right]\right\}$$

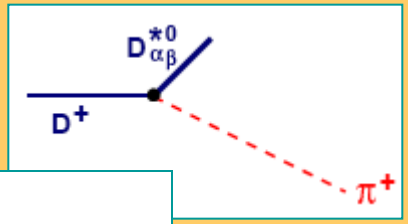
- $D^+ \to W^+ \bar{K}^0$ [FR−09|2/1]

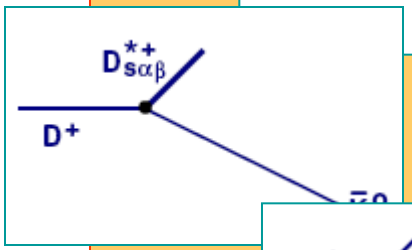


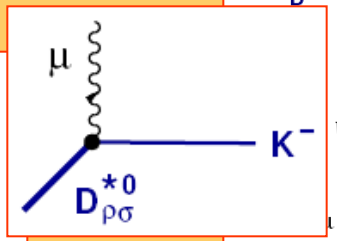
$$\to\ i\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{\sqrt{2}\,G_{wD0}}{F}\right] p^\mu_K$$

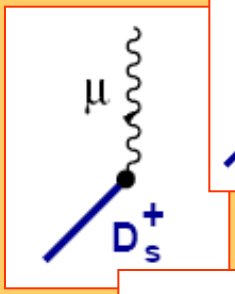




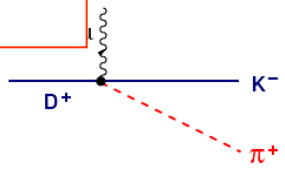




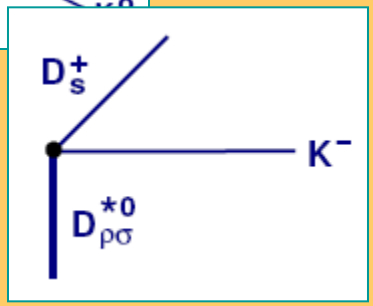


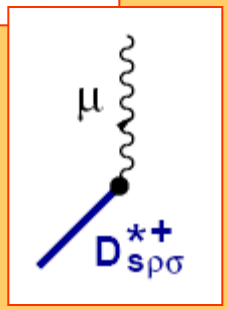


- $D^+ \to W^+ \pi^+ K^-$ [FR−09|2/1]

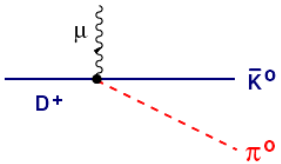


$$\to\ -\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{G_{wD0}}{F^2}\right] p^\mu_K$$





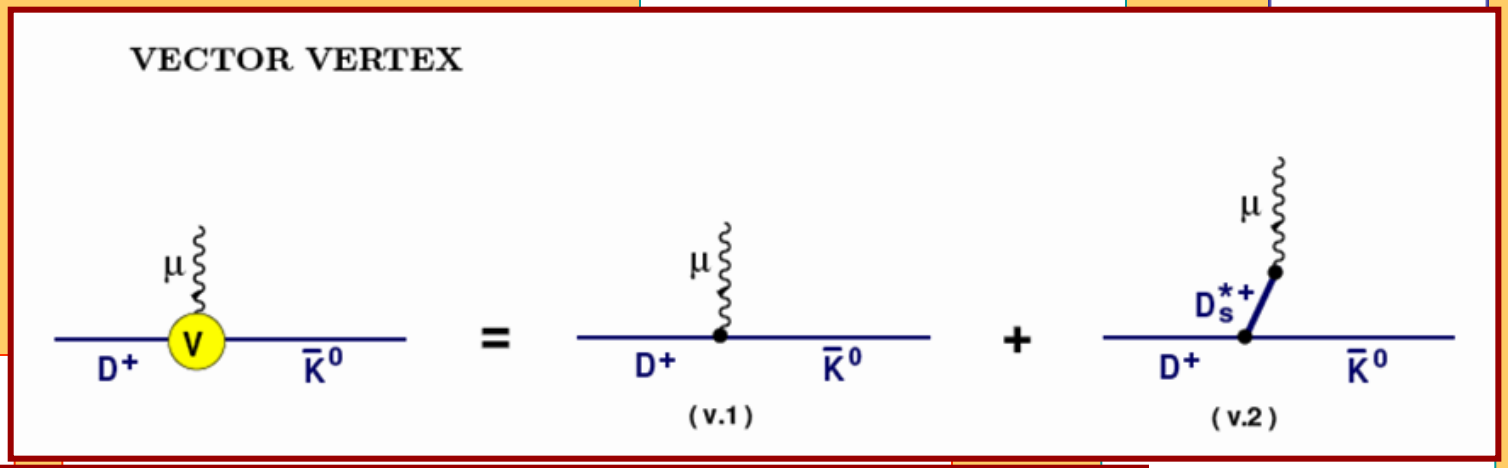
- $D^+ \to W^+ \pi^0 \bar{K}^0$ [FR−09|2/1]

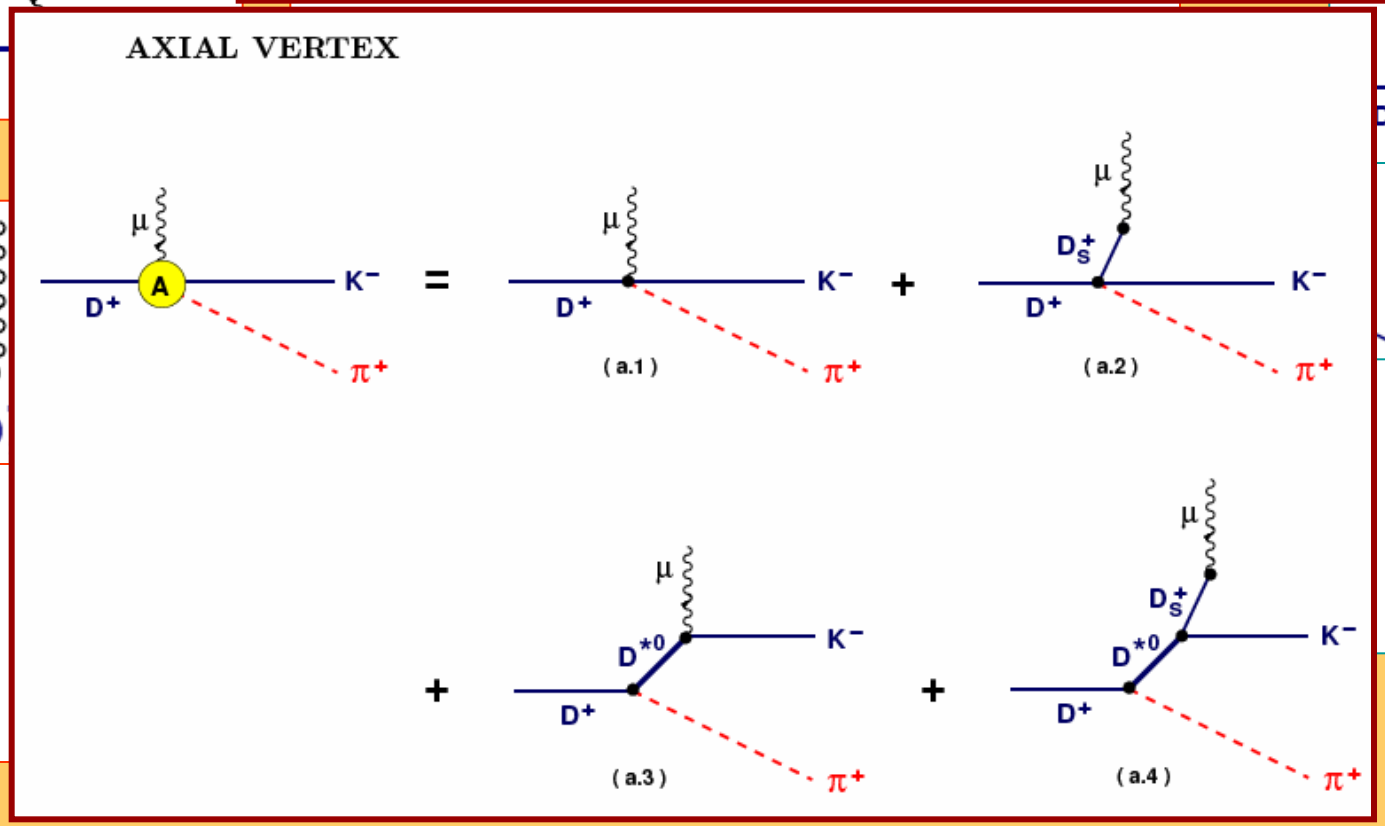


$$\to\ \frac{1}{\sqrt{2}}\left[\frac{g}{2\sqrt{2}}\cos\theta_C\right]\left[\frac{G_{wD0}}{F^2}\right] p^\mu_K$$

THEORY

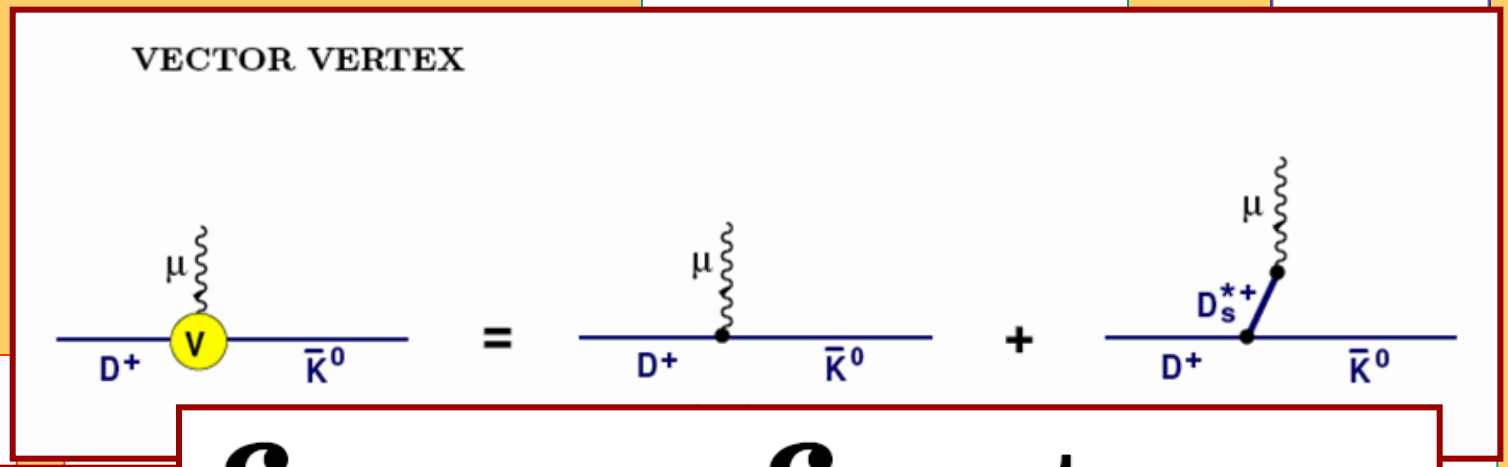
weak vertex



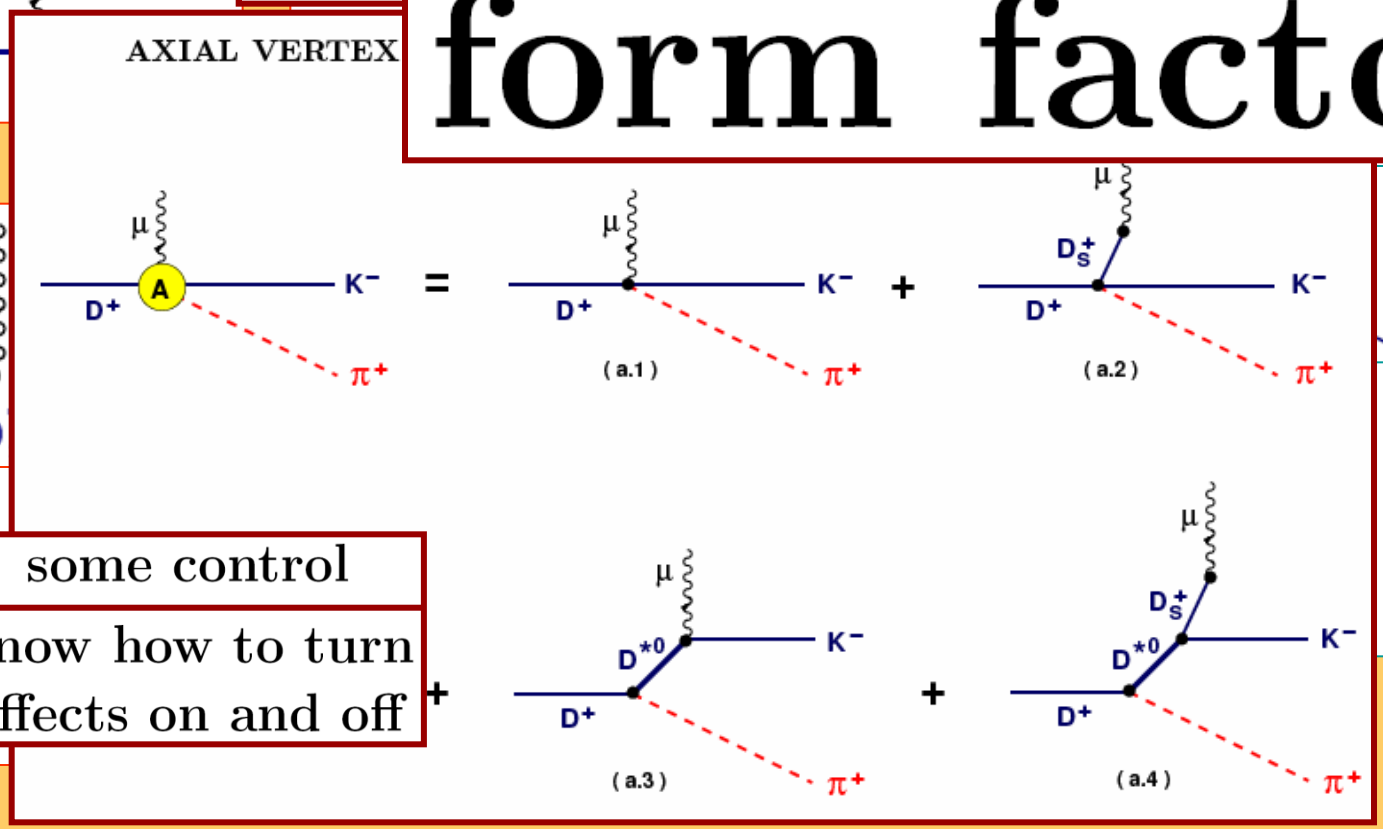


THEORY

weak vertex



form factors

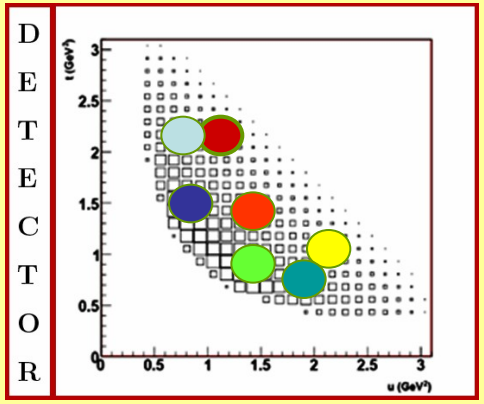


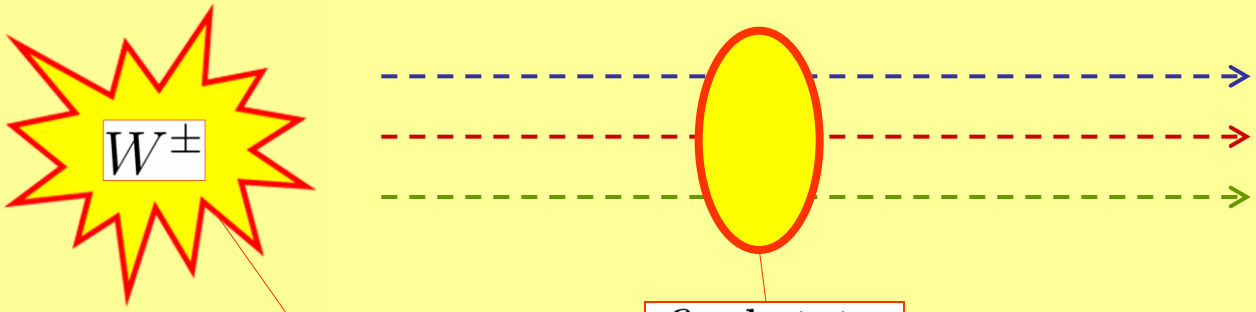
some control

know how to turn effects on and off

THEORY

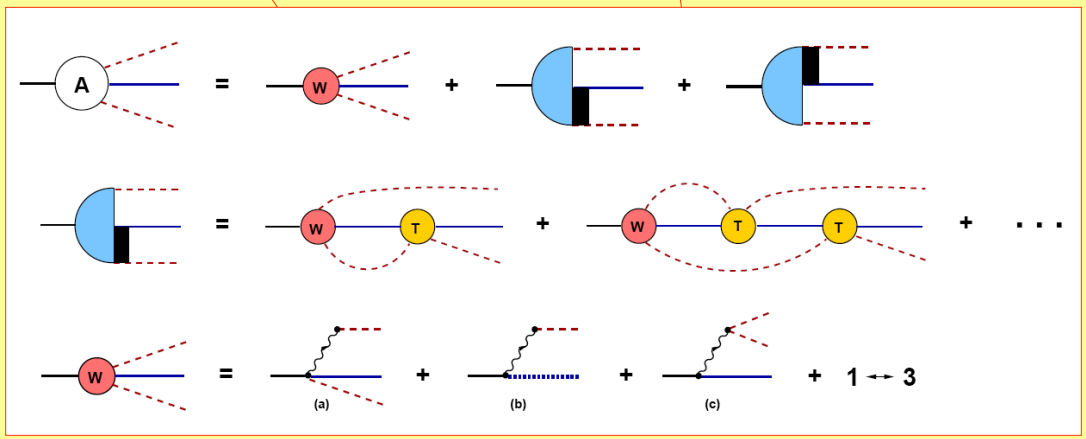
final state interactions





weak vertex

final state interactions

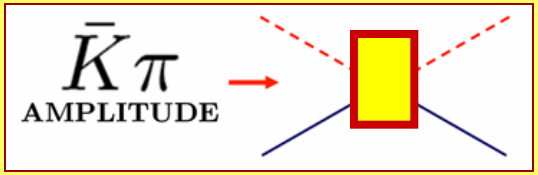


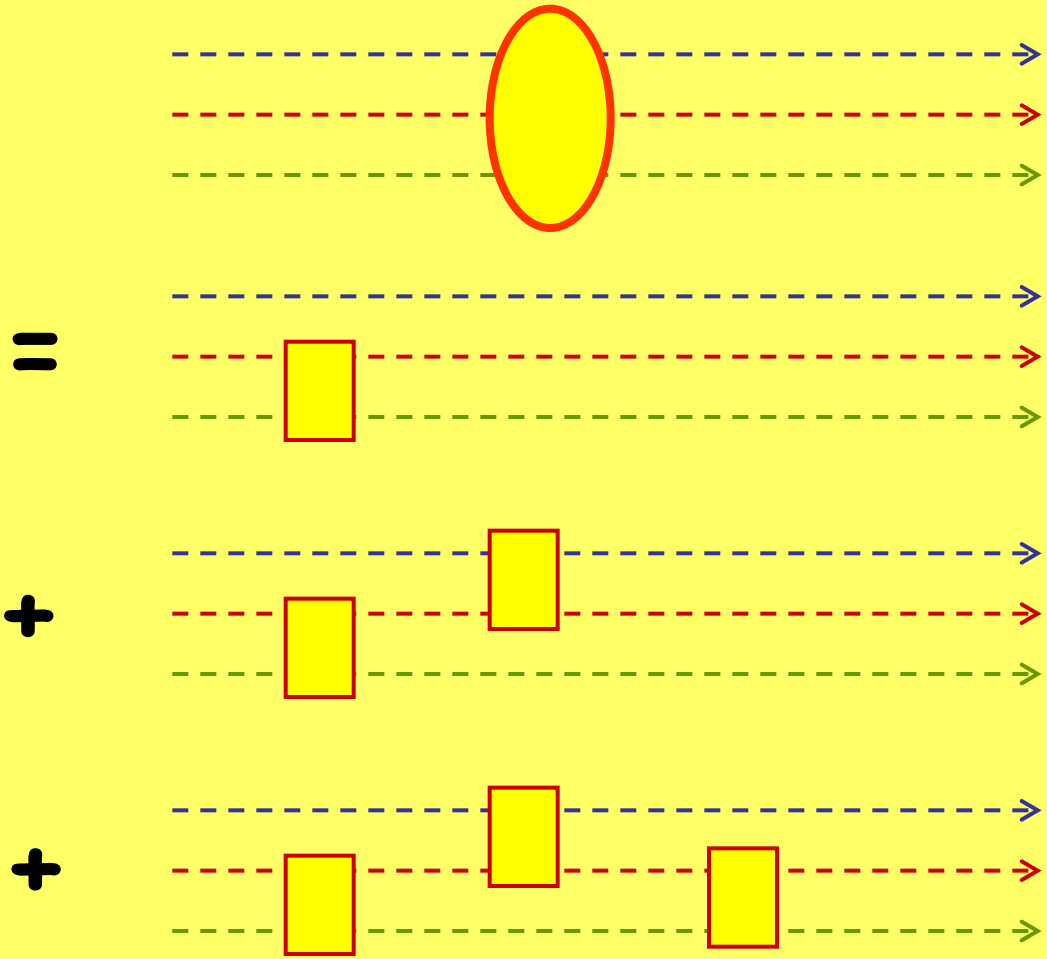
×

THEORY

final state interactions

$D^+ \to K^-\pi^+\pi^+$



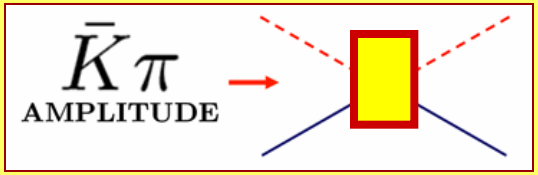


Fadeev series

THEORY

final state interactions

$D^+ \to K^-\pi^+\pi^+$



effective lagrangians

light mesons $\to$ chiral symmetry $\to$ $SU(3) \times SU(3)$

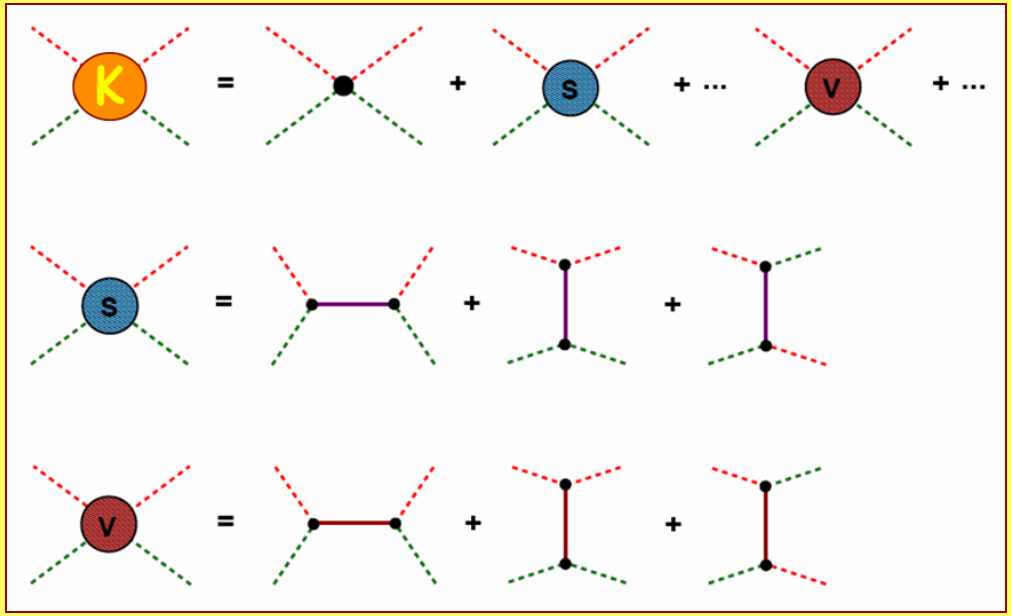
quark condensates

THEORY

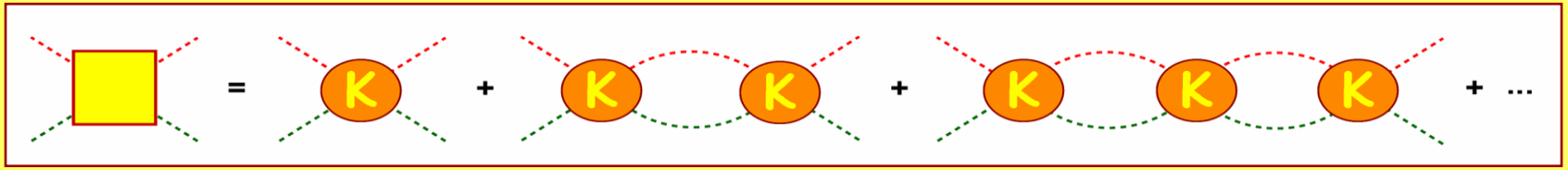
final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE



Bethe-Salpeter equation



THEORY

final state interactions

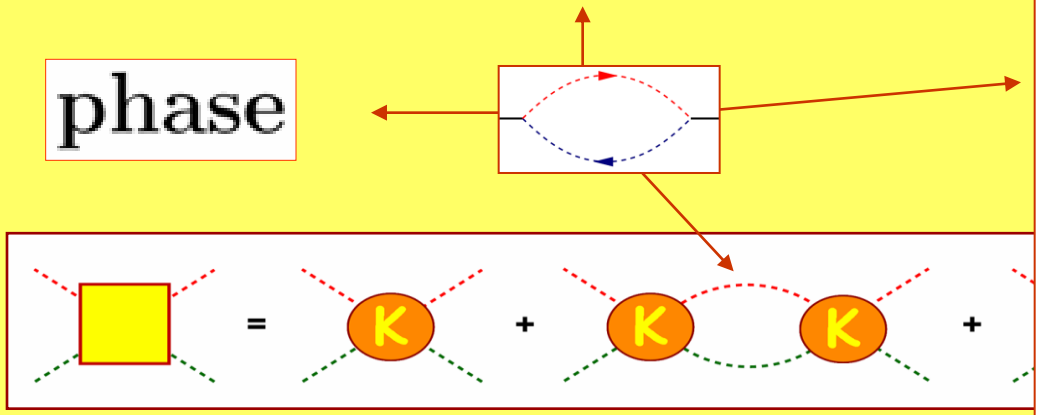
$D^+ \to K^-\pi^+\pi^+$

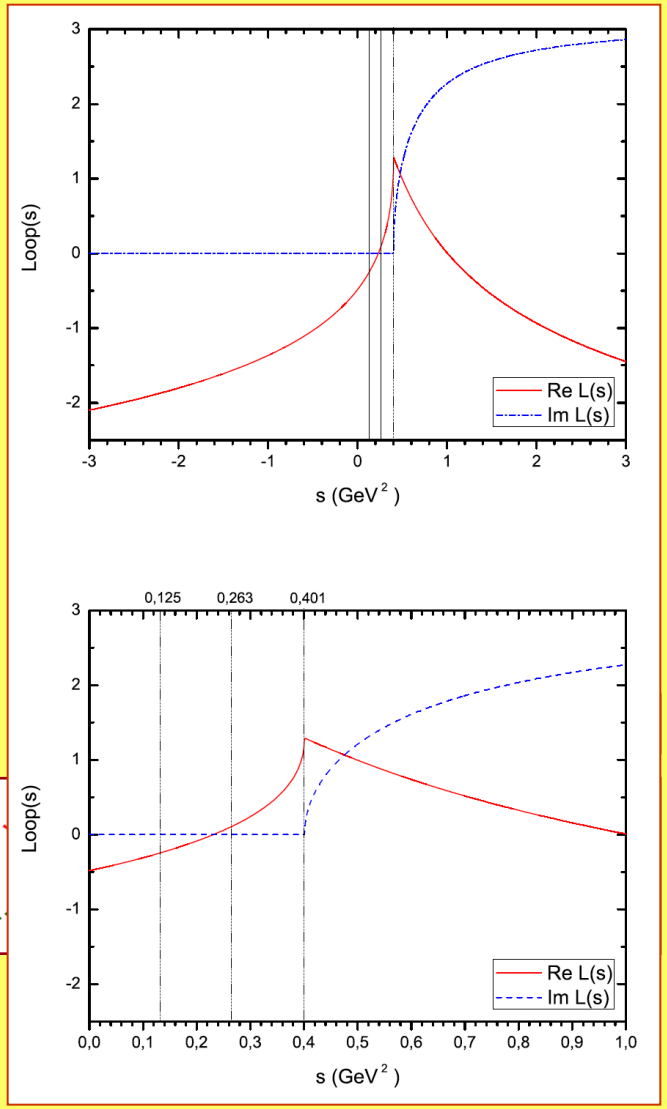
$\bar{K}\pi$ AMPLITUDE

two-body unitarity

$$= \frac{\mathcal{K}}{1 + \bar{\Omega}\mathcal{K}}$$

phase

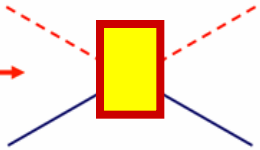




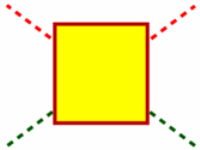
THEORY

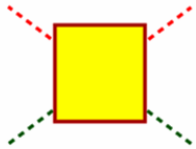
final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE 

two-body unitarity

 $= -\left[\dfrac{16\pi}{\rho}\,\dfrac{m_r\,\Gamma}{s-\mathcal{M}^2+i\,m_r\,\Gamma}\right]$

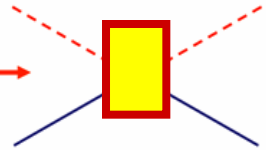
 $= \dfrac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$

Breit-Wigner + chiral symmetry

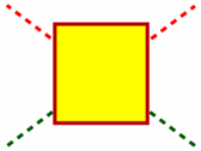
THEORY

final state interactions

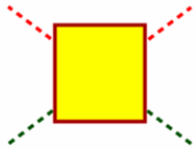
$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE 

two-body unitarity



$$= -\left[\frac{16\pi}{\rho}\,\frac{m_r\,\Gamma}{s-\mathcal{M}^2+i\,m_r\,\Gamma}\right]$$



$$= \frac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$$

Breit-Wigner + chiral symmetry

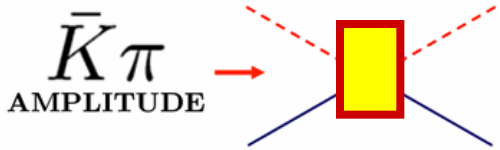
isobar model is crude

THEORY

final state interactions

$D^+ \to K^-\pi^+\pi^+$

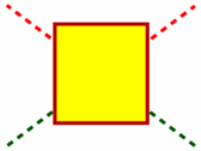
$\bar{K}\pi$ AMPLITUDE



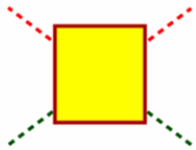
K-matrix approximation

$\mathcal{M} \longrightarrow m^*$

two-body unitarity



$$= -\left[\frac{16\pi}{\rho}\,\frac{m_r\,\Gamma}{s-\mathcal{M}^2+i\,m_r\,\Gamma}\right]$$



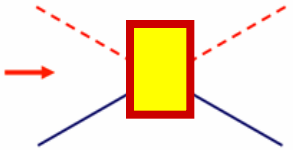
$$= \frac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$$

Breit-Wigner + chiral symmetry

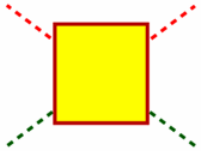
THEORY

final state interactions

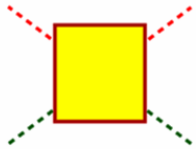
$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE 

two-body unitarity



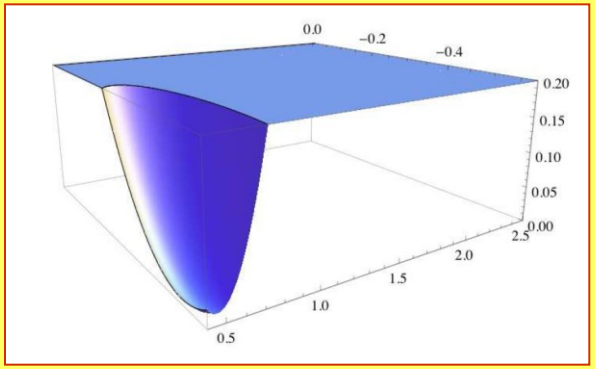
$$= -\left[\frac{16\pi}{\rho}\,\frac{m_r\,\Gamma}{s-\mathcal{M}^2+i\,m_r\,\Gamma}\right]$$



$$= \frac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$$

poles

kappa dynamical

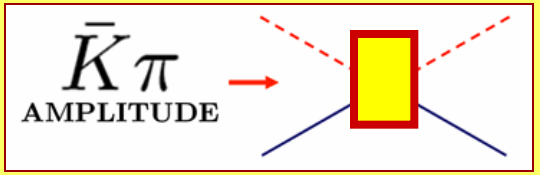


THEORY

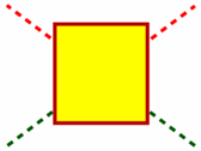
final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE



two-body unitarity



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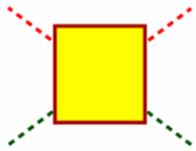
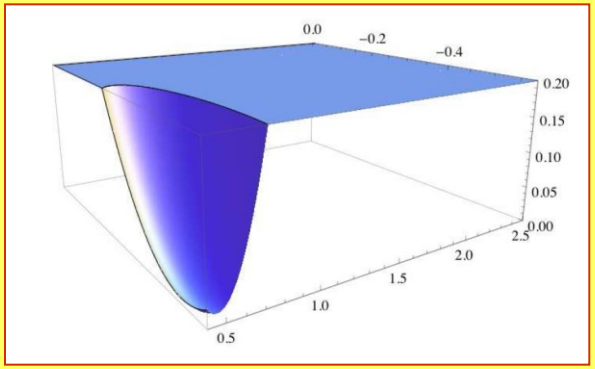


$$= \frac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$$

light sector of QCD $\to$ chiral symmetry
sigma & kappa
cannot be accommodated into chiral lagrangians

poles

kappa dynamical

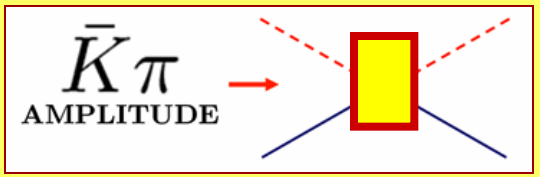


THEORY

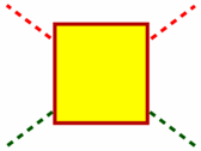
final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE



two-body unitarity



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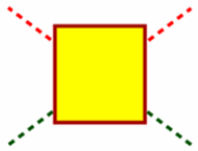

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light sector of QCD $\to$ chiral symmetry
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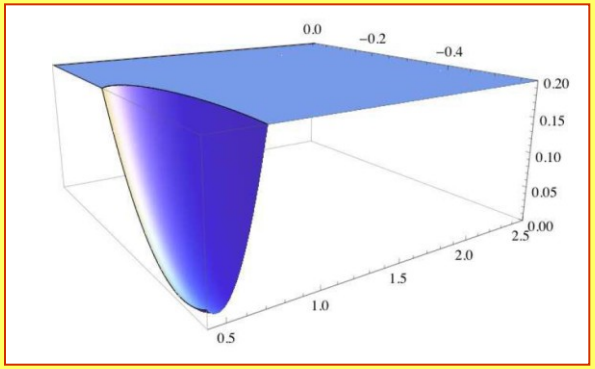
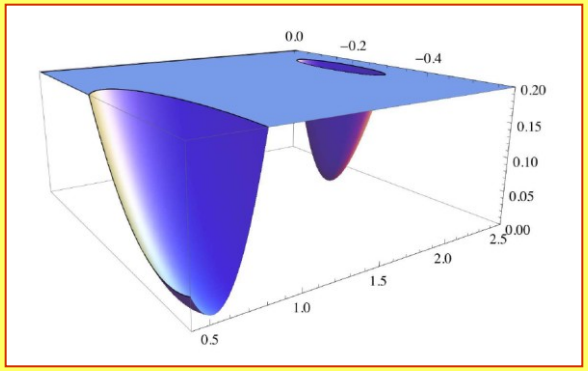
poles

kappa

dynamical



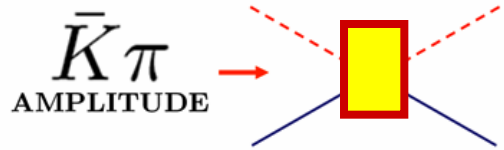


THEORY

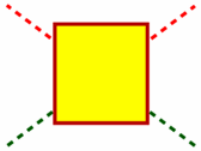
final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE



two-body unitarity



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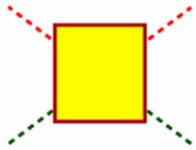
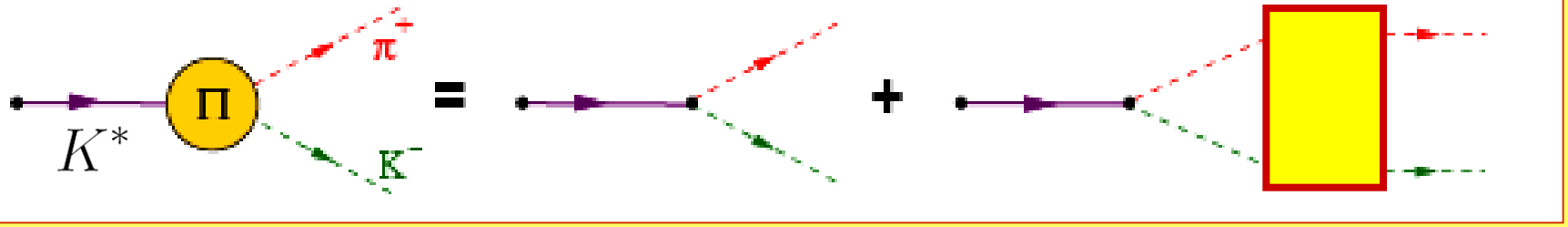


$$= \frac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$$

dynamical width

production amplitude

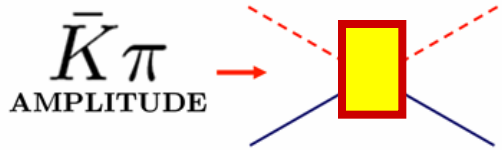


THEORY

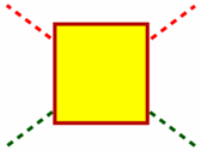
final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE



two-body unitarity



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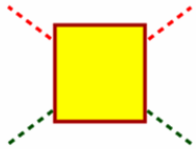


$$= \frac{\mathcal{K}}{1+\bar{\Omega}\,\mathcal{K}}$$

CONSISTENT theoretical scheme

order in the MESS

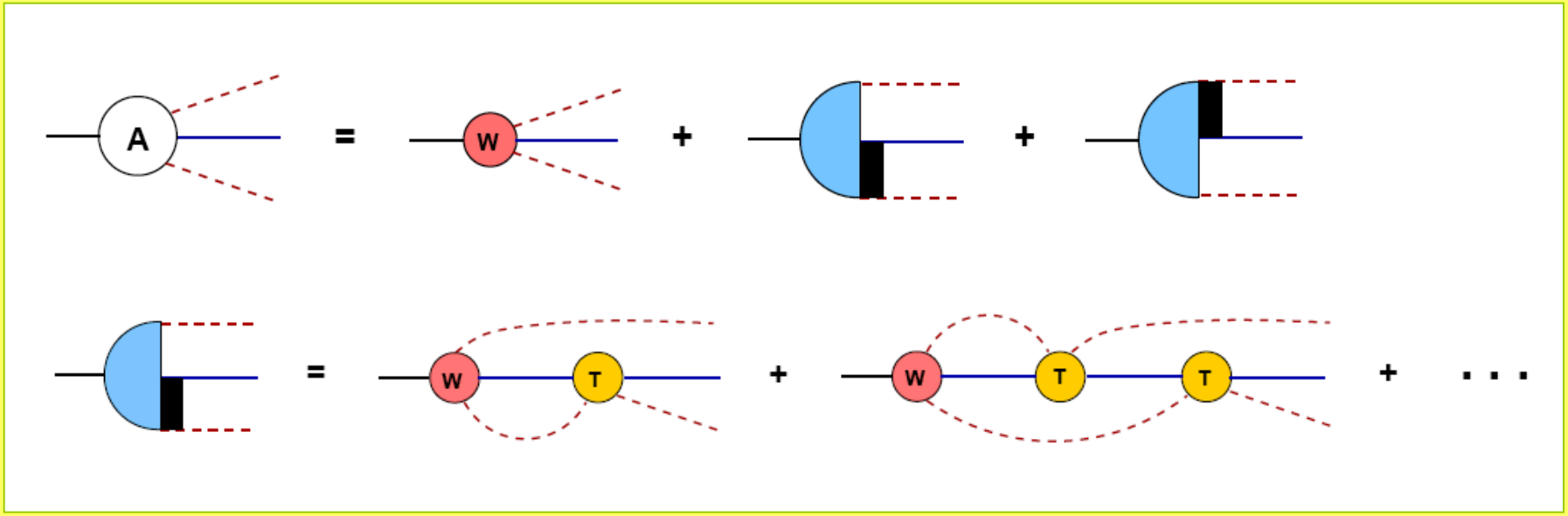
THEORY

final state interactions

$D^+ \to K^-\pi^+\pi^+$

$\bar{K}\pi$ AMPLITUDE

Fadeev series

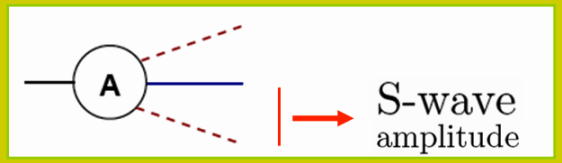


Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

PHYSICAL REVIEW D **84**, 094001 (2011)

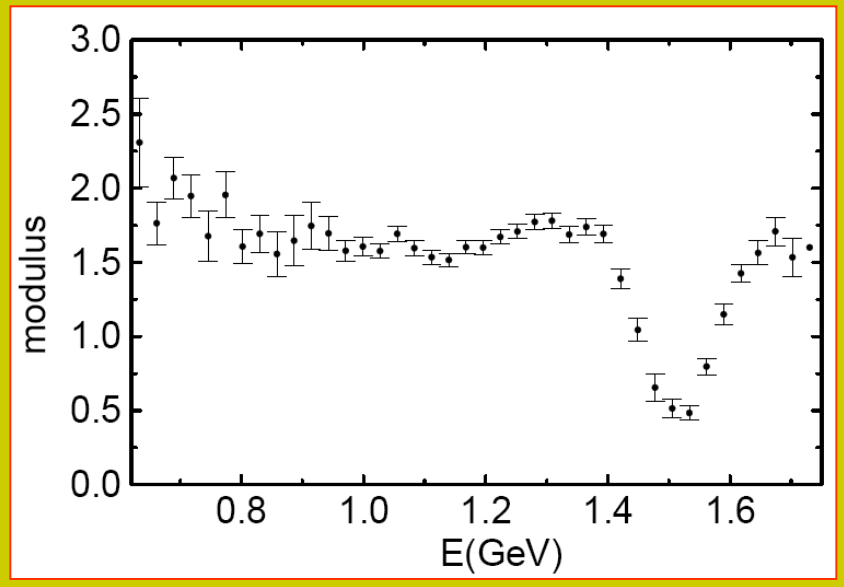
P.C. Magalhães, M.R. R., K.S.F.F. Guimarães, T. Frederico, W.S. de Paula, I. Bediaga, A.C. dos Reis, C.M. Maekawa and G.R.S. Zarnauskas

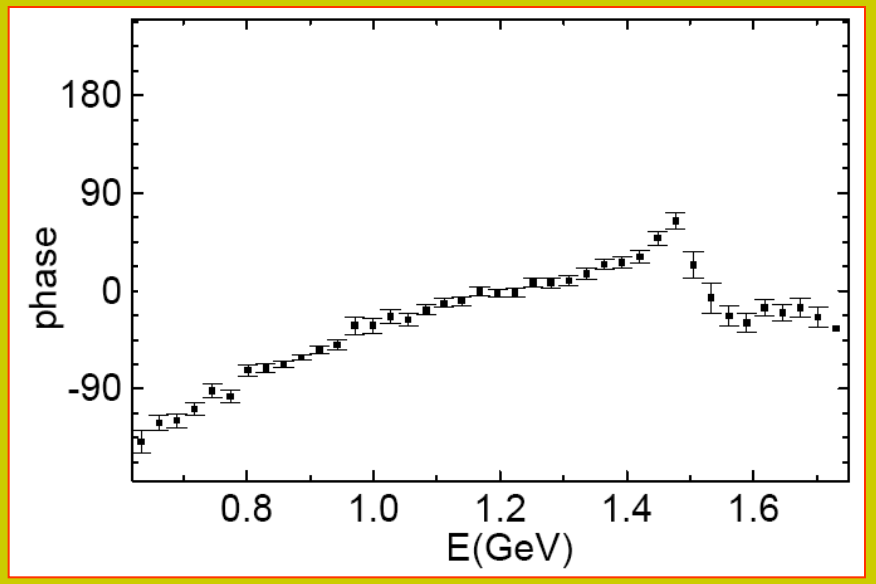
data



E.M. Aitala *et al.* [E791 Collaboration], Phys. Rev. D**73**, 032004 (2006); Erratum-ibid. D **74**, 059901 (2006)

J.M. Link *et al.* [FOCUS Collaboration], Phys. Lett. B **681**, (2009) 14





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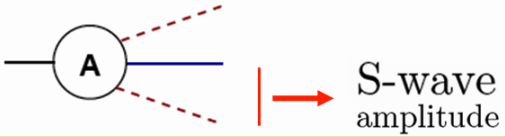
data



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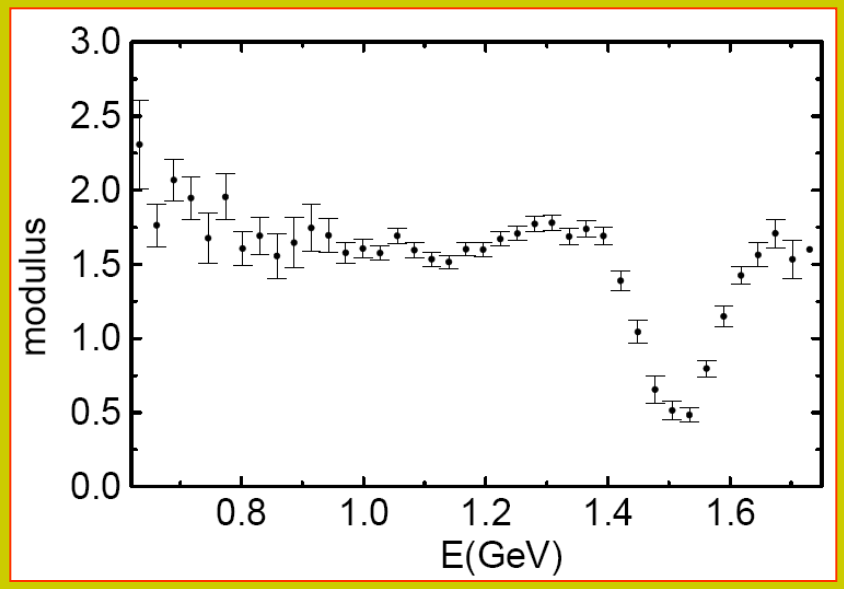

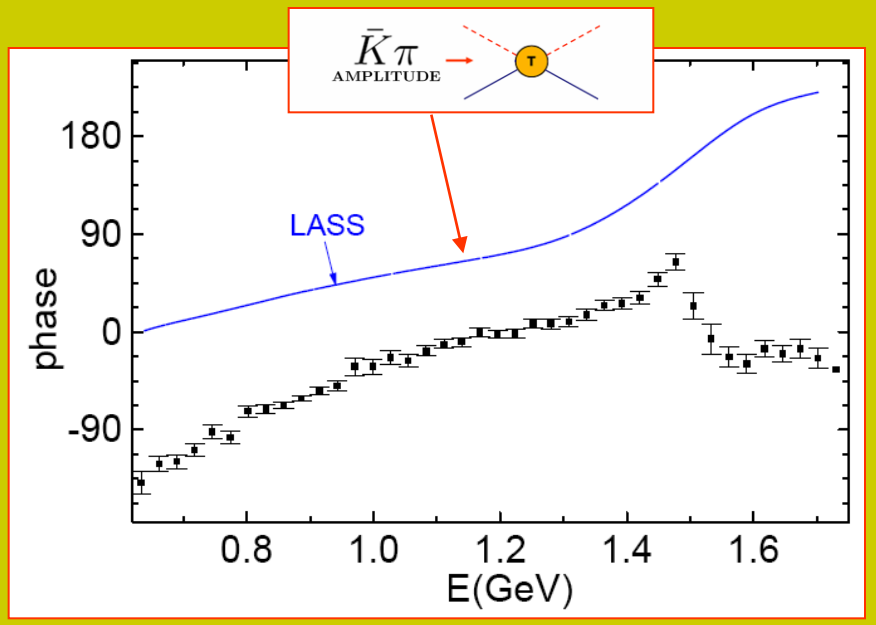

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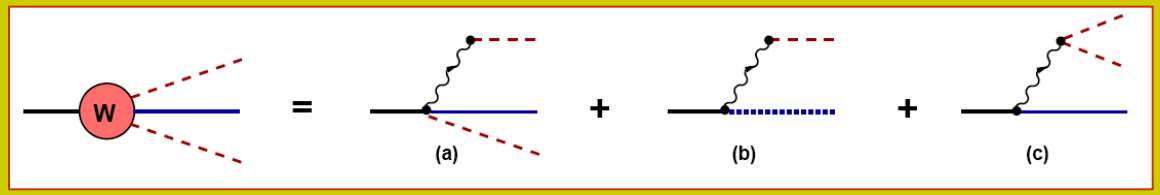

topologies without structure

Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

PHYSICAL REVIEW D **84**, 094001 (2011)

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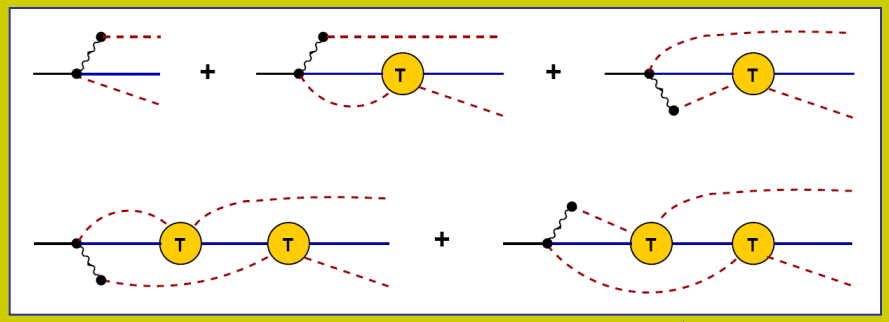

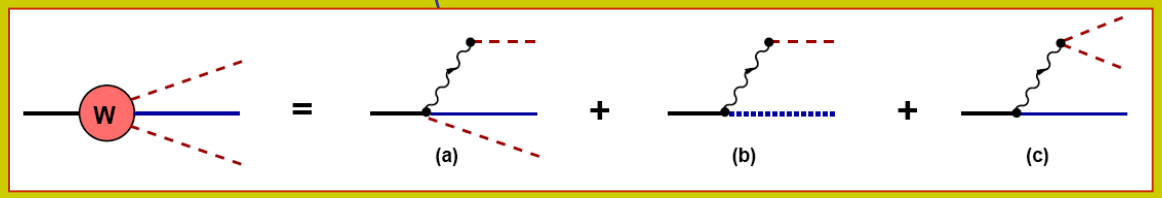

topologies without structure

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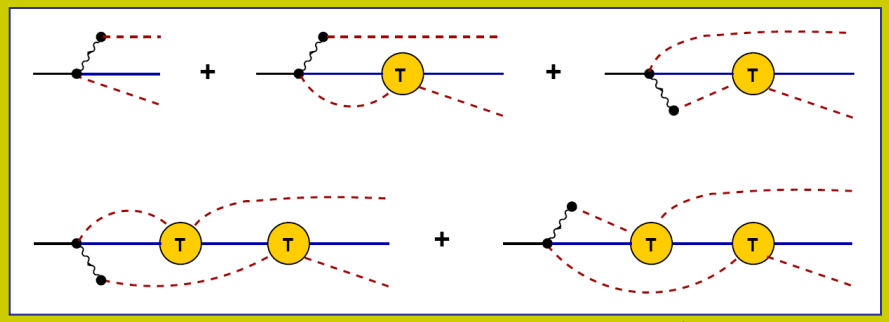

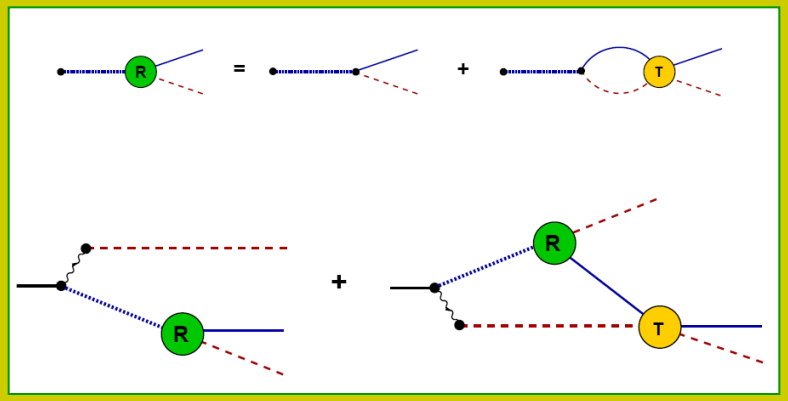

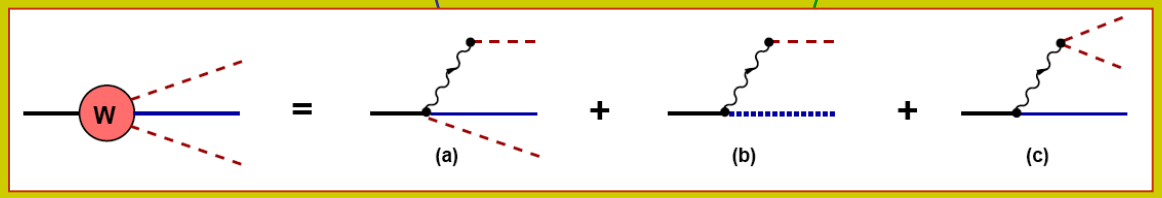

topologies without structure

Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

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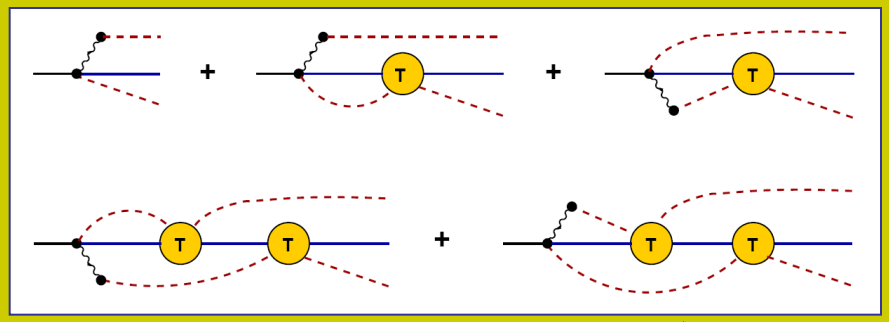
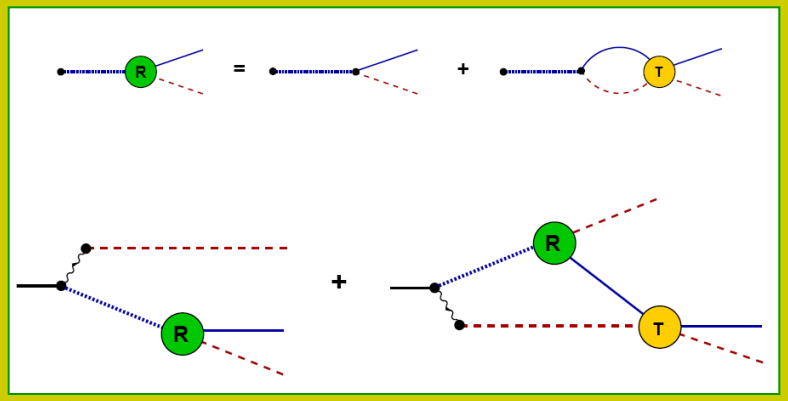
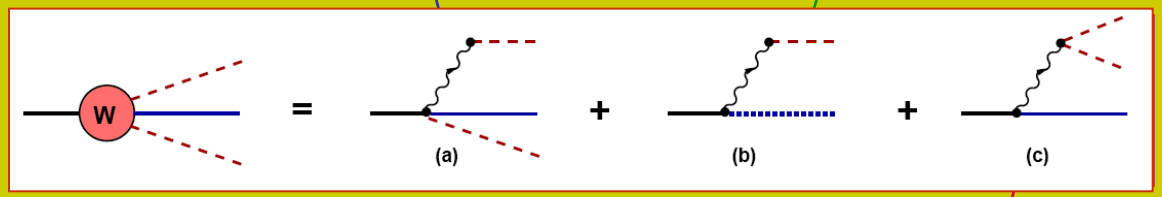
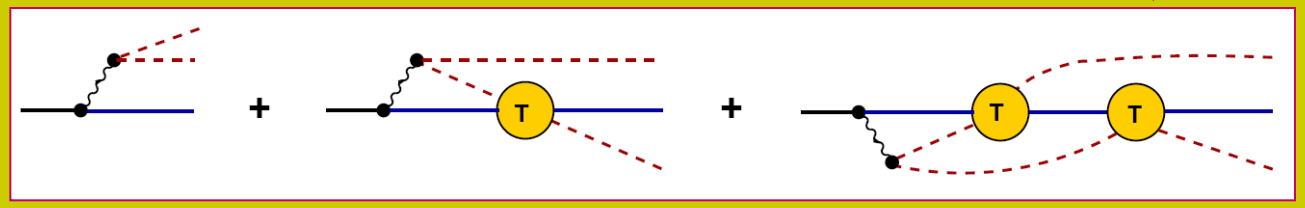






topologies without structure



Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

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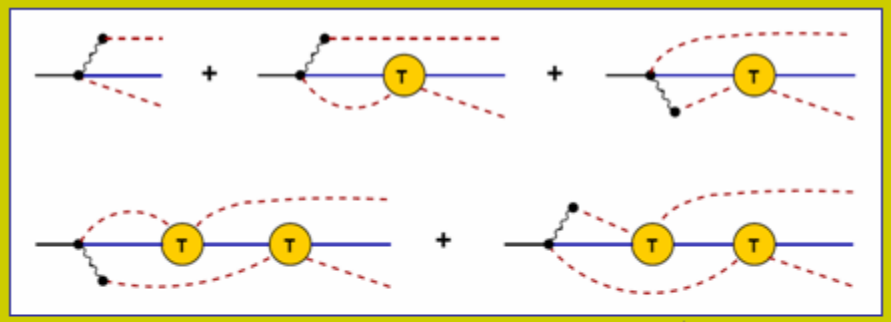
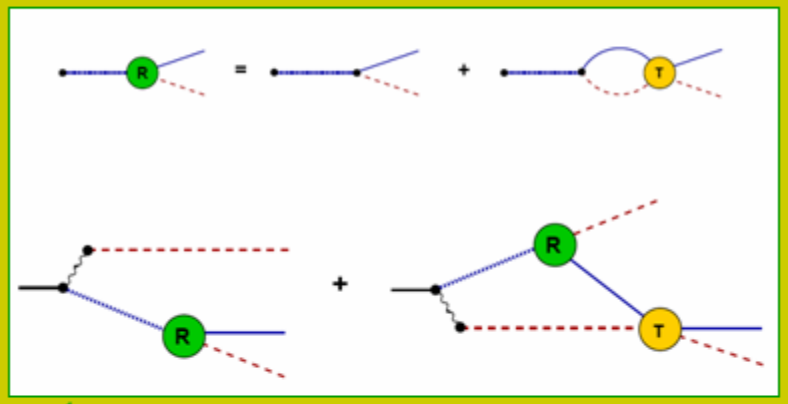
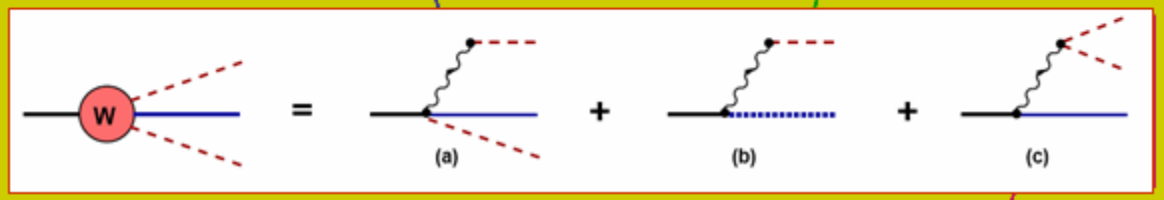
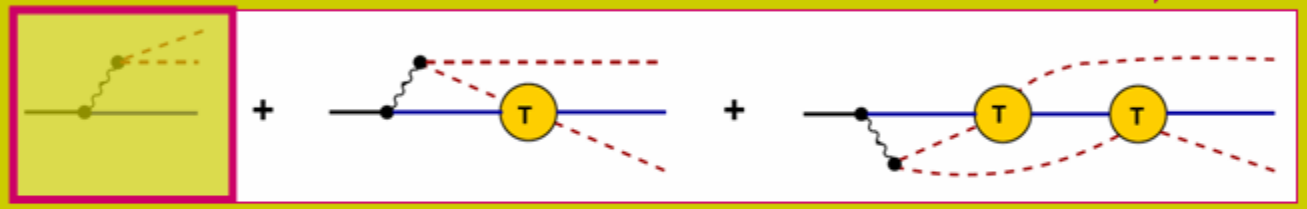






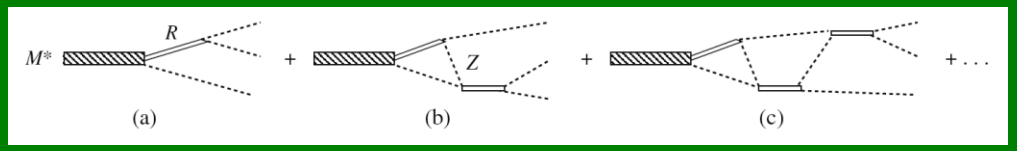
topologies without structure

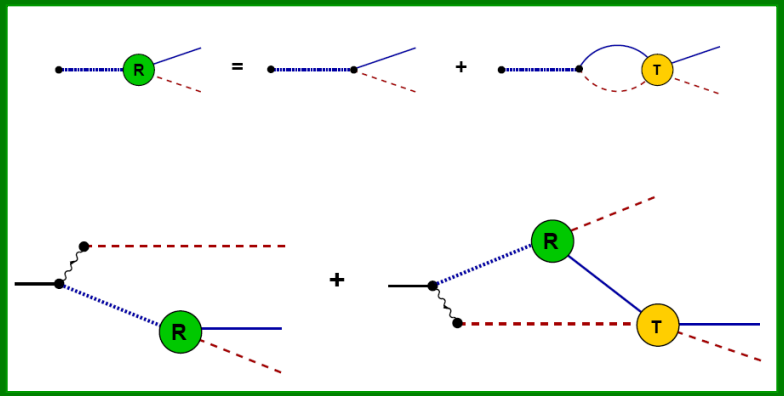


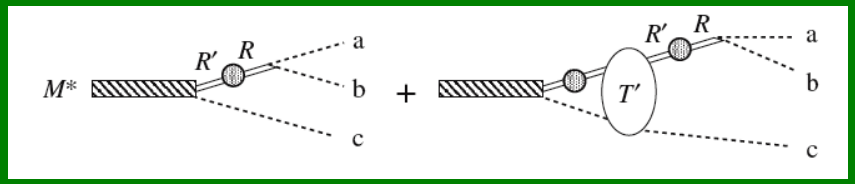
PHYSICAL REVIEW D **84,** 114019 (2011)

Unitary coupled-channels model for three-mesons decays of heavy mesons

H. Kamano,[1,2] S. X. Nakamura,[2] T.-S. H. Lee,[3,2] and T. Sato[4,2]







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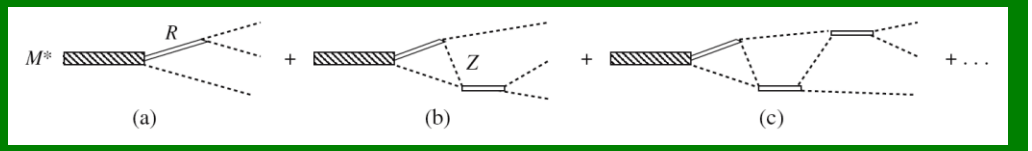

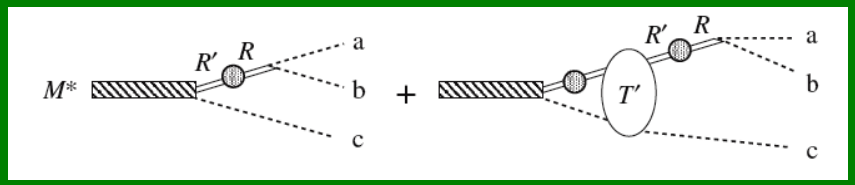

PHYSICAL REVIEW D **84,** 114019 (2011)

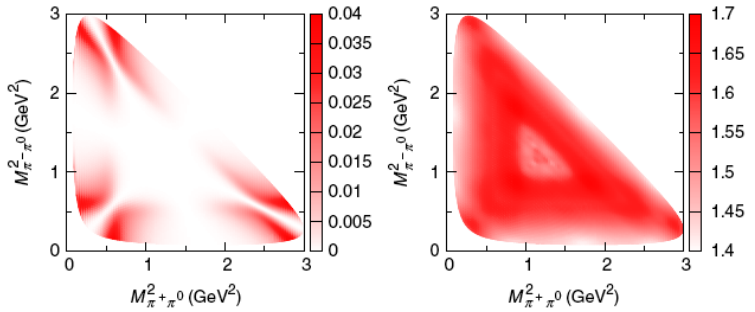

FIG. 4 (color online). (left) Dalitz plot of $D^0 \to \pi^+\pi^-\pi^0$ decay; (right) ratio of Dalitz plot distributions with and without the Z graphs.

Dalitz plot distributions by as much as 30% in magnitudes in the regions where $f_0(600)$, $\rho(770)$, and $f_2(1270)$ dominate the distributions. Also, by fitting to the same Dalitz plot distributions, we demonstrate that the decay amplitudes obtained with the unitary model and the isobar model can be rather different, particularly in the phase that plays a crucial role in extracting the Cabibbo-Kobayashi-Maskawa CP-violating phase from the data of B meson decays. Our results indicate that the commonly used isobar-model analysis must be extended to account for the final state interactions required by the three-body unitarity to reanalyze the three-mesons decays of heavy mesons, thereby exploring hybrid or exotic

Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

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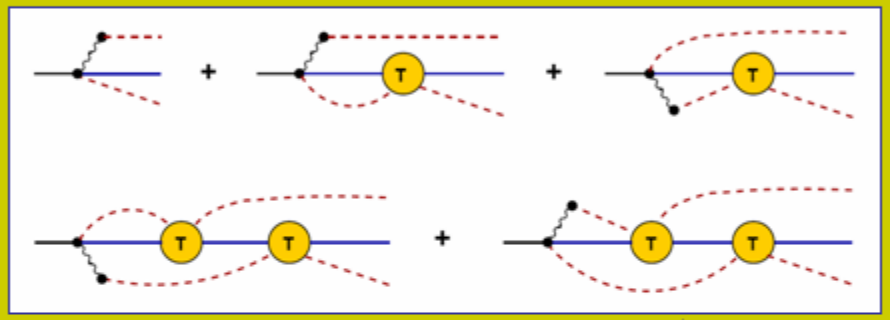
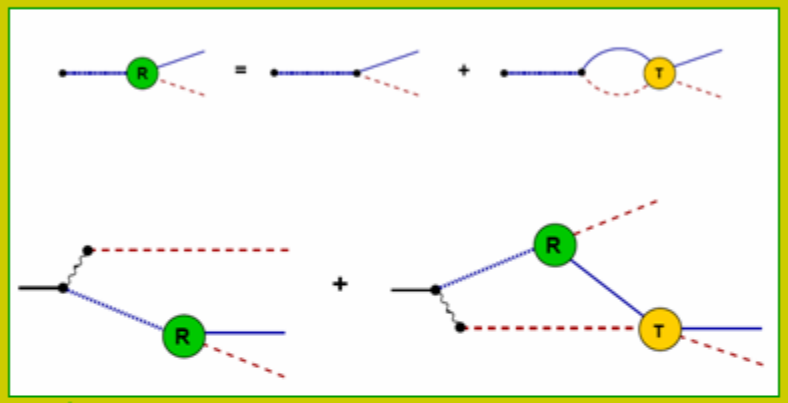
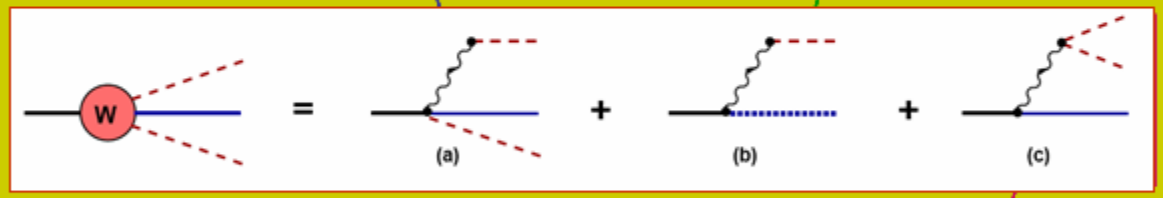
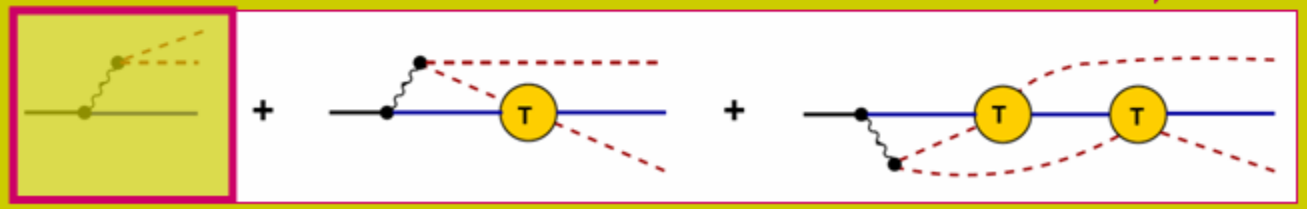
I. Bediaga, A.C. dos Reis, C.M. Maekawa and G.R.S. Zarnauskas



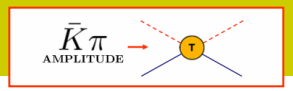


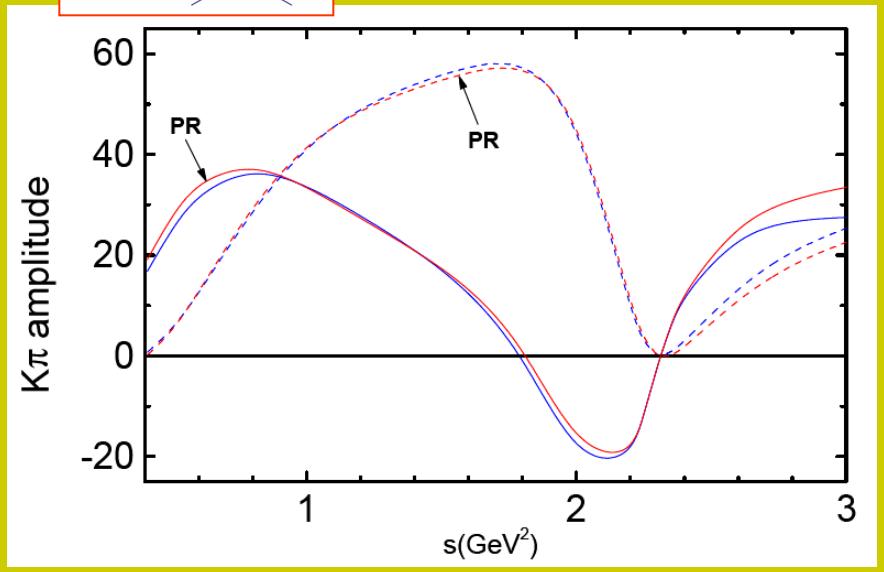


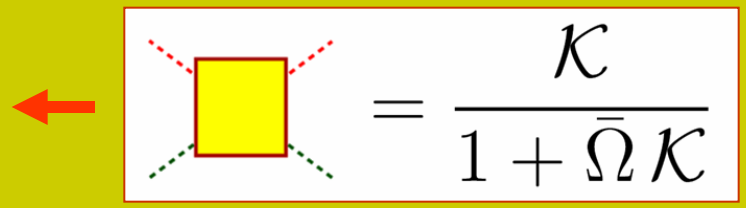
topologies without structure

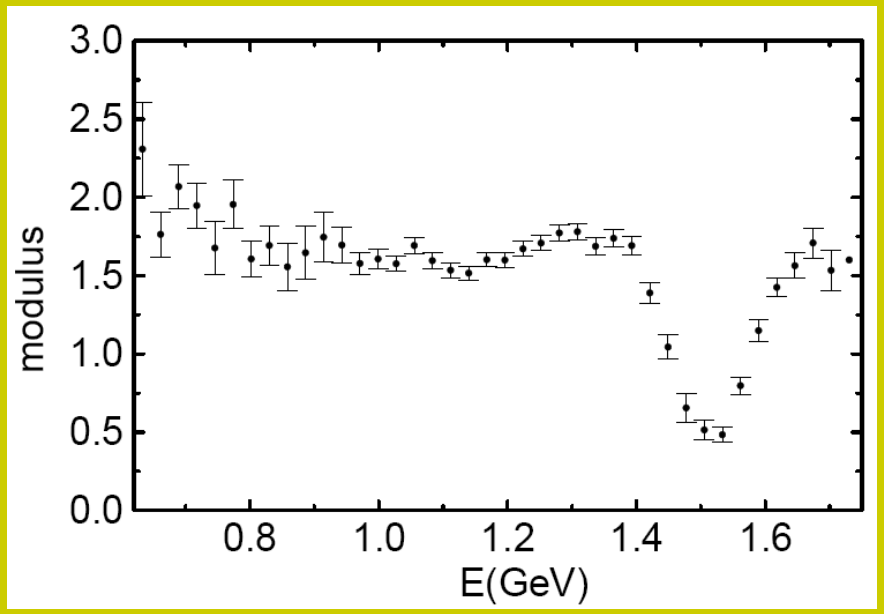


Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

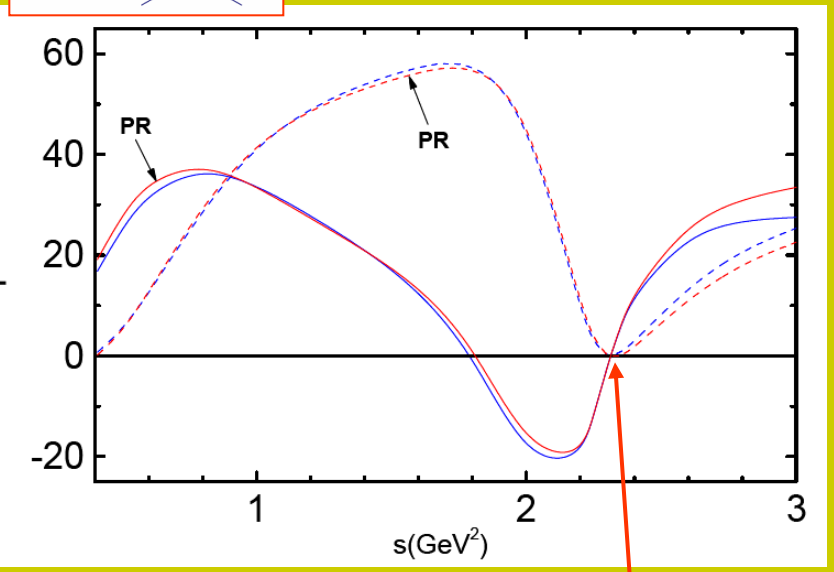


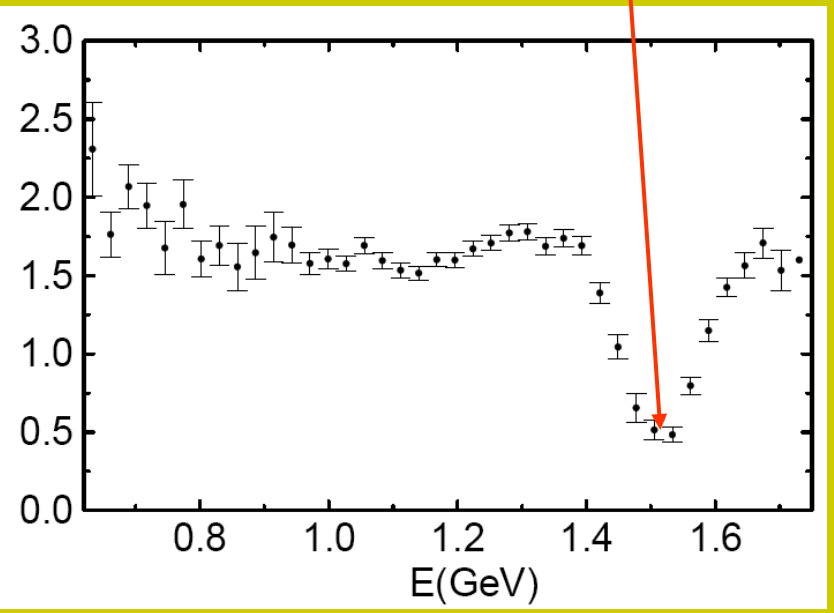






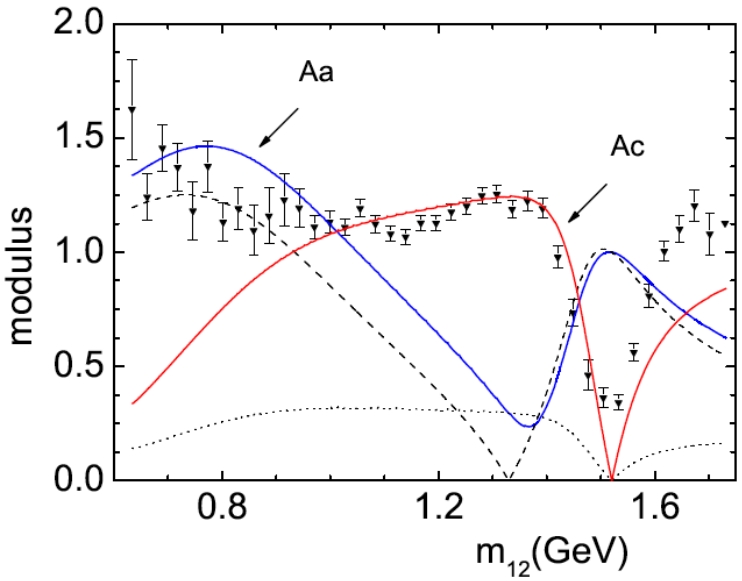
Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

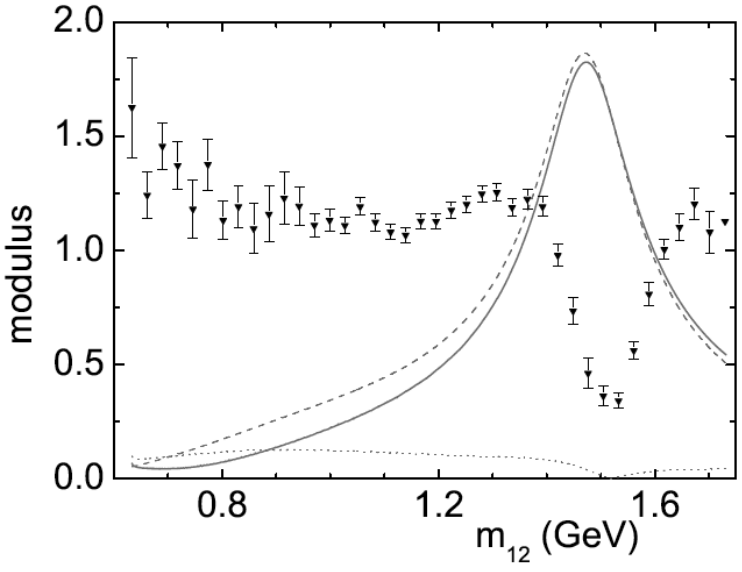


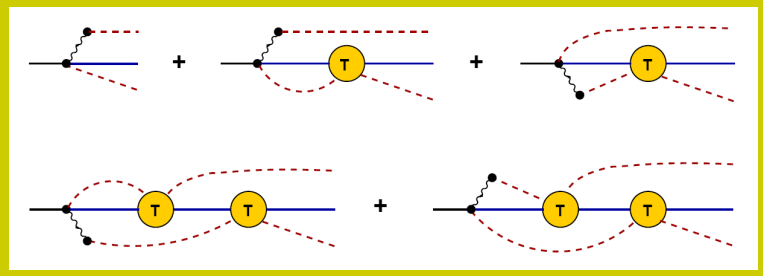


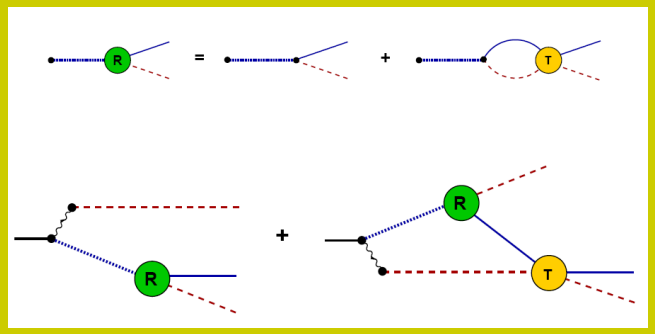
$$= \frac{\mathcal{K}}{1 + \bar{\Omega}\mathcal{K}}$$

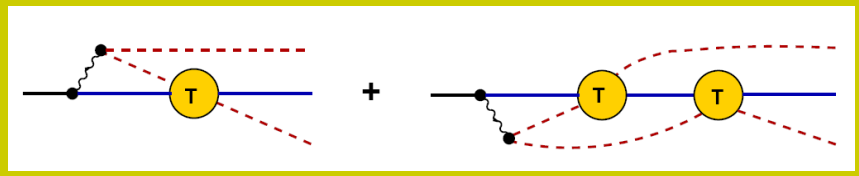
Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$



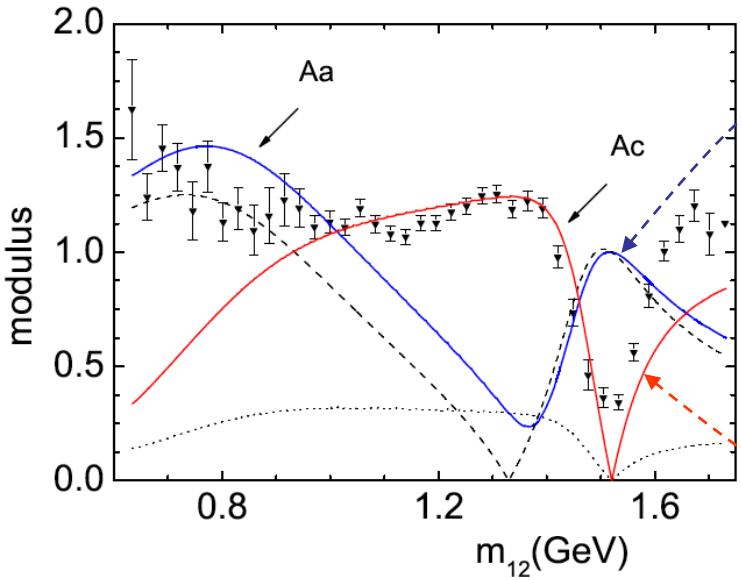


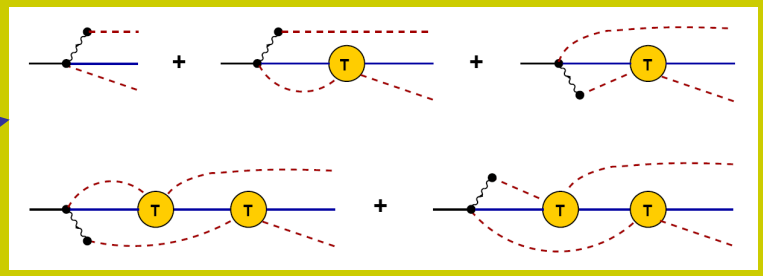


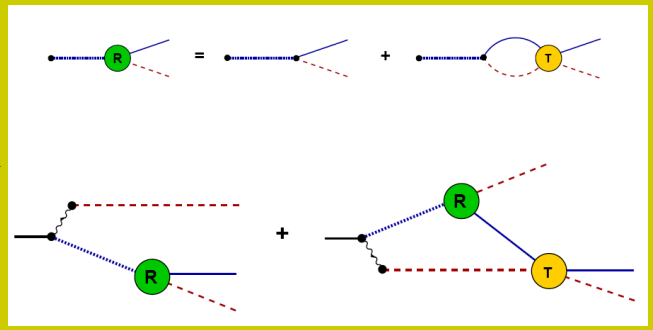


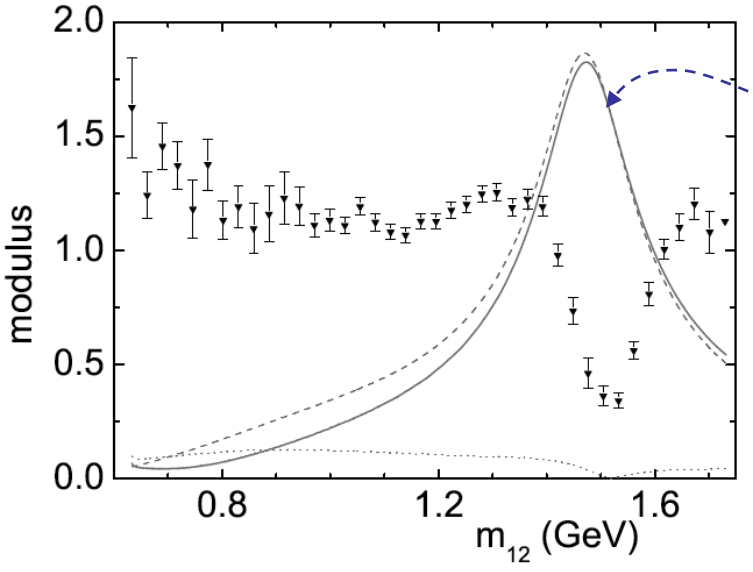


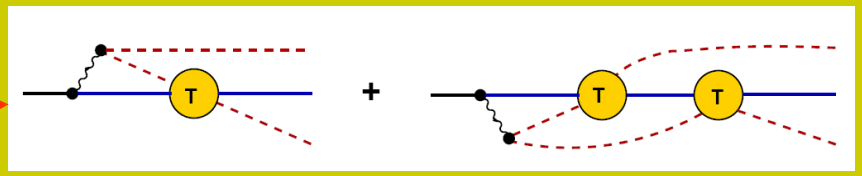
Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$



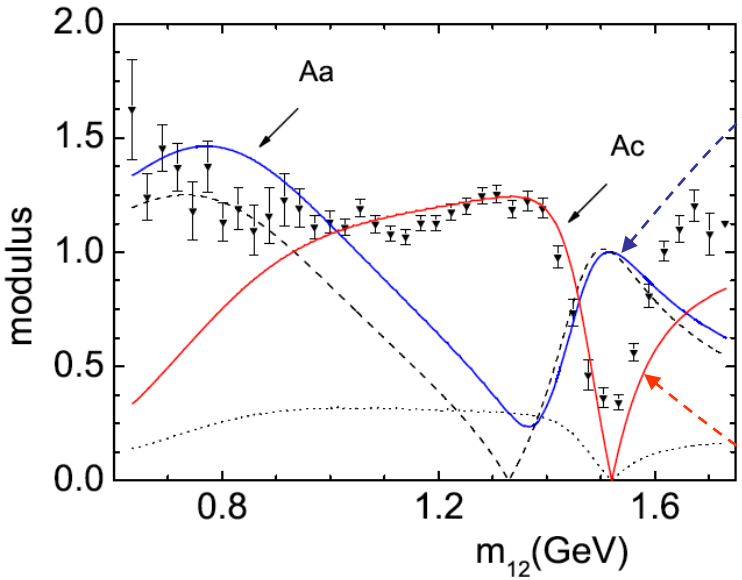


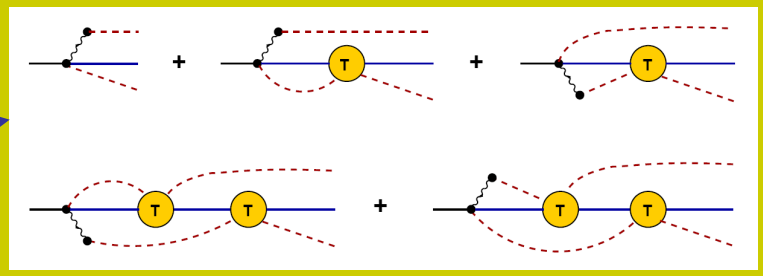


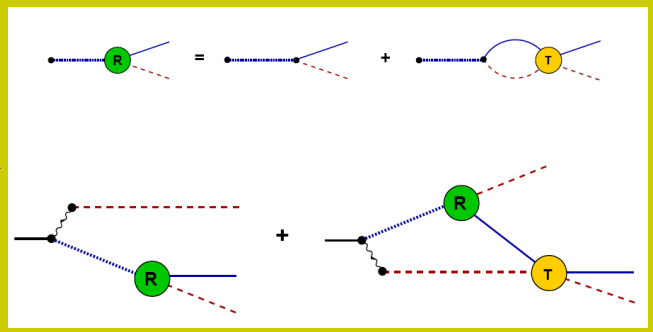


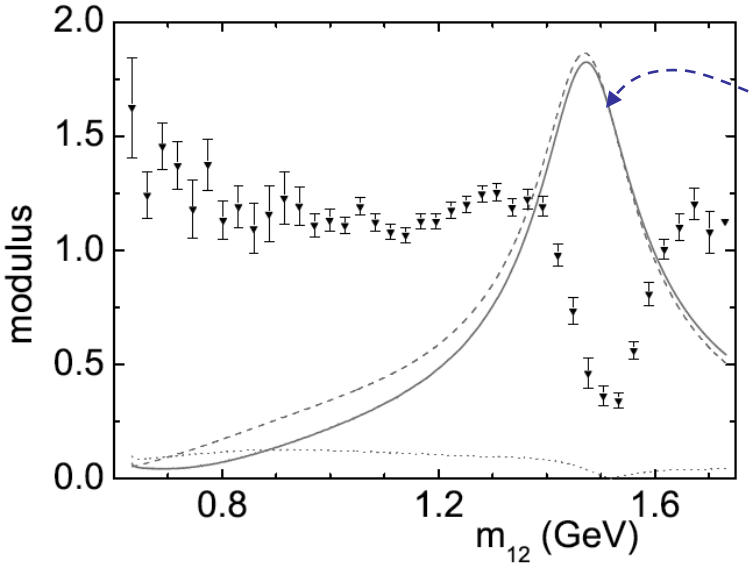


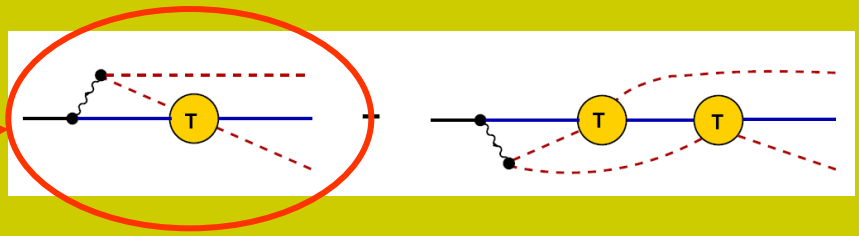
Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$



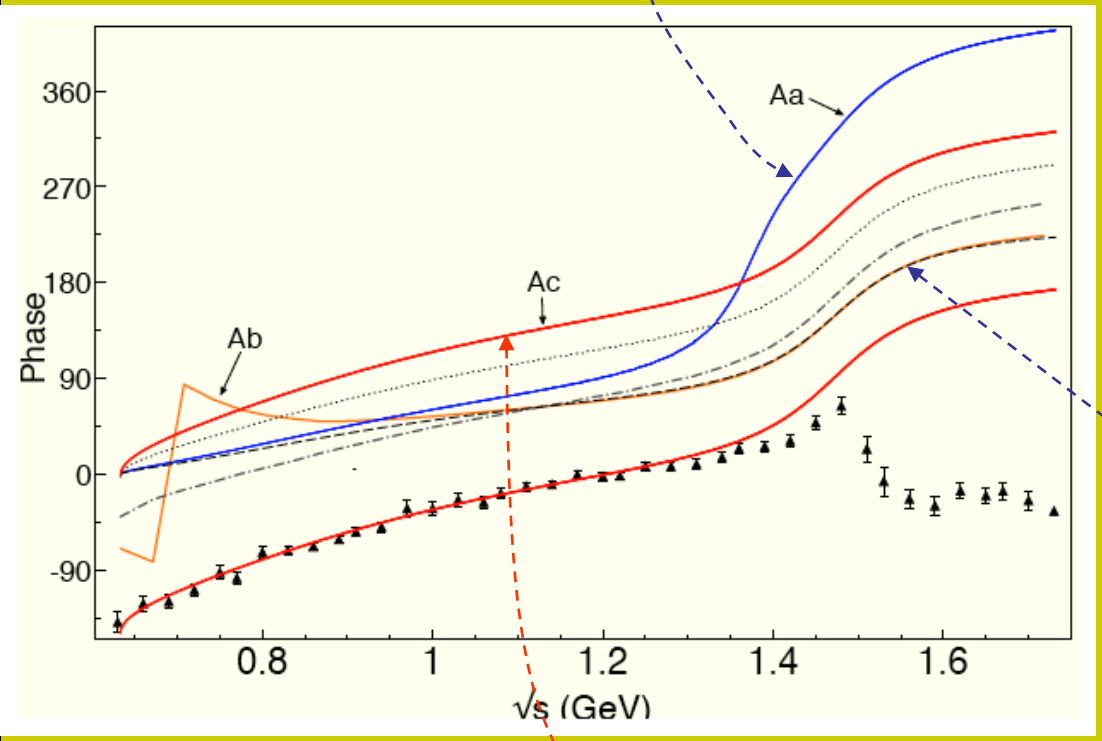


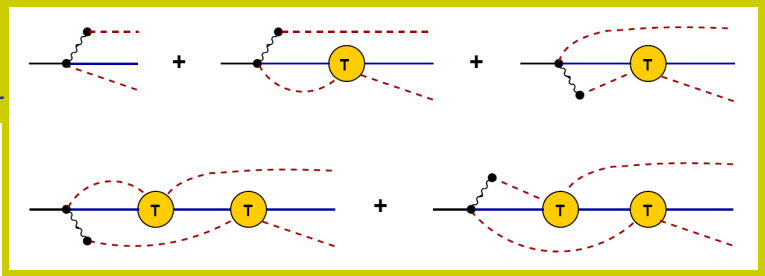


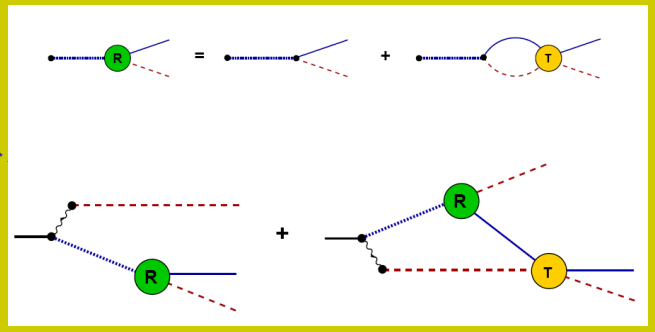


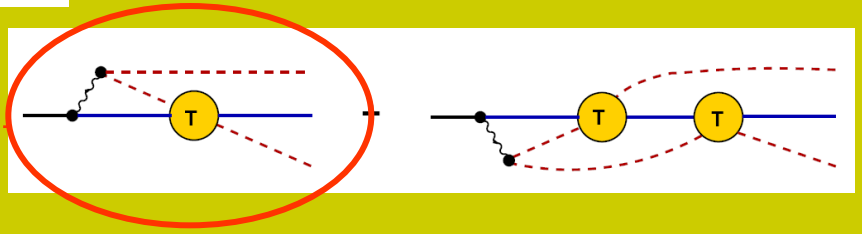


Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

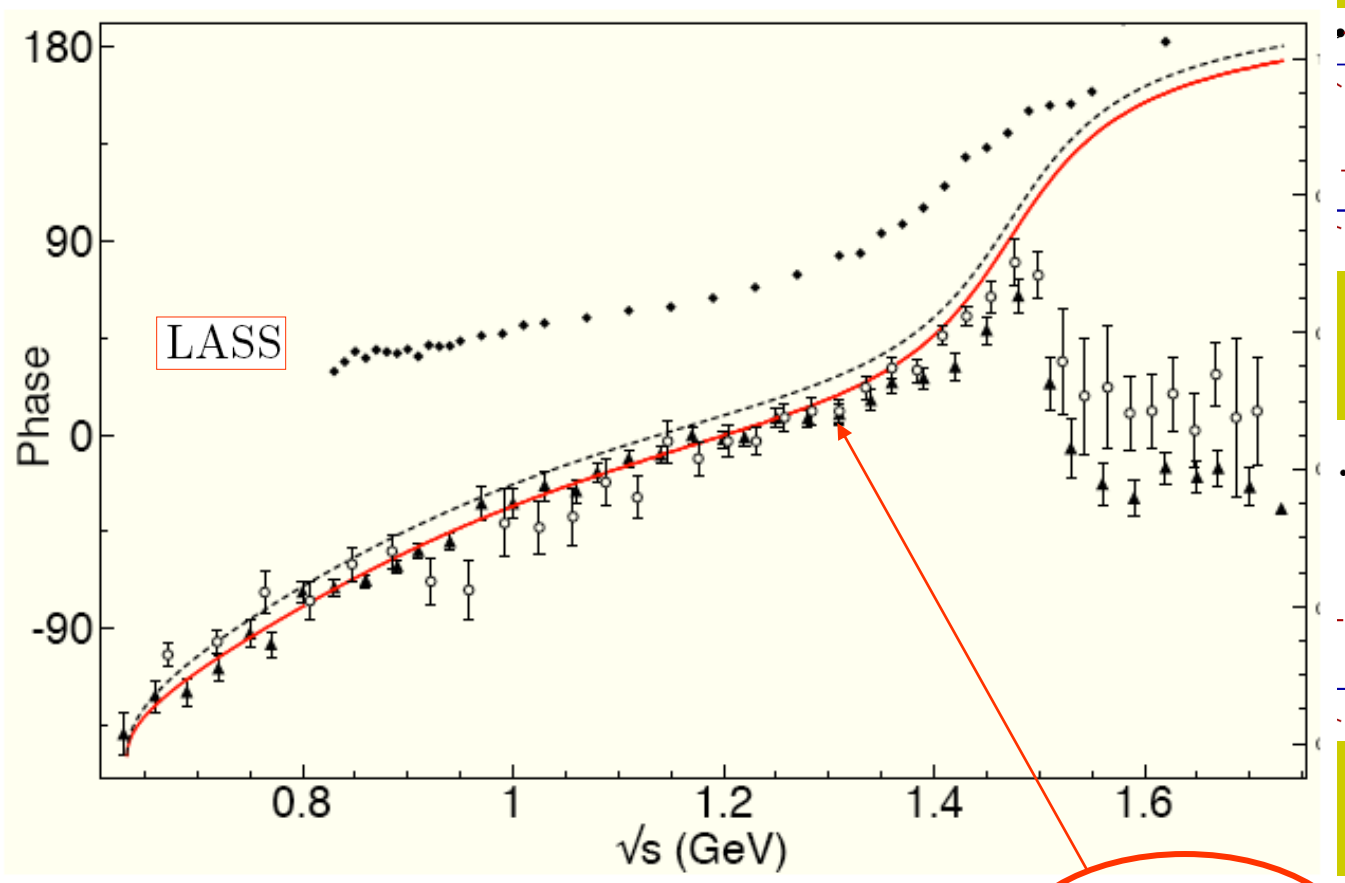


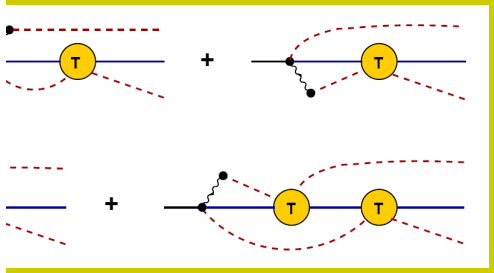


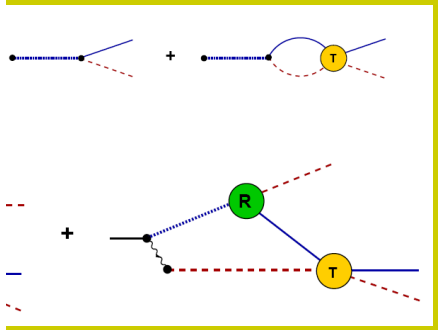


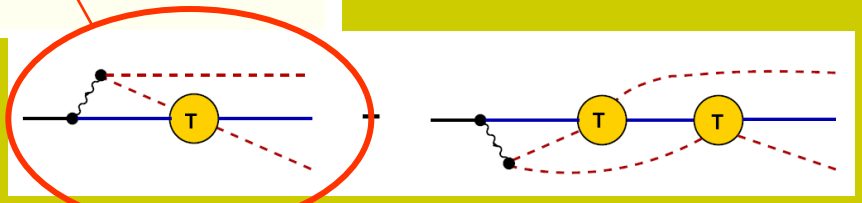


Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$









Towards three-body unitarity in $D^+ \to K^-\pi^+\pi^+$

conclusions

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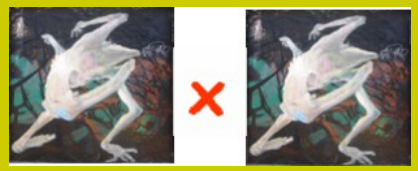
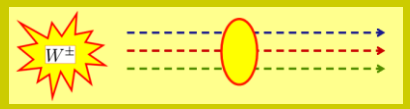
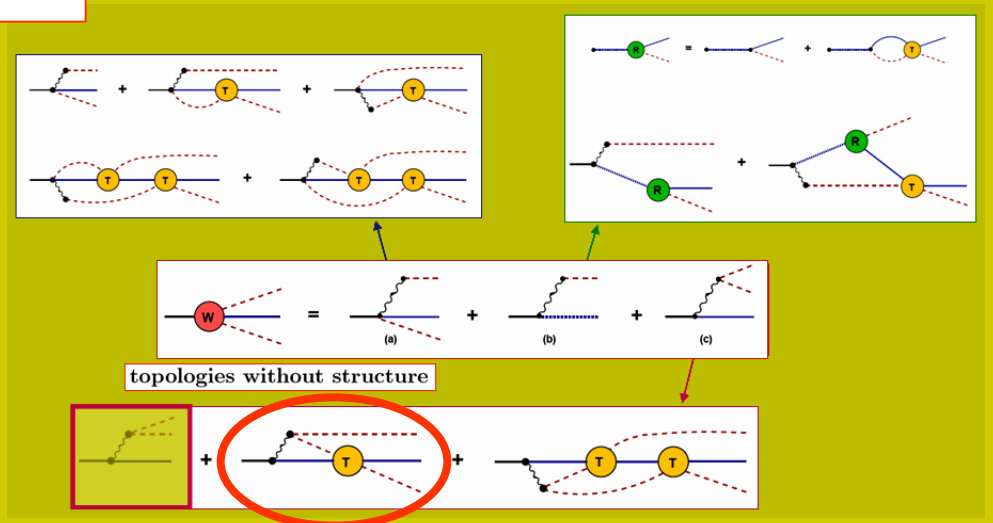




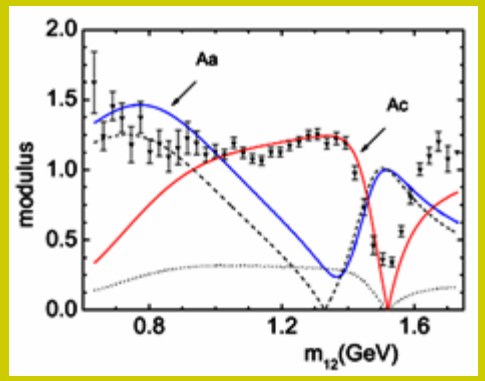


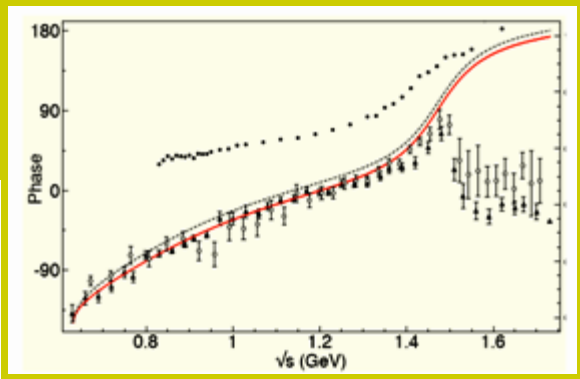
- strong series converges rapidly
- dynamics:

→ leading mechanism identified
→ vector weak vertex
→ no tree contribution

- Watson's theorem does not apply

→ two phases around





conclusions

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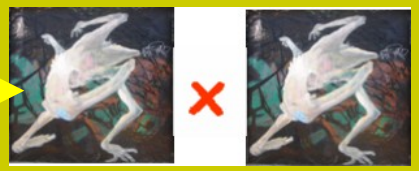
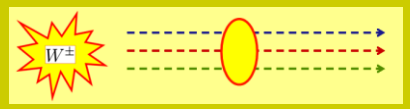
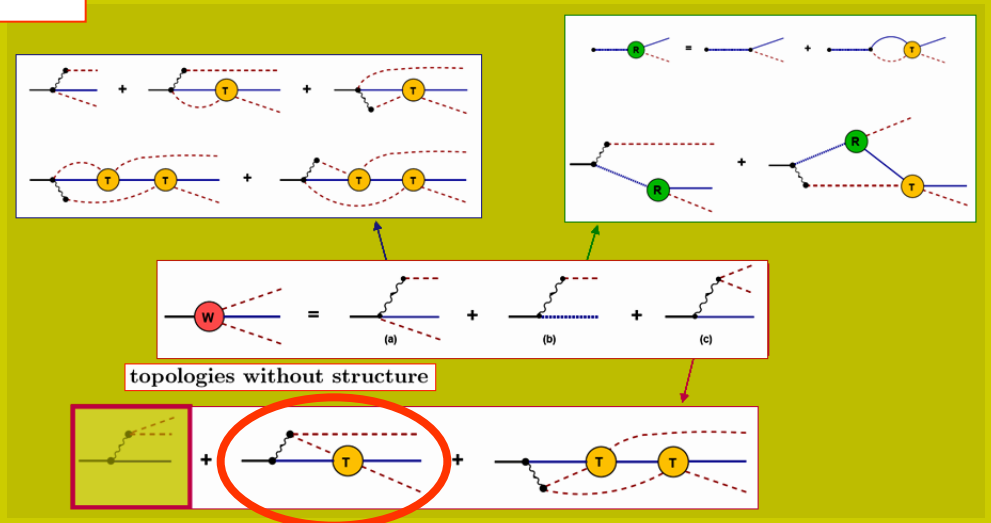
BUT...







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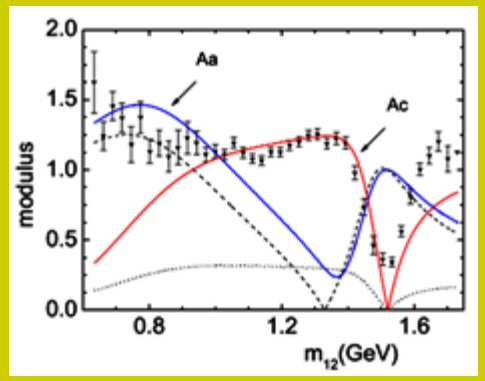
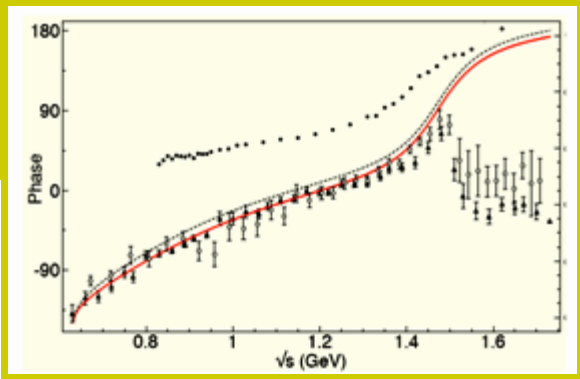


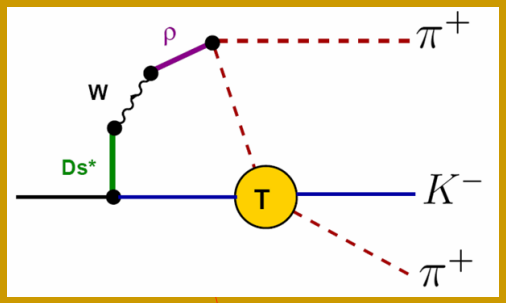


research programme

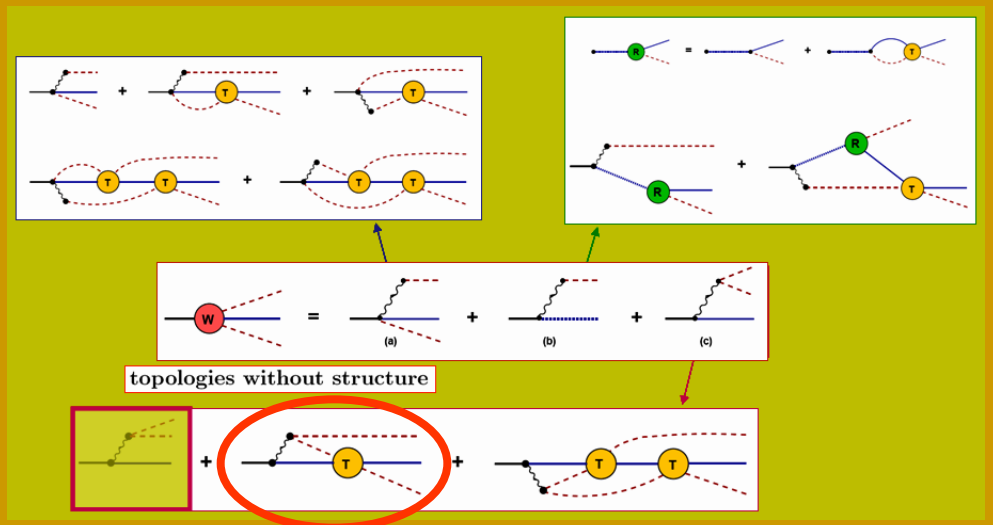
recent progress

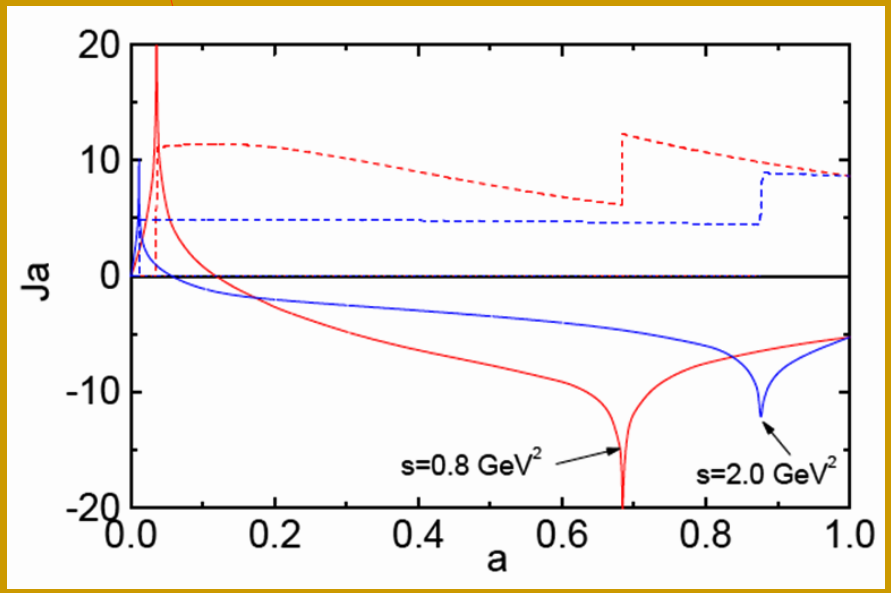
$D^+ \to K^-\pi^+\pi^+$

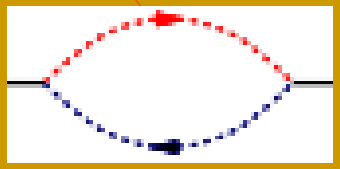
Patricia C. Magalhães → Ph.D.



rich phase structure



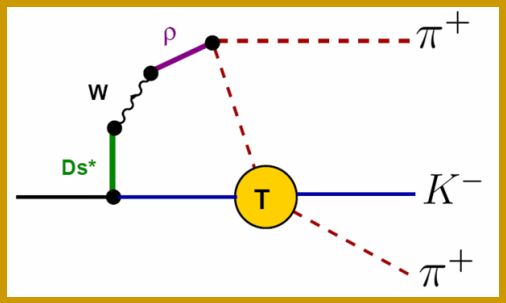




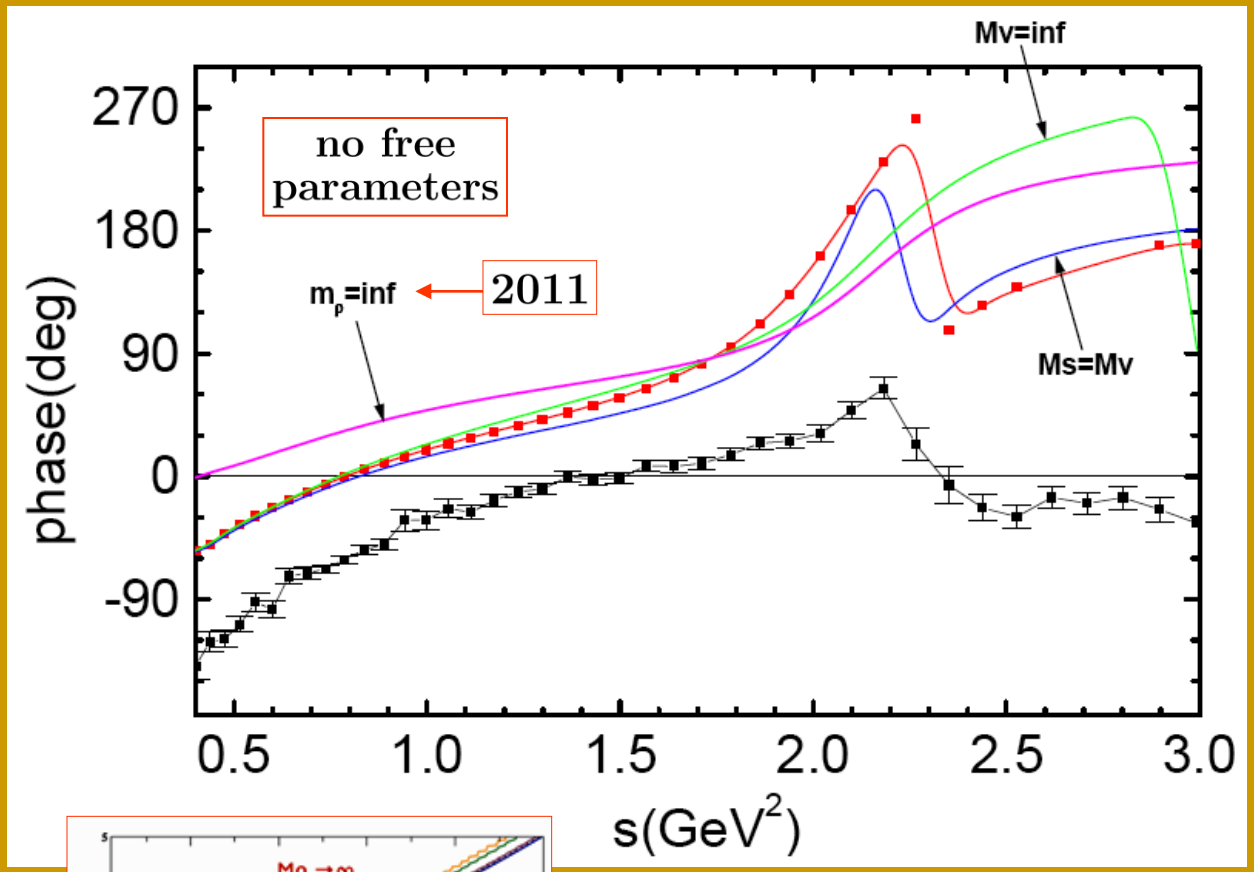
research programme

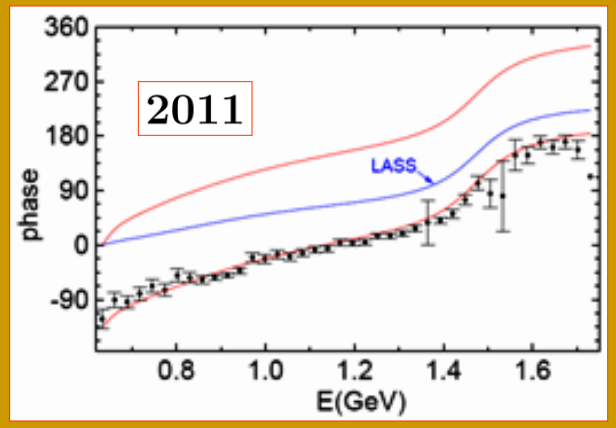
recent progress

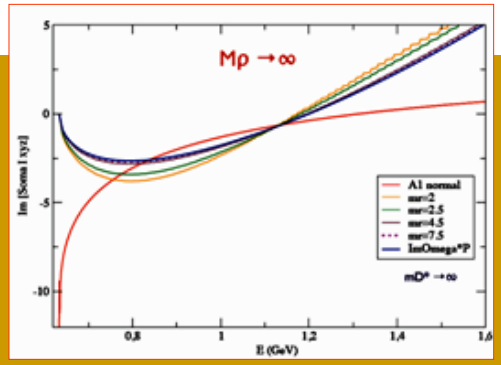
$D^+ \to K^-\pi^+\pi^+$



rich phase structure



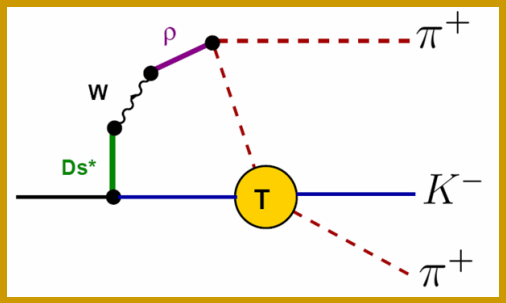




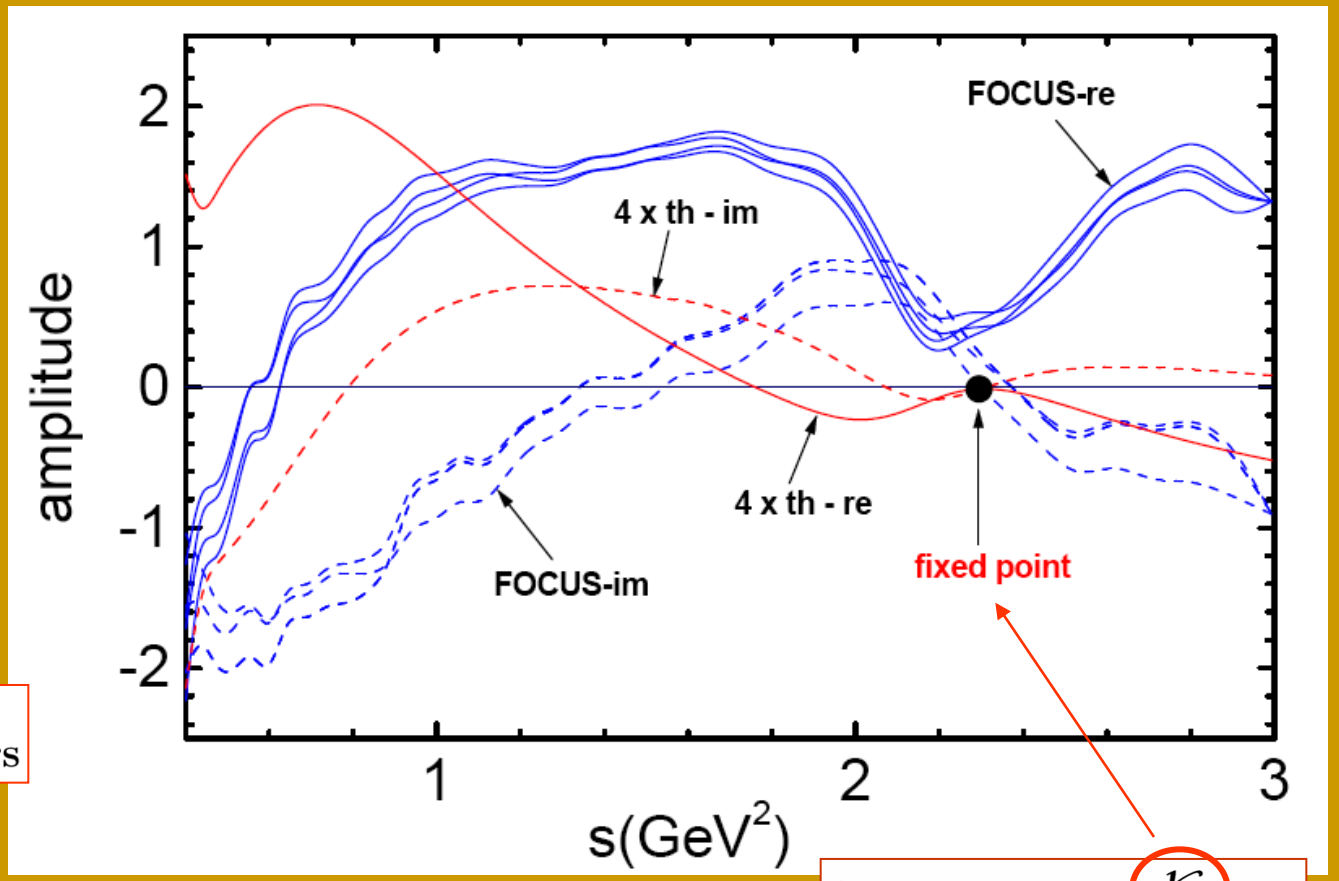
research programme

recent progress

$D^+ \to K^-\pi^+\pi^+$



rich phase structure



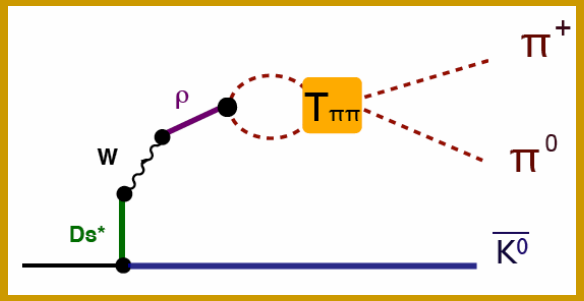
no free parameters

$$= \frac{\mathcal{K}}{1 + \bar{\Omega}\,\mathcal{K}}$$

research programme

recent progress

$D^+ \to K^0 \pi^0 \pi^+$



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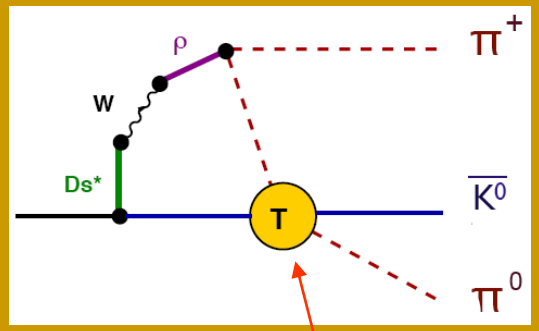
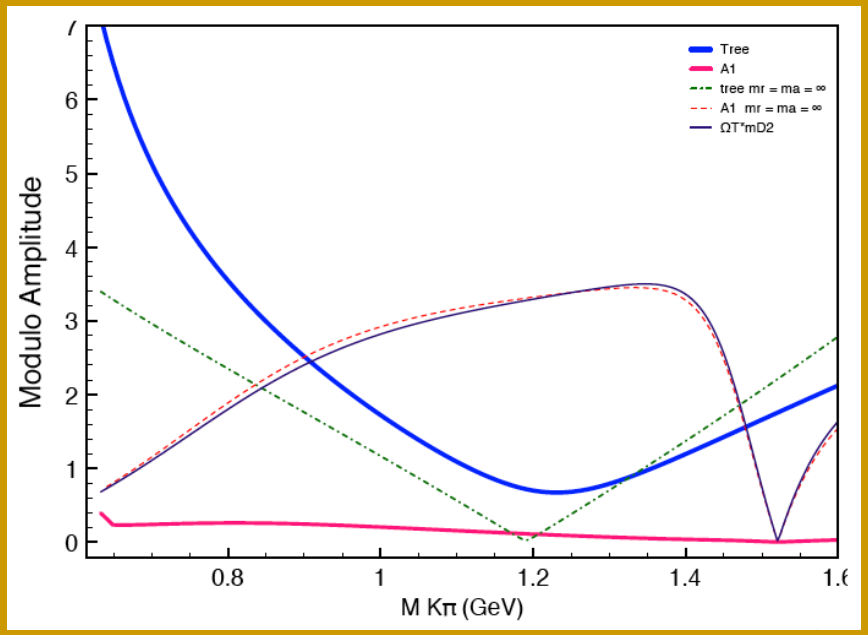


empirical amplitude

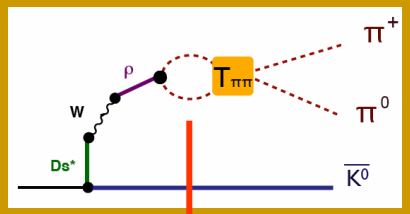
get rid of model dependence

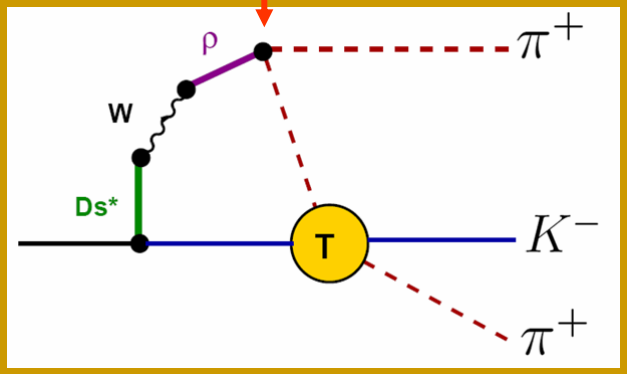


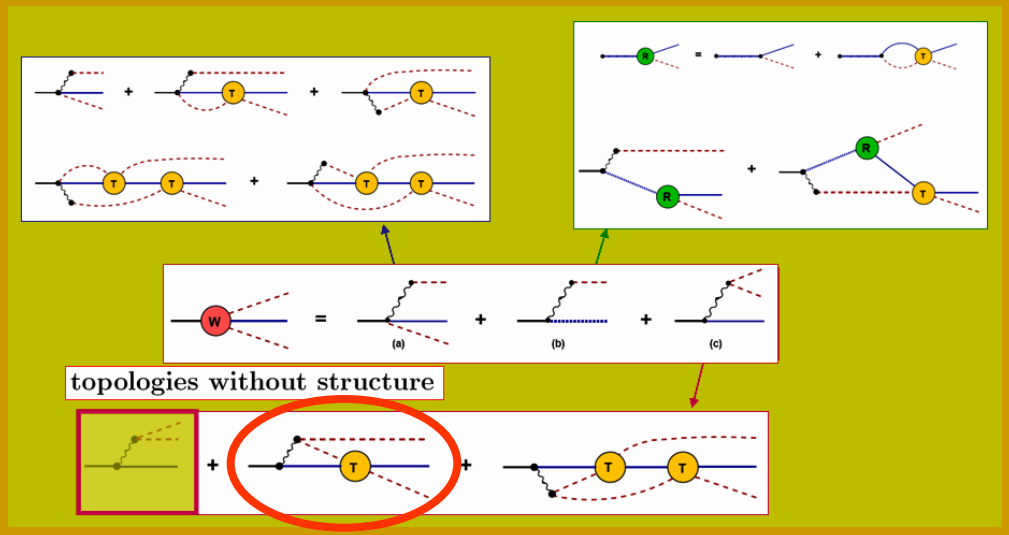
research programme

next steps

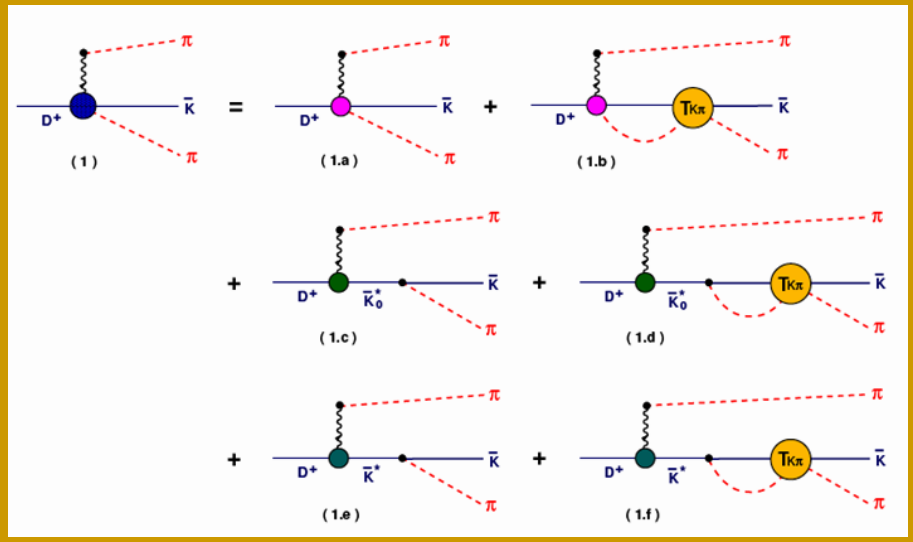
$D^+ \to K^-\pi^+\pi^+$







QM - everything that is not forbidden, is compulsory



subject is crawling ...