Measurement of the angle γ (ϕ_3) at the e⁺e⁻ B-factories

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The angle γ

CKM is 3x3 unitary matrix → 4 parameters (after ad hoc choice of quark field phases): 3 real and 1 CP violating phase

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$



Measurement of γ with B \rightarrow DK



 $A_1 \propto V_{ch} V_{\mu s}^* \sim A \lambda^3$



color suppressed $A_2 \propto V_{ub}V_{cs}^* \sim A\lambda^3(\rho + i\eta)$

- γ is measured in the interference of the two amplitudes $|A_{tot}|^2 = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos\phi$ $\phi = -\gamma + \delta$
- unknowns: γ , $(r_b, \delta_b) + \delta_D$
- theoretically clean

B hadronic parameters extracted with γ measured at charm factories

 $r_b \equiv \left| \frac{A(B^+ \to D^0 K^+)}{A(B^+ \to \overline{D^0} K^+)} \right| \sim 0.1$

 $\delta_b + \delta_D = \frac{\text{strong phase}}{\text{from B and D decay}}$

- most sensitive method to constrain γ at present
- similar principle applies to several other processes: $B^0 \rightarrow D^{(*)+}\pi^-$, $B^0 \to D^0 K^{(*)0}, B_s \to D_s K, ...$

Extraction of γ using B \rightarrow DK decays

Different methods depending on D final state:

- GLW [M. Gronau, D. London, D. Wyler, PLB253,483 (1991); PLB 265, 172 (1991)]
 - D⁰ to two-body CP eigenstates K⁺K⁻, $\pi^+\pi^-$ (even), K_s π^0 , K_s ω (odd)
- ADS [D. Atwood, I. Dunietz, A. Soni, PRL 78, 3357 (1997)]
 - D⁰ to doubly Cabibbo suppressed decays $K^+\pi^-$, $K^+\pi^-\pi^0$, ...
- GGSZ (Dalitz) [D. Atwood et al., PRL78, 3257 (1997); A. Giri et al., PRD68, 054018 (2003)]
 - D⁰ to 3-body decays $K_s \pi^+ \pi^-$, $K_s K^+ K^-$, $\pi^+ \pi^- \pi^0$, etc.
 - Dalitz plot fitted to determine how the strong phase of D⁰ decay amplitude varies over the Dalitz plane
- most powerful method nowadays

- model independent analysis
- Different B decays D⁰K[±], D^{*0}K[±], D⁰K^{*±} and flavour-tagged D⁰K^{*0}. They depend on mode-dependent hadronic factors (r_b, δ_b)
- Strategy: combine as many channels as possible to improve the overall sensitivity

The B-factories dataset



1998/1 2000/1 2002/1 2004/1 2006/1 2008/1 2010/1 2012/1

The GGSZ method

The interference varies as function of the position in the D⁰ Dalitz plot



Reconstructed decay modes





both BaBar and Belle fit signal vs Dalitz plot position using likelihood(ΔE , M_{bc}, event shape vars)

Measurement of x_{\pm}, y_{\pm}

• Extract the *cartesian coordinates* instead of γ , r_b , δ_b (likelihood unbiased and Gaussian-shaped using x,y)

 $\Gamma(B^{+}) \propto |f_{+}|^{2} + (x_{+}^{2} + y_{+}^{2})|f_{-}|^{2} + 2x_{+} \operatorname{Re}(f_{+}f_{-}^{*}) + 2y_{+} \operatorname{Im}(f_{+}f_{-}^{*})$ $\Gamma(B^{-}) \propto |f_{-}|^{2} + (x_{-}^{2} + y_{-}^{2})|f_{+}|^{2} + 2x_{-} \operatorname{Re}(f_{-}f_{+}^{*}) + 2y_{-} \operatorname{Im}(f_{-}f_{+}^{*})$ $f_{\mp} \equiv A_{D}(m_{\mp}^{2}, m_{\pm}^{2})$

$$x_{\mp} = r_b \cos(\delta_b \mp \gamma)$$

$$y_{\mp} = r_b \sin(\delta_b \mp \gamma)$$

 (x_{\pm}, y_{\pm}) 4 variables, 3 indep. $x_{+}^{2}+y_{+}^{2}=x_{-}^{2}+y_{-}^{2}$



the contours do not include the Dalitz model errors

From x_{\pm}, y_{\pm} to γ : BaBar

- Combine $B \rightarrow DK$, $B \rightarrow D^*K$, $B \rightarrow DK^*$ with $D \rightarrow K_s \pi^+ \pi^-$ and $D \rightarrow K_s K^+K^-$
- Use frequentist method to derive the physical parameters γ , r_b , δ_b from (x_{\pm} , y_{\pm})



	total error	exp. sys.	Dalitz model sys.
Parameter	68.39	% Q.L.	95.4% C.L.
γ (°)	68^{+15}_{-14}	{4, 3}	[39, 98]
$r_B (\%)$	9.6 ± 2.9	0 {0.5, 0.4	4} [3.7, 15.5]
<i>r</i> * <i>B</i> (%)	$13.3^{+4.2}_{-3.9}$	{1.3, 0.3	[4.9, 21.5]
кr _s (%)	$14.9^{+6.6}_{-6.2}$	{2.6, 0.6	§ <28.0
δ_B (°)	119^{+1}_{-2}	${}^{9}_{0}$ {3, 3}	[75, 157]
δ^*_B (°)	$-82 \pm$	21 {5, 3}	[-124, -38]
$\tilde{\delta_s}$ (°)	111 ± 3	2 {11, 3}	} [42, 178]

 $\gamma = (68 \pm 14 \pm 4 \pm 3)^{\circ} \pmod{180^{\circ}}$

stat syst model



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From x_+, y_+ to γ : Belle

- Combine $B \rightarrow D^0 K$ and $B \rightarrow D^{*0} K$ with $D \rightarrow K_s \pi^+ \pi^-$
- Use frequentist method to derive the physical parameters γ , $r_{\rm b}$, $\delta_{\rm b}$ from (x₊,y₊)



Model independent analysis

Model-independent measurement of γ . Proposed by A. Giri et al. [Phys Rev. D68 054018 (2003)]. Pioneered by Belle.

Bin index i



- divide the D \rightarrow K_s $\pi\pi$ Dalitz plot in 2k bins (symmetric w.r.t. the m_+^2 vs m_-^2 axis)
- express the $B^{\pm} \rightarrow DK^{\pm}$ yields in each bin *i* in terms of x_{\pm} , y_{\pm} and 2 parameters c_i , s_i
- c_i, s_i are measured by CLEO exploiting the quantum coherence in $\psi(3770) \rightarrow \overline{D}{}^0 D^0$
- extract x_{\pm} , y_{\pm} from ML fit to $B^{\pm} \rightarrow DK^{\pm}$ yields in all bins



Model independent measurement



GLW method

- D⁰ to K⁺K⁻, $\pi^+\pi^-$ (CP+) and Ks π^0 , Ks ω , Ks ϕ (CP-)
- measure B⁺ and B⁻ yields to determine the GLW observables:

$$R_{CP\pm} \equiv \frac{\Gamma(B^- \to D_{CP\pm}^0 K^-) + \Gamma(B^+ \to D_{CP\pm}^0 K^+)}{2\Gamma(B^- \to D^0 K^-)} = 1 \pm 2r_b \cos \gamma \cos \delta_b + r_b^2$$

$$A_{CP\pm} = \frac{\Gamma(B^- \to D_{CP\pm}^0 K^-) - \Gamma(B^+ \to D_{CP\pm}^0 K^+)}{\Gamma(B^- \to D_{CP\pm}^0 K^-) + \Gamma(B^+ \to D_{CP\pm}^0 K^+)} = \pm 2r_b \sin\gamma \sin\delta_b / R_{CP\pm}$$

• 4 observables, 3 independent unknowns: γ , δ_{b} , r_{b}

GLW reconstructed decay modes

- $B \rightarrow D^{0}K, B \rightarrow D^{*0}K (D^{*0} \rightarrow D^{0}\pi^{0} \text{ and } D^{*0} \rightarrow D^{0}\gamma), B \rightarrow D^{0}K^{*}$
- $D^0 \rightarrow K^+K^-$, $\pi^+\pi^-$ (CP+), $D^0 \rightarrow Ks\pi^0$, $Ks\omega$, $Ks\phi$, $Ks\eta$ (CP-)



GLW results



Why "not" γ from GLW alone

 The GLW observables can be expressed in terms of the cartesian coordinates x_±,y_± : good constraint on x₊

$$x_{\pm} = (R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-}))/4$$
 (comparable to GGSZ method
when in same size dataset)
$$r_{b}^{2} = x_{\pm}^{2} + y_{\pm}^{2} = (R_{CP+} + R_{CP-})/2$$
 very loose constraint on y_{\pm}

ADS method

• D⁰ to K⁺ π^- , K⁺ $\pi^-\pi^0$, K⁺ $\pi^+\pi^+\pi^-$, ... (doubly-Cabibbo-supp.)



Measures B⁺ and B⁻ yields to determine the ADS observables:

$$R_{ADS} = \frac{\Gamma(B^{-} \to D[\to f]K^{-}) + \Gamma(B^{+} \to D[\to \overline{f}]K^{+})}{\Gamma(B^{-} \to D[\to \overline{f}]K^{-}) + \Gamma(B^{+} \to D[\to f]K^{+})} = r_{b}^{2} + r_{D}^{2} + 2r_{b}r_{D}\cos(\delta_{b} + \delta_{D})\cos\gamma$$

$$A_{ADS} = \frac{\Gamma(B^{-} \to D[\to f]K^{-}) - \Gamma(B^{+} \to D[\to \overline{f}]K^{+})}{\Gamma(B^{-} \to D[\to f]K^{-}) + \Gamma(B^{+} \to D[\to \overline{f}]K^{+})} = 2r_{b}r_{D}\sin(\delta_{b} + \delta_{D})\sin\gamma/R_{ADS}$$

$$r_{D} = \left|\frac{A(\overline{D}^{0} \to f)}{A(D^{0} \to f)}\right|$$

$$(r_{0}(K^{+}\pi)=0.06)$$

$$\delta_{D} = \arg\left[\frac{A(\overline{D}^{0} \to f)}{A(D^{0} \to f)}\right]$$

ADS reconstructed decay modes



ADS $B \rightarrow D^{(*)}K^{(*)}$ results



* $D^{(*)}\pi$ results available in backup slide

LHCb dominates the $B \rightarrow DK$, $D \rightarrow K\pi$ mode.

Final states with neutrals difficult in hadronic environment

Why "not" γ from ADS alone

 The ADS observables can be expressed in terms of the cartesian coordinates x_±,y_± :

$$(x_{\mp} + r_D \cos\delta_D)^2 + (y_{\mp} - r_D \sin\delta_D)^2 = \frac{\Gamma(B^{\mp} \to \left[K^{\pm} \pi^{\mp}\right]_D K^{\mp})}{\Gamma(B^{\mp} \to \left[K^{\mp} \pi^{\pm}\right]_D K^{\mp})}$$



Note: uncertainty on the values of r_D and δ_D are neglected

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BaBar GLW+ADS+GGSZ combination

Combination of GGSZ+GLW+ADS in two stages

- combine the GGSZ, GLW and ADS $B^{\pm} \rightarrow D^{(*)}K^{(*) \pm}$ observables (34 in total) to extract the combined x_{\pm}, y_{\pm} (4 for each B mode)
- transform the combined x_{\pm}, y_{\pm} into the physically relevant quantities $\gamma, \{r_b, \delta_b\}_{D(*)K(*)}$ Π.



results:

sults:			Real part $(\%)$	Imaginary part $(\%)$
$\bar{z} \equiv x + iy$	$\mathbf{\Sigma}$	\overline{Z}_{-}	$8.1 \pm 2.3 \pm 0.7$	$4.4 \pm 3.4 \pm 0.5$
2 = n + v j	Δ	\overline{z}_+	$-9.3 \pm 2.2 \pm 0.3$	$-1.7 \pm 4.6 \pm 0.4$
	\mathbf{X}	\overline{Z}^*_{-}	$-7.0 \pm 3.6 \pm 1.1$	$-10.6 \pm 5.4 \pm 2.0$
	۵	\overline{z}^*_+	$10.3 \pm 2.9 \pm 0.8$	$-1.4 \pm 8.3 \pm 2.5$
	*	\overline{Z}_{s-}	$13.3 \pm 8.1 \pm 2.6$	$13.9 \pm 8.8 \pm 3.6$
	Ŋ	\overline{z}_{s+}	$-9.8 \pm 6.9 \pm 1.2$	$11.0 \pm 11.0 \pm 6.1$

external input required for the D hadronic parameters $r_{K\pi}$, $\delta_{K\pi}$, $r_{K\pi\pi0}, \delta_{K\pi\pi0}, k_{K\pi\pi0}$

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BaBar GLW+ADS+GGSZ combination

stage II

• $(x_{\pm}, y_{\pm})_{D(*)K(*)} \rightarrow \gamma, \{r_b, \delta_b\}_{D(*)K(*)}$ with frequentist stat procedure

 $\gamma = (69^{+17}_{-16})^{\circ}$ (modulo 180°) exp+DP model sys = $\pm 4^{\circ}$

CPV significance: **5**. **9** σ (GGSZ alone 4.0 σ . GGSZ+GLW 5.4 σ)





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Measurement of γ at e⁺e⁻ B-factories - FPCP 2013

Belle GLW+ADS+GGSZ combination

Combination of GGSZ (mod. dep.)+GLW+ADS $B \rightarrow D^{0}K+B \rightarrow D^{*0}K$ (8+8+6 observables)

- frequentist stat procedure



GGSZ: $\gamma = (82^{+18}_{-23})^{\circ}$ **GGSZ + ADS :** $\gamma = (68 \pm 22)^{\circ}$

GGSZ + ADS + GLW: $\gamma = (68^{+15}_{-14})^{\circ}$

Measurem

 $B \rightarrow D^0 K$:

 $r_b = 0.168^{+0.063}_{-0.064}$

 $r_b = 0.104^{+0.020}_{-0.021}$

 $r_b = 0.112^{+0.014}_{-0.015}$

Belle and BaBar combination

Combination of BaBar and Belle: GGSZ mod. dep.+GLW+ADS with $B \rightarrow D^{0}K$, $B \rightarrow D^{*0}K$ and $B \rightarrow D^{0}K^{*}$ (BaBar only). Frequentist stat procedure. BaBar and Belle model errors assumed uncorrelated.



D mixing and CPV in $B \rightarrow DK$ (and $B \rightarrow D\pi$)

Several interesting studies on the effect of D mixing and CPV in the extraction of γ with B $\rightarrow D^{(*)}K^{(*)}$ (and B $\rightarrow D^{(*)}\pi$)

- Effect of D mixing
 - Y. Grossman, Z. Ligeti, A. Soffer, PRD67, 071301 (2003); PRD72, 031501 (2005)
 - A. Bondar, A. Poluektov, V. Vorobiev, PRD82, 034033 (2010) (Dalitz mod ind)
- Effect of CPV in D decays
 - W. Wang, PRL110, 061802 (2013) (GLW)
 - M. Martone and J. Zupan, arXiv:1212.0165 (GLW)
 - B. Bhattacharya, D. London, M. Gronau, J. L. Rosner, arXiv:1301.5631 (GLW)
 - A. Bondar, A. Dolgov, A. Poluektov, V. Vorobiev arxiv:1303.6305 (Dalitz)

Corrections for D mixing and CPV not considered in BaBar and Belle $B \rightarrow D^{(*)}K^{(*)}$ combinations

- effects expected to be small at present B-factories (although some may not be completely negligible)
- effects more and more important at LHCb and at Belle2

Summary

- Sensitivity to γ (ϕ_3) dominated by the $B^{\pm} \rightarrow D^{(*)0} K^{(*)\pm}$ decays so far
- BaBar and Belle have reconstructed all the most sensitive decay modes using all or almost all their final datasets
- BaBar GLW+ADS+GGSZ combination:

 $\gamma = (69^{+17}_{-16})^{\circ} \pmod{180^{\circ}}$ exp+DP model sys = $\pm 4^{\circ}$

- Belle GLW+ADS+GGSZ combination: $\gamma = (68^{+15}_{-14})^{\circ} \pmod{180^{\circ}}$
- B-factories average:

 $\gamma = (67 \pm 11)^{\circ} \pmod{180^{\circ}}$

BACKUP

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Dependence of $\sigma(\gamma)$ on r_b

- the error on γ (at fixed x,y uncertainty) scales roughly as $1/r_{b}$



$B \rightarrow D^{(*)}\pi$ ADS measurements



