FPCP
Buzios . Rio . Brasil 2013

Review of lattice flavour physics

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Summary

Generalities

- The role and status of lattice QCD in flavour physics
- Uncertaininities in lattice calculations
- Averages and FLAG

Recent results:

- Decay constants of light/heavy mesons
- Charmed decays
- Neutral meson mixing
- (Quark masses)

The future:

- Lattice results coming soon

On lattice QCD

Some (approximate) quotes from FPCP2013

- “...I wouldn’t trust the lattice result.”
- “Can we believe the error?”
- “...very small errors.”
- “...lattice is not always right.”
- “I have no idea what they are doing.”

On lattice QCD

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Lattice QCD allows first principles determination of hadron masses, matrix elements, quark masses

- Inputs are: a few hadron masses (for $m_{u/d}, m_s, m_c, m_b$) + one parameter to set overall scale
- Systematic errors can be estimated and improved
- Lots of phenomenological checks

Lattice QCD in one slide

1. Take QCD, discretise it on a lattice

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BlueGene/Q, safe.epcc.ed.ac.uk

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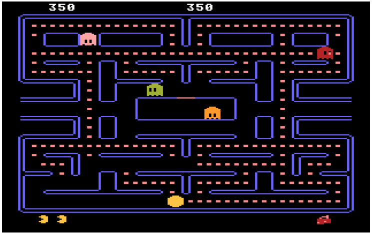
$\exists$ ∞ number of ways to discretise QCD - all bad

Progress in computing

1980s



videogamecritic.com, pcgamer.com, flickr.com, technobuffalo.com

Progress in computing

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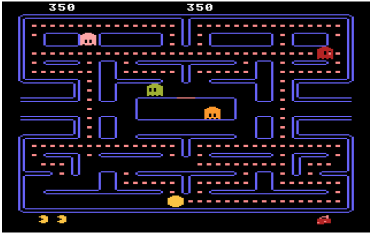	
	

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Progress in lattice QCD

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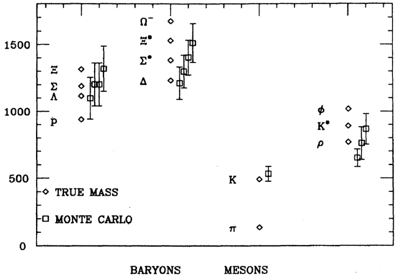

- $\sim 10^4$ lattice
- Quenched - no sea quarks
- $\sim$ 20 gauge configurations
- Large unquantified systematics
- Limited quantities calculated

Plots: Loft and DeGrand, 1989; Kronfeld, 2012

Progress in lattice QCD

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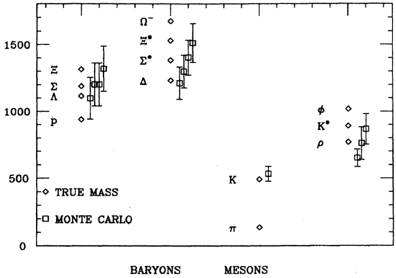

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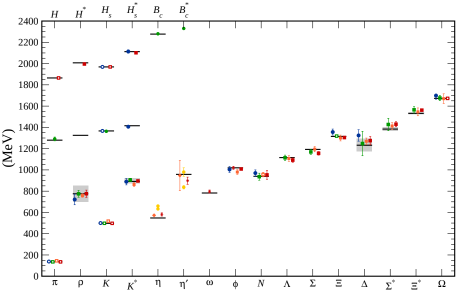
2013

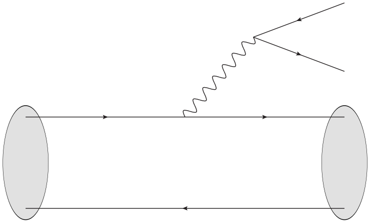


- 96^4 lattices, volumes $\sim$ 5 fm
- Multiple scales 0.15 – 0.045 fm
- 1000s of gluon configurations
- Improved actions and algorithms
- Physical quark masses!

Plots: Loft and DeGrand, 1989; Kronfeld, 2012

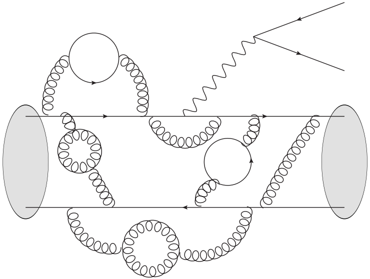
What lattice can do for flavour physics

A typical hadronic decay



What lattice can do for flavour physics

What a typical hadronic decay really looks like



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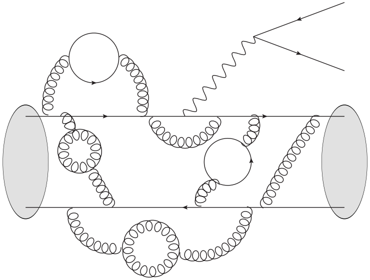


- QCD corrections can be significant
- Vacuum saturation not good enough for precision calculations
- In general, using an OPE

$$\text{Experiment} = (\text{CKM elements}) \times (\text{Perturbative}) \times (\text{Nonperturbative})$$

- Hadronic piece is often the dominant uncertainty

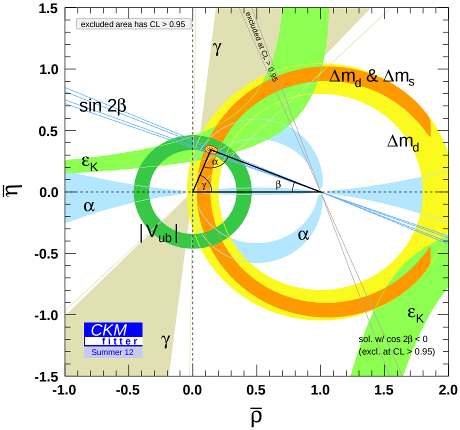
What do we use it for?

- Hadronic corrections to rare B decays
- Important input for precise CKM matrix determination via the decays:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ \pi^+ \to l^+\nu_l & K^+ \to l^+\nu_l & B^0 \to \pi^- l^+\nu_l \\ & K^0 \to \pi^- l^+\nu_l & \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ D^+ \to l^+\nu_l & D_s^+ \to l^+\nu_l & B^0 \to D^- l^+\nu_l \\ D^0 \to \pi^- l^+\nu_l & D^0 \to K^- l^+\nu_l & \\ |V_{td}| & |V_{ts}| & |V_{tb}| \\ B_d^0 \to \bar{B}_d^0 & B_s^0 \to \bar{B}_s^0 & \end{pmatrix}$$

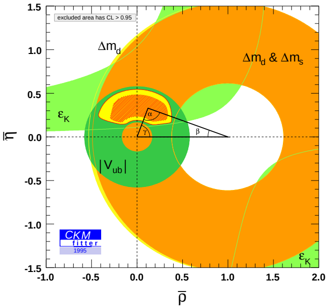
plus phase via K decays

CKM matrix determination



Vital in indirect search for new physics

- CKM unitarity triangle
- Lattice calculations enter into:
  - $\Delta M_d, \Delta M_s$
  - ϵ_K
  - $|V_{ub}|$
- Improvement since 1995 (not just lattice!):



What are the uncertainties in a lattice calculation

Robust calculation should address:

Statistics: Error from Monte-Carlo average

Scaling violations: RG equations only satisfied approximately

$$a \frac{d}{da} G(a) = \mathcal{O}(a^n, \alpha_s^m a^p)$$

Must use multiple lattice spacings to fit this away.

Chiral extrapolation: If M_π is too heavy, results must be extrapolated

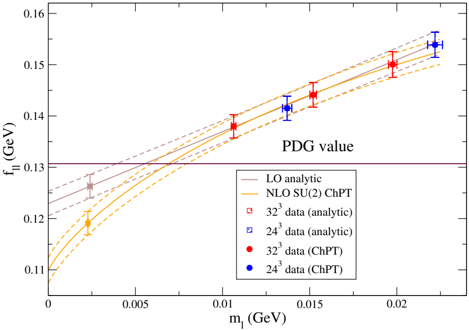
Renormalisation: Very few conserved currents on the lattice $\Longrightarrow$ $J_{QCD} = Z_J J_{latt}$

Others:
- Excited state contamination
- Finite volume effects
- Scale setting
- Quark mass tuning
- Higher orders in EFT

- Dominant source of error depends on the quantity and the action.
- Some lattice papers are exploratory and won't address all of these

The trouble with chiral extrapolations

- Cost for generating configurations grows with lighter quark masses
- Error from chiral extrapolation large for mesons with valence light quark



- Some ambiguity in what chiral fit form to use:
  - $SU(2)$ vs $SU(3)$
  - NLO, NNLO terms, analytic, priors?
- Now we can simulate at the physical point!

RBC/UKQCD 1011.0892

Averages

Several mature calculations for some quantities

- Which lattice result should be used?
- How should they be averaged (correlations)?
- Can we trust a lone result?
- Should we just use the one with the biggest error?:
  No! It might just be a rubbish/old calculation

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Flavour Lattice Averaging Group (PDG for lattice results)

- FLAG quality criterion ★, ●, ■ :
  - Chiral extrapolation
  - Continuum extrapolation
  - Finite volume effects
  - Renormalisation
- Published results with no red tags are averaged
- Not gospel, but generally a reasonable guide to a good calculation
- $N_f = 2$ and $N_f = 2 + 1$ calculations averaged separately
- See: itpwiki.unibe.ch/flag/
- FLAG-2 update due before July 2013

Summary of recent results

Results

- Focus on full calculations completed in the last few years
- Preliminary results will be obviously marked - don't quote them!
- Results available from most big lattice collaborations:
  ALPHA, BMW, ETMC, Fermilab/MILC, HPQCD, PACS-CS, RBC/UKQCD, ...
- Don't have space for everything, in particular
  - Spectroscopy: see S.Ryan FPCP2012
  - Heavy to light semileptonics: $B \to D^{(*)} l\nu$, $B \to K l^+ l^-$, $B \to \pi l \nu$ for $|V_{ub}|, |V_{cs}|, |V_{cd}|$: see A. El-Khadra FPCP2012
- A bit more detail on some calculations (HPQCD)

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In brief:

- Several results with physical pion masses
  $\Longrightarrow$ significant reduction in error for key quantities

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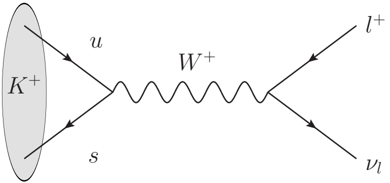
Decay constants and form factors of light mesons

f_π, f_K and $|V_{us}|$

- Decay constant:

$$\langle 0|A_\mu(0)|P\rangle = if_P p_\mu$$

easily obtained from lattice psuedoscalar correlators

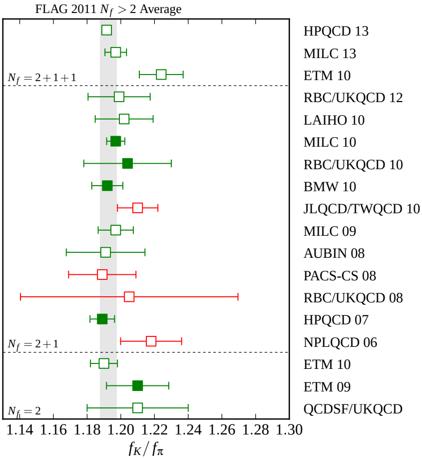


- Appears in decays $\pi^+ \to l^+\nu_l$, $K^+ \to l^+\nu_l$
- Standard model rate

$$\Gamma(K \to l\nu) = \frac{G_F^2|V_{us}|^2}{8\pi} f_K^2 m_l^2 M_K \left(1 - \frac{m_l^2}{M_K^2}\right)(1 + \mathrm{E.M.})$$

- Cannot compete with $|V_{ud}|$ from β decay. f_π is used for scale setting.
- Exp values for ratio $\Gamma(K^+ \to l^+\nu_l)/\Gamma(\pi^+ \to l^+\nu_l)$ and masses gives $\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}}$
- Determining f_{K^+}/f_{π^+} on the lattice gives $\frac{|V_{us}|}{|V_{ud}|}$ precisely
- Error previously dominated by chiral extrapolation

f_π, f_K and $|V_{us}|$



New determinations

- RBC/UKQCD: Near physical M_π
  $f_{K^+}/f_{\pi^+} = 1.199(12)_{\rm stat}(14)_{\rm sys}$
- MILC: Physical M_π
  $f_{K^+}/f_{\pi^+} = 1.1970(29)_{\rm stat}(57)_{\rm sys}$
- HPQCD: Physical M_π
  $f_{K^+}/f_{\pi^+} = 1.1916(21)$

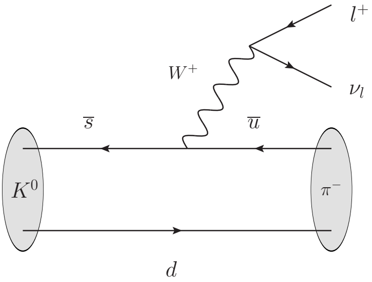
- e.g. HPQCD result gives
  $|V_{us}| = 0.22564(28)_{\mathrm{Br}(K^+)}(20)_{EM}(40)_{latt}(5)_{V_{ud}}$
- Test of first row unitarity of CKM
  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0.00009(51)$

$f_+(0)$ and $|V_{us}|$

- $|V_{us}|$ from Kaon semileptonic vector form factor
- Determined from vector current matrix element

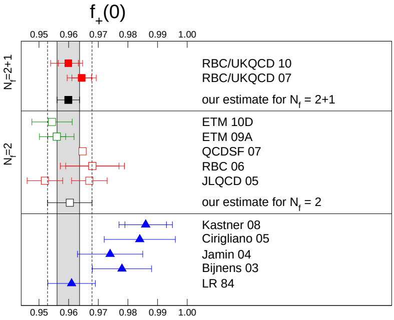
$$\langle\pi(p_\pi)|V^\mu|K(p_K)\rangle = f_+(q^2)\left[p_K^\mu + p_\pi^\mu - \frac{M_K^2 - M_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_K^2 - M_\pi^2}{q^2}q^\mu$$

- Expmt: $|V_{us}|f_+(0) = 0.2163(5)$
- Much more difficult than f_π, f_K



$f_+(0)$ results

Status pre 2011: Room for improvement

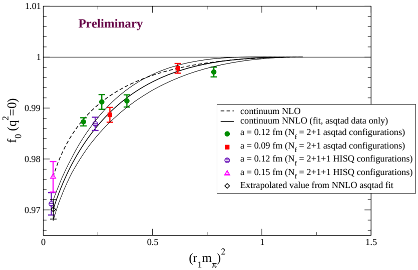


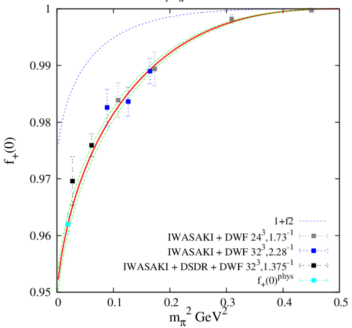
New results:

- MILC:

$$f_+(0) = 0.9667(23)(33)$$

Preliminary results at or near phys point from MILC and RBC/UKQCD →





E.Gamiz et al. arXiv:1211.0751, P.Boyle et al. arXiv:1212.3188

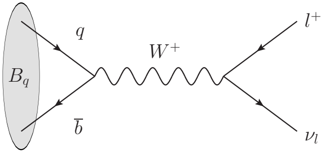
Decay constants of heavy mesons

f_B, f_{B_s}

Bottom:

- Decay constant for meson B_q, $q = d, s$

$$\langle 0 | A_\mu(0) | B_q \rangle = f_{B_q} p_\mu$$

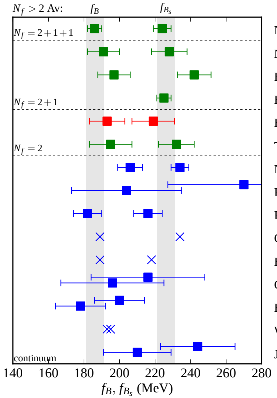


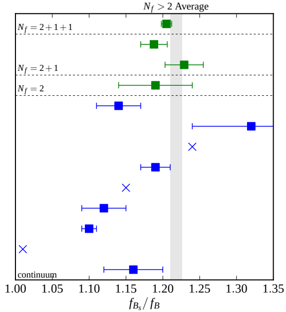
- Need to use lattice EFT or extrapolate in m_b
- Systematics cancel in the ratio f_{B_s}/f_B - used for CKM determination
- Error in SM rate for $\mathrm{Br}(B^+ \to \tau^+ \nu)$ comes from $f_B^2 |V_{ub}|^2$
- Appears in SM prediction for $\mathrm{Br}(B_q \to \mu^+ \mu^-)$

$$\begin{aligned} \mathrm{Br}^{\mathrm{SM}}(B_s \to \mu^+\mu^-) &= 3.17 \pm 0.15 \pm 0.09 \times 10^{-9} \\ \mathrm{Br}^{\mathrm{LHCb}}(B_s \to \mu^+\mu^-) &= 3.2^{+1.5}_{-1.2} \times 10^{-9} \end{aligned}$$

evidence found by LHCb 2012 See rare B-decay talks...

f_B, f_{B_s} results





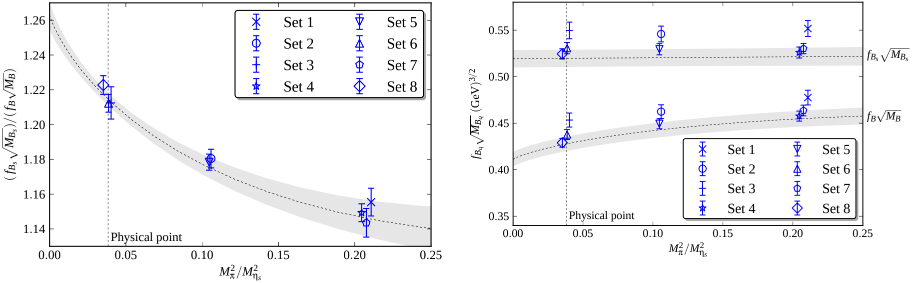
- Several consistent results with very different methods
- Competitive with most precise continuum methods

Recent results

- HPQCD: Physical M_π NRQCD/HISQ $N_f = 2+1+1$
- HPQCD: NRQCD/HISQ $N_f = 2+1$
- FNAL/MILC: Fermilab method
- ALPHA: HQET
- ETM: extrapolation

HPQCD f_B, f_{B_s} at the physical point PRL XXX(2013) XXXXXXX

Described as “optimistic” error by a previous speaker



- First results for f_B, f_{B_s} with physical M_π
- NRQCD with radiative/relativistic corrections
- $\sim$ 198k correlators for each data point
- Allowed error 10$\times$ 1-loop renormalisation
- Two consistent analyses (chiral and phys pt only), took larger error
- 0.6% error in ratio, no longer comes from chiral extrapolation

arXiv:1302.2644

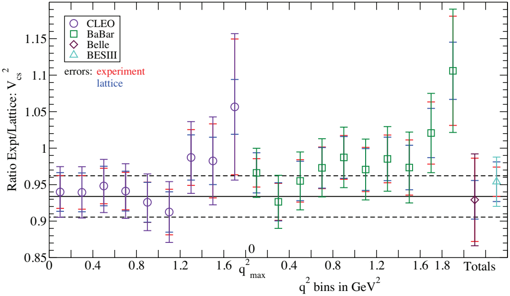
D-meson semileptonic decays

$|V_{cs}|$ from the shape of the $D \to Kl\nu$ form factor

- SM rate given as function of momentum transfer q^2

$$\frac{d\Gamma(D \to Kl\nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} p^3 |f_+(q^2)|^2$$

- $|V_{cs}|$ previously determined from form factor $f_+(0)$
- First study by HPQCD using all experimental bins
- Access all q^2 values with twisted boundary conditions
- Most precise $|V_{cs}|$, 1.5% error

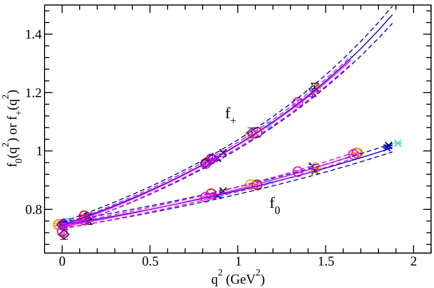


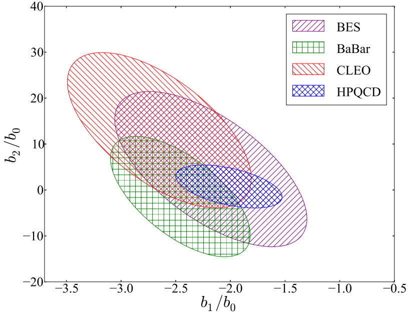
$|V_{cs}|$ from the shape of the $D \to K l \nu$ form factor

- Model independent parameterisation of $f_+(q^2)$ in z-space

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}, \quad t_\pm = (m_D \pm m_K)^2, \quad f(q^2) = \frac{1}{P(q^2)\Phi(q^2)} \sum_{n=0}^{N} b_n z^n$$

- Compare the shape to experiment using 68% C.L. of ratios $b_2/b_0, b_1/b_0$





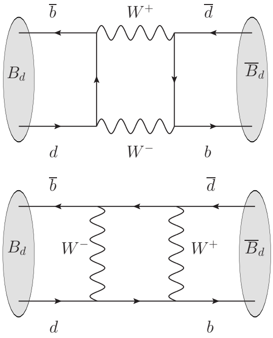
J.Koponen et al. arXiv:1305.1462

B-mixing

B meson mixing

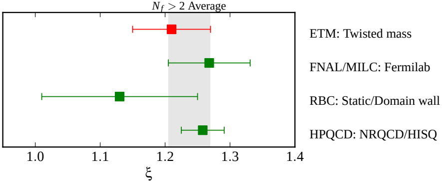
- Measure nonperturbative QCD corrections to mass difference $\Delta M_q, \Delta\Gamma_q$
- Quoted as bag-parameter $\hat{B}_q$ in OPE $\langle \overline{B}_q | \mathcal{O} | B_q \rangle = \frac{8}{3} f_{B_q}^2 m_{B_q}^2 \hat{B}_q(\mu)$ measures deviation from vacuum saturation
- Gives CKM matrix elements via

$$\frac{\Delta M_d}{\Delta M_s} = \frac{M_{B_d}}{M_{B_s}} \frac{\hat{B}_d}{\hat{B}_s} \frac{f_{B_d}^2}{f_{B_s}^2} \left| \frac{V_{td}}{V_{ts}} \right|^2$$



B meson mixing results

Most accurate results are for $SU(3)$ breaking parameter $\xi = \frac{f_{B_s}\sqrt{\hat{B}_s}}{f_{B_d}\sqrt{\hat{B}_d}}$



New results:

- Fermilab/MILC
  $\xi = 1.268(63)$
- ETM: PRELIMINARY
  $\xi = 1.21(6)$

- Current unquenched average: ξ =1.237(32)
- Error on ξ dominated by stats & chiral extrapolations - room for improvement
  $\implies$ tighter constraint on $\Delta M_s/\Delta M_d$ curve in unitarity triangle
- Expect new HPQCD results for $\hat{B}_s, \hat{B}_d$ at phys pt. soon
  - RBC/UKQCD, ETM, FNAL/MILC underway
- BSM operators also being calculated

$$H^{\Delta B=2} = \sum_{i=1}^{5} C_i O_i + \sum_{i=1}^{3} C_i^{\mathrm{BSM}} O_i^{\mathrm{BSM}}$$

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Bottom hadron decays

Fragmentation fraction for $B_s \to \mu^+\mu^-$

- Branching fraction normalised via

$$\mathrm{Br}(B_s \to \mu^+\mu^-) = \mathrm{Br}(B_q \to X) \frac{f_q}{f_s} \frac{\epsilon_X N_{\mu\mu}}{\epsilon_{\mu\mu} N_X}$$

- Measure f_q/f_s using $B \to D$ semileptonics Fleischer et al. 1004.3982

$$\frac{f_s}{f_d} \propto \frac{1}{\mathcal{N}_F}, \quad \mathcal{N}_F = \left[\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)} \right]^2$$

- LHCb use sum rule $\mathcal{N}_F = 1.24(8)$
- Measure using lattice $B_{(s)} \to D_{(s)} l\nu$ form factor ratio

$$\langle D(p')|V^\mu|B(p)\rangle = f_+(q^2)\left[(p+p')^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu\right] + f_0(q^2) \frac{M_B^2 - M_D^2}{q^2} q^\mu$$

Fragmentation fraction for $B_s \to \mu^+\mu^-$

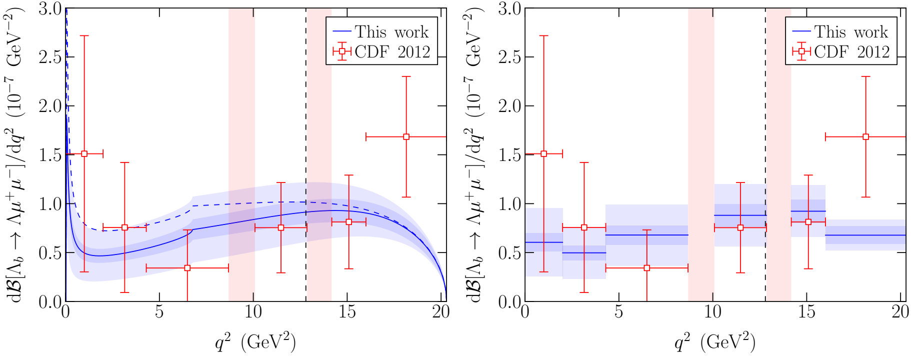
- 1st calculation by FNAL/MILC arxiv:1202.6346
- 2 lattice spacings, $\sim$ 250 MeV pion mass
- $$\frac{f_0^{(s)}}{f_0^{(d)}} = 1.046(44)_{\rm stat}(15)_{\rm sys}, \quad \Longrightarrow \mathcal{N}_F = 1.09(10)$$

  2σ from sum rules - $\mathcal{N}_F = 1.24(8)$
- 4.5% error from stats/chiral extrapolation
- Potential to reduce by $\sim 1/2$
- Limited by experimental statistics

$$\frac{f_s}{f_d} = 0.283(27)_{\rm stat}(19)_{\rm sys}(24)_{\rm theo}$$

$\Lambda_b \to \Lambda\mu^+\mu^-$

- First lattice calculation Detmold et al. 1212.4827
- Static limit, 10 form factors
- Calculate at high q^2, extrapolate with mono/di-pole formula
- Preliminary LHCb results in agreement M.O. Bettler talk



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Kaon mixing

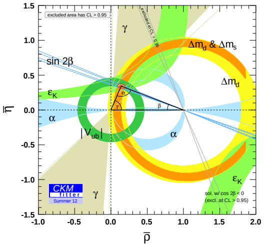
CP violation, ϵ_K and B_K

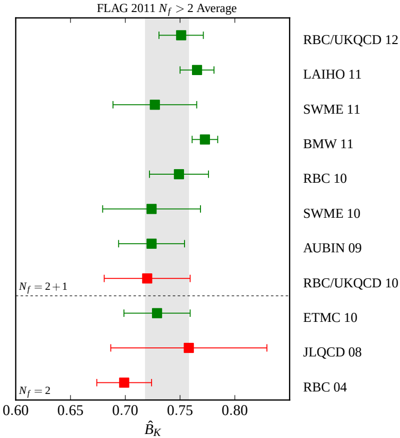
- Indirect CP violation

$$\epsilon_K = \frac{\mathcal{A}(K_L \to \pi\pi_{I=0})}{\mathcal{A}(K_s \to \pi\pi_{I=0})}$$

- Only unitarity constraint testing light-CP violation - hyperpola in $\bar{\eta}, \bar{\rho}$
- In SM:

$$\epsilon_K = \frac{G_F^2 f_K^2 m_K^2 m_W^2}{12\sqrt{2}\pi^2 \Delta m_K} \hat{B}_K \{\mathrm{CKM} + \mathrm{pert.}\}$$

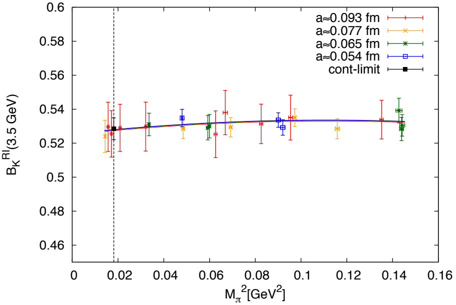




$\hat{B}_K$ no longer dominant uncertainty

BMW collaboration B_K at the physical point

- First result for $\hat{B}_K$ at Physical M_π
- Clover action
- 4 lattice spacings
- Chiral and scale dependence under good control
- Error dominated by statistics

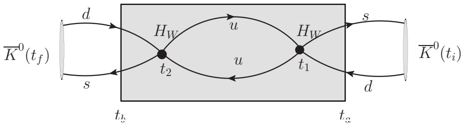


Durr et al. arXiv:1106.3230

Long distance contributions to $\epsilon_K, \Delta M_K$

- Long distance contribution to $\Delta M_K = M_{K_L} - M_{K_S} \sim 20 - 30\%$
- Long dist contribution to $\epsilon_K \sim 5\%$,
- Theoretical progress: proposal to calculate non-local contributions like

$$\int d^4x \int d^4y \langle \bar{K}^0 | T\{H_W(x) H_W(y)\} | K^0 \rangle$$



- Difficult!
  - Many diagrams to calculate
  - Finite volume method
  - GIM mechanism to control divergences
- Exploratory calculation by RBC/UKQCD gives reasonable result

N.Christ arXiv:1201.2065, N.Christ et al. arXiv:1212.5931

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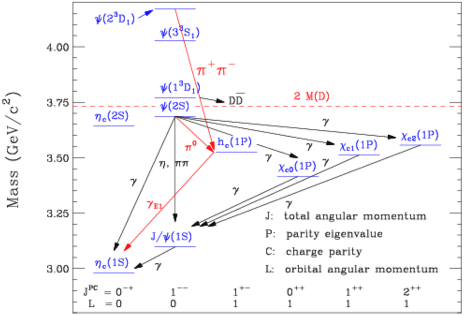
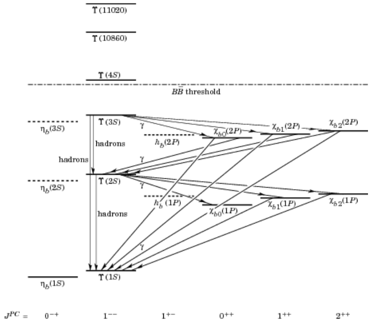
Radiative and leptonic decays in quarkonium

Radiative and leptonic decays in quarkonium





Rosner, arXiv:1107.2023

PDG

- Large number of known decays available - tests of (lattice) QCD
- e.g. form factor

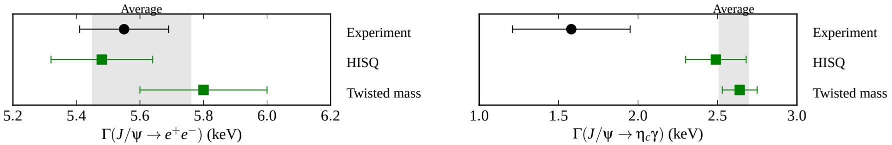
$$\Gamma(J/\psi \to \eta_c \gamma) = \frac{64}{27} \frac{\alpha_{\rm em} |\mathbf{q}|^3}{(M_{J/\psi} - M^2_{\eta c})^2} |V(0)|^2$$

- Leptonic width - via decay constant:

$$\Gamma(J/\psi \to e^+ e^-) = \frac{4\pi}{3} \alpha_s^2 Q_c^2 \frac{{f_{J/\psi}}^2}{M_{J/\psi}}$$

Radiative and leptonic decays in charmonium - results

- First study by Dudek et. al. (2009), with only one lattice spacing
- Full results from HPQCD and ETM



- Excited state decays are harder on the lattice
- HPQCD study of similar decays of bottomonium with NRQCD in progress

Quark masses

Quark masses

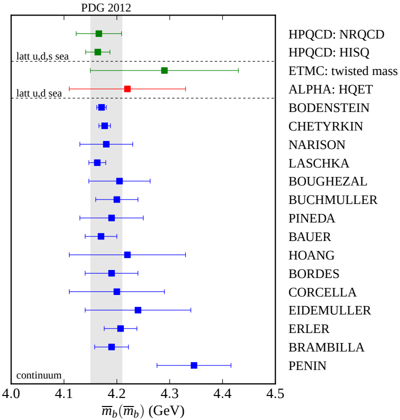
- Key standard model parameter but cannot be directly measured
- Lattice: simply tune quark mass till meson mass is correct e.g.: $M_\pi, M_K, M_{\eta_c}, M_\Upsilon$
- Must renormalise to a continuum scheme, e.g. $\overline{\mathrm{MS}}$ at scale μ

$$m^{\overline{\mathrm{MS}}}(\mu) = Z_m m^{\mathrm{latt}}(a)$$

- Dominant error usually from Z_m
- Naively, one would calculate Z_m in latt perturbation theory - difficult
- Various schemes exist to match to high order (α_s^3) continuum pert. theory
- Alternatively: calculate ratios like m_s/m_c. Z_m cancels if same action used $\Longrightarrow$ accurate to $\sim 1\%$

b quark: $\overline{m}_b(\overline{m}_b)$

Quoted in $\overline{\mathrm{MS}}$ scheme at $\mu = \overline{m}_b$



Several recent lattice determinations

- HPQCD: EFT energy shift from 2-loop NRQCD latt PT
  $\overline{m}_b(\overline{m}_b) = 4.166(43)$ GeV
- HPQCD: Matching moments of lattice correlators to α_s^3 continuum PT
  $\overline{m}_b(\overline{m}_b) =$ GeV
- ETM: Extrapoln matched to static limit
  $\overline{m}_b(\overline{m}_b) = 4.29(14)$ GeV
- ALPHA: PRELIMINARY
  Nonperturbative NLO HQET
  $\overline{m}_b(\overline{m}_b) = 4.22(10)(4)$ GeV

Comparison plot: Most accurate determinations are high order continuum PT

Summary

- Lattice QCD is a key input in rare decays and CKM fits
- Results at physical pion masses
  - Reduced error from chiral extrapolation
  - More robust results
- Consistent lattice results with very different methods
- Approaching/beating other errors in some quantities

What to expect from lattice QCD in the future

- Results at physical pion masses from all collaborations
- Electromagnetic and isospin effects included
- Long distance mixing effects
- B-physics:
  $B \to K^{(*)} ll$, $B \to \pi l \nu$, $B \to D^{(*)} l \nu$, $R(D)$

THE END

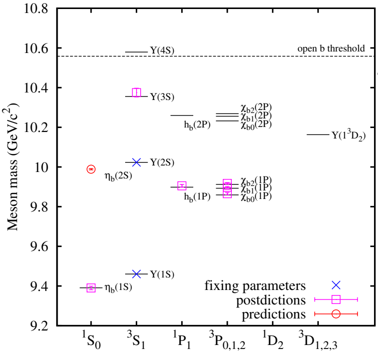
Thanks for listening

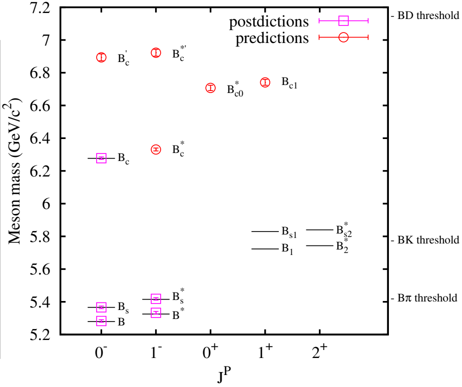
Further reading

- Some recent reviews:
  - A.Kronfeld, Twenty-first Century Lattice Gauge Theory: Results from the QCD Lagrangian, Ann.Rev.Nucl.Part.Sci. 62 (2012) 265-284
  - Z.Fodor, C Hoelbling, Light Hadron Masses from Lattice QCD, Rev.Mod.Phys. 84 (2012) 449
- Up to date plenary talks at lattice 2012:
  `http://www.physics.adelaide.edu.au/cssm/lattice2012/program.php`
- Other lattice conferences: Google "Lattice 20XX"

Spectroscopy

Good agreement for B and Υ spectrum

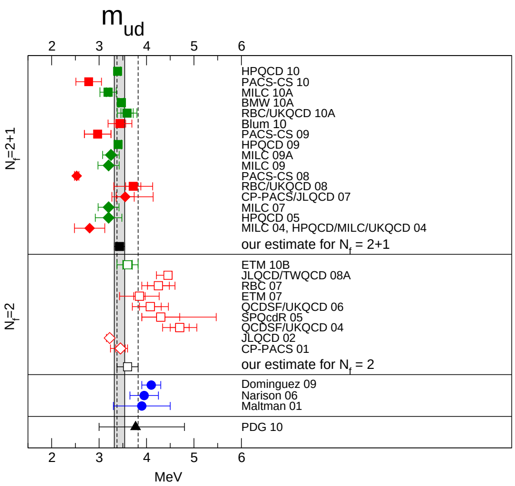




Light quark masses

Few recent results, but no update at recent FPCP conferences

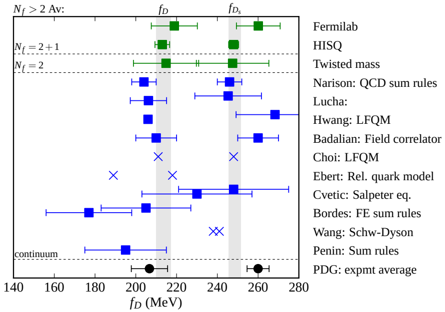
Status in 2011 FLAG review

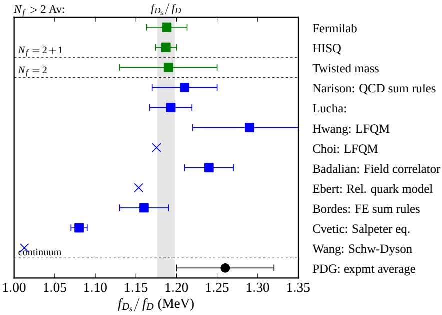


- Lattice calculations done in isospin limit $m_{ud} = (m_u + m_d)/2$
- $\overline{\text{MS}}$ scheme at $\mu = 2$ GeV
- Isospin breaking and EM effects estimated afterwards
- BMW have results at physical point - no chiral extrapolation
- FLAG average $m_{ud} = 3.43(11)$ MeV

f_D, f_{D_s}

- f_D, f_{D_s} used to get $|V_{cd}|, |V_{cs}|$ via $D \to \mu\nu$, $D_s \to \mu\nu$ $D_s \to \tau\nu$
- *or* assume unitarity and compare f_{D_q} direct to experiment
- Previous discrepancy no longer significant





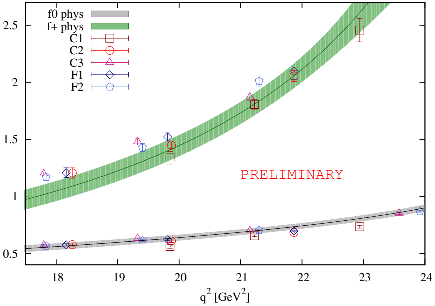
- Physical point calculations underway

Recent results

- ETM: twisted mass

$B \to \pi l \nu$

- Old results from FNAL/MILC(2011) and HPQCD(2006)
  - FNAL: $|V_{ub}| = 3.38(36) \times 10^{-3}$
  - HPQCD: $|V_{ub}| = 3.55(25)(50) \times 10^{-3}$
- Updates due soon, plus results from ALPHA
- Preliminary results from e.g. C. Bouchard:



$R(D)$

- FNAL/MILC arxiv:1206.4992
- $R(D) = \frac{\mathrm{Br}(B \to D\tau\nu)}{\mathrm{Br}(B \to Dl\nu)} = 0.316(12)(7)$
- More statistics and $R(D^*)$ coming soon...