Energy Frontier Physics: the Big Questions

1: Theory

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There is no shortage of things that we do not understand about the fundamental interactions.

It is true that we have a Standard Model that does an excellent job of accounting for current data. However, there are things about this model that are unexplained, and even strange:

Why three distinct interactions ?

Why these fermion quantum numbers and not others ?

Why three generations ?

Why flavor mixing, small for quarks, large for leptons ?

How does gravity fit in ?

Why is the cosmological constant small but nonzero ?

It may take us more centuries to solve all of these problems.

I am primarily interested in three questions that are plausibly solved by particles of mass 100 GeV - 3 TeV, the mass scale that we are now exploring directly at accelerators.

What makes up the dark matter of the universe ?

Why is there more matter than antimatter ?

and, behind these two questions -- and all questions on the previous page,

Why does the Higgs field condense and fill the universe ?

I need to define the Higgs field before I describe its central role.

The Lagrangian of the Standard Model contains two parts:

Gauge part:

$$\mathcal{L} = \sum_a -\frac{1}{4g_a^2}(F^a_{\mu\nu})^2 + \sum_f \overline{\psi}_f i \not{D}_f \psi_f$$

$$\mathcal{D}_{\mu f} = \partial_\mu - i \sum_a A^a_\mu Q^a_f$$

Higgs part:

$$\mathcal{L}_H = |D_\mu \Phi|^2 - V(\Phi) - \sum_{f_L f_R} \lambda_{f_L f_R} \overline{\psi}_{f_L} \cdot \Phi \ \psi_{f_R} - h.c.$$

The structure of the gauge part is completely determined by local gauge invariance. There are only 3 parameters. These will be reduced to 1 if the gauge interactions can be grand-unified.

The gauge part is extremely well tested. The fermions of the Standard Model have a large range of values of their SU(3) x SU(2) x U(1) quantum numbers.

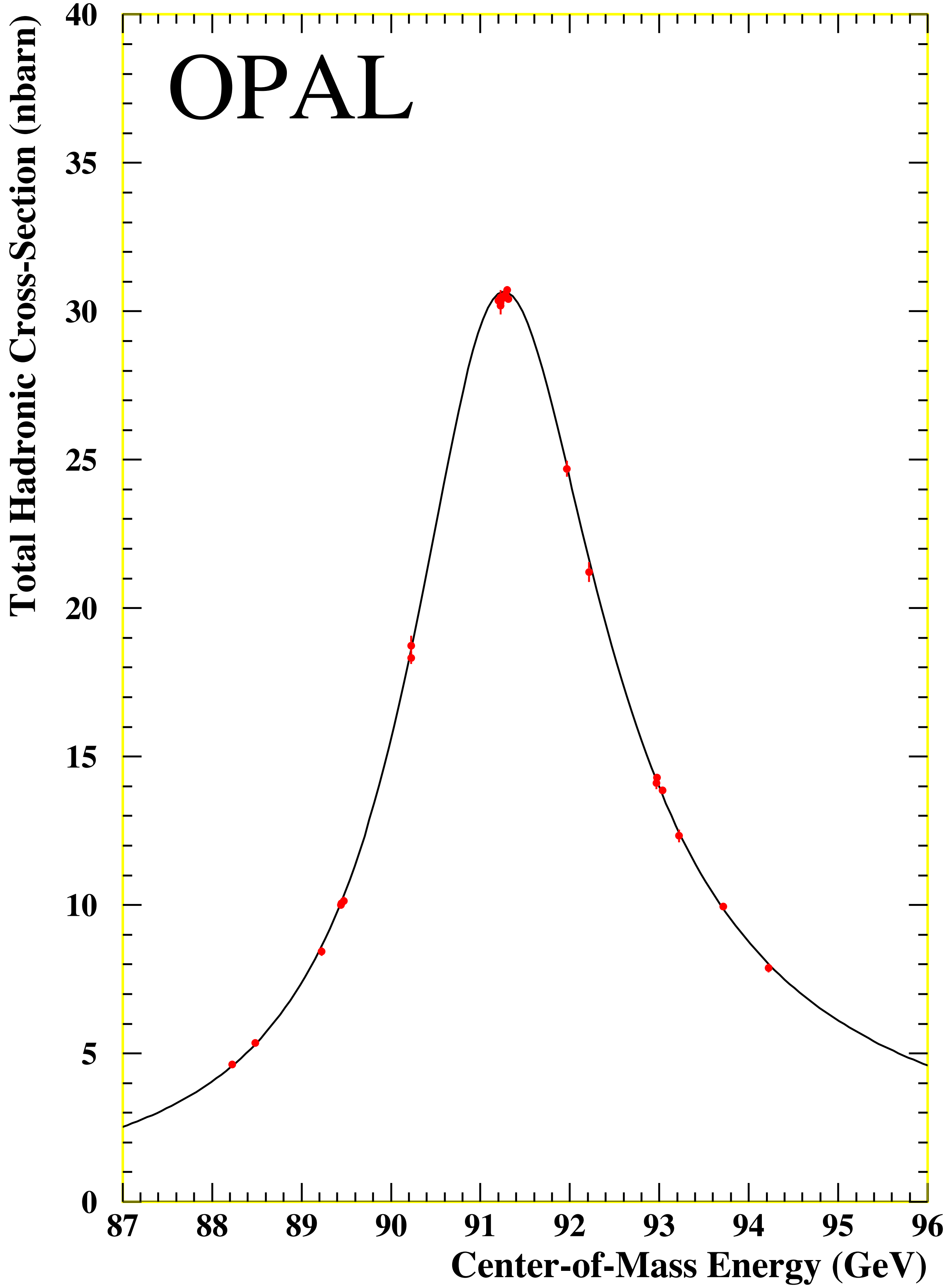
$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad e_R \qquad \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad u_R \qquad d_R$$

$$(\frac{1}{2}, -\frac{1}{2}) \qquad (0, -1) \qquad (\frac{1}{2}, \frac{1}{6}) \qquad (0, \frac{2}{3}) \qquad (0, -\frac{1}{3})$$

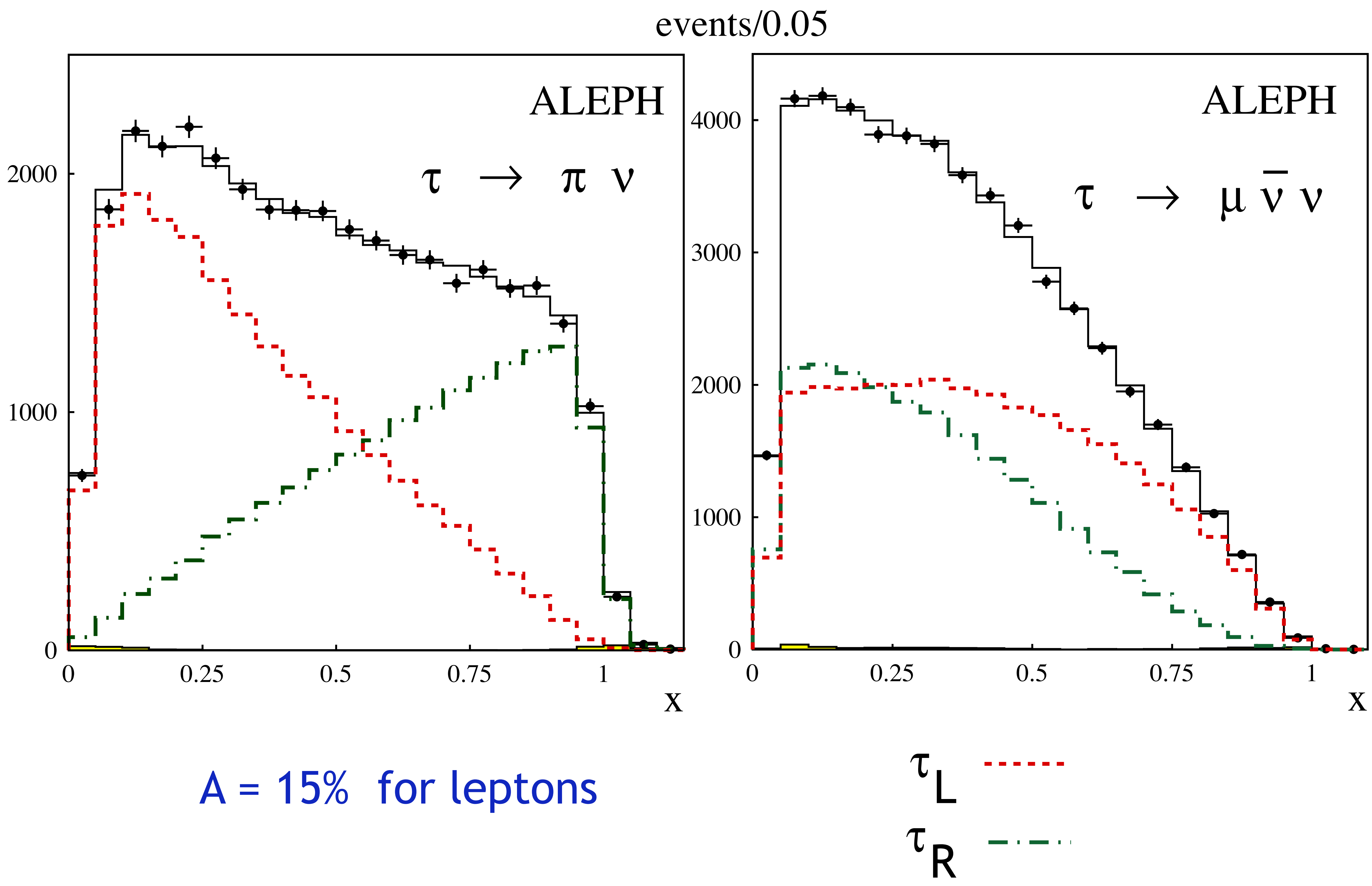
The couplings to the photon and Z are:

$$Q = I^3 + Y \qquad\qquad Q_Z = I^3 - \sin^2\theta_w Q$$

These lead to precise predictions of rates and left-right asymmetries in Z decays, tested in the precision electroweak experiments of the 1990's.

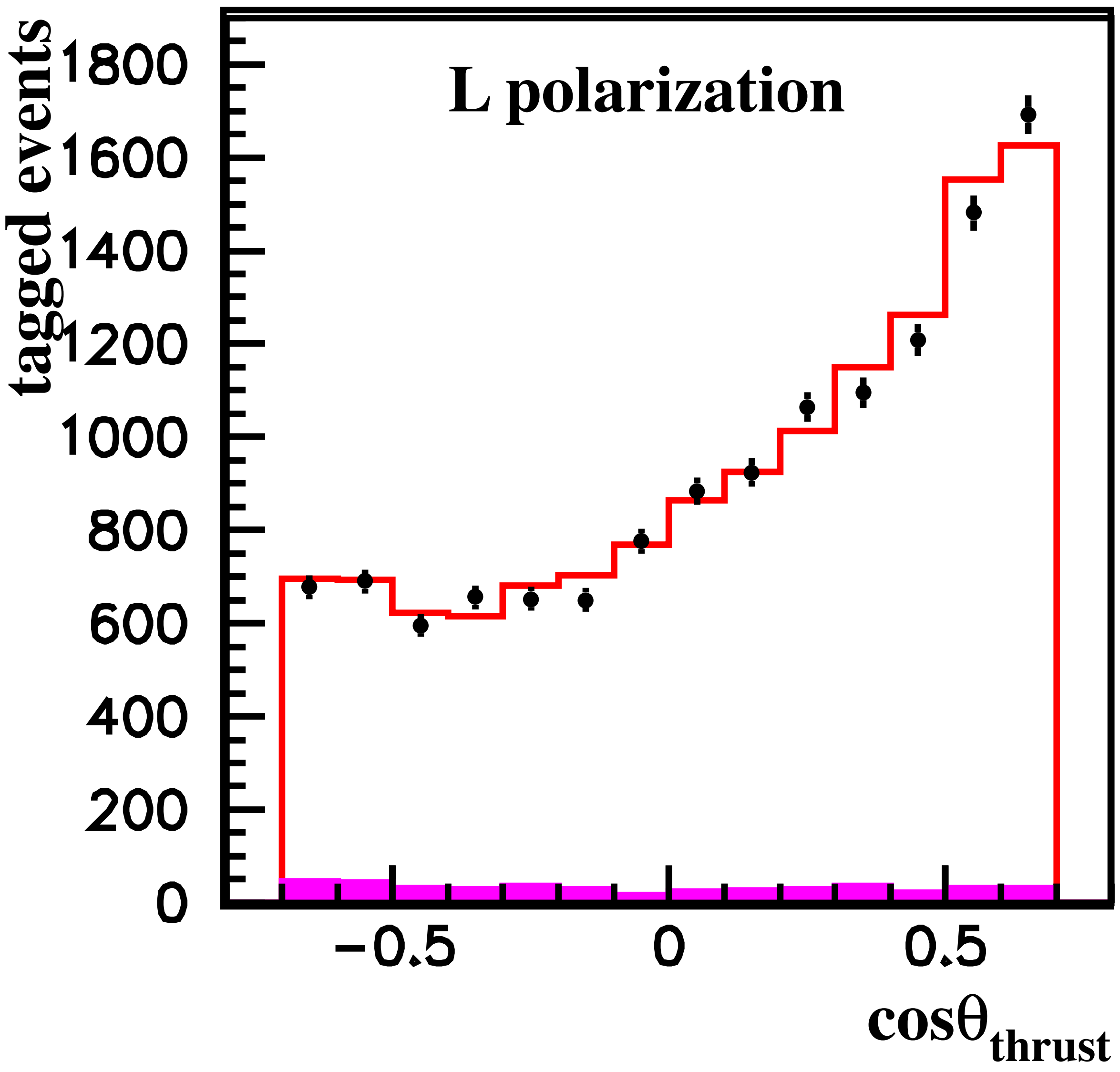

OPAL
Total Hadronic Cross-Section (nbarn)
40
35
30
25
20
15
10
5
0
87
88
89
90
91
92
93
94
95
96
Center-of-Mass Energy (GeV)

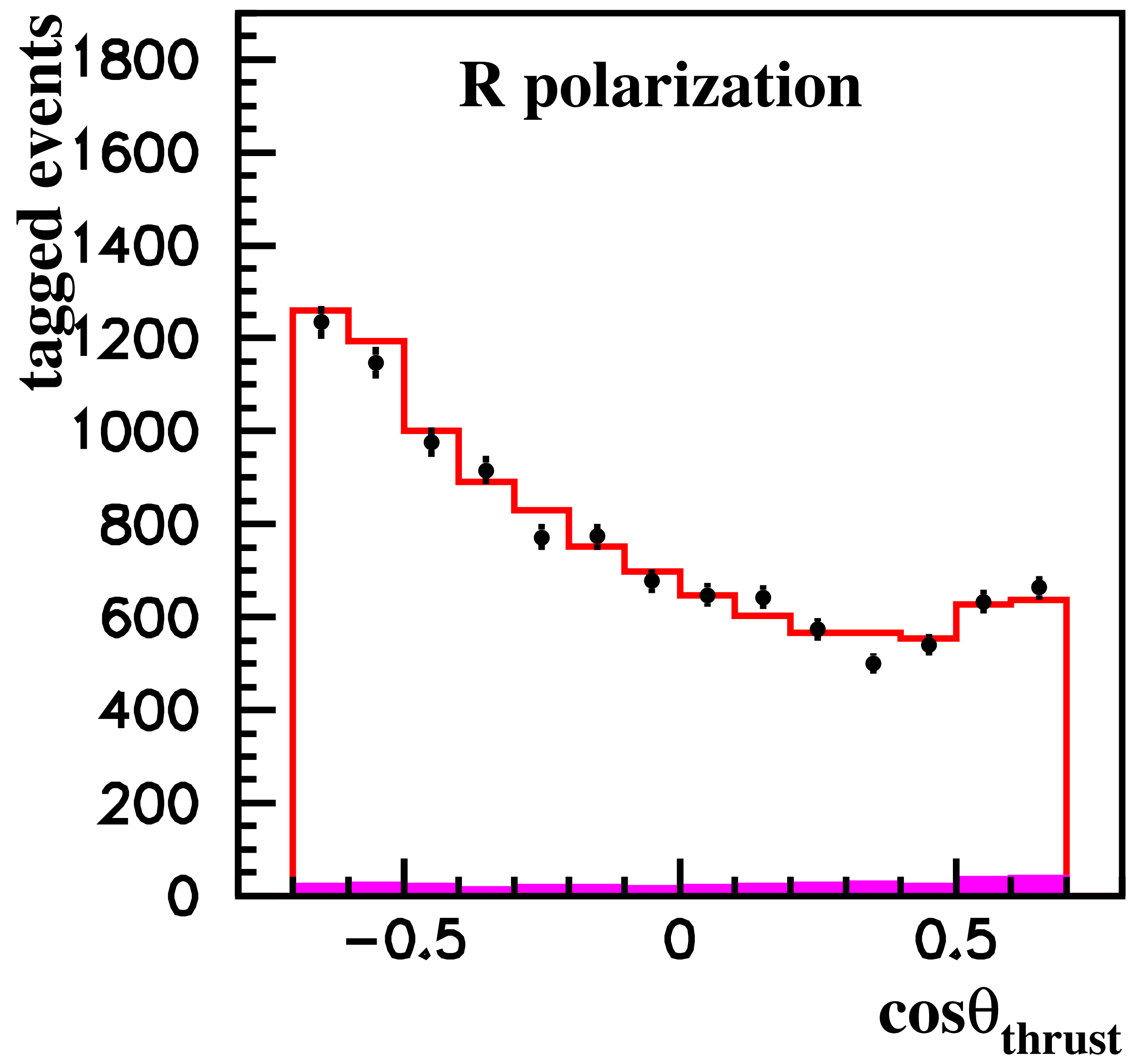
Energy distribution of tau decay products



A = 15% for leptons

Angular distribution of b jets using polarized electrons



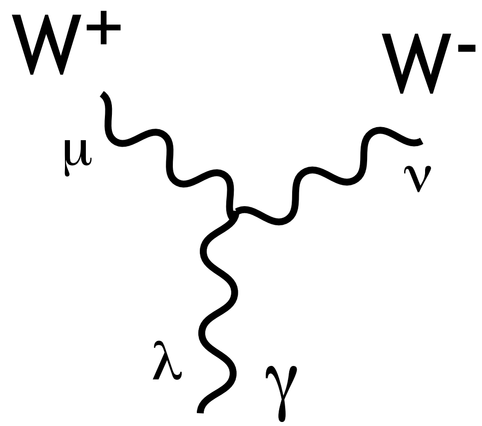


A = 94% for b quarks

SLD

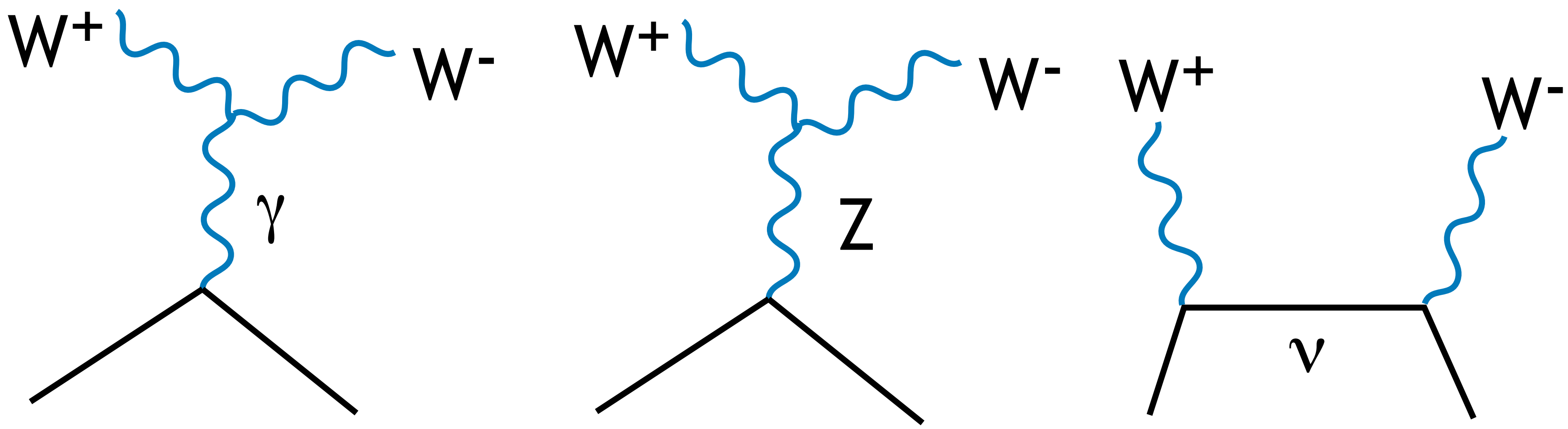
Finally, the LEP measurements of $e^+e^- \to W^+W^-$ allow measurement of the $WW\gamma$ and WWZ couplings.

The Standard Model makes a specific prediction, for example:



$$= \quad ie[g^{\mu\nu}(k_+ - k_-)^\lambda + g^{\nu\lambda}(k_- - k_\gamma)^\mu + g^{\lambda\mu}(k_\gamma - k_+)^\nu]$$

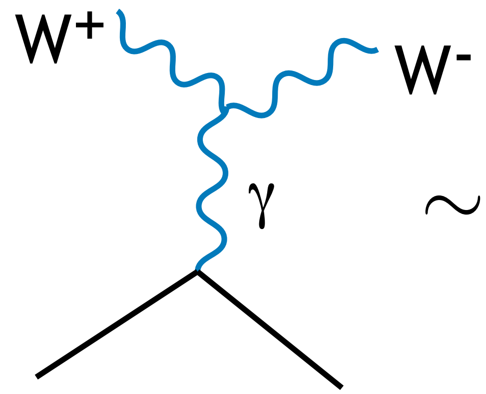
W pair production involves the three diagrams



and thus is sensitive to these vertices.

There is a famous subtle feature in this analysis.

Estimate:



$$\sim e^2 \overline{v}\gamma^\mu u \frac{1}{s}(k_+ - k_-)_\mu \ \epsilon_+^* \cdot \epsilon_-^*$$

For W bosons with longitudinal polarization

$$\epsilon^\mu = \frac{1}{m_W}(|k|, E\hat{k})^\mu \approx \frac{k^\mu}{m_W}$$

then

$$\epsilon_+^* \cdot \epsilon_-^* \approx \frac{s}{2m_W^2}$$

But, multiplying the amplitude by this factor violates unitarity in the l=1 partial wave. All three diagrams have this improper behavior.

However, with specifically the Yang-Mills structure of the vertices, these dangerous contributions cancel.

The final prediction for the cross section for

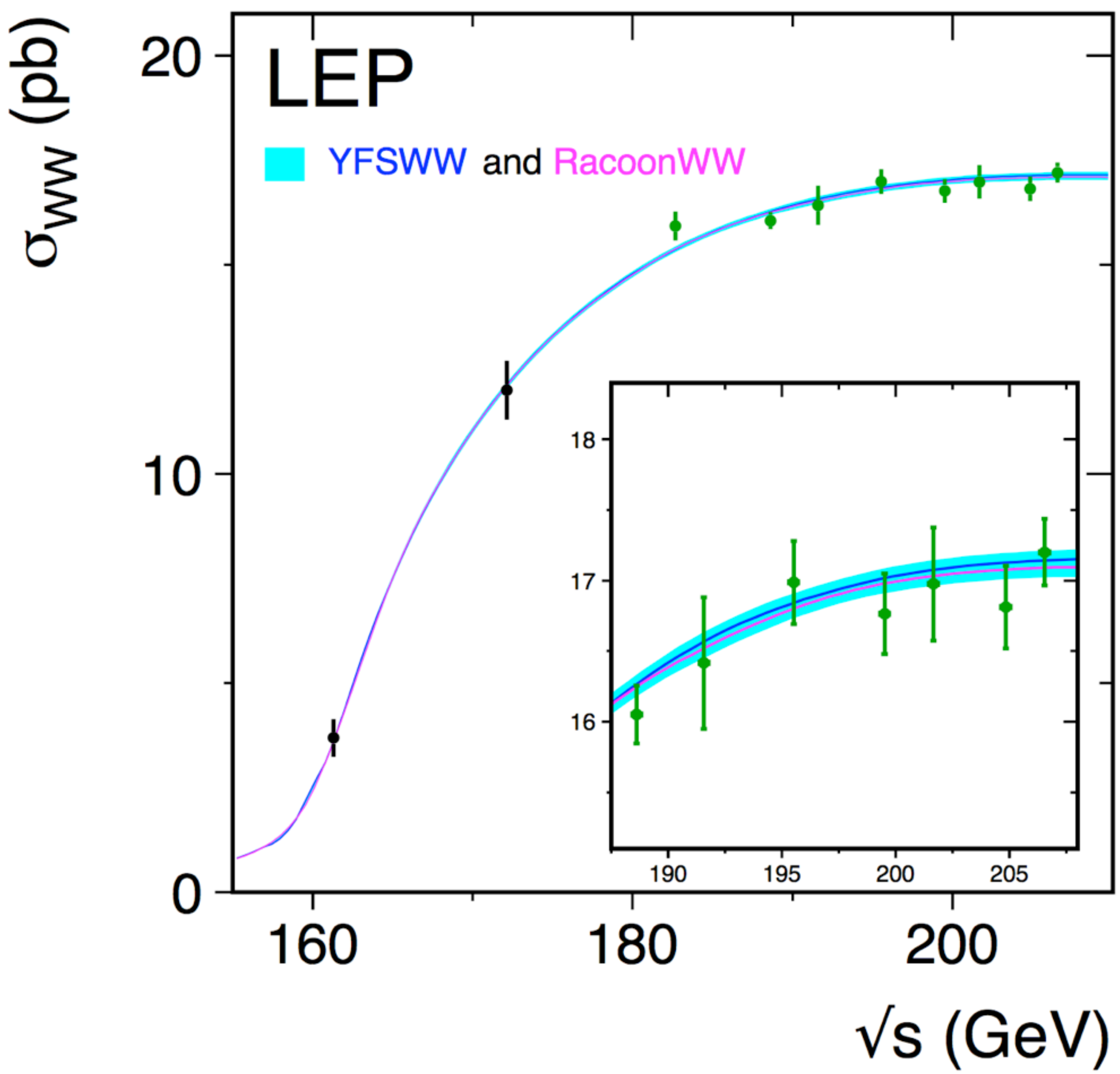
$$e^+e^- \to W_0^+W_0^-$$

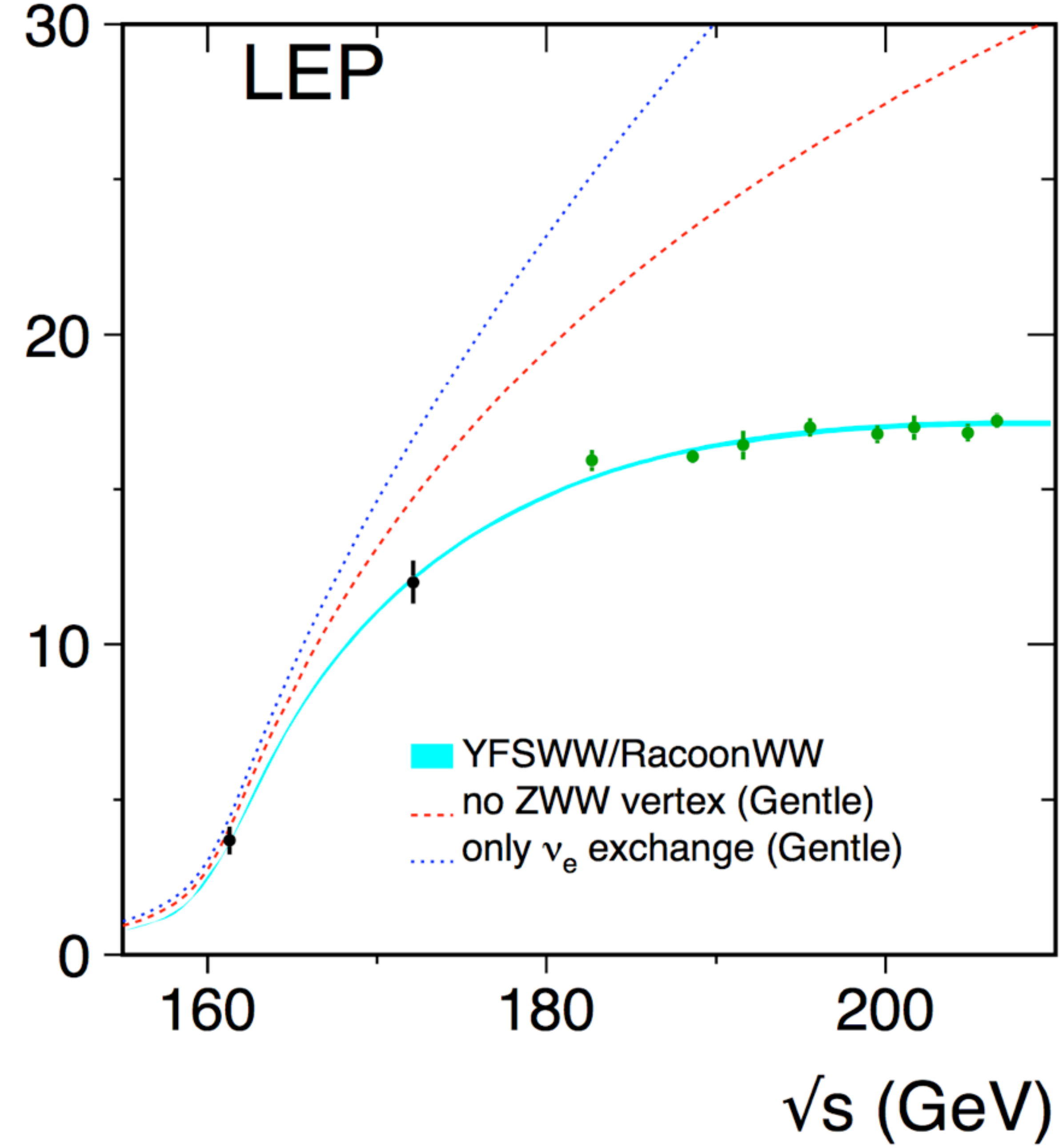
is approximately, at high energy, that for production of states of the Higgs sector.

$$e^+e^- \to \pi^+\pi^-$$

This result is called "Goldstone boson equivalence".

What does experiment have to say ?





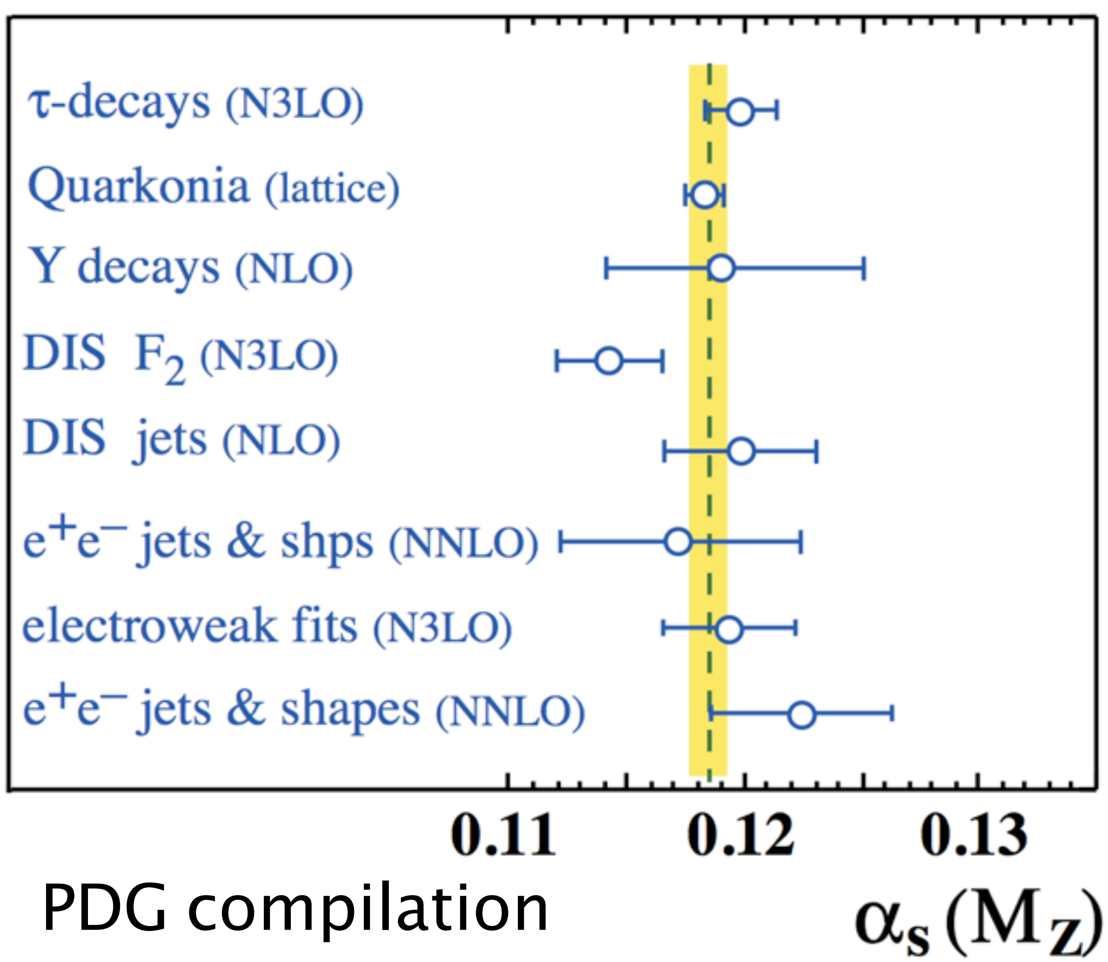
ALEPH, DELPHI, OPAL, L3

QCD, the gauge theory of the strong interactions, is similarly tested at a significant level.

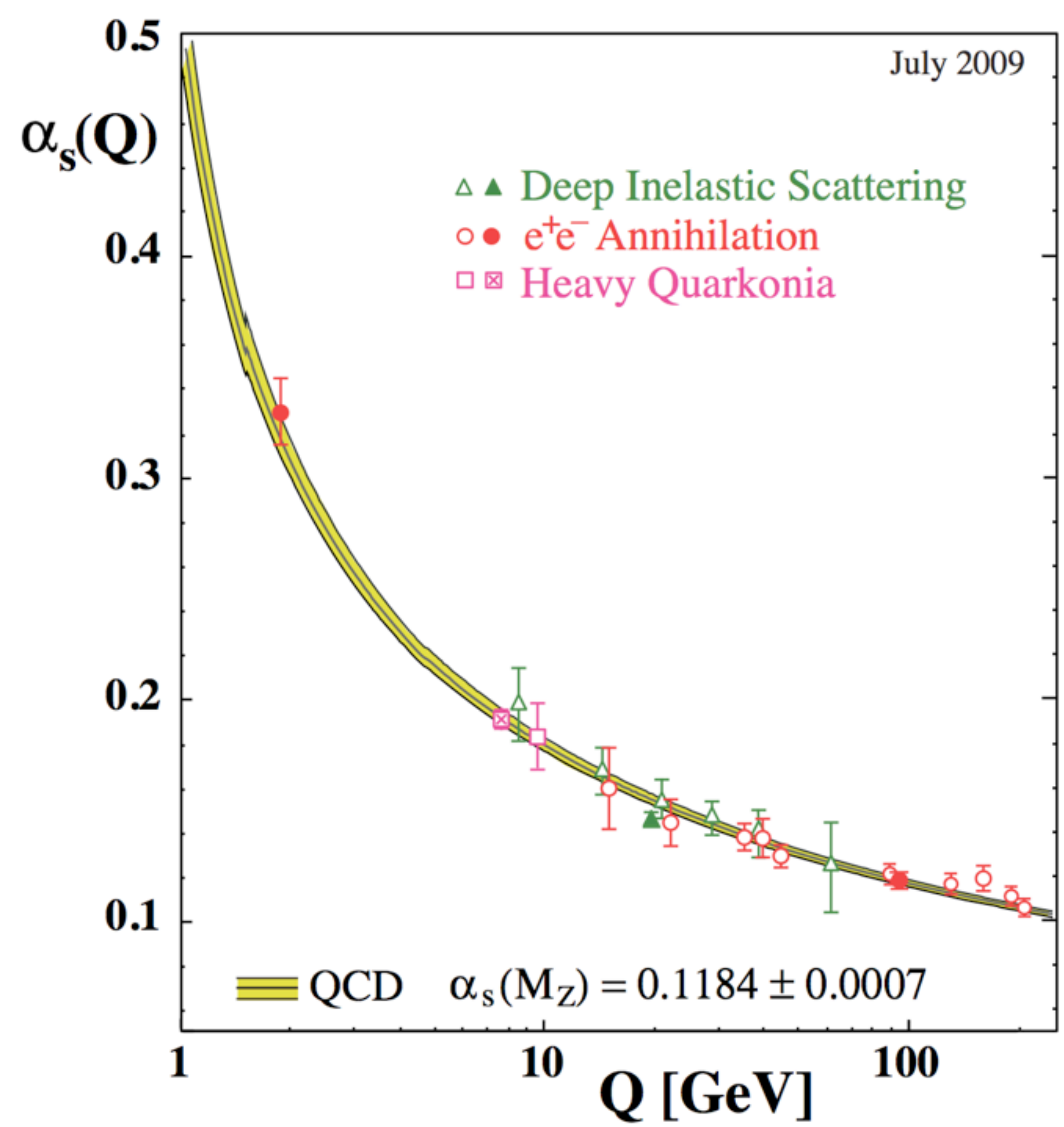
For example,

values of α_s extracted from completely different processes are in good agreement

the scale-dependence of α_s predicted from asymptotic freedom is verified



PDG compilation



It is not surprising that, in describing tests of the SU(2)xU(1) weak interaction theory, we encountered the Higgs boson.

The SM weak interaction theory is full of apparent contradictions:

The W and Z bosons obey a set of equations closely related to Maxwell's Equations. The argument that the photon is massless also implies that W and Z are massless.

The L and R fields of the various fermion species have different SU(2)xU(1) quantum numbers. For example,

$$u_L : (\frac{1}{2}, \frac{1}{6}) \qquad u_R : (0, \frac{2}{3})$$

This forbids a mass terms for all fermions, since a mass term has the structure

$$\Delta\mathcal{L} = -m_u(\overline{u}_L u_R + \overline{u}_R u_L)$$

Both problems are addressed if SU(2)xU(1) symmetry is a symmetry of the full Lagrangian that is spontaneously broken.

We call the fields responsible for this spontaneous symmetry breaking the Higgs sector.

The simplest model of the Higgs sector is that it contains a single complex doublet scalar field

$$\Phi \qquad\qquad (\frac{1}{2}, \frac{1}{2})$$

After symmetry breaking,

$$\Phi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} + \begin{pmatrix} \pi^+(x) \\ (h(x) + i\pi^0(x))/\sqrt{2} \end{pmatrix}$$

Then we find both types of needed mass term,

$$D_\mu \Phi^\dagger D_\mu \Phi \to (\frac{g^2 v^2}{4}) W^+_\mu W^{-\mu} + \cdots$$

$$\lambda_d (\overline{u}_L, \overline{d}_L) \cdot \Phi \; d_R \to (\lambda \frac{v}{\sqrt{2}}) \overline{d}_L d_R$$

with

$$m_W = \frac{gv}{2} \qquad m_d = \lambda_d \frac{v}{\sqrt{2}} \qquad v = 246 \text{ GeV}$$

I must emphasize that the choice of one scalar doublet is just a guess. Any set of fields with the same symmetry-breaking pattern will give the same results for the mass spectrum and for all Standard Model tests discussed up to this point.

The fields $(\pi^+\pi^-, \pi^0)$ are massless before the gauge couplings are turned on. After the Higgs sector is coupled to gauge fields, these Goldstone boson fields are absorbed into the massive vector bosons. In highly boosted massive vector bosons, these fields give the longitudinal components.

The presence of 3 Goldstone bosons and their absorption in this way is also a general property of the symmetry breaking

$$SU(2) \times U(1) \to U(1)$$

There is one technical qualification: In the one-Higgs-doublet case, the W and Z masses obey the relation

$$m_W^2/m_Z^2 = \cos^2\theta_w$$

which is well satisfied experimentally. It can be shown that this relation follows from the presence of an unbroken SU(2) symmetry in the Higgs sector. This is called custodial SU(2). Not all models have this, but it is easy to arrange.

With this introduction, I pose the "Big Question" about the Higgs sector:

Why is the ground state of the Higgs sector asymmetric with respect to SU(2)xU(1) ?

The spontaneous symmetry breaking of the Higgs sector is responsible for all elementary masses of quarks, leptons, and gauge bosons, which in turn give most of the phenomenology of elementary particles. So this question deserves an answer.

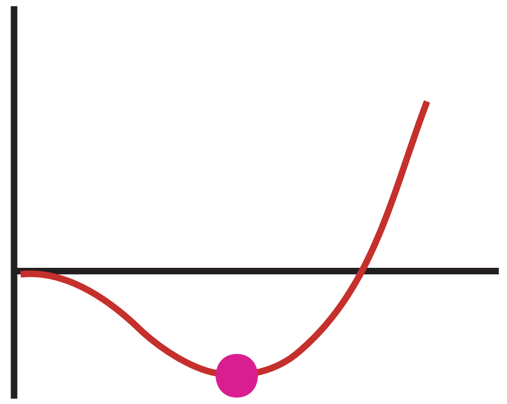
Here is the answer given in the Standard Model:

The answer is determined by the form of the Higgs potential $V(\Phi)$.
The most general form consistent with renormalizability is

$$V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$$

Assume that $\mu^2 < 0$.

Then this potential has the form

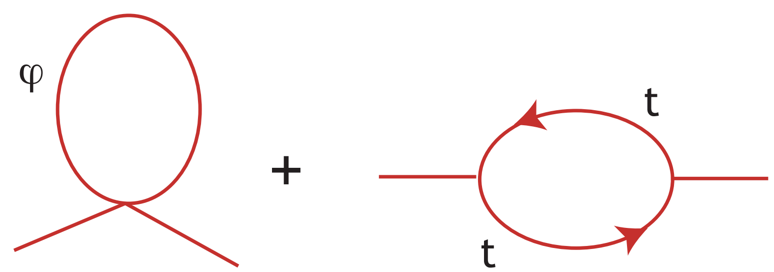


Hmm... Not very satisfying ?

This would make more sense if it were possible to compute μ^2 and find a negative value.

Unfortunately, it is not possible to compute μ^2 within the Standard Model.

The two largest Standard Model diagrams in lowest order are



They give

$$\mu^2 = \mu^2_{\text{bare}} + \frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \cdots$$

That is, both diagrams are strongly ultraviolet divergent, with opposite signs. We need knowledge of a larger theory at short distances to know how to resolve these divergent terms.

We cannot postpone this larger theory to very small distances or very high energies. A quadratic divergence is truly large. If Λ is put at the Planck scale, this equation reads

$$(100\ \text{GeV})^2 = \mu^2_{\text{bare}} + A \cdot 10^{36}\ \text{GeV}^2 - B \cdot 10^{36}\ \text{GeV}^2$$

which makes no sense. That problem is called the "gauge hierarchy problem". This is sometimes discussed as an independent problem, but I prefer to view it as a facet of the "no clue about the physics" problem.

There is some history here that is worth recalling.

The form of $V(x)$ discussed above was first written down by Landau and Ginzburg in their 1950 proposal for a phenomenological theory of superconductivity.

Landau and Ginzburg wrote the Higgs field mass term as

$$b(T - T_c)|\Phi|^2$$

to incorporate phenomenologically the fact that the superconducting transition occurs at the temperature T_c . They did not give an explanation from underlying physics.

Landau-Ginzburg theory explained many aspects of superconductivity, including the thermodynamics of the phase transitions and the possibility of magnetic flux tubes.

It was always understood that there must be a mechanical explanation for superconductivity based on atomic physics. So, after Landau and Ginzburg, people kept searching for it, and eventually, it was found.

Later, in 1957, Bardeen, Cooper, and Schrieffer developed a theory of electron-electron attraction through phonons that explained the symmetry breaking. This led to a predictive theory with much more insight.

In elementary particle physics, there is sometimes the idea that, if we can write down an appropriate equation, that is sufficient, and we are done.

We have been there before. Particle physicists had this attitude in the 1960's concerning the strong interactions. Go back and read some of the literature on "the analytic S-matrix and nuclear democracy". QCD, a mechanical theory based on the demonstrable insight of asymptotical freedom, is much more satisfying.

My opinion is that we have to keep asking why.

This leads us to ask,

What does a theory that predicts Higgs sector symmetry breaking actually look like?

Such a theory must have two components:

First, there must be a way to eliminate the quadratic divergences. This requires either new physics very close to the Higgs energy scale, or a symmetry that forbids the quadratic divergence from being generated very short distances. In the latter case, the diagrams shown earlier still exist and so must be cancelled. New particles are required to do this.

Second, there must be a calculation that leads to $\mu^2 < 0$. This calculation must involve some particles or fields not present in the Standard Model.

So, any predictive theory of Higgs symmetry breaking requires that there must be physics beyond the Standard Model.

An important question is, how far can this new physics be above the Higgs mass scale

$$v = 246 \text{ GeV}$$

A criterion called "naturalness" says that the particles responsible for the shape of the Higgs potential must be not so far away, at energies of at most a few TeV. Otherwise, the argument goes, we rely on large cancellations between terms in the equations, which also destroy our physical understanding.

It would be nice to turn naturalness into an upper bound on new particle masses, but this is not so easy. I will give some examples later.

There is a very attractive theory that explains Higgs symmetry breaking through a clearly understood mechanism and also solves the gauge hierarchy problem. This follows quite precisely the analogy with superconductivity.

This theory, called "technicolor", states that there is a new strong interaction theory, similar to QCD, for which the analog of α_s becomes strong at about 2 TeV. We postulate technifermion multiplets

$$\begin{pmatrix} U \\ D \end{pmatrix}_L \qquad U_R \qquad D_R$$

and couple these straightforwardly to SU(2)xU(1). The strong interactions cause the technifermions to pair up and the pairs to condense into the vacuum -- exactly as is known to happen in QCD. The condensates

$$\langle \overline{U}_L U_R \rangle = \langle \overline{D}_L D_R \rangle \neq 0$$

generate the correct pattern of W and Z masses. (The analog of isospin plays the role of the custodial symmetry.)

Unfortunately,

Mixing of the analog of the ρ and a_1 mesons with the Z generate large (many %) corrections to precision electroweak observables, in disagreement with experiment.

It is very difficult to generate quark and lepton masses. The simplest ways to do this lead to processes such as

$$K_L^0 \to \mu^+ e^-$$

at unacceptable levels ($BR \sim 10^{-10}$).

It is very difficult to generate the large top quark mass without making the top a technifermion, leading to large corrections to $\sigma(pp \to t\bar{t})$.

The theory does not contain a light Higgs particle. The first 0^+ resonance in the Higgs sector is at the technicolor scale of 2 TeV.

We now know that there is a particle at 125 GeV that strongly resembles the Higgs boson of the Standard Model. There is strong -- not yet decisive -- evidence that this particle is a scalar, either elementary or composite.

This points to models in which a scalar Higgs field is part of the theory of the Higgs potential. Such models only make sense as predictive theories if the quadratic divergences in the Higgs mass term cancel.

To insure this, contributions to the Higgs mass term must be forbidden by symmetry in this theory. But the Higgs mass term

$$\Delta\mathcal{L} = -\mu^2 |\Phi|^2$$

is invariant under every symmetry most of us can think of. So how can this be possible ?

In fact, there are three known symmetries that forbid a scalar field mass term.

Invariance under a shift: $\delta\Phi = \epsilon$

Rotation into a vector: $\delta\Phi = \epsilon_\mu A^\mu$

Rotation into a fermion: $\delta\Phi = \bar{\epsilon}\psi$

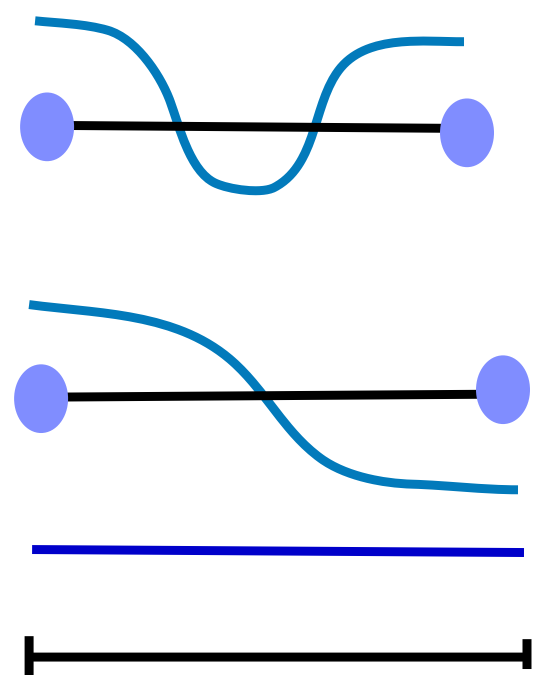
The shift symmetry is characteristic of a Goldstone boson. In this case, the Higgs doublet is a Goldstone boson of another level of symmetry breaking at a higher energy scale. Perhaps this is a strong interaction theory and the Higgs is composite. Models with this symmetry are called "Little Higgs".

The rotation into a vector appears in theories with extra space dimensions. In this case, the Higgs fields are the 5th components of gauge fields A^a_M .

The rotation into a fermion is supersymmetry. In this case, the underlying theory must be completely supersymmetric to avoid quadratic divergences to all orders. This brings many new constraints.

Each type of symmetry leads to a spectrum of new particles.

This is especially obvious for models with extra dimensions. Each mode of the field in the extra dimensions is realized as a particle in our 4d space.

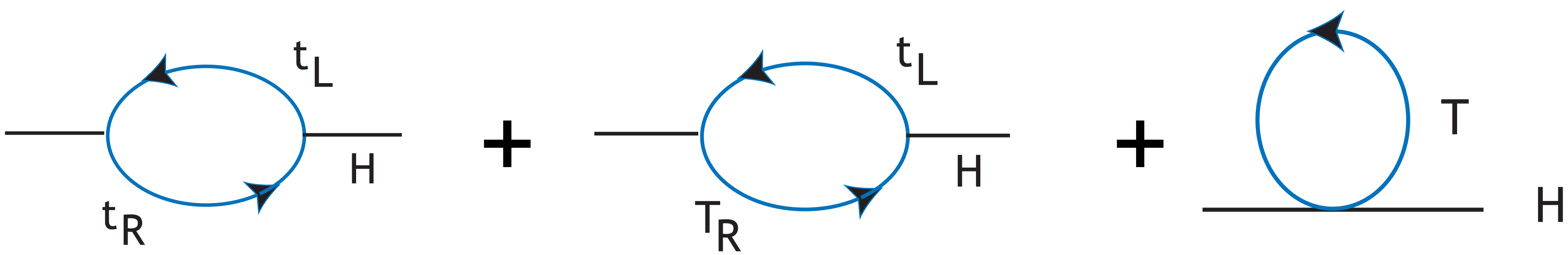


The new massive particles are called Kaluza-Klein excitations. Notice that they receive mass without needing the Higgs mechanism. For fermions, each SM representation leads to a Dirac fermion with vectorlike coupling. For example, the KK excitations of t_R are singlet top quarks

$$T_L + T_R \qquad (0, \frac{2}{3})$$

With a flat extra dimension, integrating out a fermion coupled to a gauge field generates a potential with $\mu^2 < 0$ for the 5th component of that gauge field. This is the Hosotani mechanism.

Little Higgs models also require extended gauge symmetries and new vectorlike third-generation quarks. These are required to cancel the quadratic divergences of the Higgs mass term, e.g.



= (finite)

By explicit calculation, the residual finite (log-divergent) term is negative in the simplest cases.

Supersymmetry is a space-time symmetry like the rotation into extra dimensions. For similar reasons, supersymmetry requires that every particle of the SM has a partner with spin differing by 1/2.

The formal argument for this is based on the fact that

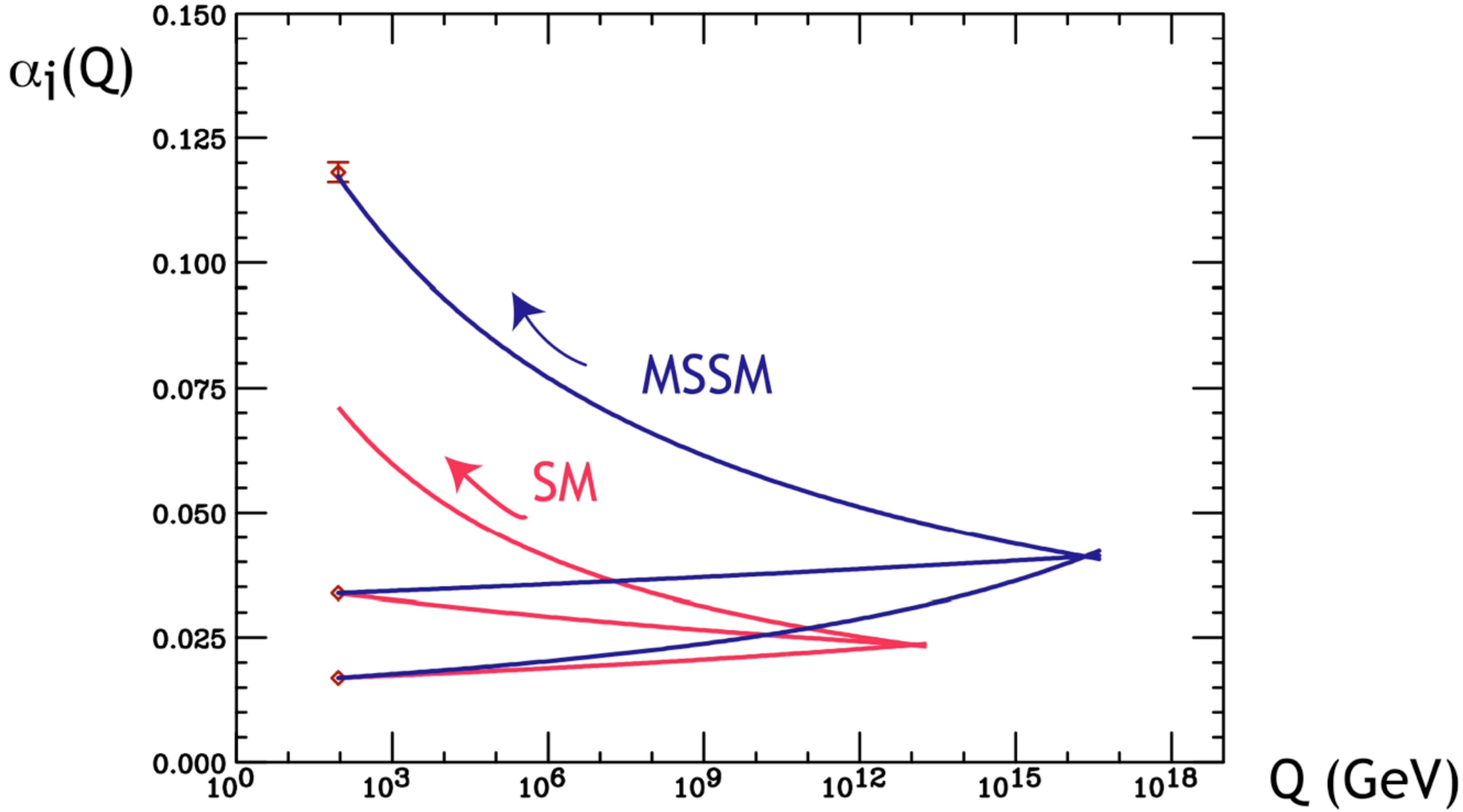
$$\{Q_\alpha, Q_\beta\} = 2\gamma^\mu_{\alpha\beta} P_\mu$$

is the unique extension of the Poincare algebra to an algebra with spin-1/2 charges.
is the energy-momentum of everything.

The partners must be new, as yet undiscovered particles

$$e_L \to \widetilde{e}_L \qquad e_R \to \widetilde{e}_R \qquad g \to \widetilde{g}$$

The presence of these particles has an interesting implication. They change the coefficients in the renormalization group formulae for the scale-dependent of the SU(3)xSU(2)xU(1) couplings.

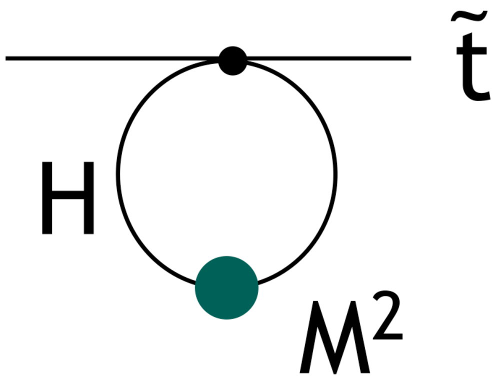


$\alpha_i(Q)$
0.150
0.125
0.100
0.075
0.050
0.025
0.000
MSSM
SM
10^0
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}
Q (GeV)

If the new particles have not yet been seen, they must be massive. These masses come from the spontaneous breaking of supersymmetry. The logic of the model is

first, supersymmetry is spontaneously broken,
then, this symmetry breaking is communicated to the partners of the Standard Model particles,
then, this gives us information to calculate the Higgs potential
finally, the Higgs field condenses and gives mass to quarks, leptons, and gauge bosons

It works. Diagrams such as



give negative μ^2 to the Higgs field and the scalar partner fields of the top quarks.

The Higgs receives the largest of these negative contributions; this drives Higgs condensation

So far, I have only discussed the problem of understanding the physics of the Higgs boson.   Now I would like to circle back and discuss the two other questions that I highlighted.

The problem of the origin of the dark matter of the universe is the most striking evidence that there is new physics beyond the Standard Model.

Many sources of evidence, from galaxies, galaxy clusters, and the cosmic microwave background, imply that 80% of the mass in the universe is composed of a weakly interacting neutral particle (or particles) not contained in the Standard Model.

Mark Kamionkowski will describe this evidence in his lectures.

There are many models for the origin of dark matter, with masses spanning 30 orders of magnitude. Here, I would like to start from the following generic, plausible, idea:

Dark matter is composed of a weakly interacting neutral massive particle that is absolutely stable and was once in thermal equilibrium in the hot early universe.

Such a dark matter candidate particle is called a WIMP.

Under the WIMP hypothesis, the initial condition for the density of dark matter particles is set by thermal equilibrium. When

$$T \sim m_N$$

more dark matter particles annihilate than are produced, and the number decreases relative to photons. However, by hypothesis, dark matter particles can annihilate only in pairs, so as the universe expands, some of these particles will not be able to find partners. Solving the Boltzmann equation, one finds

$$\Omega_N = \frac{s_0}{\rho_c} \left(\frac{45}{\pi g_*} \right)^{1/2} \frac{1}{\xi_f m_{\rm Pl}} \frac{1}{\langle \sigma v \rangle}$$

We know all terms in this equation except for the WIMP annihilation cross section. Solving for this gives

$$\langle \sigma v \rangle = 1 \text{ pb}$$

If the interaction strength is electroweak, we can estimate

$$\sigma v = \frac{\pi \alpha^2}{2 m_N^2}$$

then $m_N \sim 200 \text{ GeV}$. From purely astrophysical data, we find that the WIMP mass must be at the TeV scale.

Our models of the Higgs potential contain many particles that could play the role of the WIMP. In supersymmetry, the partner of the photon, Z, or Higgs is a candidate. In extra dimensional models, the KK partners of these particles are candidates.

We need a symmetry the keeps the lightest new particle absolutely stable. This is easily arranged. We need a Z_2 symmetry under which the SUSY generators are odd, or a reflection symmetry in the extra dimensions.

In principle, the WIMP or other dark matter candidate can come from a new source, unconnected to other questions in fundamental physics. However, the models of the Higgs potential provide a raison d'etre for the WIMP.

Further, we can search for these particles directly in collider experiments. If we find them, we can measure their properties and build a microscopic understanding of dark matter.

The last of my questions was the origin of the baryon asymmetry of the universe.

Above the phase transition to Higgs condensation, the Standard Model contains a source of baryon number violation, through a nonperturbative effect in the SU(2) gauge theory called the sphaleron.

To generate a baryon asymmetry, we also need CP violation. It is known that the CP violation in the Standard Model from the Kobayashi-Maskawa phase is not sufficient to generate the observed baryon asymmetry.

The new particles in models of the Higgs potential have couplings that can be CP-violating. Again, if we discover these particles, we can measure the couplings and build a complete theory of TeV-scale baryogenesis.

Among all of the Big Questions, the question of the Higgs potential is the only one that must be solved at the TeV scale. We have seen that the solution to this problem necessarily brings in new particles and forces.

Once we expand our list of particles and forces, the new particles will be actors in all other elementary particle processes that we are interested in

flavor mixing, flavor-changing neutral currents

theories of flavor

models of non-WIMP dark matter

grand unification

models of inflation

The understanding of any of these questions begins from this one:

What is the complete spectrum of particles at the TeV energy scale ?

This is the problem of our era in fundamental physics. Experiments at high-energy accelerators give the path to solving this problem.

I will discuss the opportunities for new particle discovery and exploration in tomorrow's lecture.