Toward a set of 2HDM benchmarks

Howard E. Haber

Santa Cruz Institute for Particle Physics

University of California, Santa Cruz, CA 95064, USA

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Abstract

In these notes, the theoretical details of the two-Higgs doublet model (2HDM) are described with an eye toward developing benchmark scenarios for LHC Higgs studies. In particular, the Type-I and Type-II 2HDM parameters are formulated subject to the condition that the lightest CP-even Higgs boson is identified with the 126 GeV boson discovered on 4 July 2012. Some preference is given to the region of parameter space in which the couplings of this scalar to WW and ZZ approach the Standard Model expectations.

1 Introduction

The most general two-Higgs doublet model (2HDM) consists of a Higgs scalar potential with two real squared-mass parameters, four real self-coupling parameters, one complex squared-mass parameter and three complex self-coupling parameters. In addition, all possible dimension-four Higgs-fermion Yukawa coupling matrices are present. Such a model yields new CP-violating neutral Higgs boson couplings, which imply that none of the three neutral Higgs states have definite CP properties. In addition, this most general model leads to tree-level flavor changing neutral currents (FCNCs) mediated by neutral Higgs bosons, which is very strongly constrained by particle physics data.

In order to avoid these potential phenomenological problems, it is standard practice to impose a symmetry on the Higgs-fermion couplings, which if chosen correctly will eliminate all tree-level Higgs-mediated FCNCs. A possible symmetry that achieves this goal is a discrete $\mathbb{Z}_2$ symmetry, in which one of Higgs doublet fields changes sign. If we focus for the moment on the Higgs-quark couplings, there are two distinct implementations that leads to the so-called Type-I and Type-II Higgs-quark interactions. Another possibility is to impose supersymmetry. In its minimal implementation, the corresponding Higgs-fermion interactions corresponds to Type-II.

In order to consistently impose the required symmetry, one must also enforce the symmetry on the scalar Higgs potential. In the case of the discrete symmetry, the imposition of the symmetry on the Higgs potential is actually more strict than necessary. In particular, one can relax the symmetry requirement by imposing it only on the quartic Higgs interactions. In this case, a quadratic term in the Higgs potential that mixes the two Higgs doublets is allowed even though it explicitly breaks the discrete symmetry.[1] One can also show that

[1]This generalization is also useful in that it allows one to simultaneously treat the MSSM Higgs sector which allows this quadratic Higgs mixing term.

although the inclusion of the quadratic Higgs mixing term will generate Higgs-mediated FC-NCs at one-loop, these effects are small enough so as not to be in conflict with observed data.

Hence, for the rest of this note, I will focus on the 2HDM subject to a discrete $\mathbb{Z}_2$ symmetry, which enforces either Type-I or Type-II Higgs-fermion couplings, and is at most softly broken by a quadratic term in the Higgs potential that mixes the two Higgs doublets. This provides a well-defined framework for experimental 2HDM studies.

2 Details of the 2HDM model

We begin with the most general 2HDM and impose a discrete $\mathbb{Z}_2$ discrete symmetry on the quartic Higgs self-interactions and the Higgs-fermion interactions. In this section, we focus on the scalar Higgs potential. Given two hypercharge-one, weak doublet fields, Φ_1 and Φ_2, we impose a symmetry on the quartic Higgs self-interactions where $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to \Phi_2$. For simplicity, we also impose the requirement of CP conservation. In this case, the most general scalar Higgs potential is given by

$$\begin{aligned}\mathcal{V} = {}& m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\} ,\end{aligned}$$

where the two potentially complex parameters m_{12}^2 and λ_5 are taken to be real. This scalar potential is explicitly CP-conserving, although we must impose one further condition to avoid a possible CP-violating vacuum that is a global minimum. Note that only the $\Phi_1^\dagger \Phi_2 + \text{h.c.}$ term breaks the discrete $\mathbb{Z}_2$ symmetry, as advertised.

We now require that the minimum of the potential correspond to a CP-conserving vacuum that does not break $\text{U}(1)_{\text{EM}}$. In this case, the corresponding vacuum expectation values are: $\langle \Phi_i^0 \rangle = v_i/\sqrt{2}$, with $\tan\beta \equiv v_2/v_1$ and $v^2 \equiv v_1^2 + v_2^2 = (246\ \text{GeV})^2$. Thus, we trade in the two parameters m_{11}^2 and m_{22}^2 (via the potential minimum conditions) for v^2 and $\tan\beta$. This leaves six free parameters—m_{12}^2 and the five real Higgs self-couplings, λ_1, $\lambda_2, \ldots, \lambda_5$. From these six parameters, one can compute the four physical Higgs masses (m_h, m_H, m_A and $m_{H^\pm}$) and the neutral CP-even Higgs mixing angle α obtained by diagonalizing the $2\times$ CP-even Higgs squared-mass matrix. This leaves one free parameter left over.

The parameter $\cos(\beta-\alpha)$ is a critical parameter of the model, as it controls the approach to the decoupling limit. In particular, if we set $\cos(\beta-\alpha) =$, then the tree-level couplings of h^0 coincide exactly with the tree-level couplings of the Standard-Model Higgs boson. Since current LHC data suggests that the observed Higgs boson is "Standard-Model-like," we shall assume in these notes that the value of $\cos(\beta-\alpha)$ is not all that far away from 0.[2]

[2]Strictly speaking, one could also consider an alternative scenario in which $\sin(\beta-\alpha)$ is close to 0, in which case the heavier H^0 would be identified with the newly discovered boson. At present, this scenario cannot be ruled out, and should be presented as one of the benchmark points for further studies. I will come back to this possibility later in these notes.

In order to establish a strategy for benchmark scenarios, it is important to derive relations between physical Higgs observables and the basis parameters that appear in the Higgs scalar potential. Here, we shall follow Ref. 1. It is convenient to define the following four linear combinations of the λ_i:

$$\lambda \equiv \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \tfrac{1}{2}\lambda_{345} s_{2\beta}^2\,, \tag{1}$$

$$\widehat{\lambda} \equiv \tfrac{1}{2} s_{2\beta}\left[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}\right]\,, \tag{2}$$

$$\lambda_A \equiv c_{2\beta}(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2) + \lambda_{345} s_{2\beta}^2 - \lambda_5 \tag{3}$$

$$\lambda_F \equiv \lambda_5 - \lambda_4\,, \tag{4}$$

where $\lambda_{345} \equiv \lambda_3+\lambda_4+\lambda_5$, and c and s stand for the cosine and sine of the angle that appears as a subscript. The significance of these particular linear combinations will be addressed later.

The physical Higgs squared-masses can be expressed in terms of these coupling combinations and $\beta - \alpha$ as follows:

$$m_h^2 = v^2\left[\lambda - \frac{\widehat{\lambda}\, c_{\beta-\alpha}}{s_{\beta-\alpha}}\right]\,, \tag{5}$$

$$m_H^2 = v^2\left[\lambda + \frac{\widehat{\lambda}\, s_{\beta-\alpha}}{c_{\beta-\alpha}}\right]\,, \tag{6}$$

$$m_A^2 = v^2\left[\lambda_A + \widehat{\lambda}\left(\frac{s_{\beta-\alpha}}{c_{\beta-\alpha}} - \frac{c_{\beta-\alpha}}{s_{\beta-\alpha}}\right)\right]\,, \tag{7}$$

$$m_{H^\pm}^2 = m_A^2 + \tfrac{1}{2}v^2\lambda_F\,. \tag{8}$$

These are exact expressions. Note that eqs. (7) and (6) yield

$$m_H^2 - m_h^2 = \frac{\widehat{\lambda}}{s_{\beta-\alpha}c_{\beta-\alpha}}\,. \tag{9}$$

By definition, $m_h < m_H$,[3] which imposes the constraint:

$$\widehat{\lambda}\, s_{\beta-\alpha}c_{\beta-\alpha} > 0\,. \tag{10}$$

In light of eq. (10), we see from eq. (5) that

$$m_h^2 \leq \lambda v^2\,. \tag{11}$$

We now impose a key assumption that the Higgs self-coupling parameters satisfy

$$\frac{\lambda_i}{4\pi} \lesssim \mathcal{O}(1)\,. \tag{12}$$

[3]In principle, it is possible to have $m_h = m_H$, but this is an isolated point of the 2HDM parameter space, so we will neglect it.

This is necessary to maintain tree-level unitarity and the perturbativity of the theory. Of course, since the observed Higgs boson has a mass of about 125 GeV, eq. (11) already tells us that λ satisfies eq. (12). We simply extend this rough upper bound to all Higgs self-coupling parameters that appear in the Higgs scalar potential (which of course also applies to the self-coupling combinations given in eqs. (1)–(4).

As previously noted, if $c_{\beta-\alpha}=0$, then the couplings of h match those of the SM Higgs boson, whereas if $s_{\beta-\alpha}$, then the couplings of H match those of the SM Higgs boson. For the present analysis, let us assume that it is h that should be identified with the SM-like Higgs boson. In this case, it will be convenient to eliminate the parameter $\widehat{\lambda}$ in favor of m_h^2. In particular, eq. (5) yields[4]

$$\widehat{\lambda}=\left(\lambda-\frac{m_h^2}{v^2}\right)\frac{s_{\beta-\alpha}}{c_{\beta-\alpha}}\,. \tag{13}$$

Plugging this back into eqs. (6) and (8) yields

$$m_H^2=\frac{1}{c^2_{\beta-\alpha}}\left(\lambda v^2-m_h^2 s^2_{\beta-\alpha}\right)\,, \tag{14}$$

$$m_A^2=\lambda_A v^2+\left(\lambda v^2-m_h^2\right)\left(\frac{s^2_{\beta-\alpha}}{c^2_{\beta-\alpha}}-1\right)\,, \tag{15}$$

$$m^2_{H^\pm}=(\lambda_A+\tfrac{1}{2}\lambda_F)v^2+\left(\lambda v^2-m_h^2\right)\left(\frac{s^2_{\beta-\alpha}}{c^2_{\beta-\alpha}}-1\right)\,. \tag{16}$$

As we approach the decoupling limit ($c_{\beta-\alpha}\to 0$), we interpret the above equations by noting that eq. (5) implies that $\lambda-m_h^2/v^2\sim\mathcal{O}(c_{\beta-\alpha})$. This implies that $m_H^2\sim m_A^2\sim m^2_{H^\pm}\sim\mathcal{O}(1/c_{\beta-\alpha})\gg m_h^2$.

Finally, the following result, which is easily obtained from eqs. (5) and (9):

$$\lambda v^2-m_h^2=(m_H^2-m_h^2)c^2_{\beta-\alpha}\,. \tag{17}$$

One can use this relation to obtain

$$m_A^2=m^2_{H^\pm}-\tfrac{1}{2}\lambda_F v^2=\lambda_A v^2+(m_H^2-m_h^2)(s^2_{\beta-\alpha}-c^2_{\beta-\alpha})\,. \tag{18}$$

The couplings of the Higgs bosons to vector bosons are well-known (see e.g. Appendices A1–A3 of Ref. 2), and can be expressed directly in terms of gauge couplings, $s_{\beta-\alpha}$ and $c_{\beta-\alpha}$. In the decoupling limit where $c_{\beta-\alpha}\to 0$ and $s_{\beta-\alpha}\to 1$, one can easily verify that the couplings of h^0 to gauge bosons reduce to the corresponding Standard Model values. The Higgs self-couplings can be expressed in terms of $s_{\beta-\alpha}$, $c_{\beta-\alpha}$, the coupling combinations λ, $\widehat{\lambda}$, λ_A, λ_F and three additional coupling combinations that only appear in the tri-linear and quadra-linear couplings of the physical Higgs bosons (as discussed in Ref. 1). Perhaps the only relevant Higgs self-coupling for Higgs phenomenology in the near term is the H^+H^-h

[4] Keep in mind that $\widehat{\lambda}/4\pi\lesssim\mathcal{O}(1)$. This means that in the decoupling limit, as $c_{\beta-\alpha}\to 0$, we also have $m_h^2\to\lambda v^2$.

coupling, as this enters in the analysis of the charged Higgs loop contribution to $h \to \gamma\gamma$ decay. However, this contribution tends to be quite small, and we will neglect it in the present discussion.

We end this section with a brief discussion of the Higgs-fermion interaction. The most general Yukawa Lagrangian, in terms of the quark mass-eigenstate fields, is:

$$-\mathscr{L}_{\rm Y} = \sum_{a=1}^{2} \left\{ \overline{U}_L \widetilde{\Phi}_a^0 \eta_a^U U_R + \overline{D}_L K^\dagger \widetilde{\Phi}_a^- \eta_a^U U_R + \overline{U}_L K \Phi_a^+ \eta_a^{D\,\dagger} D_R + \overline{D}_L \Phi_a^0 \eta_a^{D\,\dagger} D_R + \text{h.c.} \right\}, \quad (19)$$

where $a = 1, 2$, $\widetilde{\Phi}_a \equiv (\widetilde{\Phi}^0\,, \widetilde{\Phi}^-) = i\sigma_2 \Phi_a^*$ and K is the CKM mixing matrix. The $\eta^{U,D}$ are 3×3 Yukawa coupling matrices.

We now extend the $\mathbb{Z}_2$ symmetry of the Higgs scalar potential to the Higgs-fermion Yukawa Lagrangian. In addition to $\Phi_1 \to -\Phi_1$, we assume that Φ_2 and all the fermion fields are unchanged, then it follows that $\eta_1^U = \eta_1^D = 0$. Thus, only Φ_2 couples to the fermions. This corresponds to the Type-I Higgs-fermion interaction. The diagonalization of the up-type and down-type fermion mass matrices automatically diagonalizes the Yukawa coupling matrices η_2^U and η_2^D, which yields flavor-diagonal couplings of the fermions to the neutral Higgs bosons. The corresponding Higgs-fermion couplings depend on α and β as indicated below:

Table 1: Type-I Yukawa couplings: $\eta_1^U = \eta_1^D = 0$.

	h^0	A^0	H^0
up-type quarks	$\cos\alpha/\sin\beta$	$\cot\beta$	$\sin\alpha/\sin\beta$
down-type quarks and leptons	$\cos\alpha/\sin\beta$	$-\cot\beta$	$\sin\alpha/\sin\beta$

The couplings of the neutral CP-even Higgs bosons to the up and down-type fermions can be conveniently re-expressed by employing the trigonometric identities,

$$\frac{\cos\alpha}{\sin\beta} = s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta\,, \quad (20)$$

$$\frac{\sin\alpha}{\sin\beta} = c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta\,, \quad (21)$$

One can check easily that in the decoupling limit where $c_{\beta-\alpha} \to 0$ and $s_{\beta-\alpha} \to 1$, the h^0 couplings to fermions reduce to the corresponding Standard Model values.

Alternatively, one can extend the $\mathbb{Z}_2$ discrete symmetry such that $\Phi_1 \to -\Phi_1$ and $D_R \to -D_R$ (with Φ_2 and all other fermion fields unchanged). In this case, Φ_1 couples exclusively to D_R whereas Φ_2 couples exclusively to U_R. This corresponds to the Type-II Higgs-fermion interaction. In this case, the diagonalization of the up-type and down-type fermion mass matrices automatically diagonalizes the Yukawa coupling matrices η_2^U and η_1^D, which again yields flavor-diagonal couplings of the fermions to the neutral Higgs bosons. The corresponding Higgs-fermion couplings depend on α and β as indicated below:

Table 2: Type-II Yukawa couplings: $\eta_1^U = \eta_2^D = 0$.

	h^0	A^0	H^0
up-type quarks	$\cos\alpha/\sin\beta$	$\cot\beta$	$\sin\alpha/\sin\beta$
down-type quarks and leptons	$-\sin\alpha/\cos\beta$	$\tan\beta$	$\cos\alpha/\cos\beta$

In this case, eqs. (20) and (23) provide the couplings of the neutral CP-even Higgs bosons to the up-type fermions. Likewise, the couplings of the neutral CP-even Higgs bosons to the down-type fermions can be conveniently re-expressed by employing the trigonometric identities,

$$-\frac{\sin\alpha}{\cos\beta} = s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta\,, \tag{22}$$

$$\frac{\cos\alpha}{\cos\beta} = c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta\,, \tag{23}$$

One can again check easily that in the decoupling limit where $c_{\beta-\alpha} \to 0$ and $s_{\beta-\alpha} \to 1$, the h^0 couplings to fermions reduce to the corresponding Standard Model values.

3 Benchmarks and Parameter scans

Eqs. (14)–(16) provide exact expressions for the masses of H, A and $H^\pm$ as a function of of the known value of m_h, the angle parameter $\beta-\alpha$ and the three self-coupling combinations λ, λ_A and λ_F. To interpret searches for 2HDM scalars, we will need to fix certain parameters in order to make the analysis tractable. Eventually, when the Higgs coupling data becomes more precise (and assuming that no significant deviation from SM-like Higgs couplings is observed), we can restrict the scan of $c_{\beta-\alpha}$ over values close to zero. At present, one must take a less restrictive view.

My recommendations are as follows:

- Take $m_h \simeq 125$ GeV as input into the analysis.
- I recommend scanning over values of $|c_{\beta-\alpha}|$ from 0 to 1/2 (corresponding to a very rough SM-like h). The alternative case where we identify the observed 125 GeV scalar with H will be treated separately.
- One must scan in $\tan\beta$ as well, as this parameter (along with $c_{\beta-\alpha}$) controls the Higgs-fermion couplings, as noted at the end of the previous section. For the most inclusive scan, I would take $\frac{1}{2} \lesssim \tan\beta \lesssim 50$ to avoid excessively large Higgs couplings to either top or bottom respectively.

- For the coupling parameters λ, λ_A and λ_F, select a few representative values. Note that eq. (11) implies that $\lambda \gtrsim \frac{1}{4}$. Additional restrictions apply to λ_A and λ_F by demanding that $m_{H^\pm}$ satisfies the current experimental bound (and likewise for m_A, although the experimental bounds in this case are much more model-dependent). I recommend looking at a few sample values in which these coupling parameters are of $\mathcal{O}(1)$.

- When presenting results, I would provide plots of $c_{\beta-\alpha}$ vs. $\tan\beta$, for benchmark choices of λ, λ_A and λ_F.

Note that this proposal consists of a two parameter scan (over values of $c_{\beta-\alpha}$ and $\tan\beta$), with discrete benchmark choices for the coupling parameters λ, λ_A and λ_F,

It should be noted that in the decoupling limit, the specific values of these parameters become less relevant to accessing the discovery potential of the heavy Higgs states, as these coupling parameters mainly control the small electroweak corrections that split the heavy (approximately degenerate) Higgs masses. Of course, they are more relevant once $c_{\beta-\alpha}$ deviates significantly from zero.

If one is emboldened to scan over three parameters, then it is convenient to swap λ and m_H^2 using eq. (18). In this case, one would scan over possible values of m_H, $c_{\beta-\alpha}$ and $\tan\beta$ and choose benchmark values for λ_A and λ_F.

4 Significance of the coupling parameters λ, $\widehat{\lambda}$, λ_A and λ_F

To appreciate the significance of the combination of Higgs self-couplings defined in eqs. (1)–(4), we note that starting from any Higgs scalar potential, one can always define two new linear combinations of Higgs doublet fields,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^*\Phi_1 + v_2^*\Phi_2}{v}\,, \qquad\qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v}\,.$$

It follows that $\langle H_1^0\rangle = v/\sqrt{2}$ and $\langle H_2^0\rangle = 0$. This is the *Higgs basis*, which is uniquely defined up to an overall rephasing, $H_2 \to e^{i\chi}H_2$. In the Higgs basis, the scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 \\ & + \tfrac{1}{2}Z_2(H_2^\dagger H_2)^2 + Z_3(H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4(H_1^\dagger H_2)(H_2^\dagger H_1) \\ & + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + \big[Z_6(H_1^\dagger H_1) + Z_7(H_2^\dagger H_2)\big]H_1^\dagger H_2 + \text{h.c.}\right\}\,,\end{aligned}$$

where Y_1, Y_2 and $Z_1, \ldots, Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$[Y_3, Z_6, Z_7] \to e^{-i\chi}[Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi}Z_5\,.$$

In the CP-conserving 2HDM, it is always possible to choose χ such that all potentially complex parameters, Y_3, Z_5, Z_6 and Z_7 are simultaneously real.[5]

[5] In this case, the Higgs basis is unique up to an overall sign change, $H_2 \to -H_2$, which would change the signs of Y_3, Z_6 and Z_7 while leaving all other squared-masses and self-couplings unchanged.

One can then show that [see Ref. 3]:

$$\lambda_1 = Z_1 \,, \qquad \widehat{\lambda} = -Z_6 \,, \qquad \lambda_A = Z_1 - Z_5 \,, \qquad \lambda_F = Z_5 - Z_4 \,. \tag{24}$$

In particular, in the Higgs basis, the CP-even Higgs mixing angle is $\alpha - \beta$. Thus, we see that the expressions of the physical Higgs masses given in eqs. (14)–(16) depend in a very transparent way on the parameters of the Higgs basis.

Of course, the Higgs basis analysis also applies to the most general 2HDM (with no additional discrete symmetries or CP imposed). For further details, see Ref. 4.

5 The MSSM Higgs sector

The MSSM Higgs sector employs a Type-II Higgs-fermion interactions. The above analysis can also be applied to the MSSM Higgs sector which is a special case of the 2HDM analyzed above. In particular, the Higgs self-coupling parameters satisfy

$$\lambda_1 = \lambda_2 = -\lambda_{345} = \tfrac{1}{4}(g^2 + g'^2) \,, \qquad \lambda_4 = -\tfrac{1}{2}g^2 \,, \qquad \lambda_5 = 0 \,,$$

which implies that

$$\begin{aligned} \lambda &= \tfrac{1}{4}(g^2 + g'^2)\cos^2 2\beta \,, \\ \widehat{\lambda} &= \tfrac{1}{4}(g^2 + g'^2)\sin 2\beta\cos 2\beta \,, \\ \lambda_A &= \tfrac{1}{4}(g^2 + g'^2)\cos 4\beta \,, \\ \lambda_F &= \tfrac{1}{2}g^2 \,. \end{aligned}$$

However, one must keep in mind that the formulae for Higgs masses in section 2 are tree-level results. In the general 2HDM, this is not a real problem as the masses are all independent parameters. However, in the MSSM one can obtain all tree-level Higgs masses and $c_{\beta-\alpha}$ given m_A and $\tan\beta$ as input. These tree-level relations suffer significant radiative corrections that must be taken into account in any Higgs analysis.

References

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