e–Ion Collider & Crab Cavity Feasibility

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Required crab cavity RF voltage depends on the frequency:

$$U_{\text{crab}} = \frac{cE \tan \phi_{\text{crab}}}{2\pi f_{\text{RF}} \sqrt{\beta_{\text{IP}} \beta_{\text{crab}}}}.$$

Example eRHIC (crabbing of proton beam):
$U_{\text{crab}} = 16\,\text{MV}$ for 197 MHz RF, 110 MV for 28 MHz.

$\Rightarrow$ Higher frequencies are preferred to keep RF voltage low

An ideal crab cavity ($\lambda_{\text{crab}} = \infty$) provides a perfect rotation of the entire bunch:

$$x(s) = s \cdot \phi_{\text{crab}}.$$

However, due to the finite RF wavelength of the crab cavity, this is only approximately the case:

$$x(s) = \frac{\lambda_{\text{crab}}}{2\pi} \cdot \sin\left(2\pi \frac{s}{\lambda_{\text{crab}}}\right) \cdot \phi_{\text{crab}}.$$

This approximation of a linear dependence by a sine wave results in a crab crossing error

$$\delta x(s) = \frac{\lambda_{\text{crab}}}{2\pi} \left[2\pi \cdot \frac{s}{\lambda_{\text{crab}}} - \sin\left(2\pi \frac{s}{\lambda_{\text{crab}}}\right)\right] \cdot \phi_{\text{crab}}.$$

To first order, this crab crossing error can be approximated as

$$\delta x(s) \;=\; \frac{4\pi^2}{6\lambda_{\mathrm{crab}}^2} \cdot s^3 \cdot \phi_{\mathrm{crab}}.$$

Linearizing the beam-beam kick as

$$\Delta x' = (x + \delta x(s))/f = (x + bs^3)/f,$$

we get a set of coupled differential equations of the form

$$\begin{aligned}
\ddot{x} + \omega_x^2 x \;&=\; \frac{\mathrm{d}}{\mathrm{d}t}\left[(x + b \cdot s^3)/f\right], \\
&=\; \frac{x}{f^2} + \frac{bs^3}{f^2} + \frac{3bs^2\dot{s}}{f}, \\
\ddot{s} + \omega_s^2 s \;&=\; \frac{\mathrm{d}}{\mathrm{d}t}\left[\phi_{\mathrm{crab}} \cdot (x + b \cdot s^3)/f\right].
\end{aligned}$$

For weak coupling,

$$x(t) = x_0 \cdot \cos(\omega_x t)$$
$$s(t) = s_0 \cdot \cos(\omega_s t).$$

Inserting solution for $s(t)$ into right-hand side of the differential equation for x yields

$$\ddot{x} + \omega_x^2 x \;=\; \frac{x}{f^2} + \frac{b s_0^3 \cos^3(\omega_s t)}{f^2} + \frac{3 b s_0^3 \omega_s \sin(\omega_s t) \cos^2(\omega_s t)}{f}$$

Resonant coupling effects therefore occur if

$$\Omega_x^2 = \omega_x^2 - \frac{1}{f^2} \;=\; \pm\omega_s^2,$$

or

$$\Omega_x^2 = \omega_x^2 - \frac{1}{f^2} \;=\; \pm 3 \cdot \omega_s^2.$$

Taking into account the full crab crossing error

$$\alpha - \sin\alpha = \frac{\alpha^3}{3!} - \frac{\alpha^5}{5!} + \frac{\alpha^7}{7!} - \ldots,$$

the right-hand side of the differential equations has the form

$$a \cdot \left(x + \sum_{k=0}^{\infty} c_k \cdot \cos(2k+1)\omega_s t \right),$$

and the condition for resonant coupling becomes

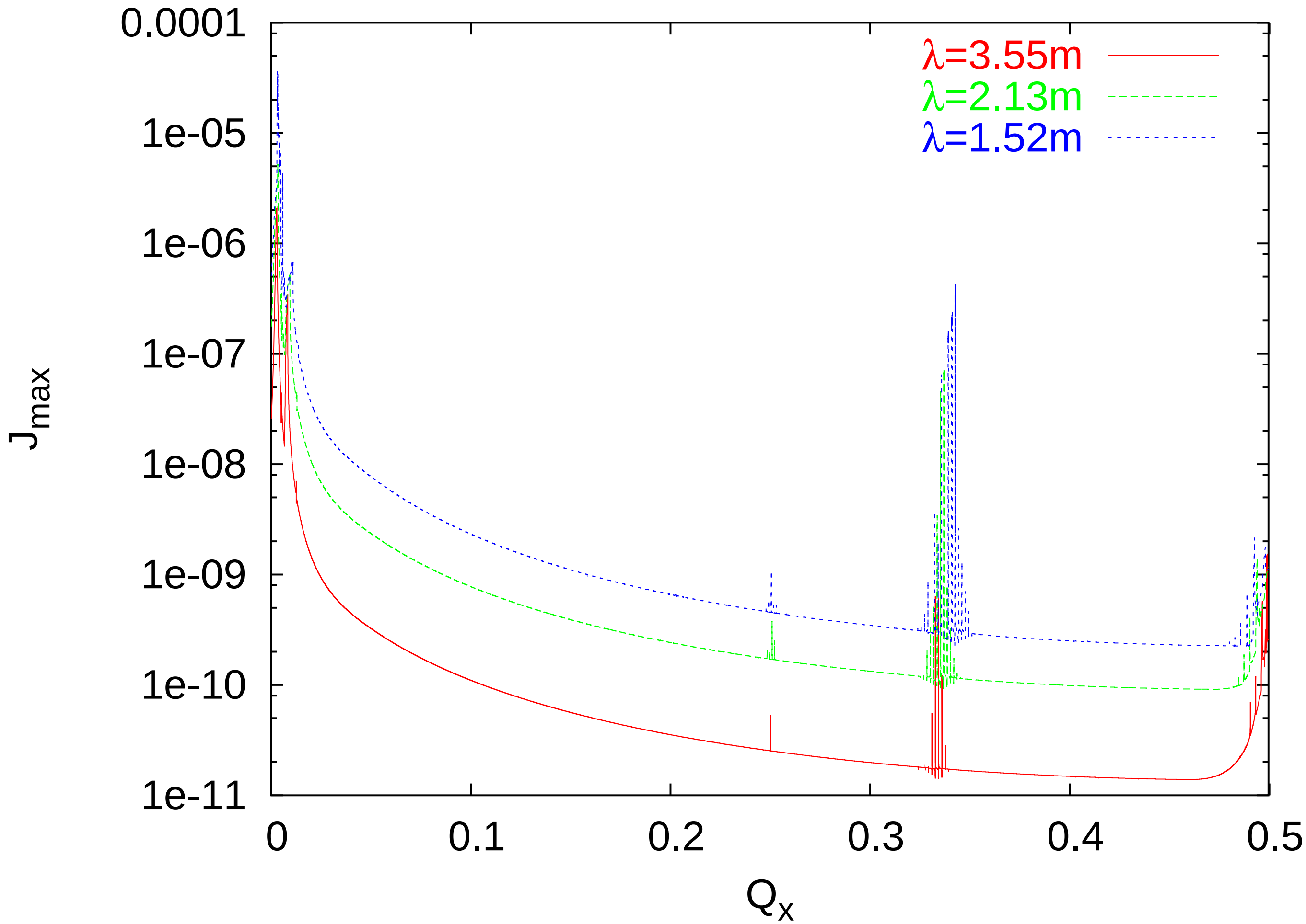
$$\Omega_x = (2k+1) \cdot \omega_s.$$

Using the nonlinear beam-beam force, this leads to synchro-betatron sidebands at the nonlinear resonances.

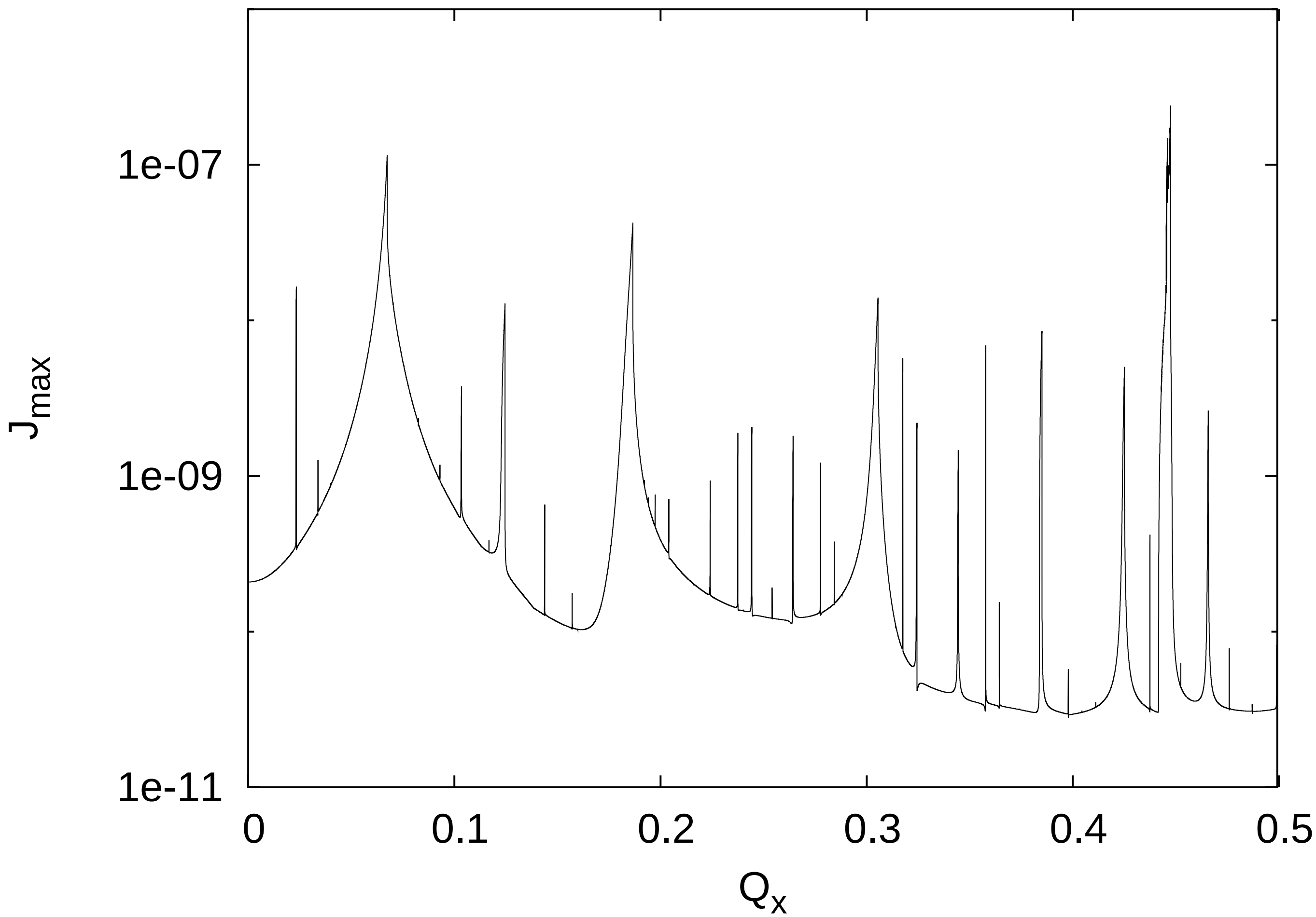
Parameter list for eRHIC and ELIC

		eRHIC	ELIC
rms transv. beam size [μm]	$\sigma_{x,y}$	50/50	4.5/4.5
ion beam emittance [nm]	$\epsilon_{x,y}$	9.5/9.5	1.0/1.0
β-function at IP [cm]	$\beta^*_{x,y}$	27.0/27.0	1.0/1.0
rms ion bunch length [cm]	σ_s	20.0	1.0
# of electrons/bunch	N_b	$3.0 \cdot 10^{11}$	$1.0 \cdot 10^{10}$
revolution frequency [kHz]	f_{rev}	78	192
collision frequency [MHz]	f_c	28	1500
total crossing angle [mrad]	$2 \cdot \phi_{\text{crab}}$	5.0	100.0
ion Lorentz factor	γ	267.0	107.0
ion transition γ	γ_t	23	10
RF voltage [MV]	U_{RF}	2	29
RF harmonic number	h	2520	7800

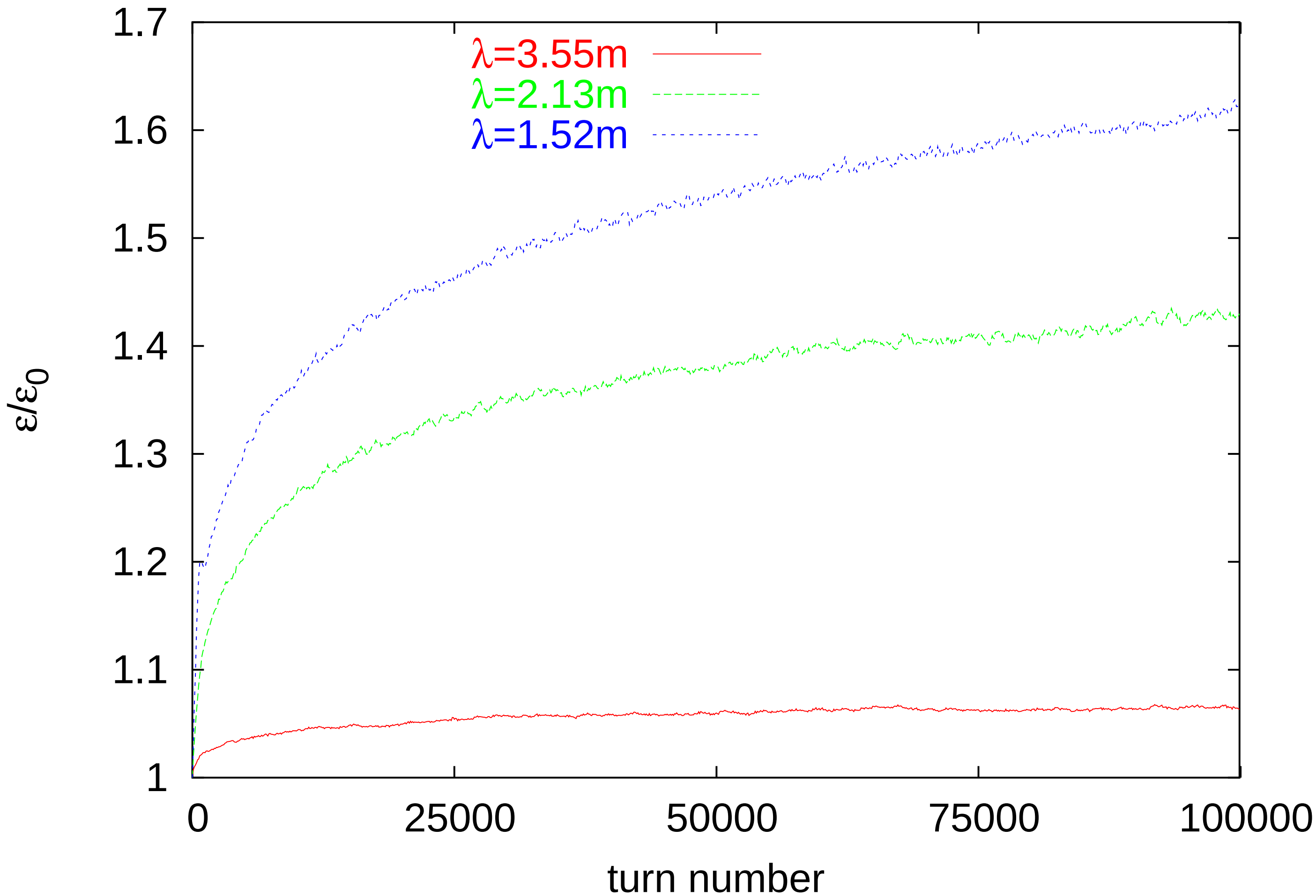
Synchro-betatron resonance spectra for three different eRHIC crab cavity harmonics



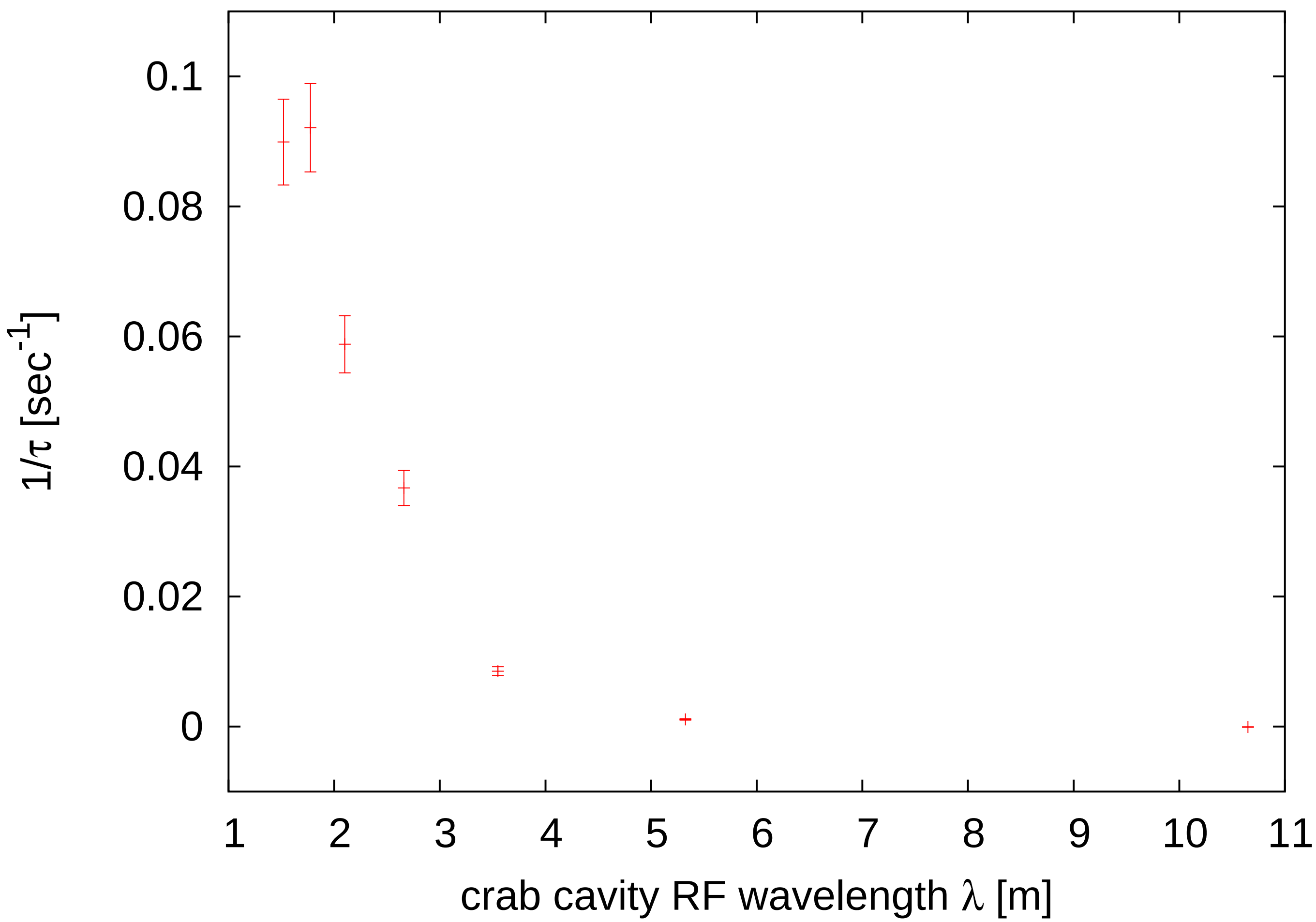
Synchro-betatron resonance spectrum for ELIC



eRHIC emittance growth at the polarized proton working point (.695/.685), for the same crab cavity harmonics



Emittance growth rates (from fits, starting at turn 35000)



Conclusion

- For eRHIC, growth rates with crab crossing (at current RHIC pp working point) are acceptable only at the lowest possible harmonic (28 MHz).
  The required RF voltage of 110 MV seems very (prohibitively?) large.
  However, installation of a copper liner in the stainless steel vacuum pipes of RHIC would allow shorter bunches, which in turn would allow shorter crab cavity wavelenths.

- For ELIC, with its very large crossing angle (100 mrad) and high synchrotron tune $Q_s = 0.06$, synchro-betatron sidebands are just about everywhere
  $\rightarrow$ no space left for large beam-beam tunespread