

# Synchro-beta resonance and noise in beam-beam interaction at LHC

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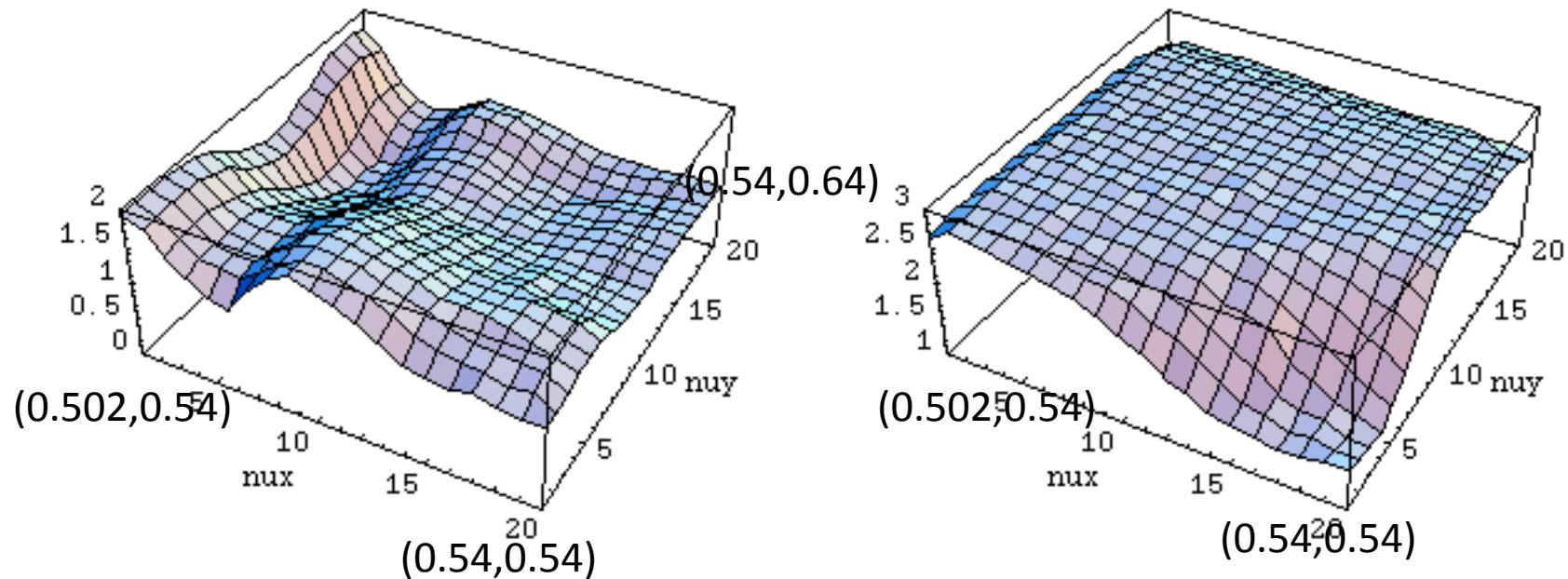
Crab 2008 @ BNL

25-26 Feb. 2008

# Synchro-beta resonance in KEKB (weak-strong simulation)

- Half Crossing angle=11mrad

0 mrad



- Clear synchro-betatron line ( $2\nu_x - \nu_s = n$ ) is seen in the finite crossing angle collision.  $\nu_s = 0.025$
- In experiments, anomalous emittance growth due to synchro-beta resonance (K.Oide et al.) is seen without beam-beam collision depending on lattice.

# Taylor map analysis of the beam-beam interaction

- Calculate beam-beam map

$$\mathbf{x} = \mathbf{f}(\mathbf{x}_0)$$

$$w(z; \sigma_x, \sigma_y) = e^{-z^2} (1 - erf(-iz)) \quad (5)$$

- Remove linear part

$$erf(z+z_0) = erf(z_0) + \frac{2}{\sqrt{\pi n!}} \sum_{n=1} (-1)^{n-1} H_{n-1} e^{-z_0^2} z^n \quad (6)$$

$$\mathbf{X} = \mathbf{f}(R^{-1}\mathbf{x}_0) = \mathbf{x}_0 + \sum a_{ij} x_{0,i} x_{0,j} + \text{3-rd order .....}$$

- Factorization , integrate polynomial

$$\mathbf{X} = \exp[-:(U_3 + U_4 + \dots):] \mathbf{x}_0$$

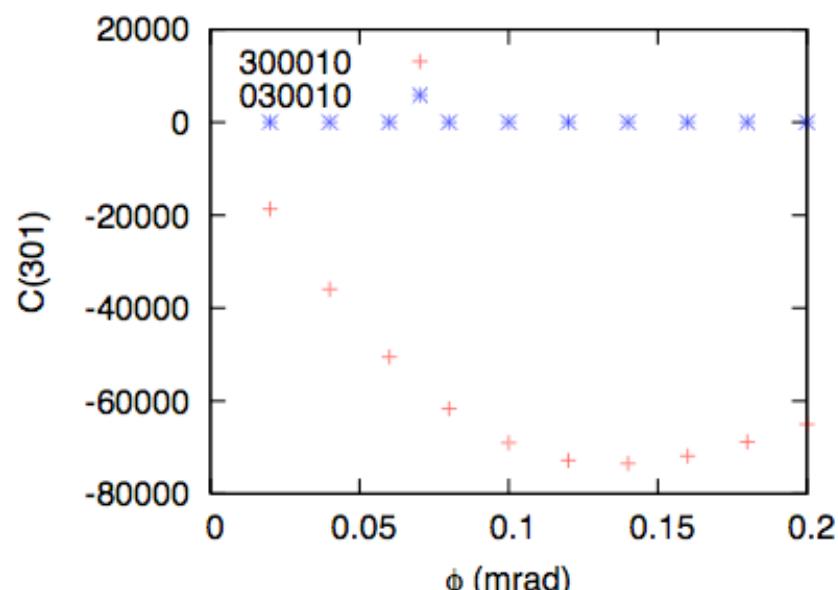
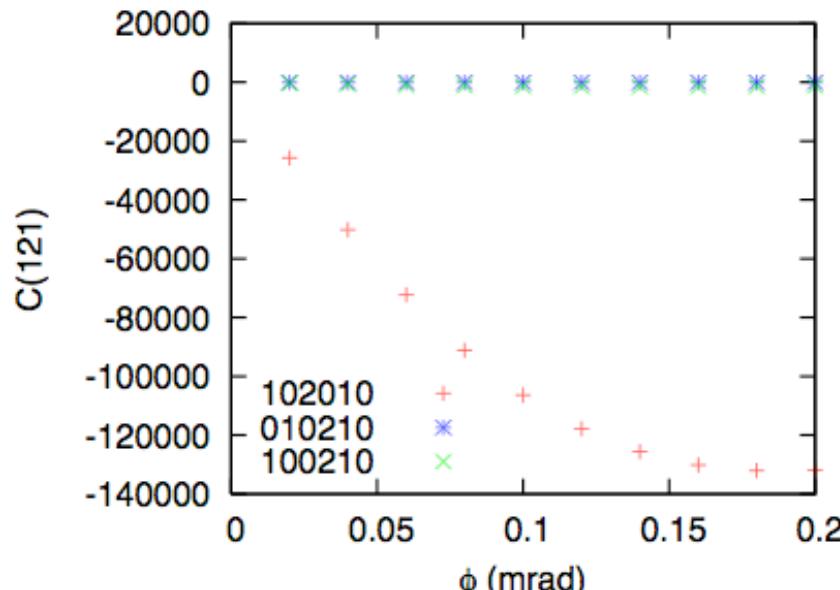
$$\sum_{ij}^6 a_{ij} x_{i,0} x_{j,0} = -:U_3:\mathbf{x}_0 = [-U_3, \mathbf{x}_0]$$

# Coefficients of beam-beam Hamiltonian

- Expression-1 ( $k_x, k_p, k_y, k_q, k_z, k_e$ )  $p=p_x, q=p_y, e=p_z$
- Expression-2 ( $n_x, n_y, n_z$ )
- 4-th order coefficients
  - C400 (400000), (310000), (220000), (130000), (040000)
  - C301 (300010), (210010), (120010), (030010)
  - C220 (202000), (112000), (022000), (201100), (111100), (021100), (200200),  
(110200), (020200)
  - C040 (004000), (003100), (002200), (000300), (000400)
  - C121 (102010), (012010), (101110), (011110), (100210), (010210)
- 3<sup>rd</sup> order coefficients (except for chromatic terms)
  - C300 (300000), (210000), (120000), (030000)
  - C210 (201000), (111000), (021000), (200100), (110100), (020100)
  - C120 (102000), (012000), (101100), (011100), (100200), (010200)
- Low order nonlinear terms are efficient in e+e- colliders, while higher order terms are efficient in proton colliders.

# Does crossing angle affect the beam-beam performance?

- The beam-beam performance is degraded at a high beam-beam parameter, for example it was degraded a half for KEKB.
- How is in LHC, low beam-beam parameter and no radiation damping?
- Crossing angle induces odd terms in Hamiltonian.
- The odd terms degrade luminosity performance in  $e^+e^-$  colliders. Tune scan shows clear resonance lines due to the terms.



# Synchro-beta resonance

$$H = H_0 + U(x, y, z)\delta(s^*)$$

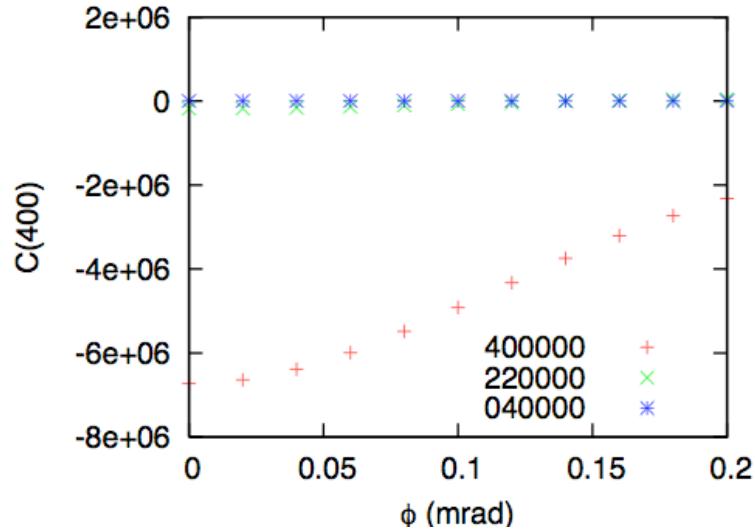
$$\nu_i = \nu_{i,0} - \frac{1}{2\pi} \frac{\partial U(x, y, z)}{\partial J_i}$$

Resonance condition

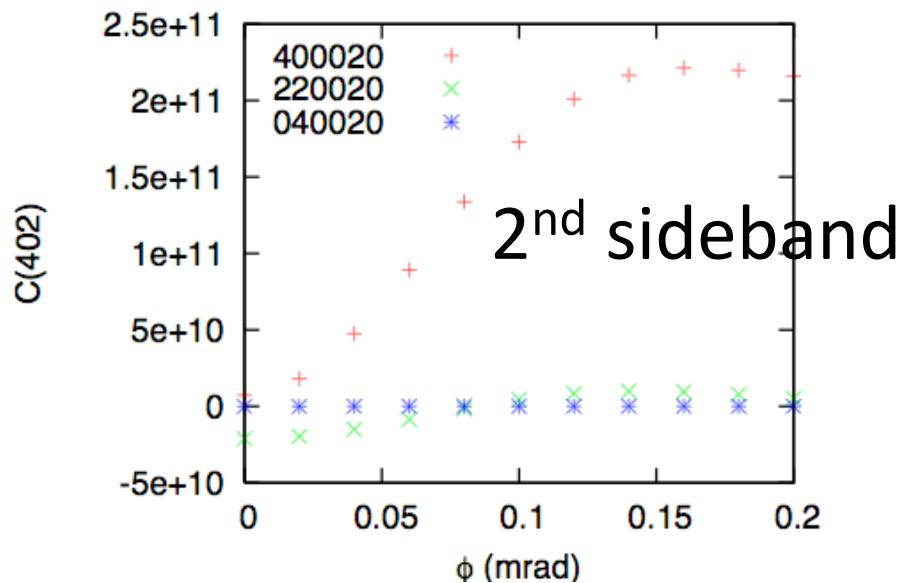
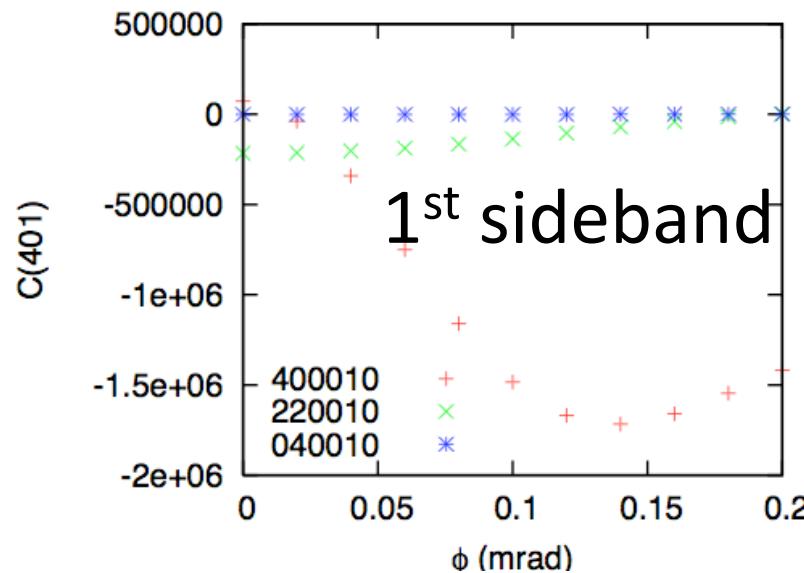
$$k_x \nu_x + k_y \nu_y + l \nu_z = n$$

Coefficients of Taylor map characterize the resonance strength

# Resonance overlapping between synchrotron modes



4<sup>th</sup> order betatron resonance



# Truncated model

Preliminary, does it make sense?

$$U = a_{xx}X^4 + 2a_{xz}X^2Z^2 + a_{zz}Z^2$$

$$U_{00} = \frac{3}{2}a_{xx}J_x^2 + 2a_{xz}J_xJ_z + \frac{3}{2}a_{zz}J_z^2$$

$$\begin{aligned} U_{kl} = & \frac{a_{xx}}{2} \left( J_x^2 \cos 4\psi_x + 4 \cos 2\psi_x + 3 \right) + \frac{a_{zz}}{2} \left( J_z^2 \cos 4\psi_z + 4 \cos 2\psi_z + 3 \right) \\ & + a_{xz}J_xJ_z \left( \cos(2\psi_x + 2\psi_z) + \cos(2\psi_x - 2\psi_z) + 2 \cos 2\psi_x + 2 \cos 2\psi_z + 2 \right) \end{aligned}$$

For LHC nominal parameter,

$\theta=140$  mrad,  $a_{xx}=-3.7\times 10^6$ ,  $a_{xz}=-275$   $a_{zz}=-0.0024$

$\theta=0$ ,  $a_{xx}=-6.7\times 10^6$ ,  $a_{xz}=-14$   $a_{zz}=-3.5\times 10^{-6}$

# Resonance approximation

$$4\nu_x(J_x, J_z) = 1 \quad a_{xx}J_x + a_{xz}J_z = \frac{1}{2}\left(\nu_x^0 + \xi - \frac{1}{4}\right)$$

$$\nu_x^0 = 0.252 \quad \frac{J_x}{\varepsilon_x} + 1.2 \frac{J_z}{\varepsilon_z} = 0.34$$

$$4\nu_x(J_x, J_z) + l\nu_s(J_x, J_z) = 1$$

$$\left(a_{xx} + \frac{l}{4}a_{xz}\right)J_x + \left(a_{xz} + \frac{l}{4}a_{zz}\right)J_z = \frac{1}{2}\left(\nu_x^0 + \xi + \frac{l\nu_s^0 - 1}{4}\right)$$

$$\frac{J_x}{\varepsilon_x} + 1.2 \frac{J_z}{\varepsilon_z} = 0.34 - 0.125l$$

- $k=4, n=1, l=1, 2\dots$

# Resonance width

$$k\nu_x(J_x, J_z) = 1$$

$$4\sqrt{U_{40}\left(\frac{\partial^2 U_{00}}{\partial J_x^2}\right)^{-1}} = \frac{4}{\sqrt{6}} J_x$$

$$k\nu_x(J_x, J_z) + l\nu_s(J_x, J_z) = 1$$

$$4k\sqrt{U_{4l}\Lambda^{-1}}$$

$$\begin{aligned}\Lambda &= k^2 \frac{\partial^2 U}{\partial J_x^2} + 2kl \frac{\partial^2 U}{\partial J_x \partial J_z} + l^2 \frac{\partial^2 U}{\partial J_z^2} \\ &= 48a_{xx} + 8la_{xz} + 3l^2a_{zz}\end{aligned}$$

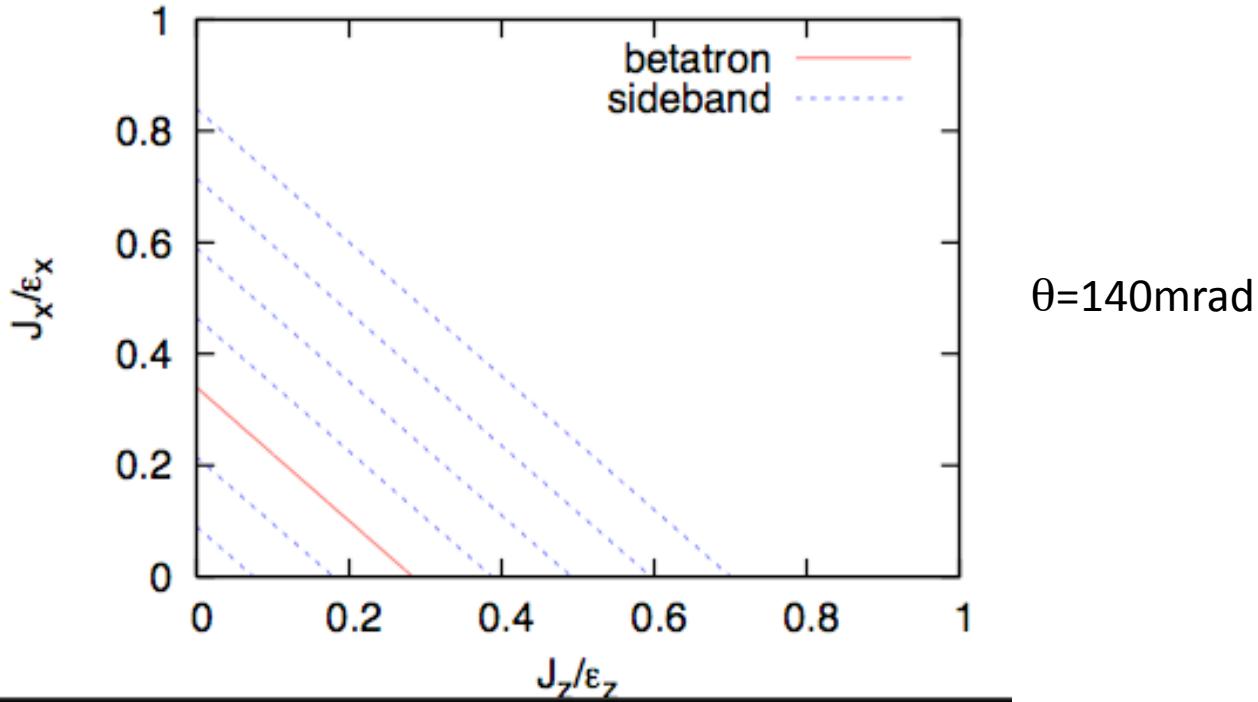
$$4k\sqrt{U_{41}\Lambda^{-1}} = 0.92J_x J_z^{1/4}$$

$$4k\sqrt{U_{42}\Lambda^{-1}} = 560J_x J_z^{1/2}$$

- $k=4, n=1, l=1, 2, \dots$      $\varepsilon_x = 5.1 \times 10^{-10} \text{m}, \varepsilon_z = 8.5 \times 10^{-6} \text{m}$

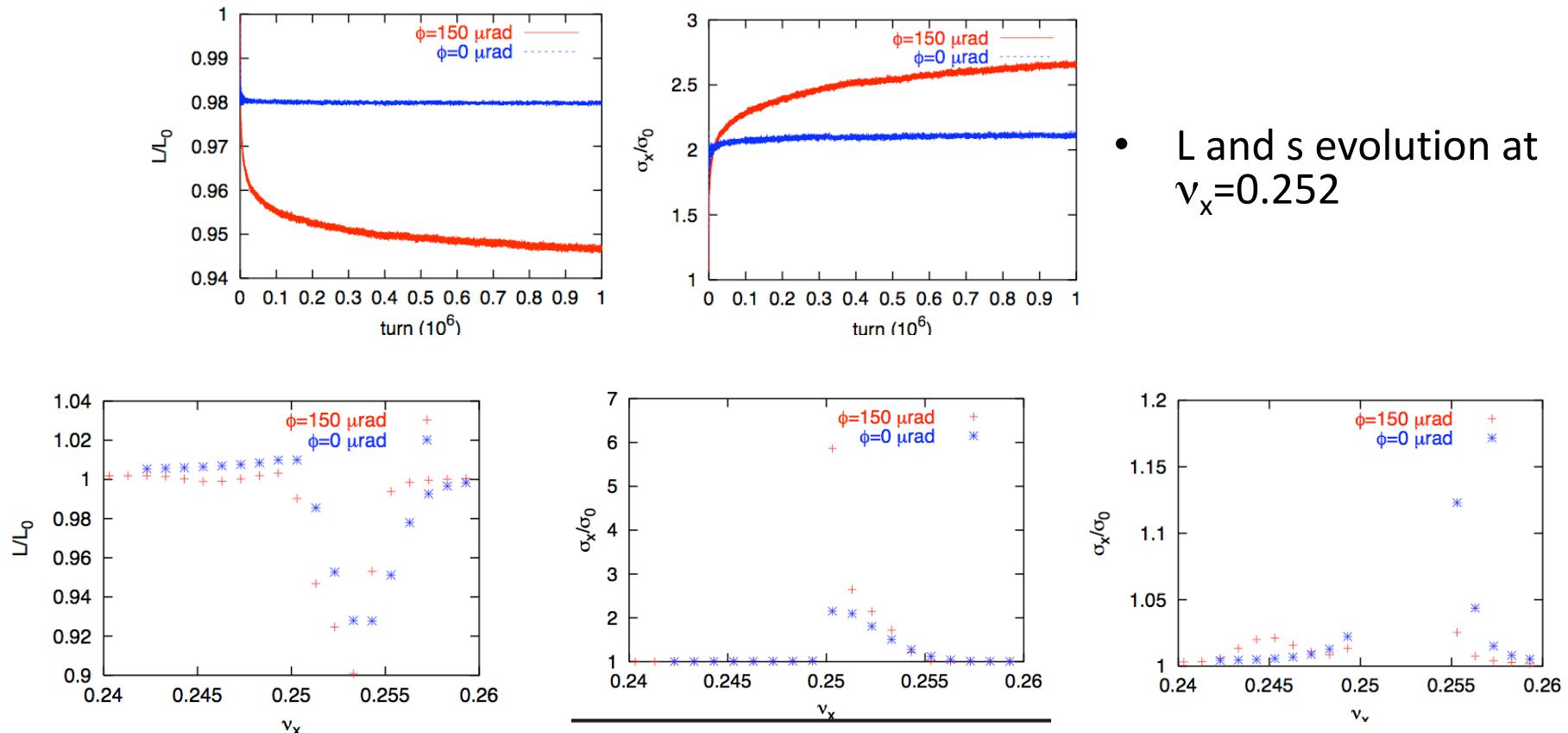
# Resonances in $J_x$ - $J_z$ plane

$(\nu_x = 0.252, \nu_s = 0.0019)$

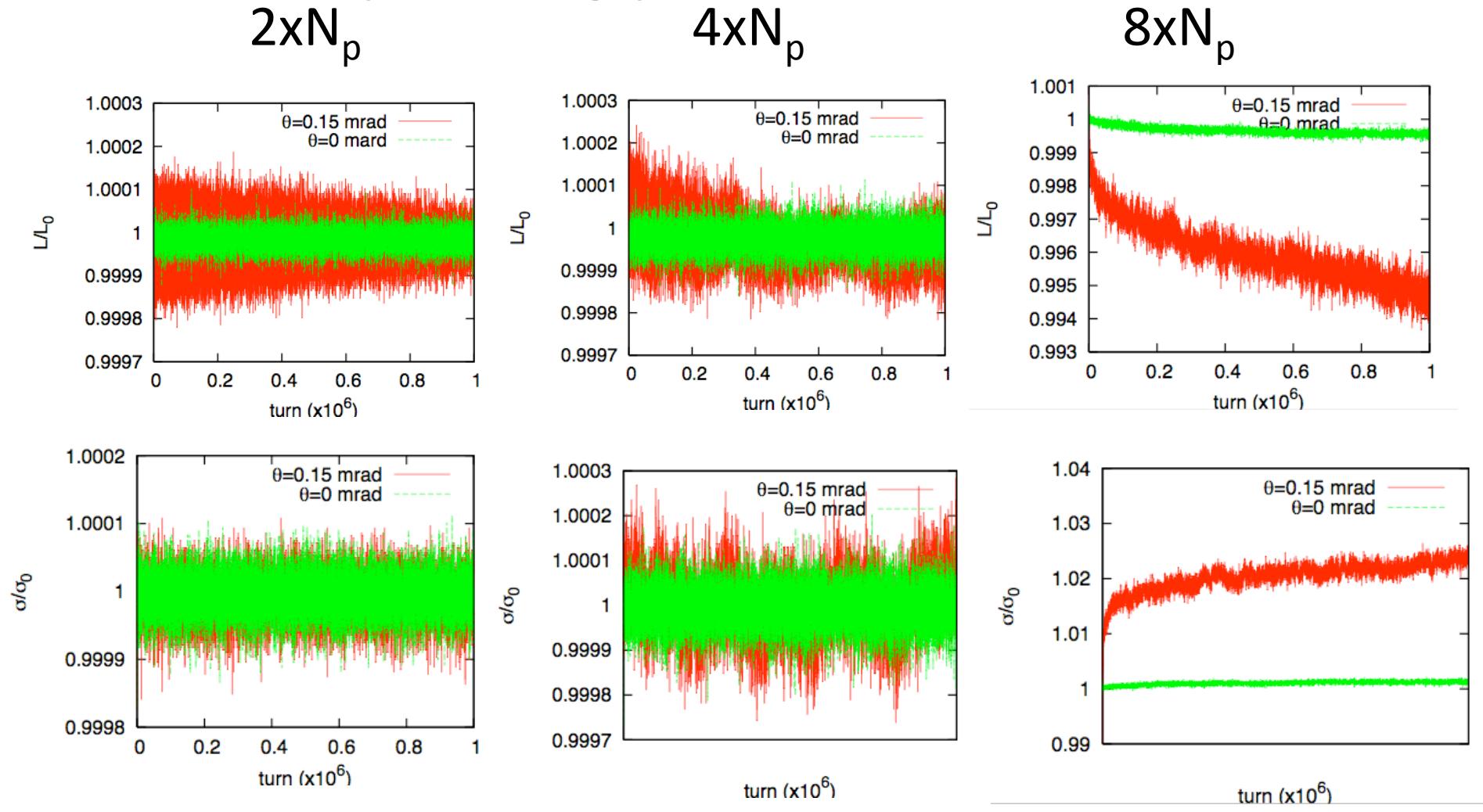


- Resonance width of the betatron is  $O(1)\varepsilon_x$ .
- 1<sup>st</sup> sideband is  $0.1\varepsilon_x$ , and 2<sup>nd</sup> is  $O(1)\varepsilon_x$ .
- The resonances are overlapped.

# Weak-strong simulation



# Effect of crossing angle at nominal operating point (0.31,0.32)

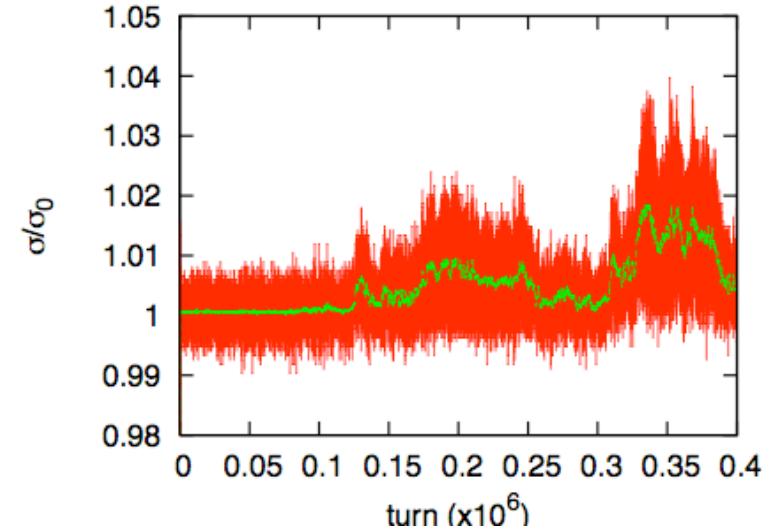
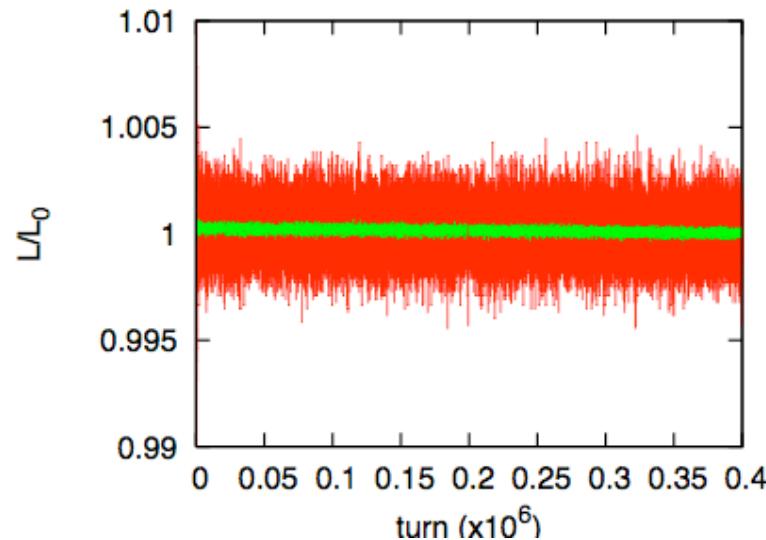


No parasitic collision

# Large Piwinski angle option-I

F. Zimmermann et al.

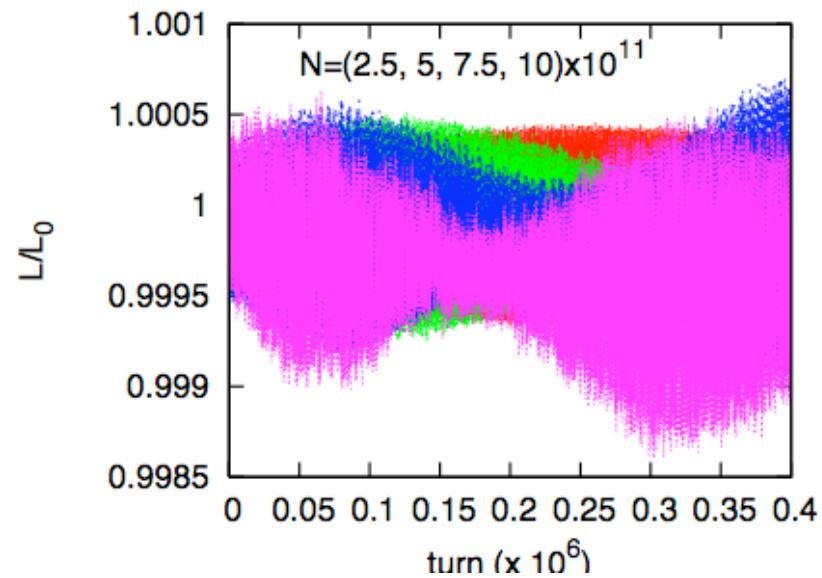
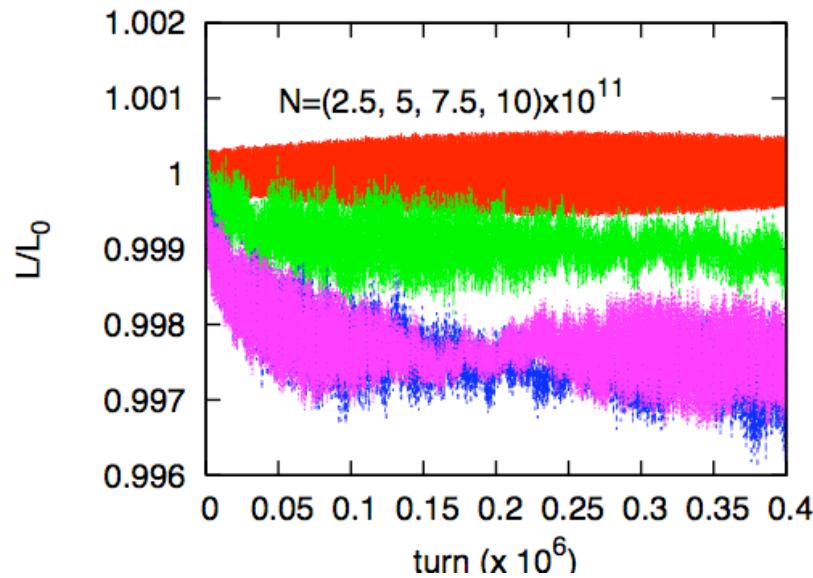
- Piwinski angle  $\phi=2(0.4)$ . Note () is nominal.
- Bunch spacing 50 (25) ns ,  $n_b=1401(2808)$ .
- Uniform longitudinal profile with  $\sigma_z=11.8(7.55)$  cm,  $L_z=41$  cm.  $\theta$  (half)=190(143)  $\mu$ rad.
- $N_p=4.9(1.15)\times 10^{11}$ ,  $\beta^*=0.25$  cm
- $L=10(1)\times 10^{34}$  cm $^{-2}$ s $^{-1}$
- Though the luminosity degradation is not seen, emittance growth occurs due to parasitic interactions. (HV collision)



# Large Piwinski angle option-II

J.P. Koutchouk et al.

- $N=2.5 \times 10^{11}$  /bunch ,  $\beta^*=14$  cm,  $\sigma_z=7.5$  cm, ,  $\phi$ (half xangle)=393  $\mu$ rad, Piwinski angle = 3.5, HV or HH (or VV) crossing, no parasitic



# Summary for synchro-beta effect

- Crossing angle induces synchro-beta resonance and other resonances with odd symmetry of x and y.
- Luminosity degrades near the  $\frac{1}{4}$  resonance in the nominal parameter.
- Synchrotron sideband is seen near  $\frac{1}{4}$  resonance.
- Luminosity degradation is seen at 8 times population,  $\xi \sim 0.03$ , for the crossing angle of  $150 \mu\text{rad}$  in the nominal collision.
- Large Piwinski angle option should work well. Cure for parasitic interaction may be required.

# Fluctuation of collision offset

- Early works by G. Stupakov (SSC), T.Sen, Y. Alexahin (LHC).
- Nonlinear system with noise enhances diffusion.
- Effects related to single particle dynamics and coherent motion.
- Analyze them with weak-strong and strong-strong simulations, respectively.
- High statistics simulation is required so as to be sensitive to the emittance growth rate  $\Delta\epsilon/\epsilon \sim 1 \times 10^9 / \text{turn}$ .
  
- Crab cavity noise, RF cavity noise
- Noise of the bunch by bunch feedback system

# Implementation of the noise

- Orbit fluctuation at the collision point

$$T(\delta) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \delta_x \\ y + \delta_y \end{pmatrix}$$

$\delta$ : random, but unique  
for every particles

- Orbit Diffusion and damping

$$\exp(-:U_{col}:) T_D(b,G) M_0$$

$$T_D(b,G) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \langle x \rangle(1 - G) + b_x \\ \langle y \rangle(1 - G) + b_y \end{pmatrix}$$

- These are equivalent for  $\delta=b/2G$  with correlation time of  $1/G$ .

# Fluctuation in collision due to the crab cavity and RF cavity noise

- Noise of RF system. Deviation of RF phase,  $\delta\phi$ .

$$\delta x = \frac{c \tan \phi}{\omega_{RF}} \delta\phi_{RF}$$

- Phase error between two crab cavities.

$$\delta x = \frac{c \tan \phi}{\omega_{RF}} \frac{\cos(\pi\nu_x - \Delta\psi(s^*, s_c))}{2 \sin \pi\nu_x} \delta\phi_c \approx \frac{c \tan \phi}{\omega_{RF}} \delta\phi_c$$

# Bunch by bunch feedback system of LHC (W. Hofle)

- 14 bit resolution,  $2^{14}=16384$ .
- Covered area is  $\Delta x = \pm 2$  mm at  $\beta=100-150$  m,  
resolution is  $\delta x_{\text{mon}} = 0.001\sigma$ .
- Effect of kick error is the same contribution, if an  
oscillation with  $\Delta x$  is damped by the damping rate of  $G$   
with 14 bit system.
- $G$ : damping rate of the feedback system (feedback  
gain).
- Beam fluctuation without beam-beam

$$\sqrt{\frac{\langle y^2 \rangle}{\sigma_y^2}} = \frac{\delta x_{\text{kick}}}{\sqrt{2G}} = \sqrt{2G} \delta x_{\text{mon}} \quad \delta x_{\text{kick}} = 2G \delta x_{\text{mon}}$$

# Weak-strong effect

## Diffusion rate due to offset noise

⊕

$$D_J(a_x, 0) = \frac{\langle J_x(N)^2 - J_x(0)^2 \rangle}{N} = \frac{(C|\delta x|)^2}{8 - 4/\tau} \sum_{k=0}^{\infty} \frac{\sinh \theta (2k+1)^2 G_k^2(a)}{\cosh \theta - \cos[2\pi(2k+1)\nu_x]}$$

$$C = \frac{N_p r_p}{\gamma_p} \quad a_x = \frac{\beta^* J_x}{2\sigma^2} \quad a_y = \frac{\beta^* J_y}{2\sigma^2} = 0$$

$$G_k = \frac{\sqrt{a}}{\sigma} [U'_{k+1} + U'_{-k}] + \frac{1}{\sqrt{a}\sigma} [(k+1)U_{k+1} - kU_k]$$

$$U_k(a) = \int_0^a \frac{1}{w} [\delta_{0k} - (2 - \delta_{0k})(-1)^k e^{-w} I_k(w)] dw$$

$$\theta = -\ln(1 - 1/\tau)$$

# Coherent (Strong-strong) effect

Y. Alexahin, NIM391, 73 (1996)

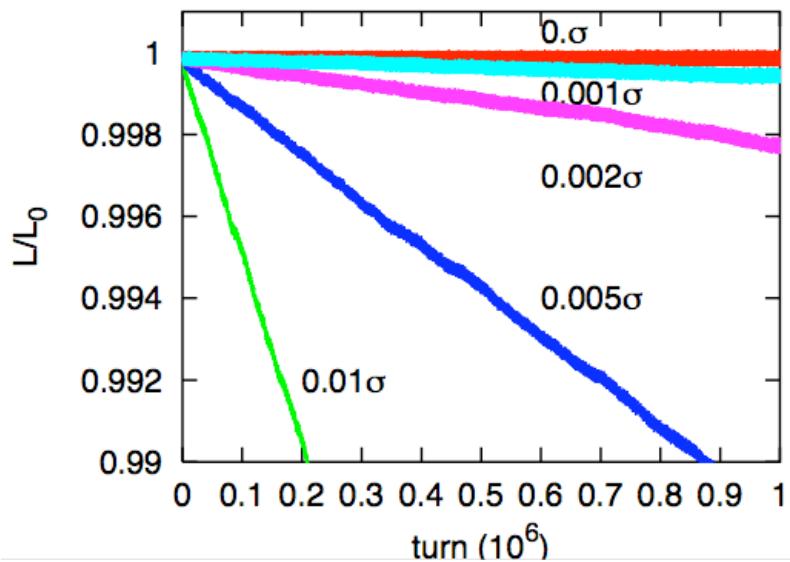
- Kick due to offset noise → coherent oscillation → decoherence → emittance growth

$$\frac{1}{\varepsilon} \frac{d\varepsilon}{dN} = 0.09 \left( \frac{\delta x}{\sigma_x} \right)^2 \frac{1}{1 + (G / 2\pi\xi)^2}$$

- $\delta x$ : kick error due to offset noise.
- $G/2\pi\xi$ : ratio of the feedback gain and nonlinear decoherence.
- Form factor, 0.09, for size increment due to decoherence

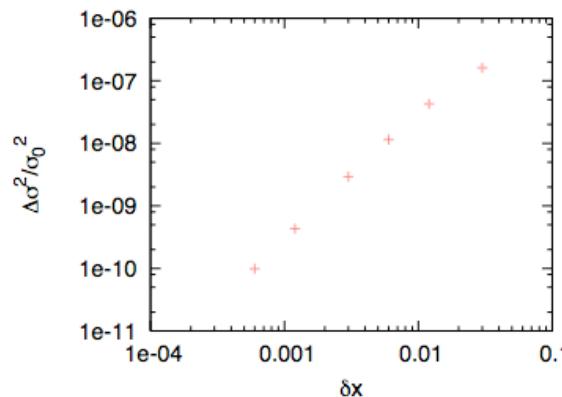
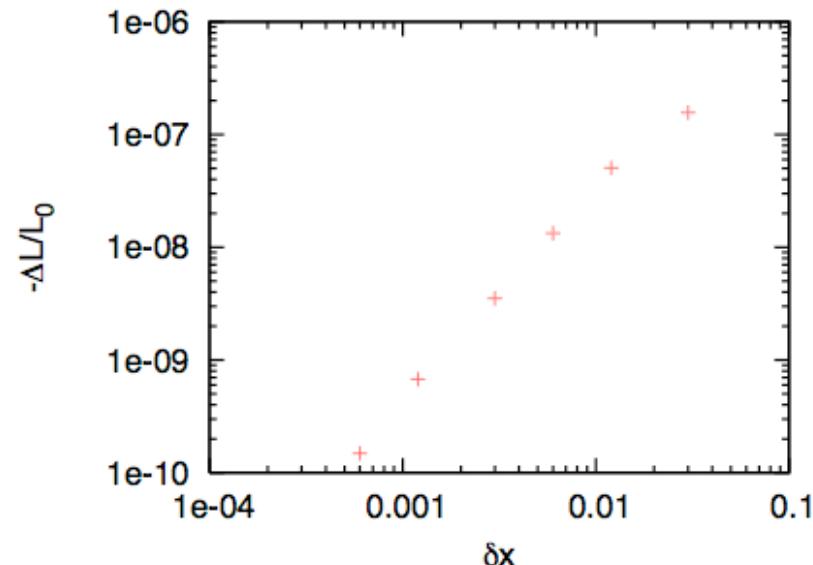
# Weak-strong model

- Luminosity degradation for turn by turn noise



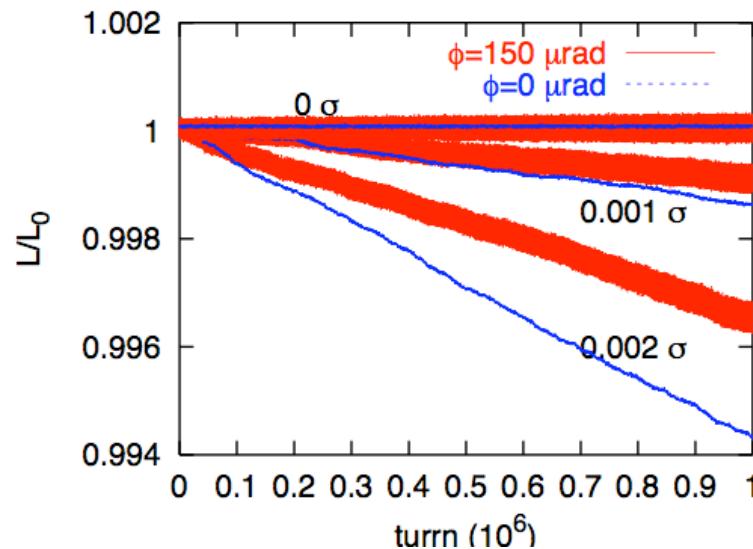
1 IP nominal LHC parameter  
Head-on

The damping rate is twice for 2 IP.  
Tolerance is 0.2 %  $\sigma$ .



# Effect of noise and crossing angle

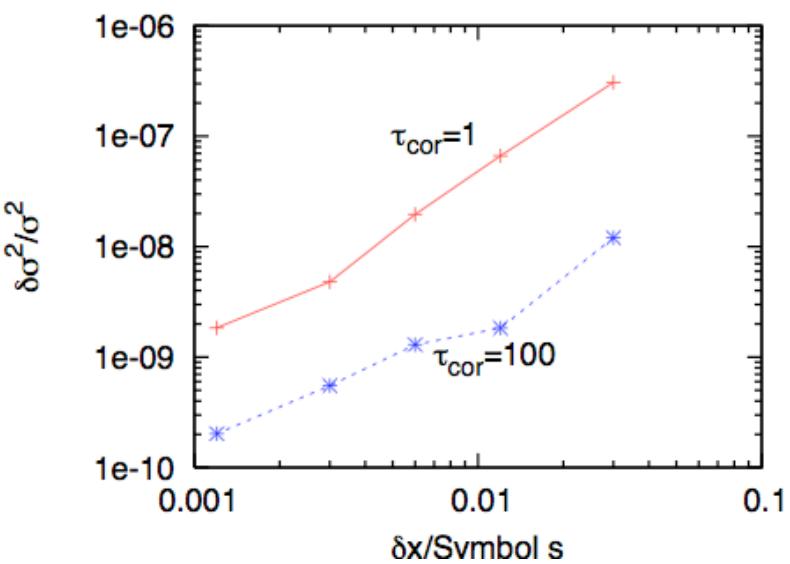
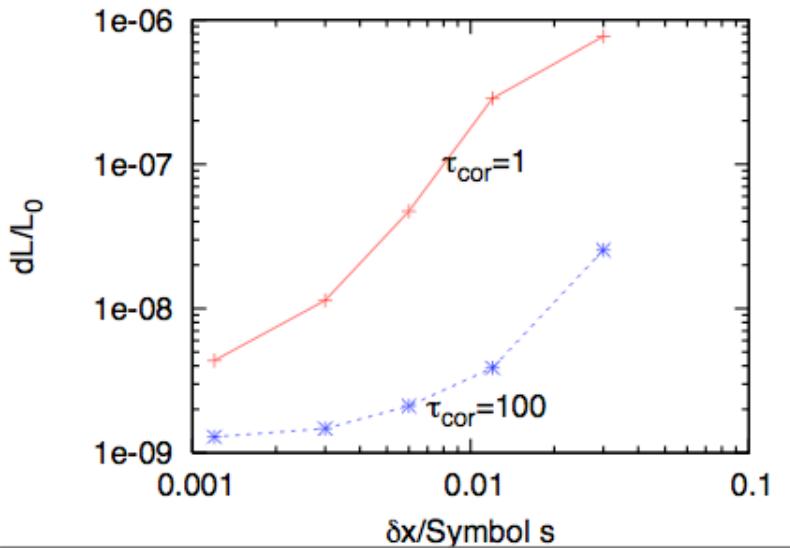
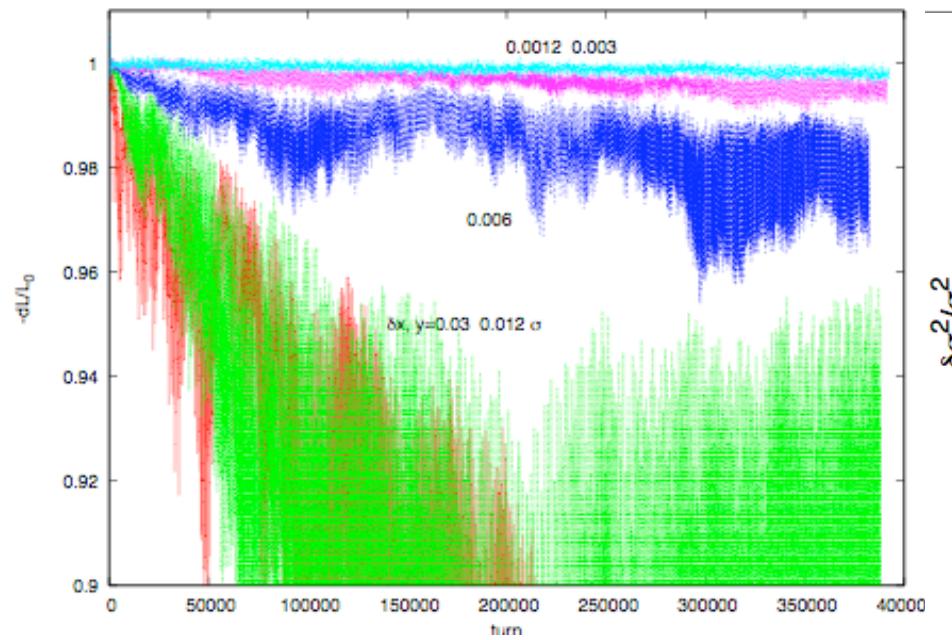
- Early separation scheme. Bunch population is  $1.7 \times 10^{11}$ .



- Head on collision is worse for noise, perhaps due to the higher beam-beam parameter.
- The diffusion rate is 2-3 times faster than the nominal case. Tolerance is  $0.1\% \sigma$ .

# Strong-strong model

- Simulation with  $10^6$  macro-particles.



# How to understand the result of the strong-strong simulation

- The result shows severe than that of the weak strong simulation. For  $10^{-9}$  degradation rate, noise level less than 0.1%  $\sigma$  is required.
- The simulation contains numerical offset noise of 0.1%  $\sigma$ . Higher statistics is hard.
- The degradation rate is saturated around 0.1%  $\sigma$ .
- The statistics of the macro-particle can affect the result.
- Coherent effect may enhance the degradation.
- It seems the results shows the tolerance is about 0.1 %  $\sigma$  for turn by turn noise (10kHz).
- The tolerance is  $\sim 1\%$  for noise with correlation of 100 turn (0.1kHz)

# Tolerance for crab cavity noise

- Fast noise, tolerance of RF phase error ( $\delta\varphi$ ) is

$$\delta\varphi = \frac{\delta x}{\sigma_x} \frac{\omega \sigma_x}{c \tan \theta}$$

- $\delta x=0.1\% \sigma$  and  $\omega=2\pi 400$  MHz corresponds to

$$\delta\varphi[\text{mrad}] = 0.14 \frac{1}{\theta[\text{mrad}]}$$