

Emittance growth due to wake force with offset orbit and crabbing K.Ohmi

- Beam with crab angle along the ring experiences z dependent kick due to wake field, with the result that it can be distorted to banana shape.
- When the center of the transverse wake deviate, similar dipole kick is induced, with the result that the beam can be distorted to banana shape.
- The vertical distortion is serious because of the small size.

Tilt due to the transverse wake force (Ikeda)

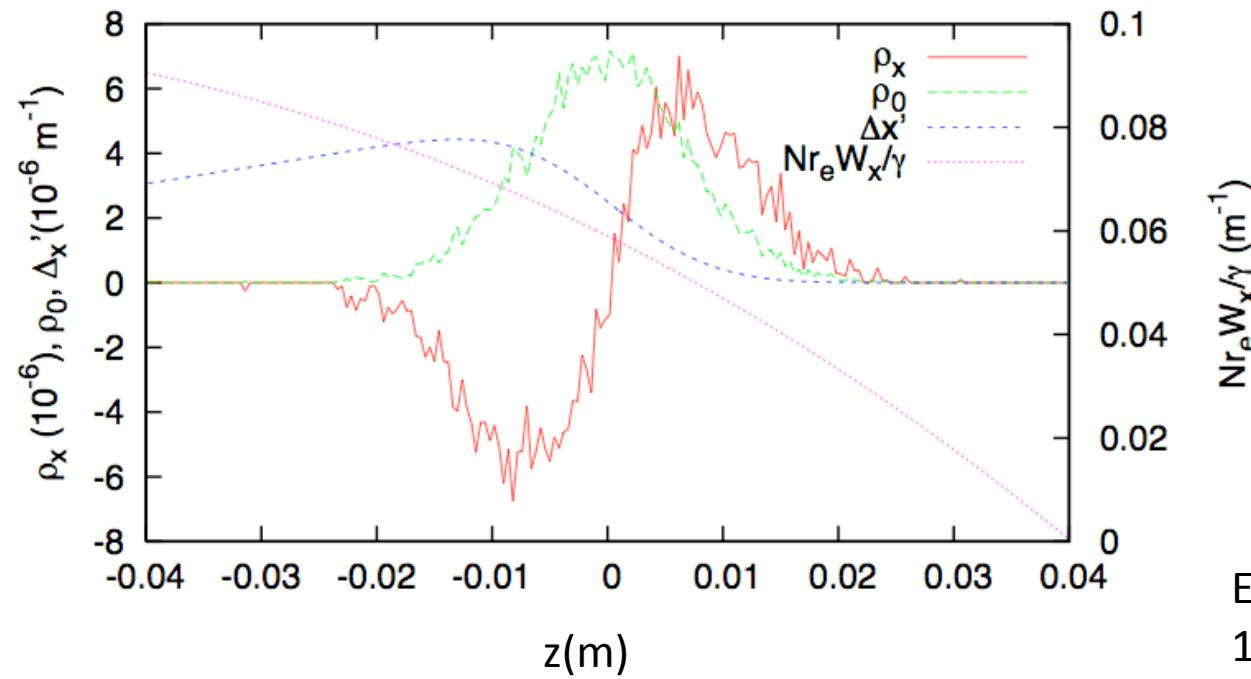
- Measured by leiri.

$$\left(\frac{d\boldsymbol{v}_x}{dI} \right)_{I=0} = -\frac{r_e W_0}{8e\gamma\omega_0} \beta$$

$$W_0 = \left(\frac{d\boldsymbol{v}_x}{dI} \right)_{I=0} \frac{8e\gamma\omega_0}{r_e \beta} \quad \left(\frac{d\boldsymbol{v}_x}{dI} \right)_{I=0} = 4A^{-1}$$
$$= 1.7 \times 10^6 \text{ m}^{-2}$$

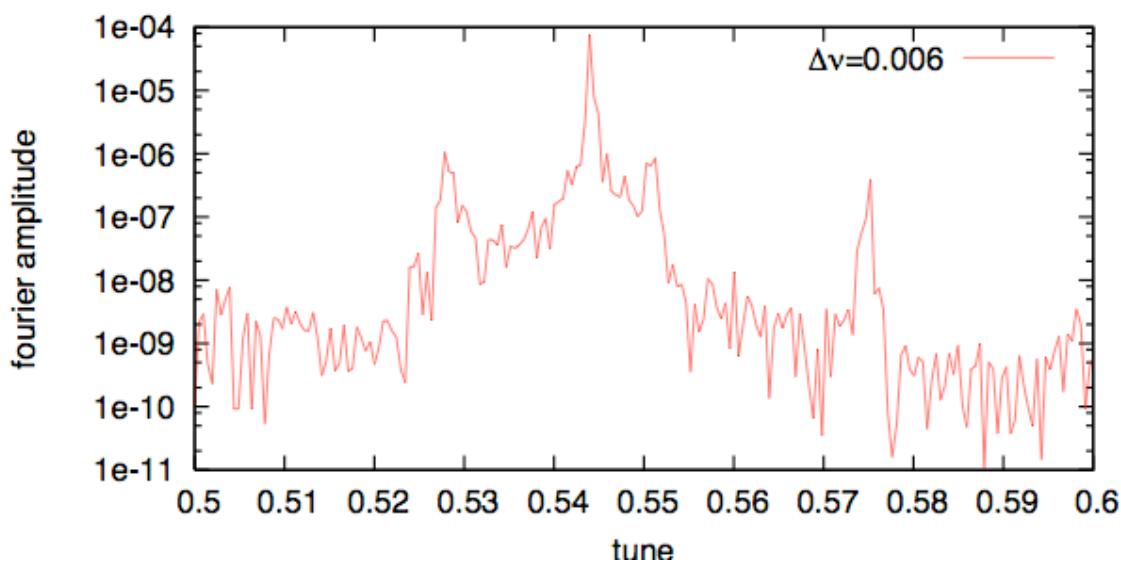
$$\Delta x'_2 = \frac{Nr_e W_0}{\gamma} \sigma_x = 7.5 \times 10^{-6}$$

$$\Delta x'_{crab} = \frac{eV'}{E} \sigma_z = \frac{eV_0}{E} \frac{\omega_{rf} \sigma_z}{c} = 1.25 \times 10^{-4}$$



*wake $z=0 \sim -0.08$

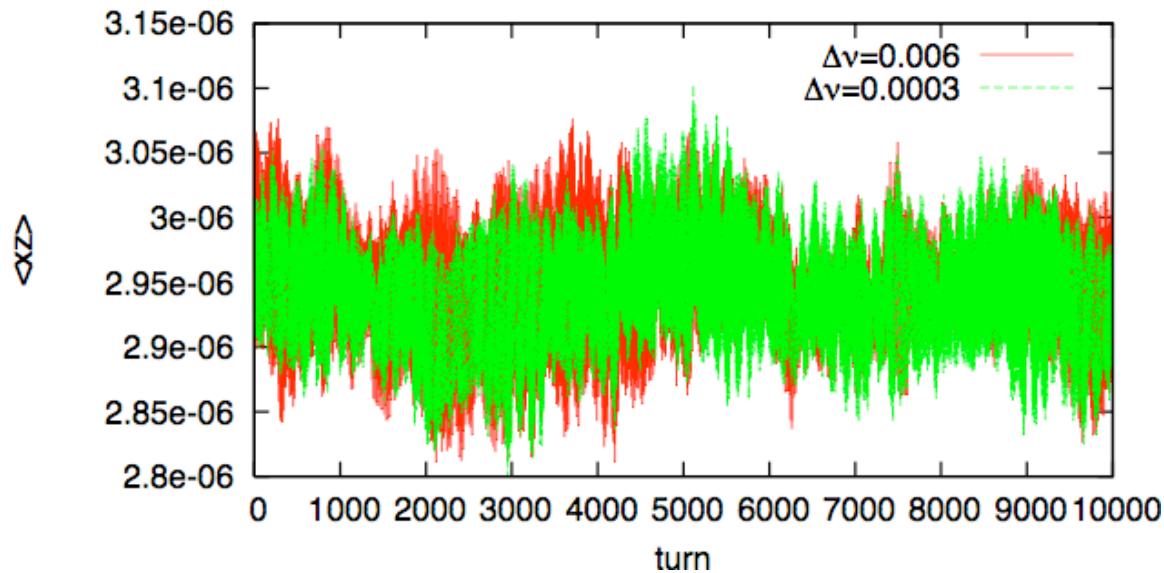
Even $\Delta v=0.006$, the kick is
1/30 of crab kick



$v_x=0.55$

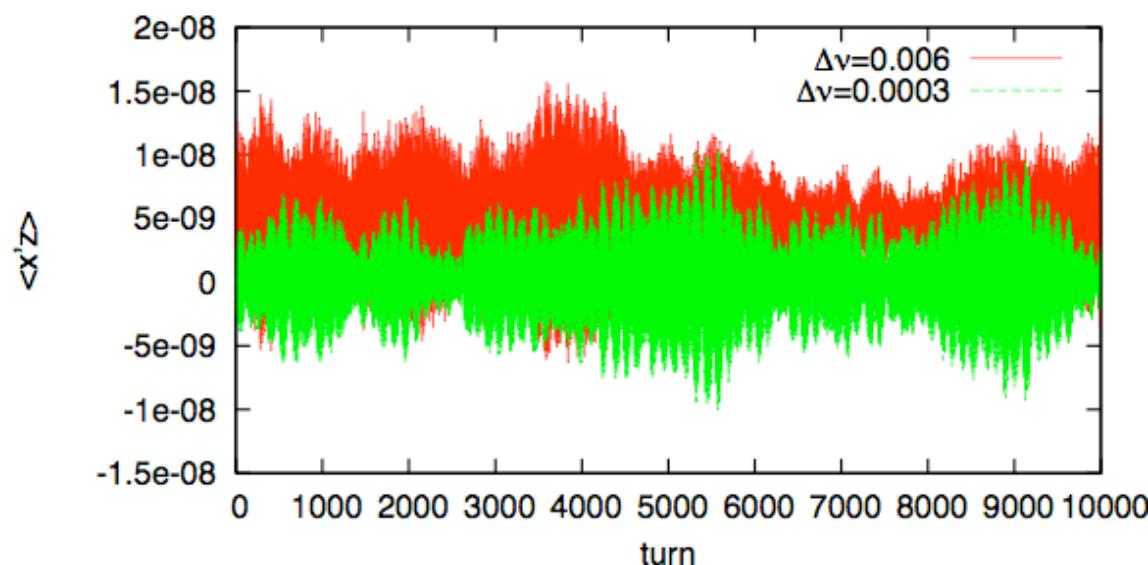
Spectrum by simulation with this
condition

$$\Delta x'_{crab} = \frac{eV'}{E} \sigma_z = \frac{eV_0}{E} \frac{\omega_{rf} \sigma_z}{c} = 1.25 \times 10^{-4}$$



Change of equilibrium distribution

- $\langle xz \rangle$ does not change.
- $\langle x'z \rangle$ change a little $\sigma_{x'} \sigma_z / 30$.



The wake effect of x-z tilt beam is weak.

Asymmetric wake

- Assume beam off set shifts 1mm.

$$\Delta p_y = \frac{Nr_e}{\gamma} \int_z^\infty W_{1y}(z - z') \rho_1(z') dz'$$

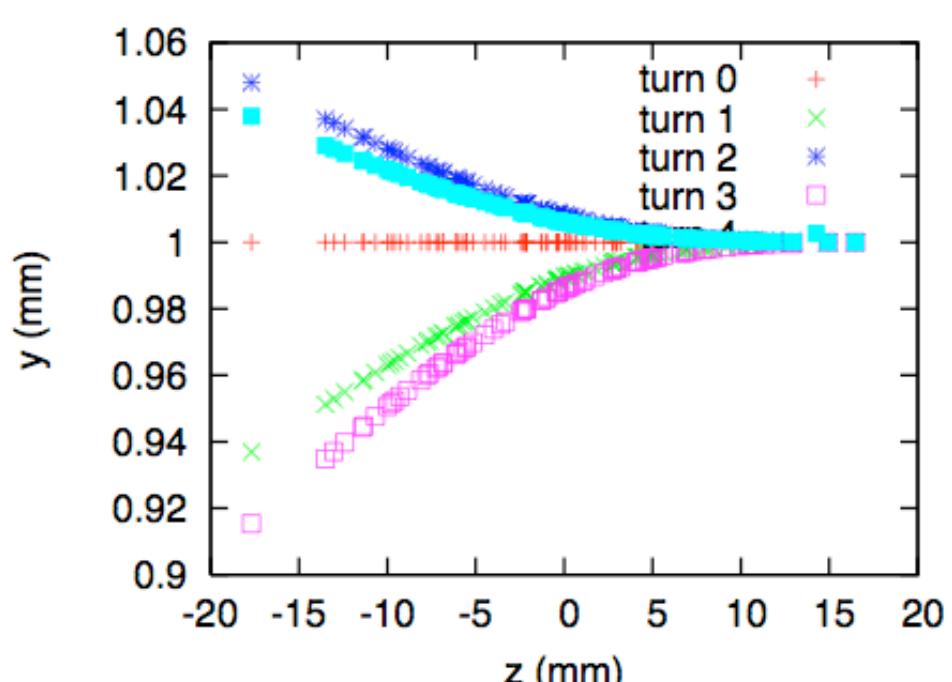
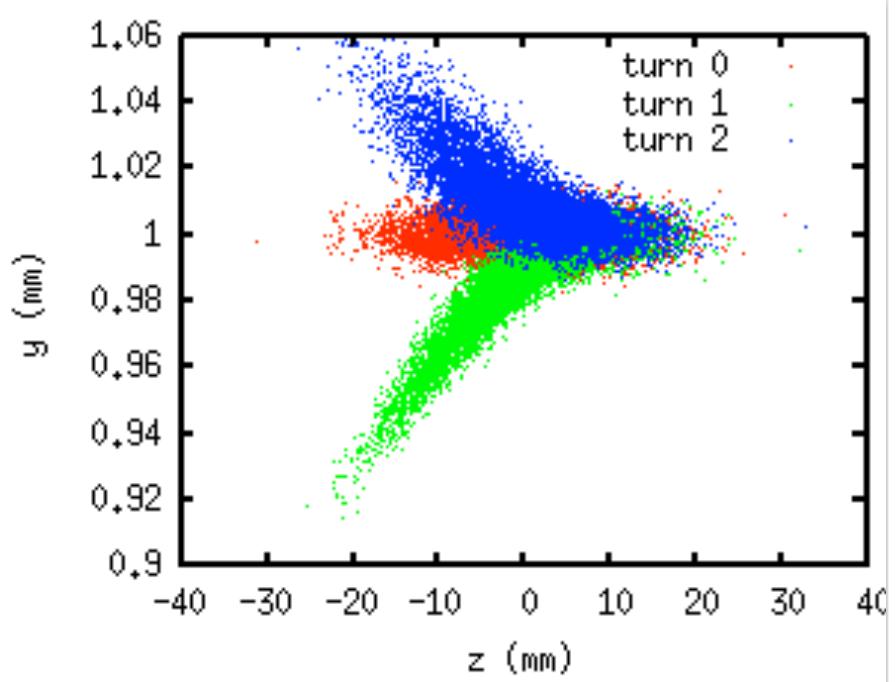
$$\rho_1(z') = \delta y \rho_0(z')$$

$$\rho_0(z) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$\Delta p_y \approx \frac{Nr_e}{\gamma} \frac{W_1 \delta y}{\sigma_z} \operatorname{Erfc}\left(\frac{z}{\sqrt{2\pi}\sigma_z}\right)$$

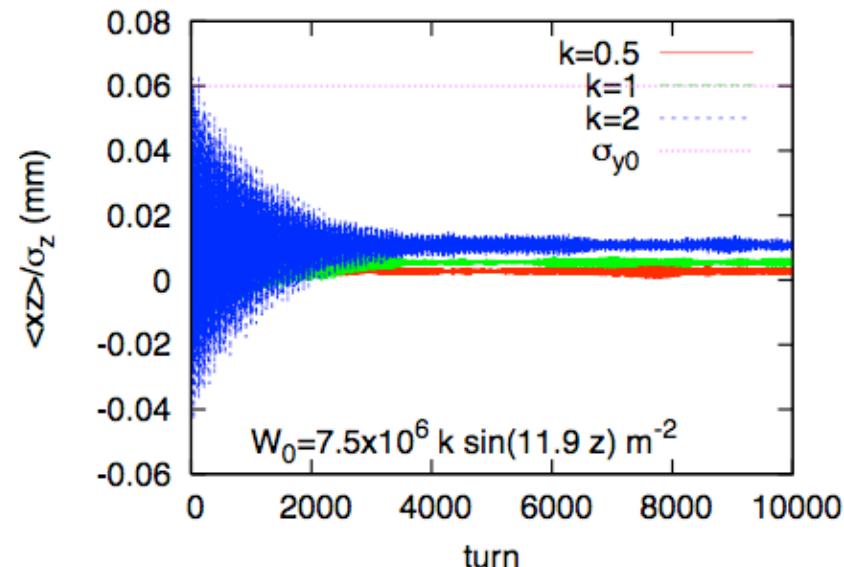
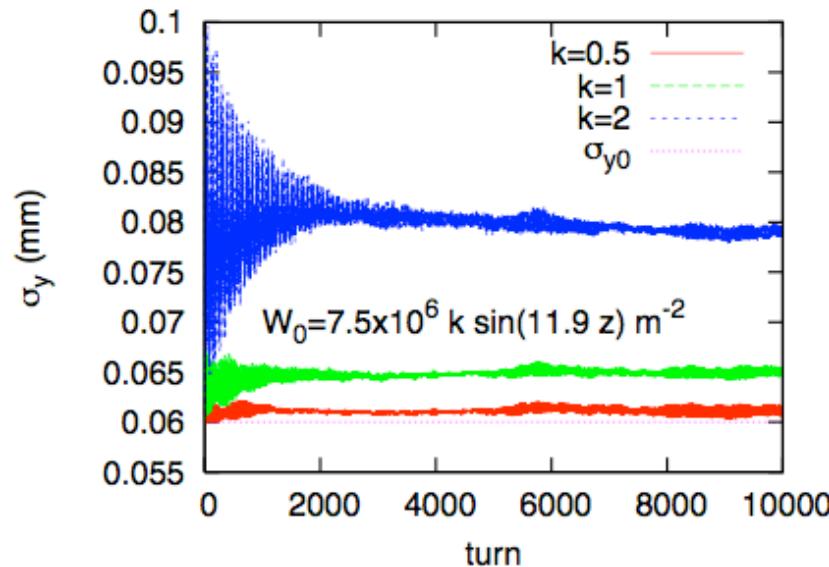
Check the code

- Motion of first 2 turns was consistent with analytic estimate.
- Obtain an equilibrium distribution for radiation damping and excitation.



Beam size and y-z tilt

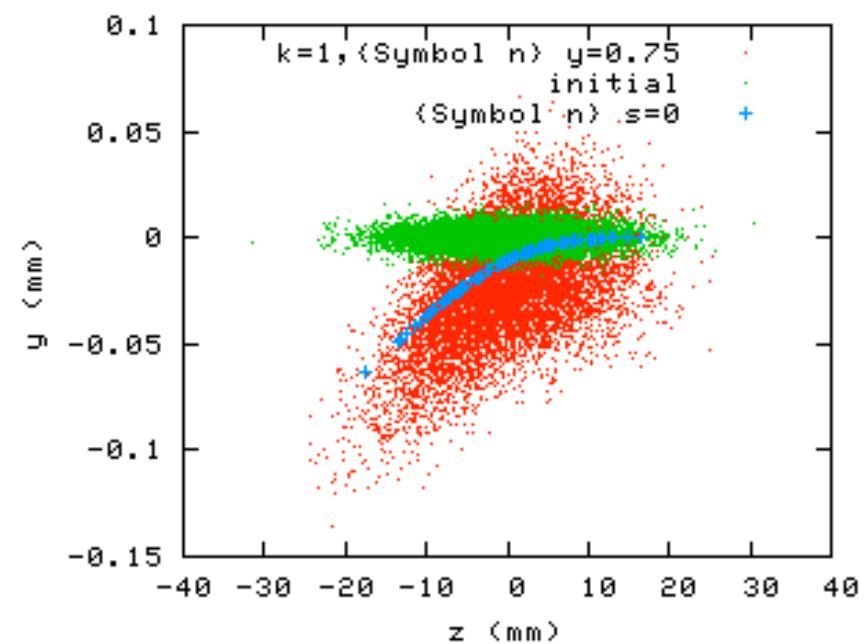
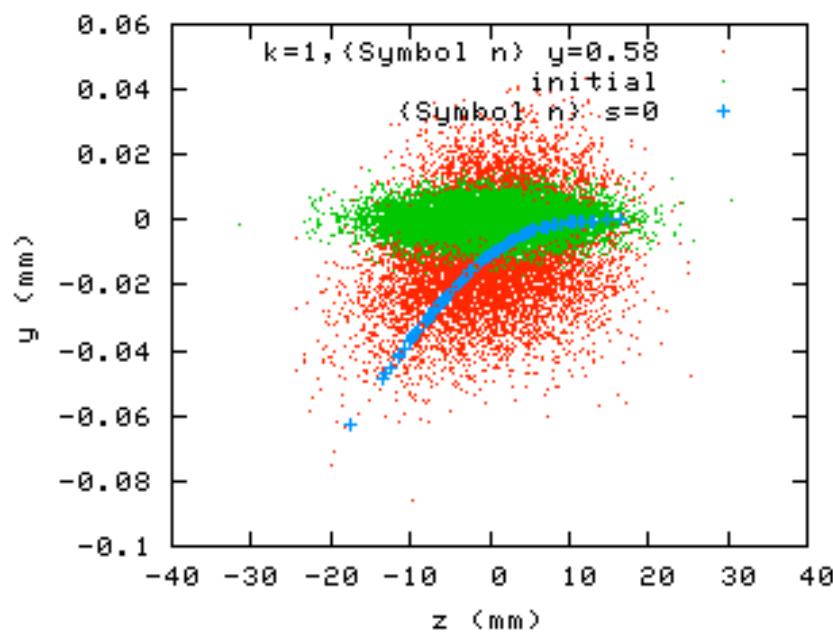
- They oscillate initial stage, but arrive at an equilibrium
- Emittance is large compare than tilt.



* $W=W_0 k$

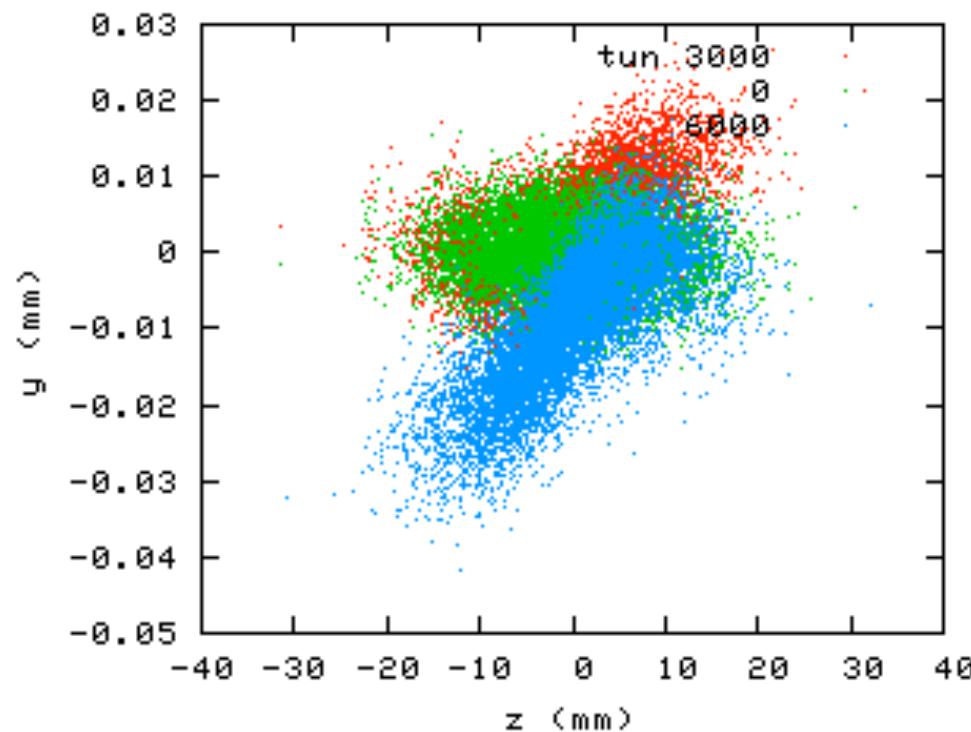
** Threshold of strong head-tail instability, $k=3$

- Emittance (phase space volume) increases.
- Equilibrium shape depends on v_y .



Radiation damping/excitation off

- Beam oscillates with banana shape, but its size does not increase.



Summary of wake effect for crabbing beam

- Asymmetry of 1mm is perhaps pessimistic.
- It does not seem to be big issue in present KEKB, though it depends on the asymmetry amplitude.
- The effect is more serious for Low emittance ring.
- What is the mechanism of the emittance growth? A kind of anomalous emittance growth.