GR as an Effective Field of a Power-Counting Renormalizable Tensor-Tensor Theory

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OUTLINE

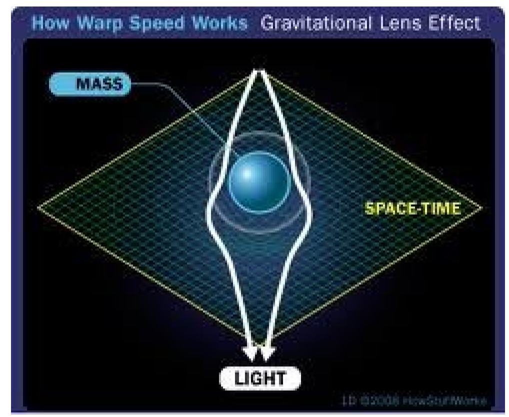
- Introduction
  - Renormalizability
  - Why GR is like that?
  - An example
  - Old ideas and no go theorems
- Constructing a Poincare Invariant Theory of Gravity
  - Tensor-Vector Field
  - Tensor-Tensor Field
- Summary

General Relativity

- A Novel Description of Space-Time.

- Explains The Low Energy Observations

Solar System,
Bending of Light,
Gravitational Redshift.



Anology With Yang-Mills

- Global symmetries are locally true as well. Both are Gauge Fields.

- Physical quantities transform covariantly in both cases.

A Deep Difference

- Yang-Mills theory is proved to be renormalizable.
- GR is Proved to be UV incomplete. It is not renormalizable.

Is Renormalizablity Important At All?

- All Other Theories of Physics are Renormalizable.

- We haven't observed non-renormalizable interactions.

- Some Interactions are Allowed by Symmetries but not by renormalizability.

  Like $\bar{\psi}[\gamma^\mu, \gamma^\nu]\psi F_{\mu\nu}$ in QED.

Why GR is Like This?

- It couples to energy-momentum tensor.
- This leads to negative coupling constant.
- It blows up the lagrangian.

An Example

A massless scalar field couples to trace of energy-momentum tensor

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \lambda T \phi\right),$$

where

$$T = T_\mu^\mu.$$

An Example

We assume a vacuum case. There is no external source

$$T_{\mu\nu}^{(1)} = 0.$$

The field itself has energy and momentum

$$T_{\mu\nu}^{(2)} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\partial_\lambda\phi\partial^\lambda\phi.$$

An Example

Field Equation is

$$\partial^2 \phi = 0.$$

Hence, energy-momentum is conserved

$$\partial^\mu T^{(2)}_{\mu\nu} = \partial^2 \phi \partial_\nu \phi = 0.$$

An Example

Energy of the field itself is also a source. Therefore

$$S^{(3)} = \int d^4x (\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \lambda T^{(2)} \phi).$$

Using Noether's theorem again

$$T^{(3)}_{\mu\nu} = T^{(2)}_{\mu\nu} - 2\lambda\phi\partial_\mu\phi\partial_\nu\phi + \lambda\eta_{\mu\nu}\phi\partial_\lambda\phi\partial^\lambda\phi.$$

Now, energy-momentum is conserved to a higher order

$$\partial^\mu T^{(3)}_{\mu\nu} = O(\phi^4).$$

An Example

To the forth order, the source term is

$$\lambda T^{(3)}_{\mu\nu}\phi,$$

where

$$T^{(3)}_{\mu\nu} = T^{(2)}_{\mu\nu} - 2\lambda\phi\partial_\mu\phi\partial_\nu\phi + \lambda\eta_{\mu\nu}\phi\partial_\lambda\phi\partial^\lambda\phi.$$

Another Problem
The Cosmological Constant

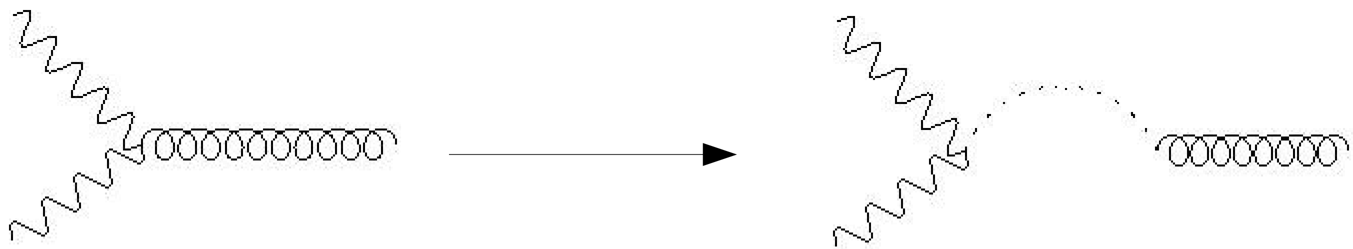
“Astronomical observations indicate that cosmological constant is many orders of magnitude smaller than estimated in modern theories of elementary particles.”

Steven Weinberg

One Possible Solution

There exist one or more heavy particle/s

$$\mathcal{L} = g^{\mu\nu}g^{\kappa\lambda}F_{\mu\kappa}F_{\nu\lambda}$$



1980's Emergent Graviton

- People started believing that graviton is a composite particle. Just like the pions.

- Pions are composites of quarks and anti quarks.

1980's Emergent Graviton

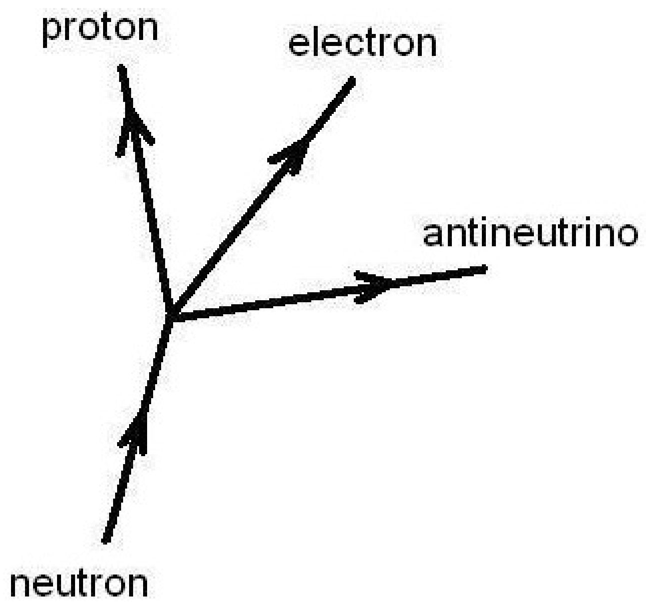
- The underlying theory of pions is QCD.
- We can effectively describe them by Chiral Perturbation Theory.

1980's Emergent Graviton

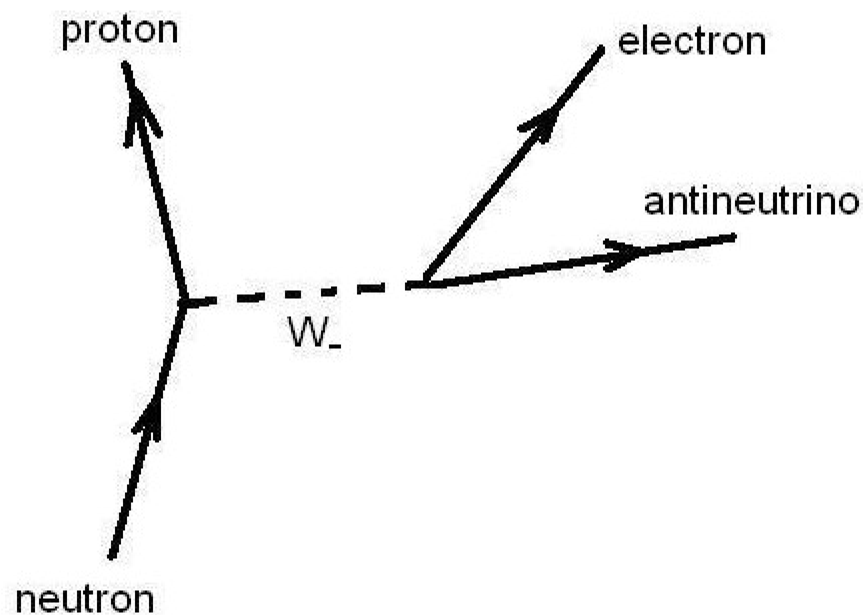
Some no go theorems exclude the possibility of constructing a composite massless spin 2 particle.

Fermi Theory of Beta Decay

Our idea to deal with gravity is analogous to the way we removed difficulties in the Fermi theory



Appearance of W and Z Particles



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    An example

    Old ideas and no go theorems
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What Kind of Fields Do We Need?

A massless spin 2 field that represent the gravitational field, h.

Can't gravity has other spins?

- Spin zero

  Light won't bend since its energy momentum is traceless.
- Spin one

  There must be repulsive forces.
- Higher integer spins

  No go theorems prohibit them.
- Non-integer spins

  There can't exist classical static forces

A Massless Spin 2 Field

A massless spin 2 field has a unique lagrangian to the quadratic order

$$\begin{aligned}\mathcal{L}_h \; &= \; \partial_\mu h^{\mu\nu}\partial_\nu h - \partial^\mu h^{\nu\sigma}\partial_\nu h_{\mu\sigma} \\ &\quad + \frac{1}{2}\partial_\mu h^{\rho\sigma}\partial^\mu h_{\rho\sigma} - \frac{1}{2}\partial_\mu h \partial^\mu h.\end{aligned}$$

It also must be symmetric. Otherwise, it propagates different spins.

What Symmetries?

- The Poincare symmetry.

- Internal symmetry?

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu.$$

$$\delta X_{(?)} = ?$$

Should it hold to all orders? Like QED?

$$\delta A_\mu = \partial_\mu \omega.$$

What Next?

We are interested in the generator of 1LPI correlation functions at tree level

$$\gamma^t[h]$$

What is this?

Effective Action

The effective action or generator of 1PI is

$$\Gamma[X,h] = W[J_X, J_h] - \int d^4x (J_X X + J_h h).$$

Here the currents come from the following

$$J_X = -\frac{\delta\Gamma[X,h]}{\delta X}, \qquad J_h = -\frac{\delta\Gamma[X,h]}{\delta h}.$$

The semi-classical expansion is

$$\Gamma[X,h] = S[X,h] + \Gamma_1[X,h]$$
$$+(all\ 1PI\ graphs\ with\ higher\ loops).$$

Generator of 1LPI

We are not interested in X-correlators. This means we can write

$$J_X = 0.$$

Or equivalently

$$\frac{\delta\Gamma[X,h]}{\delta X} = 0.$$

Hence,

$$\gamma^t[h] = \Gamma^t[h, \bar{X}].$$

The Heavy Particle

The lowest order solution to the massive field must lead to the third order of GR. This implies existence of one derivative in the interactions with gravity.

This means the field must be a tensor of rank 1 or 3.

Which one should we use?

- We don't know yet.
- It worth to study both.
- We look for a self-consistent mathematically beautiful theory.

A Heavy Vector Field

A massive spin 1 vector field looks like

$$\mathcal{L}_X = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + M^2 X^\mu X_\mu,$$

where

$$F_{\mu\nu} = \partial_\nu X_\mu - \partial_\mu X_\nu.$$

Coupling to Gravity

All Poincare invariant interactions with dimensionally non-negative coupling constants are

$$
\begin{aligned}
\mathcal{L}_{X-h} \;=\; & b_1 h_{\mu\nu} X^{\mu} X^{\nu} + b_2 h X^{\mu} X_{\mu} + b_3 h_{\mu\nu} h^{\nu\sigma} X^{\mu} X_{\sigma} \\
& + b_4 h h_{\mu\nu} X^{\mu} X^{\nu} + b_5 h_{\mu\nu} h^{\mu\nu} X^{\sigma} X_{\sigma} + b_6 h^2 X^{\sigma} X_{\sigma} \\
& - b_7 \partial_{\mu} h^{\mu\nu} X_{\nu} - b_8 X^{\mu} \partial_{\mu} h - b_9 \partial^{\mu} h_{\mu\nu} h^{\nu\sigma} X_{\sigma} \\
& - b_{10} \partial^{\mu} h h_{\mu\nu} X^{\nu} - 2 b_{11} \partial^{\sigma} h_{\mu\nu} h^{\mu\nu} X_{\sigma} - 2 b_{12} h \partial^{\sigma} h X_{\sigma} \\
& - b_{13} h_{\mu\nu} \partial^{\mu} h^{\nu\sigma} X_{\sigma} - b_{14} h \partial^{\mu} h_{\mu\nu} X^{\nu}.
\end{aligned}
$$

Total Lagrangian

The lagrangian that describes our theory is

$$\mathcal{L} = \mathcal{L}_h + \mathcal{L}_X + \mathcal{L}_{X-h},$$

where

$$\begin{aligned} \mathcal{L}_h \;&=\; \partial_\mu h^{\mu\nu}\partial_\nu h - \partial^\mu h^{\nu\sigma}\partial_\nu h_{\mu\sigma} \\ &\quad + \frac{1}{2}\partial_\mu h^{\rho\sigma}\partial^\mu h_{\rho\sigma} - \frac{1}{2}\partial_\mu h \partial^\mu h. \end{aligned}$$

Field Equations

$$\partial^\nu F_{\nu\mu} + 2M^2 X_\mu + 2b_1 h_{\mu\nu} X^\nu + 2b_2 h X_\mu$$
$$+2b_3 h_{\mu\nu} h^{\nu\sigma} X_\sigma + 2b_4 h h_{\mu\nu} X^\nu + 2b_5 h_{\sigma\nu} h^{\sigma\nu} X_\mu$$
$$+2b_6 h^2 X_\mu - b_7 \partial^\nu h_{\mu\nu} - b_8 \partial_\mu h - b_9 \partial_\nu h^{\nu\sigma} h_{\sigma\mu}$$
$$-b_{10} \partial^\nu h h_{\mu\nu} - 2b_{11} h_{\rho\sigma} \partial_\mu h^{\rho\sigma} - 2b_{12} h \partial_\mu h$$
$$-b_{13} h^{\nu\sigma} \partial_\nu h_{\sigma\mu} - b_{14} h \partial^\nu h_{\nu\mu} = 0.$$

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Leading Order Solution

$$
\begin{aligned}
X_\nu \;=\; & \frac{1}{2M^2}\Big[b_7\partial^\kappa h_{\kappa\nu} + b_8\partial_\nu h + b_9\partial_\kappa h^{\kappa\lambda}h_{\lambda\nu} \\
& +b_{10}\partial^\kappa h h_{\kappa\nu} + 2b_{11}h_{\kappa\lambda}\partial_\nu h^{\kappa\lambda} + 2b_{12}h\partial_\nu h \\
& +b_{13}h^{\kappa\lambda}\partial_\kappa h_{\lambda\nu} + b_{14}h\partial^\kappa h_{\kappa\nu}\Big] + O(\frac{1}{M^4}).
\end{aligned}
$$

The Low Energy Lagrangian

Substituting the solution back to the total lagrangian

$$
\begin{aligned}
&\mathcal{L}_{eff} = \mathcal{L}_h + \mathcal{L}^{(2)}_{h-h} \\
&+\frac{1}{2M^2}\Big[2(b_7 b_{12} + b_8 b_{10}) h^{\mu\nu} h \partial_\mu \partial_\nu h \\
&+(b_7 b_{13}) h^{\mu\nu} \partial_\mu \partial_\sigma h^{\sigma\lambda} h_{\lambda\nu} \\
&+(b_8 b_{13} - b_8 b_9 - b_7 b_{10}) \partial^\kappa h_{\nu\kappa} \partial_\lambda h h^{\nu\lambda} \\
&+(b_8 b_{13}) h^{\mu\nu} \partial_\mu \partial^\sigma h h_{\nu\sigma} + (b_8 b_{10} + 2 b_7 b_{12}) \partial_\nu h \partial_\lambda h h^{\nu\lambda} \\
&-(b_7 b_{14}) h \partial^\kappa h_{\nu\kappa} \partial_\lambda h^{\nu\lambda} + (4 b_8 b_{12}) h \partial_\nu h \partial^\nu h \\
&+(2 b_8 b_{10} + 2 b_7 b_{12} + b_8 b_{14}) h \partial_\nu h \partial_\kappa h^{\kappa\nu} \\
&+(b_7 b_{11}) h_{\kappa\lambda} h^{\kappa\lambda} \partial^\mu \partial^\nu h_{\mu\nu} + (b_8 b_{11}) h_{\kappa\lambda} h^{\kappa\lambda} \partial^2 h \\
&+(b_7 b_{12} + b_8 b_{14}) h^2 \partial^\mu \partial^\nu h_{\mu\nu} + (3 b_8 b_{12}) h^2 \partial^2 h \\
&+(b_7 b_{13} - b_7 b_9) \partial^\kappa h_{\kappa\lambda} h^{\lambda\nu} \partial^\mu h_{\mu\nu}\Big] \\
&+O(\frac{1}{M^4}, h^4).
\end{aligned}
$$

Is This GR lagrangian?

Not exactly.
One term is missing

$$-\sqrt{16\pi G}h_{\kappa\lambda}h_{\mu\nu}\partial^{\mu}\partial^{\nu}h^{\kappa\lambda}.$$

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A Heavy Rank 3 Tensor Field

The total lagrangian is

$$\mathcal{L} = \mathcal{L}_h + \mathcal{L}_X + \mathcal{L}_{X-h}.$$

Here

$$\begin{aligned}\mathcal{L}_X \ &= \sum \partial X \partial X + \frac{1}{2} M_1^2 X_{\alpha\beta\gamma} X^{\alpha\beta\gamma} \\ &+ \frac{1}{2} M_2^2 X^{\alpha}_{\alpha\gamma} X^{\beta\gamma}_{\beta}.\end{aligned}$$

A Heavy Rank 3 Tensor Field

All Poincare invariant interactions with non-negative coupling constants are

$$\begin{aligned}
\mathcal{L}_{X-h} &= a_1 X_\alpha^{\alpha\gamma}\partial_\gamma h + a_2 X_\alpha^{\alpha\gamma}\partial^\beta h_{\beta\gamma} + a_3 X^{\alpha\beta\gamma}\partial_\gamma h_{\alpha\beta} \\
&+ a_4 h\partial_\gamma h X_\alpha^{\alpha\gamma} + a_5\partial^\beta h h_{\beta\gamma} X_\alpha^{\alpha\gamma} + a_6 h\partial^\beta h_{\beta\gamma} X_\alpha^{\alpha\gamma} \\
&+ a_7\partial_\gamma h h_{\alpha\beta} X^{\alpha\beta\gamma} + a_8 h\partial_\gamma h_{\alpha\beta} X^{\alpha\beta\gamma} \\
&+ a_9\partial_\gamma h_{\kappa\lambda} h^{\kappa\lambda} X_\alpha^{\alpha\gamma} + a_{10}\partial_\gamma h_{\alpha\kappa} h_\beta^\kappa X^{\alpha\beta\gamma} \\
&+ a_{11}\partial^\beta h_{\beta\kappa} h_\gamma^\kappa X_\alpha^{\alpha\gamma} + a_{12} h_{\beta\kappa}\partial^\beta h_\gamma^\kappa X_\alpha^{\alpha\gamma} \\
&+ a_{13}\partial^\kappa h_{\kappa\gamma} h_{\alpha\beta} X^{\alpha\beta\gamma} + a_{14} h_{\kappa\gamma}\partial^\kappa h_{\alpha\beta} X^{\alpha\beta\gamma} \\
&+ O(X^2).
\end{aligned}$$

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Field Equations

$$
\begin{aligned}
&a_1\partial_\gamma h\eta_{\alpha\beta} + a_2\partial^\kappa h_{\kappa\gamma}\eta_{\alpha\beta} + a_3\partial_\gamma h_{\alpha\beta} + a_4 h\partial_\gamma h\eta_{\alpha\beta}\\
&+a_5\partial^\kappa h h_{\kappa\gamma}\eta_{\alpha\beta} + a_6 h\partial^\kappa h_{\kappa\gamma}\eta_{\alpha\beta} + a_7\partial_\gamma h h_{\alpha\beta} + a_8 h\partial_\gamma h_{\alpha\beta}\\
&+a_9\partial_\gamma h_{\kappa\lambda}h^{\kappa\lambda}\eta_{\alpha\beta} + a_{10}\partial_\gamma h_{\kappa\alpha}h^\kappa_\beta + a_{11}\partial^\kappa h_{\kappa\lambda}h^\lambda_\gamma\eta_{\alpha\beta}\\
&+a_{12}h_{\kappa\lambda}\partial^\kappa h^\lambda_\gamma\eta_{\alpha\beta} + a_{13}\partial^\kappa h_{\kappa\gamma}h_{\alpha\beta} + a_{14}h_{\kappa\gamma}\partial^\kappa h_{\alpha\beta}\\
&+M_1^2\Big(\frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\eta_{\nu\alpha}\eta_{\mu\beta} + (\frac{M_2}{M_1})^2\eta_{\mu\nu}\eta_{\alpha\beta}\Big)X^{\mu\nu}{}_\gamma\\
&= O(\frac{1}{M^2}).
\end{aligned}
$$

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The Effective Lagrangian

$$
\begin{aligned}
&\mathcal{L}_{eff} = \mathcal{L}_h + \mathcal{L}^{(2)}_{h-h} \\
&+B_1 h_{\kappa\lambda} h_{\mu\nu} \partial^\mu \partial^\nu h^{\kappa\lambda} + (B_3 + B_4) h^2 \partial^2 h \\
&+(B_5 + B_6) h_{\kappa\lambda} h^{\kappa\lambda} \partial^2 h + (2B_5 + B_7) h^{\mu\nu} \partial_\mu \partial_\sigma h^{\sigma\lambda} h_{\lambda\nu} \\
&+(-B_5 + B_8) h_{\kappa\lambda} h^{\lambda\mu} \partial_\mu \partial^\kappa h + B_9 h_{\kappa\lambda} \partial_\mu h^{\mu\kappa} \partial_\nu h^{\nu\lambda} \\
&+(-2B_3 + B_{10}) h_{\kappa\lambda} \partial_\mu h^{\mu\kappa} \partial^\lambda h + (B_3 + B_{12}) h_{\kappa\lambda} \partial^\kappa h \partial^\lambda h \\
&+(-2B_3 + B_{13}) h \partial_\mu h^{\mu\kappa} \partial^\lambda h_{\kappa\lambda} + (3B_3 + B_{14}) h \partial_\kappa h^{\kappa\lambda} \partial_\lambda h \\
&+(-B_5 + B_{17}) h_{\kappa\lambda} h^{\kappa\lambda} \partial^\mu \partial^\nu h_{\mu\nu} + O(\frac{1}{M^4}, h^4),
\end{aligned}
$$

Is This GR Lagrangian?

- It is identically third order of Einstein-Hilbert lagrangian if we choose the constants appropriately.

- Comparing with GR doesn't fix all the couplings. It only put some constraints on them. Other coupling constants must be fixed by internal symmetries or perhaps experimentally.

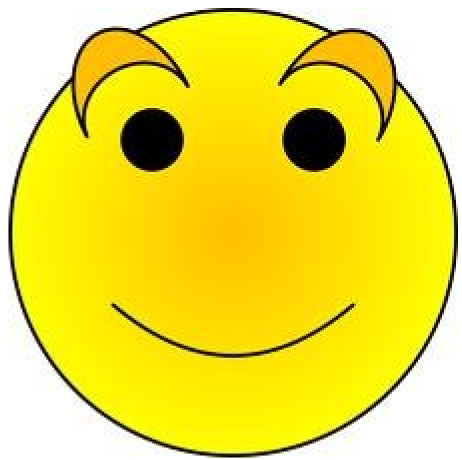
Summary

- General Relativity is not UV complete. And, hence, is only effective field of a more fundamental theory.

- If quantum gravity fits within the borders of field theory , it will be more interesting than building totally new concepts.

- A Poincare invariant rank 3 field seems to be capable of giving rise to GR at low energies.

Thanks For Your Attention



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