

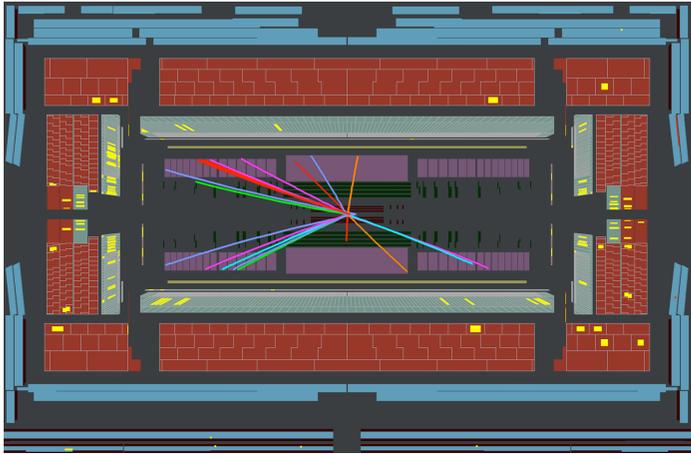


# Results from Lattice QCD

Christine Davies  
University of Glasgow

Lepton-Photon 2013  
San Francisco, June 2013

# QCD is a key part of the Standard Model but quark confinement is a complication/interesting feature.

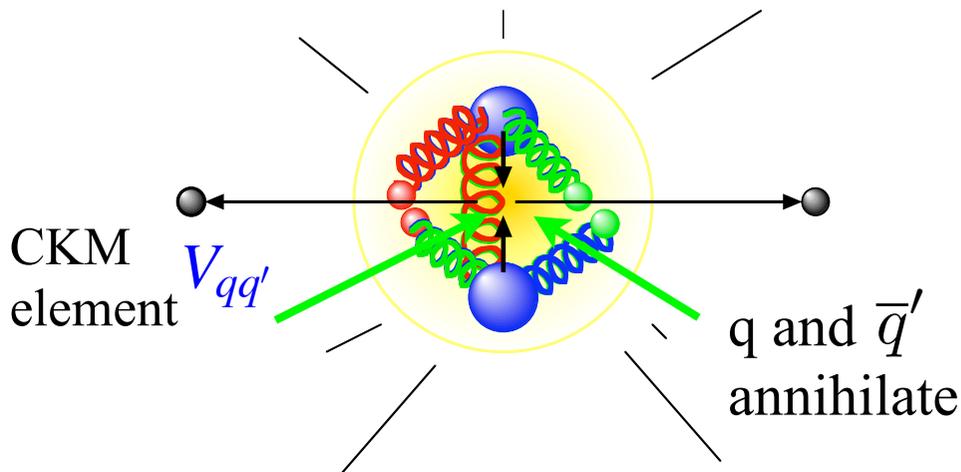


ATLAS  
@LHC

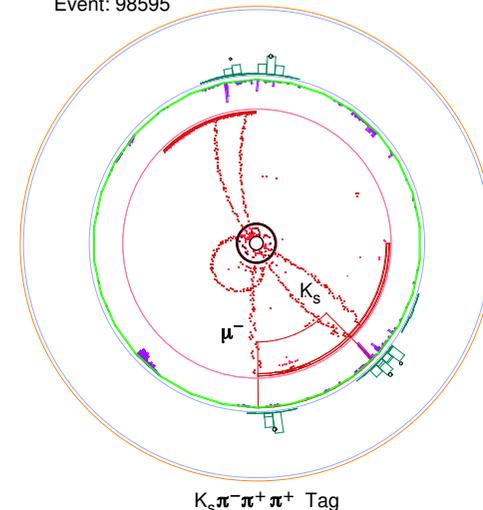
VS



Properties of hadrons calculable from QCD if fully nonperturbative calculation is done - can test QCD/search for new physics and determine parameters (to 1%).



Event: 98595



CLEO-c

$$D_s \rightarrow \mu \nu$$

$$D_s \rightarrow K \pi \pi \pi$$

# Applications of Lattice QCD/Lattice field theory

Annual proceedings of  
lattice conference:  
<http://pos.sissa.it/>

## *Particle physics*

Hadron spectrum  
Dudek, Mohler:  
Lat2012

QCD parameters  
Shintani: Lat2011

Hadron structure  
Lin: Lat2012

CKM elements  
Tarantino: Lat2012

Glueballs and exotica

Theories beyond the  
Standard Model  
Giedt, Panero: Lat2012

QCD at high temperatures  
and densities  
Lombardo, Aarts:  
Lat2012;  
Satz: Saturday

Quantum gravity  
Laiho: Lat2011

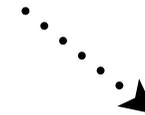
## *Astrophysics*

Young: Lat2012

## *Nuclear physics*

Nuclear masses  
and properties  
Doi: Lat2012

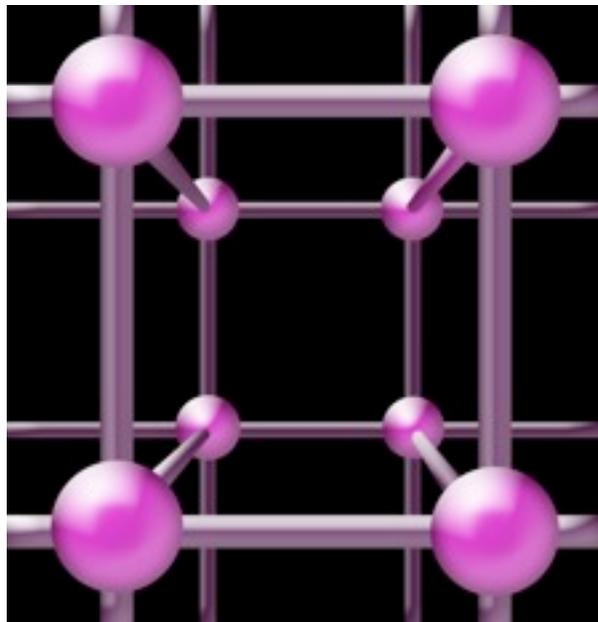
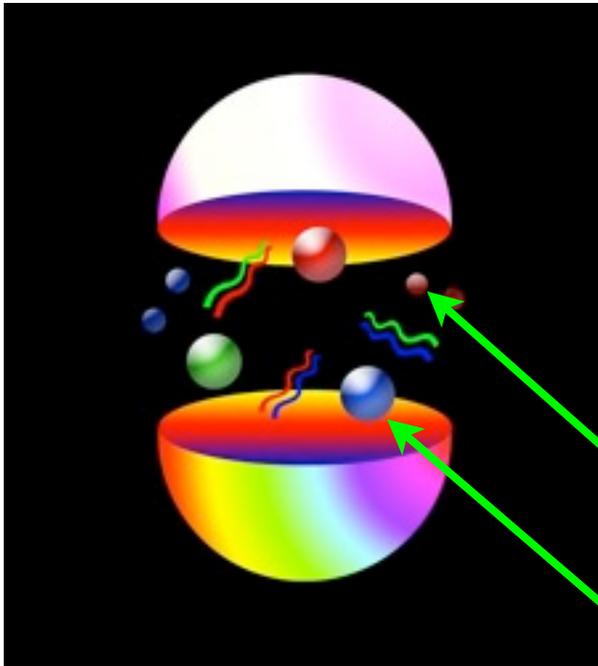
condensed matter physics  
computational physics  
computer science ...  
Boyle: Lat2012



Lattice QCD = fully nonperturbative QCD calculation

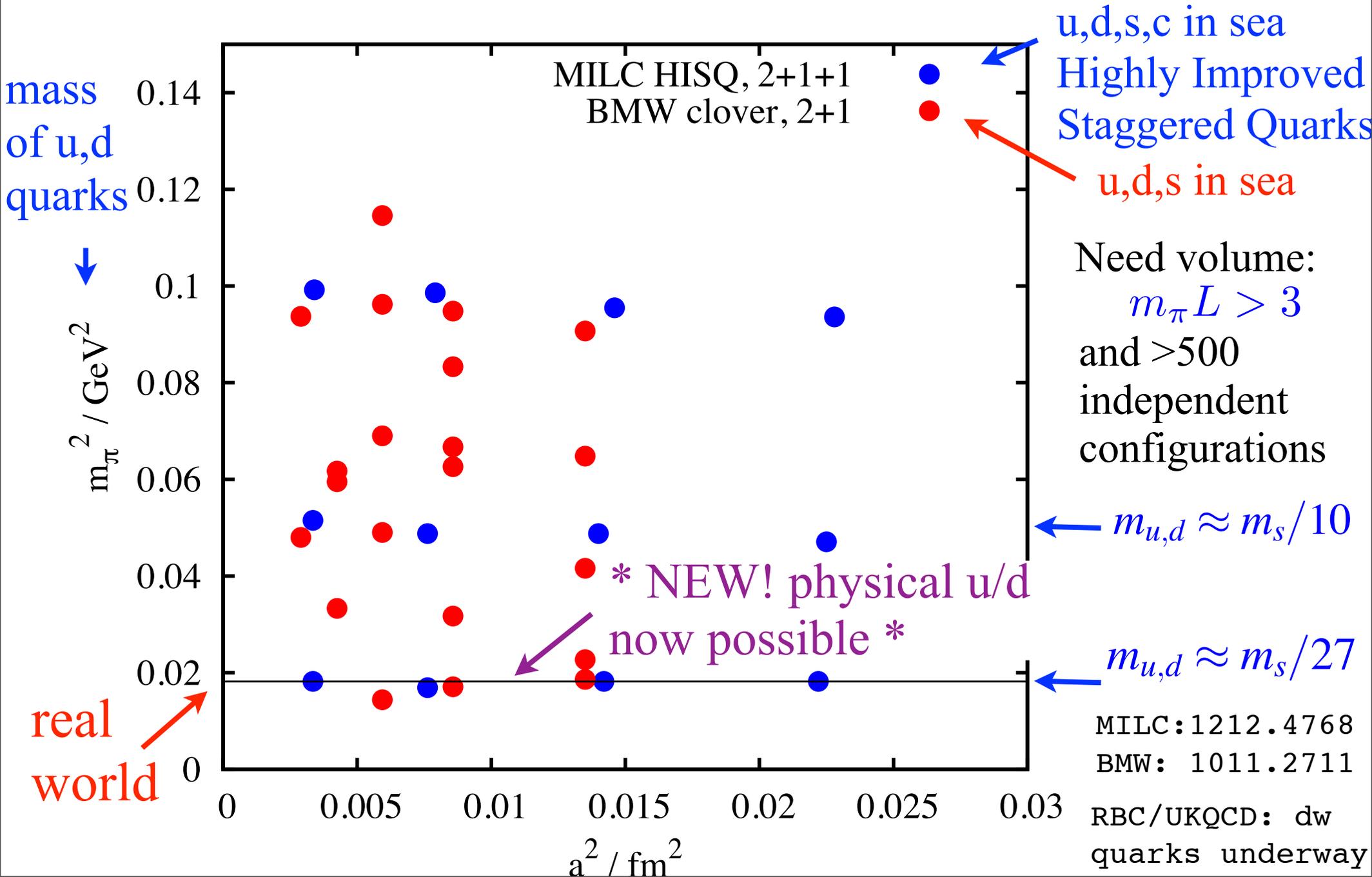
## RECIPE

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s (+ c) sea quarks)
- Calculate averaged “hadron correlators” from valence q props.
- Fit as a function of time to obtain masses and simple matrix elements
- Determine  $a$  and fix  $m_q$  to get results in physical units.
- extrapolate to  $a = 0, m_{u,d} = phys$  for real world. \*now have phys  $m_{u,d}$ \*

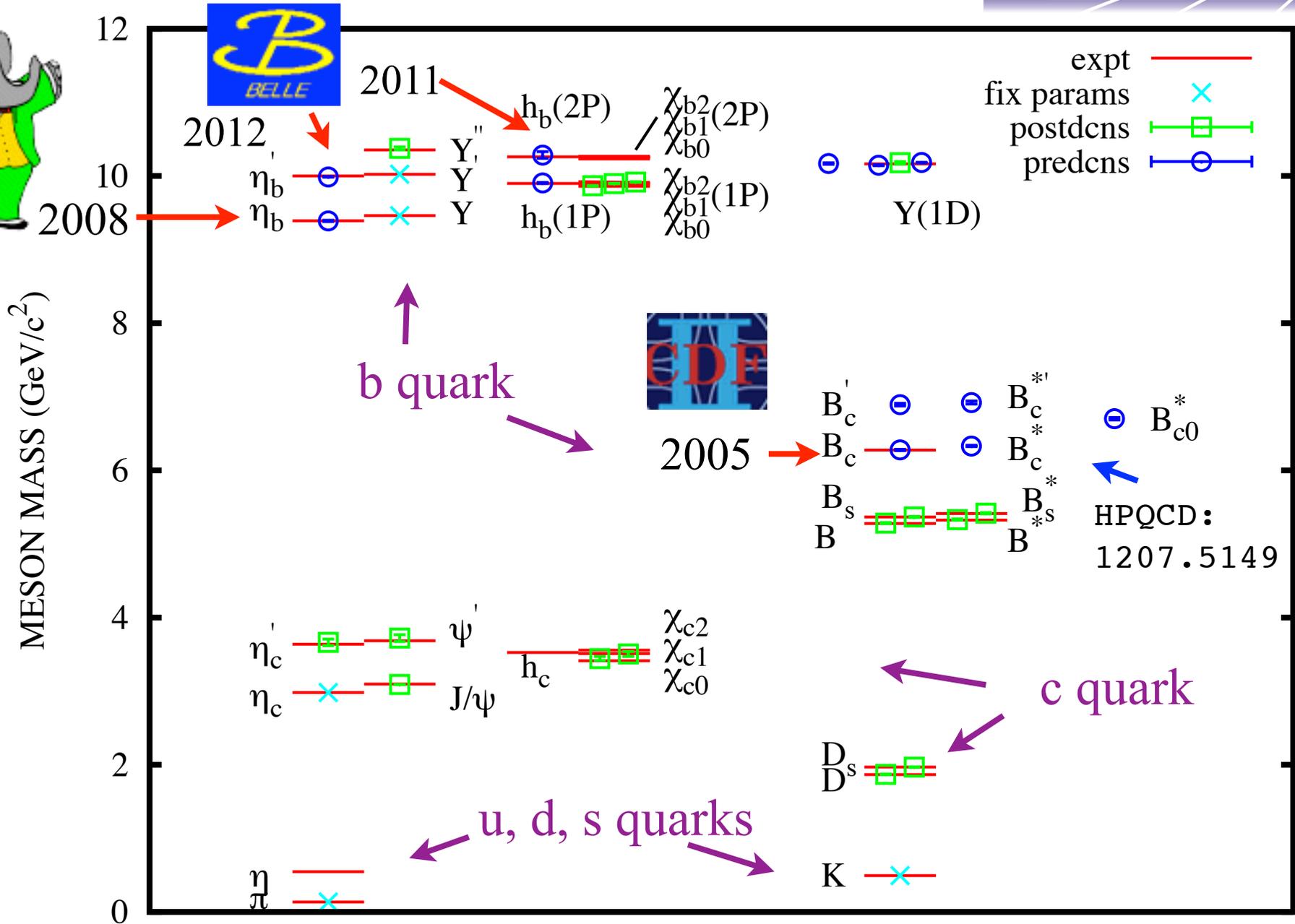


$a$

# Example parameters for gluon configurations being made using two different formalisms for handling quarks.



# Results for the masses of mesons that are long-lived and so can be well-characterised in experiment



Agreement very good - errors typically a few MeV, need to worry about em, mu-md ..

# Mapping excited states is harder ..

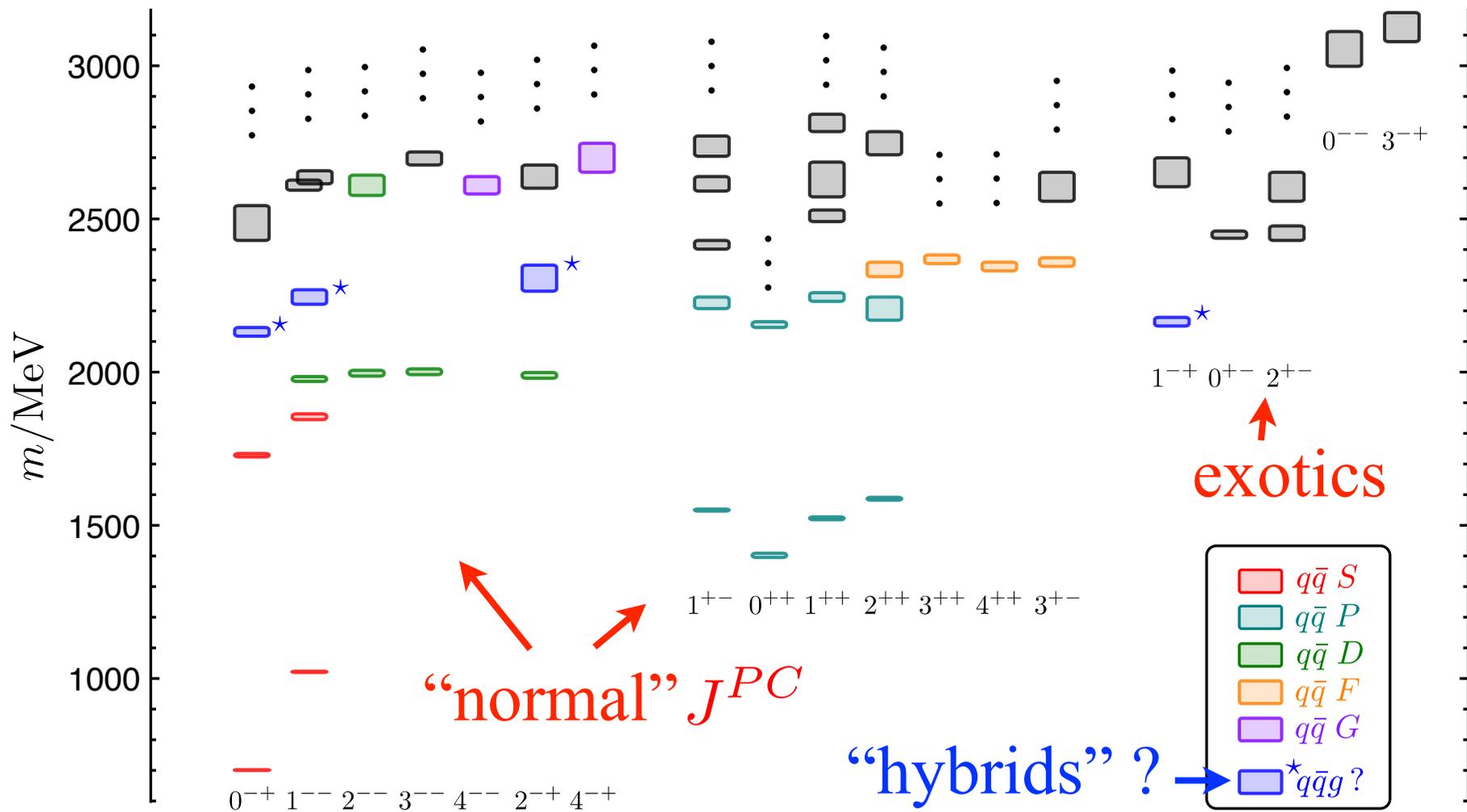
Hadron Spectrum: 1004.4930

Dudek: Lattice2012

BGR: 1112.1601

## Light isovectors: $\pi, \rho, \dots$

(huge basis of single-hadron operators, but  $a=0.12$  fm (anisotropic), mass  $\pi = 700$  MeV)



lightest hybrid multiplet =  $1^{--}, (0, 1, 2)^{-+} = S\text{wave } q\bar{q}(0^{-+}, 1^{--}) \times g(1^{+-})$   
 strong overlap with "gluey" operators  $\pi_1(1600), \pi(1800), \pi_2(1880) + 1^{--}?$

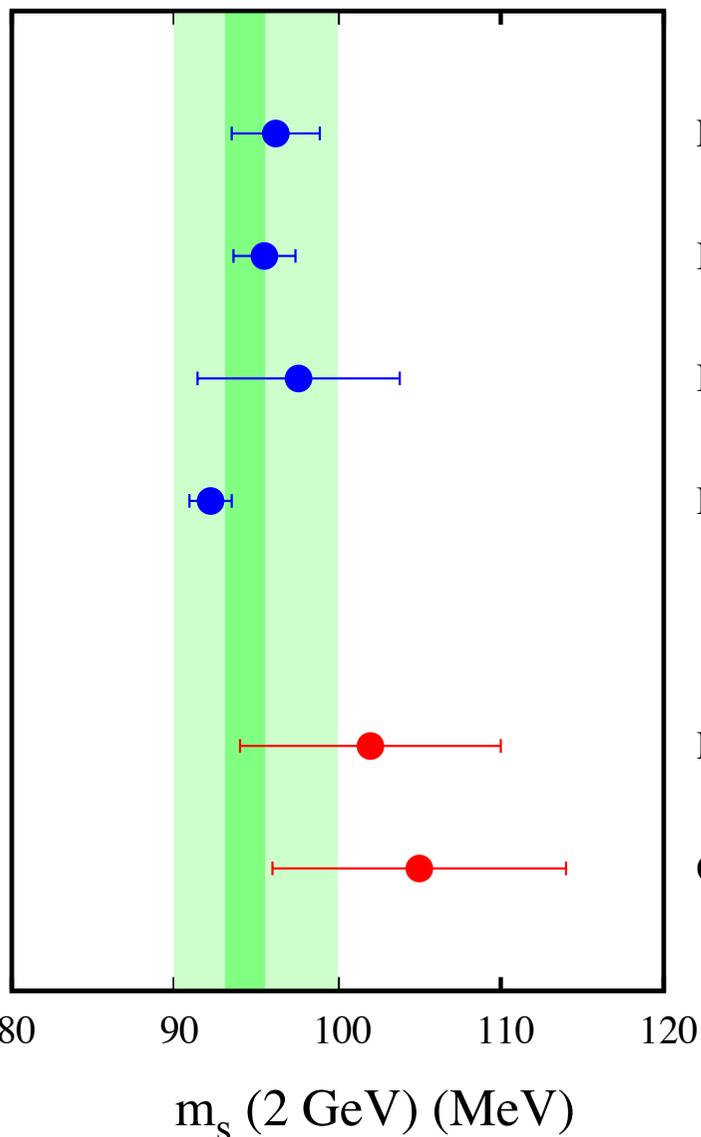
see also charmonium: Hadron Spectrum: 1204.5425

# Lattice QCD sets world averages for quark masses and $\alpha_s$

Direct access to parameters in QCD Lagrangian means systematic errors smaller

PDG av: 94.3(1.2) MeV

a variety of lattice methods agree



RBC-UKQCD 11

BMW 11

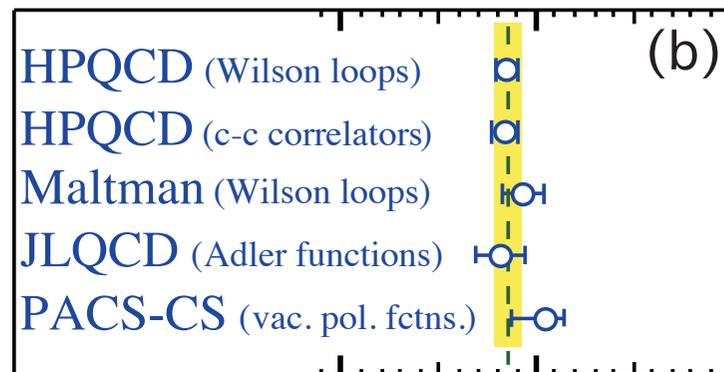
Blum 10

HPQCD 10

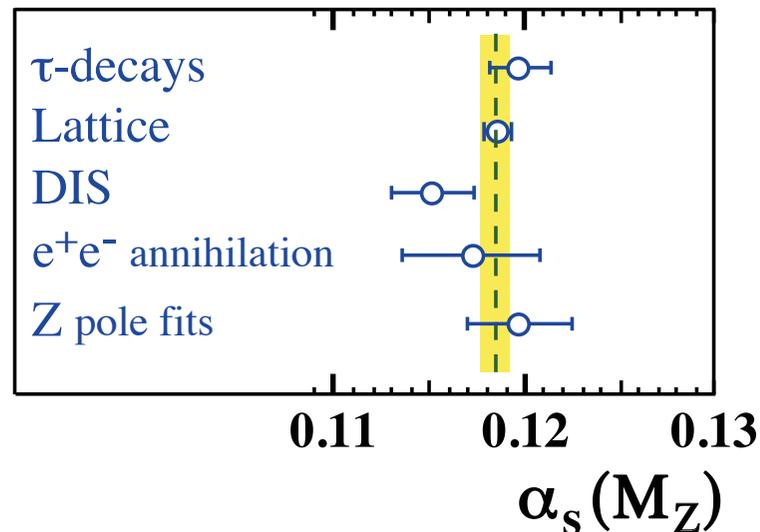
Dominiguez 08

Chetyrkin 06

non-lattice methods  
have larger errors



av: 0.1184(7) PDG

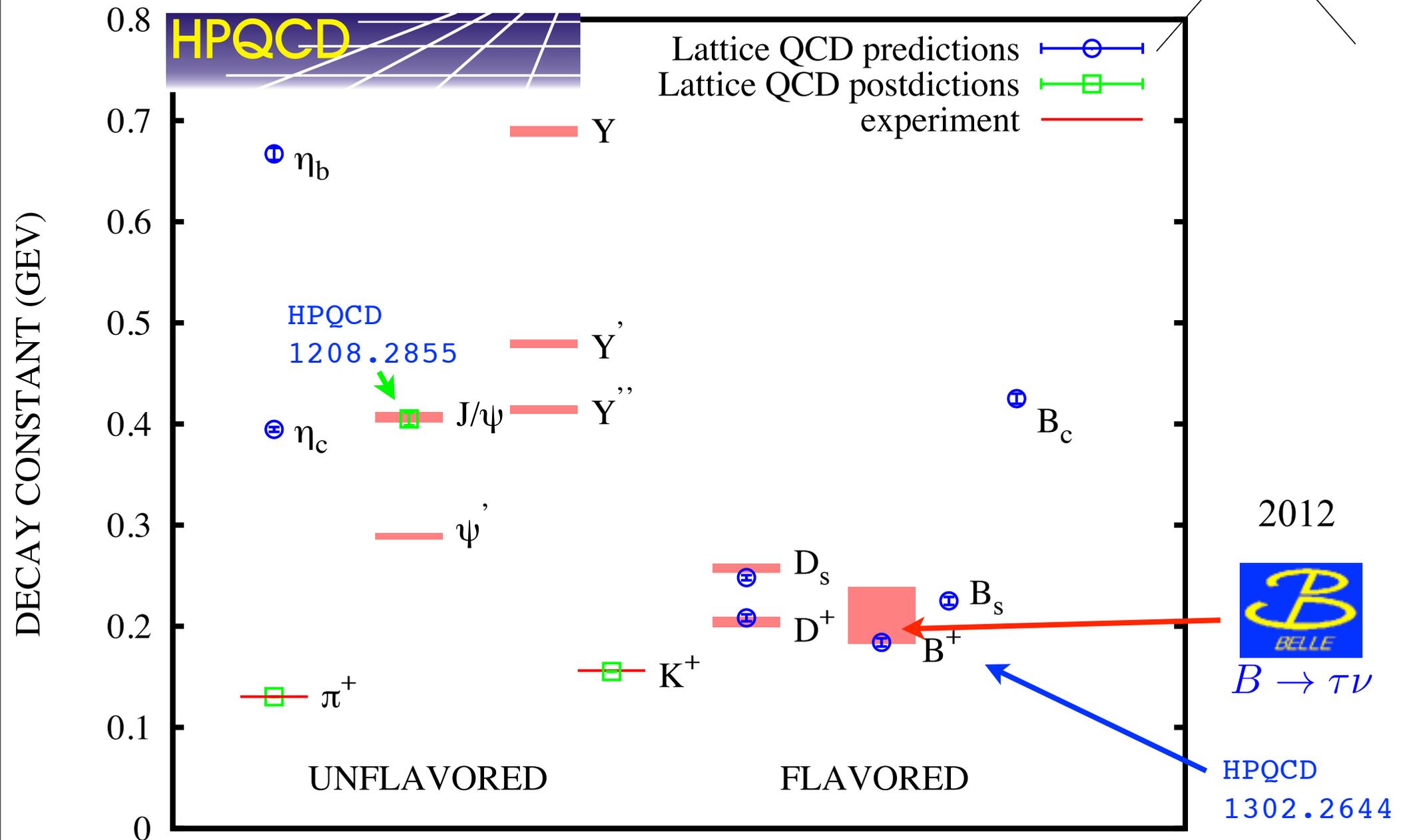
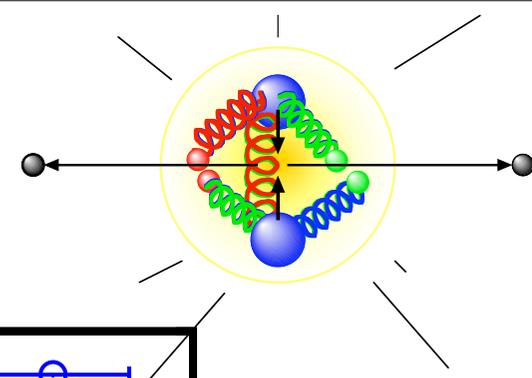


Lattice calcs now adding QED for accurate  $m_u/m_d$   
Izubuchi:Lattice2012; RM123: 1303.4896

# Meson decay constants

Parameterises hadronic information needed

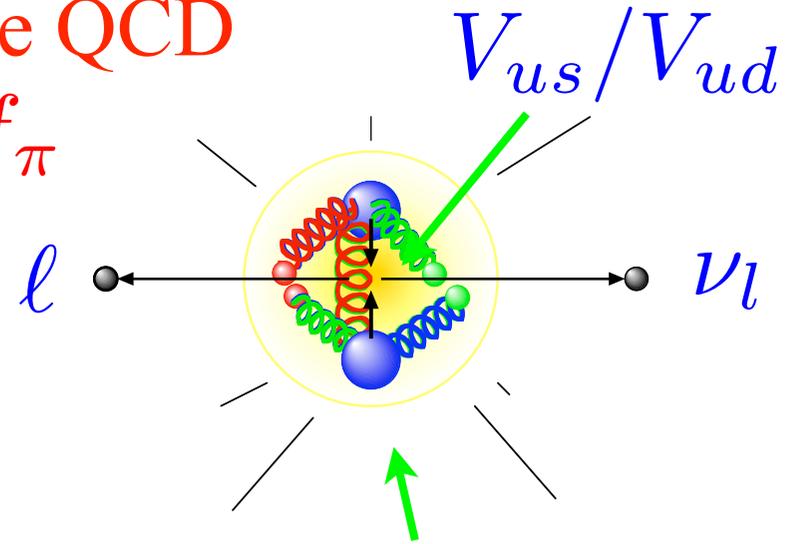
for annihilation rate to W or photon:  $\Gamma \propto f^2$



# Constraining new physics with lattice QCD

\* results at physical u/d quark masses\*

$f_K / f_\pi$

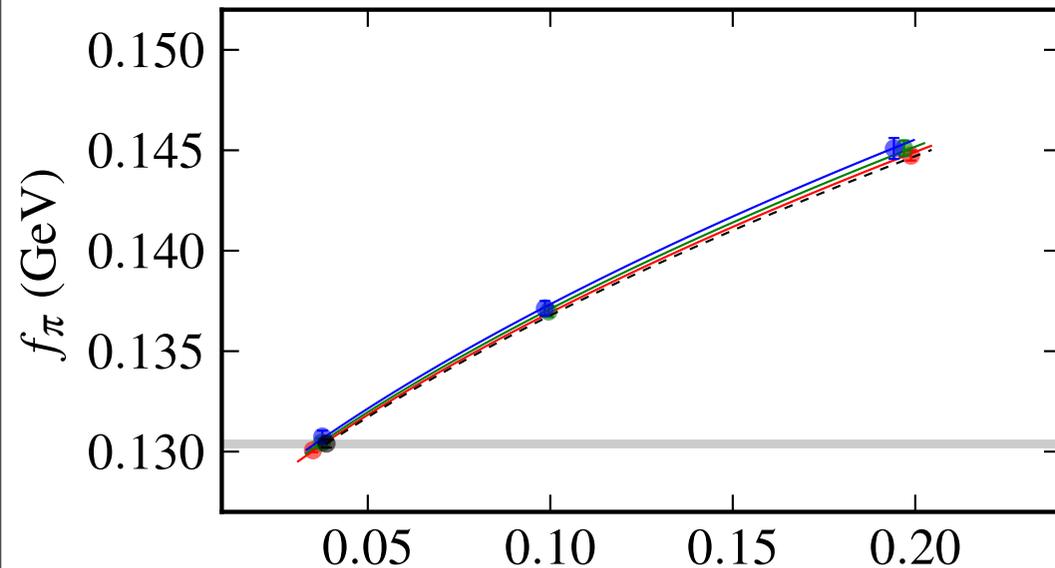
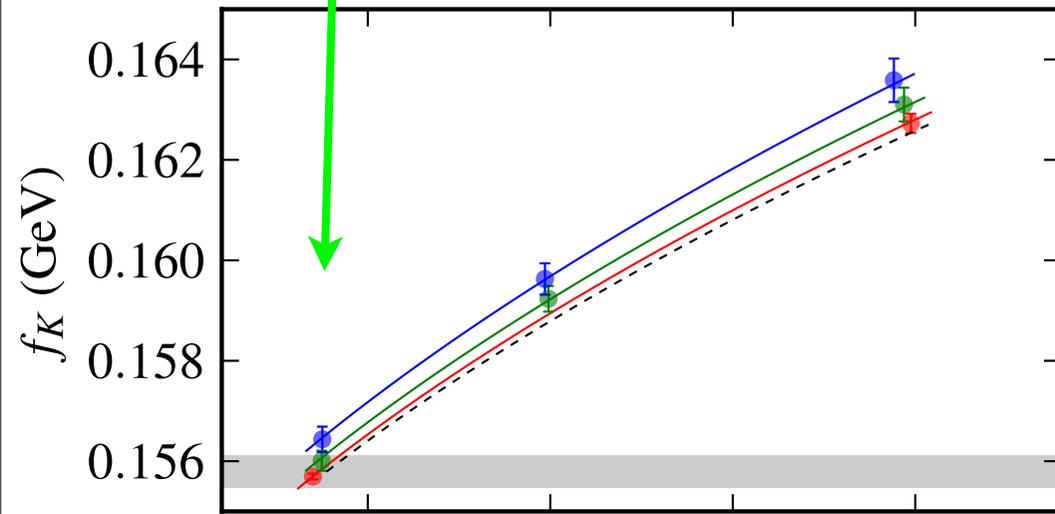


Annihilation of  $K/\pi$  to W allows CKM element determination given decay constants from lattice QCD

expt for  $\frac{\Gamma(K^+ \rightarrow l\nu)}{\Gamma(\pi^+ \rightarrow l\nu)}$

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.27598(35)_{\text{Br}(K^+)}(25)_{EM}$$

$\frac{f_{K^+}}{f_{\pi^+}}$  from lattice gives CKM



HISQ on MILC configs  
HPQCD: 1301.1670

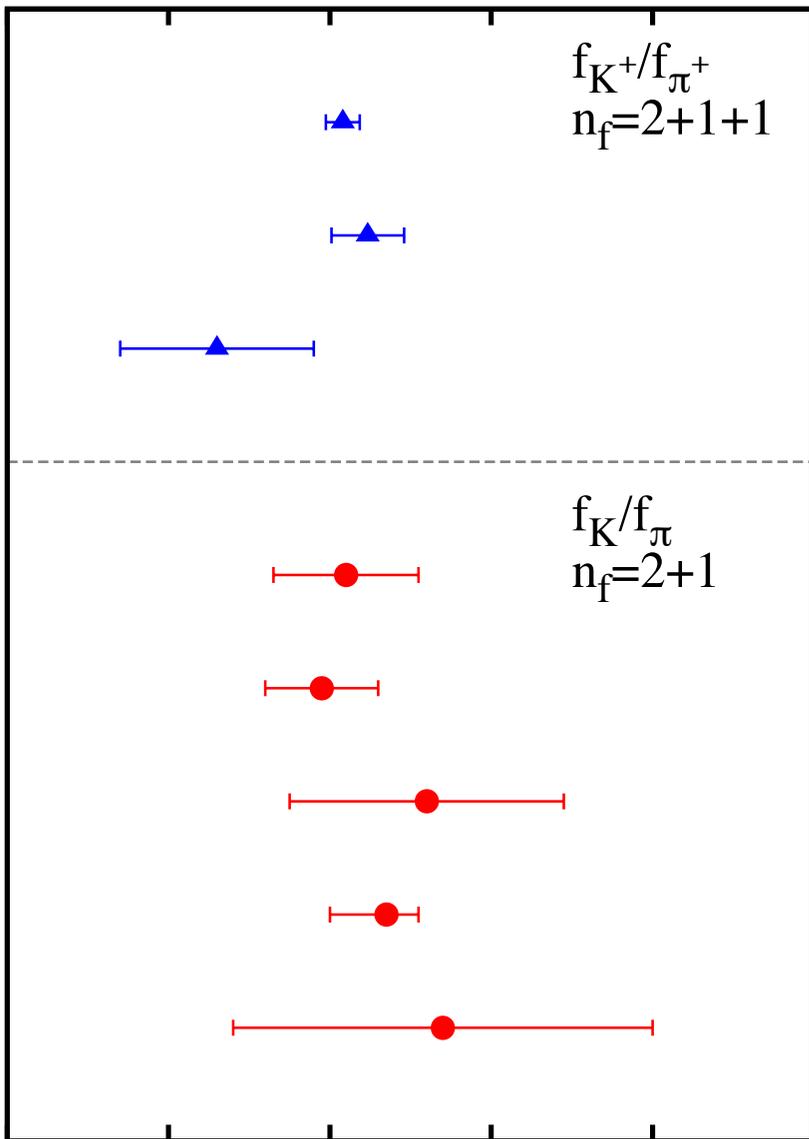
$$\frac{m_\pi^2}{2m_K^2 - m_\pi^2} = m_{u,d}/m_s$$

# Comparison of results

(note:  $f_{K^+} < f_K$ )

RM123:1303.4896  
gives by 0.40(4)%

good agreement from different formalisms



HPQCD, 1303.1670  
HISQ  
MILC, 1301.5855  
HISQ  
ETMC, Lattice2013  
twisted mass

BMW, 1001.4692  
clover  
HPQCD, 0706.1726  
HISQ  
LvW, 1112.4861  
domain-wall  
MILC, 1012.0868  
asqtad  
RBC/UKQCD  
1011.0892  
domain-wall

\* results at physical u/d  
quark masses\*

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.1916(21)$$

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23160(29)_{expt}(21)_{EM}(41)_{latt}$$

$$|V_{us}| = 0.22564$$

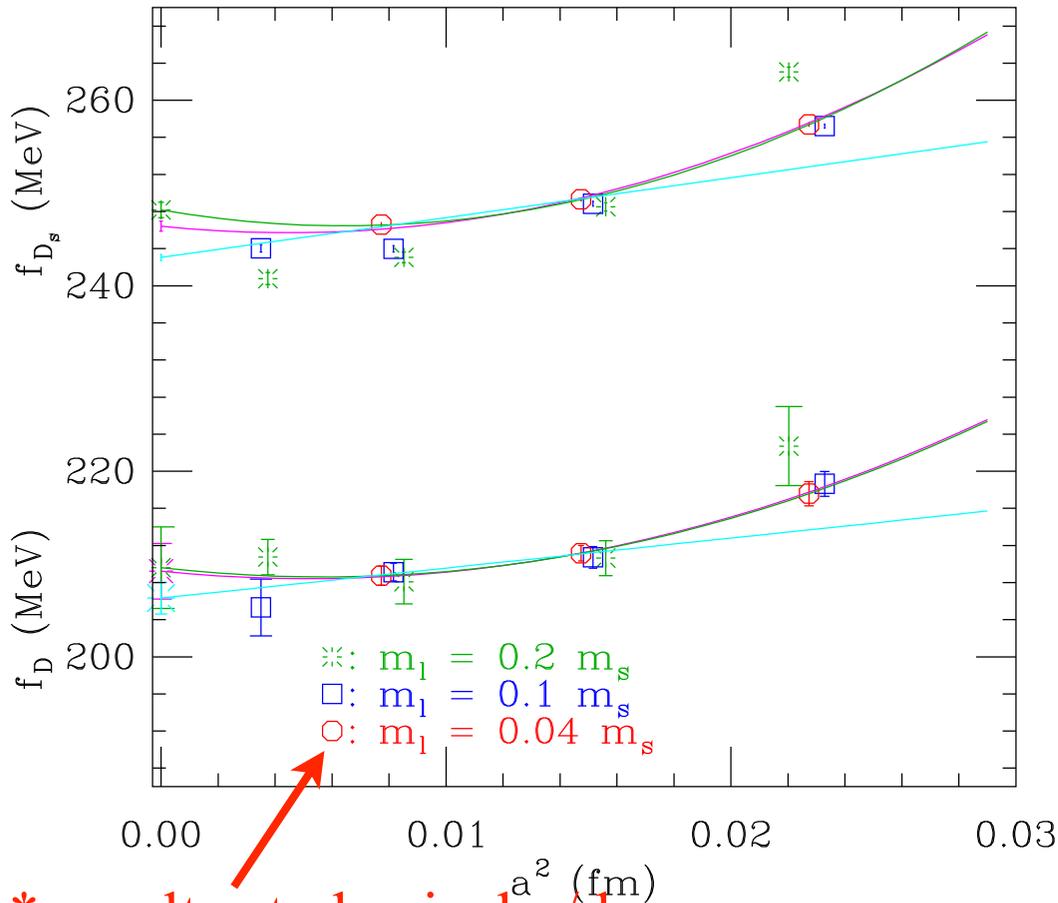
$$(28)_{Br}(20)_{EM}(40)_{latt}(5)_{V_{ud}}$$

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = -0.00009(51)$$

$V_{ud}$  from nuclear  $\beta$  decay now needs improvement for unitarity test!

# Constraining new physics with lattice QCD: $f_{D_s}, f_D$

MILC:1210.8431 new results using HISQ quarks on MILC 2+1+1 configs



\* results at physical u/d quark masses\*

experimental update: new Belle results

World av:  $f_{D_s} = 257.2(4.5)\text{MeV}$

$2\sigma$  above theory

Zupanc:charm2012

MILC (Lattice2013):

$$f_{D_s} = 247.2(2.2)\text{MeV}$$

$$f_D = 211.4(1.6)\text{MeV}$$

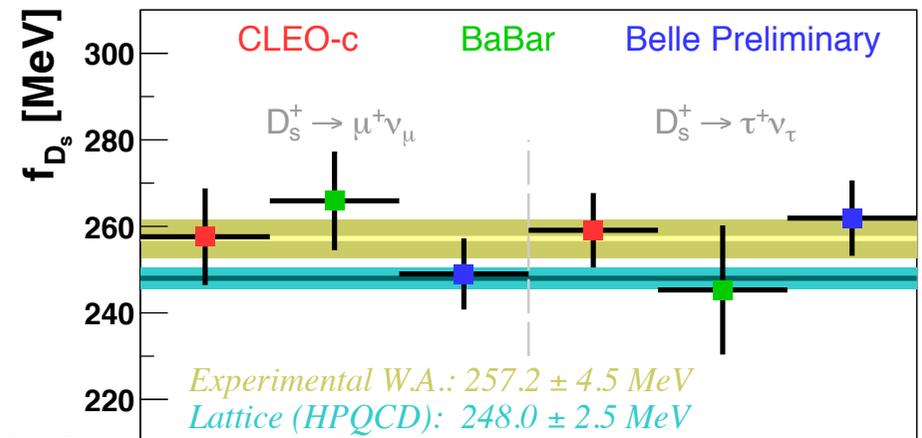
agree well with previous HPQCD:

$$f_{D_s} = 248.0(2.5)\text{MeV}$$

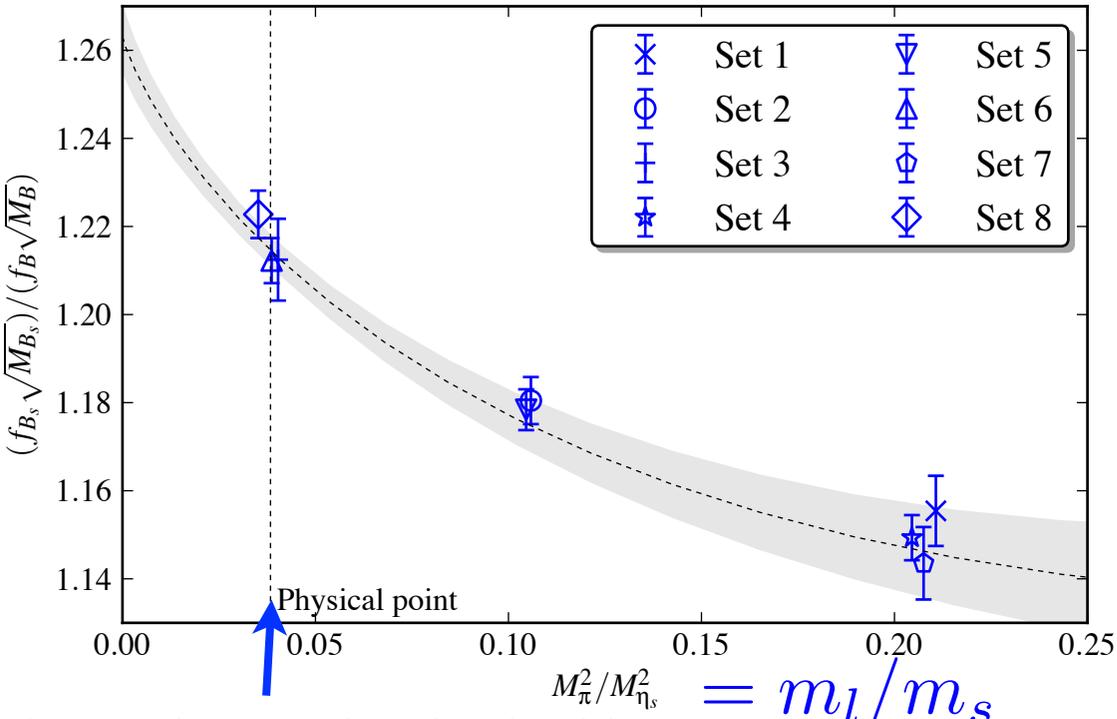
HPQCD:1008.4018

$$f_D = 208.3(3.4)\text{MeV}$$

HPQCD:1206.4936



# Constraining new physics with lattice QCD: $f_{B_s}, f_B$



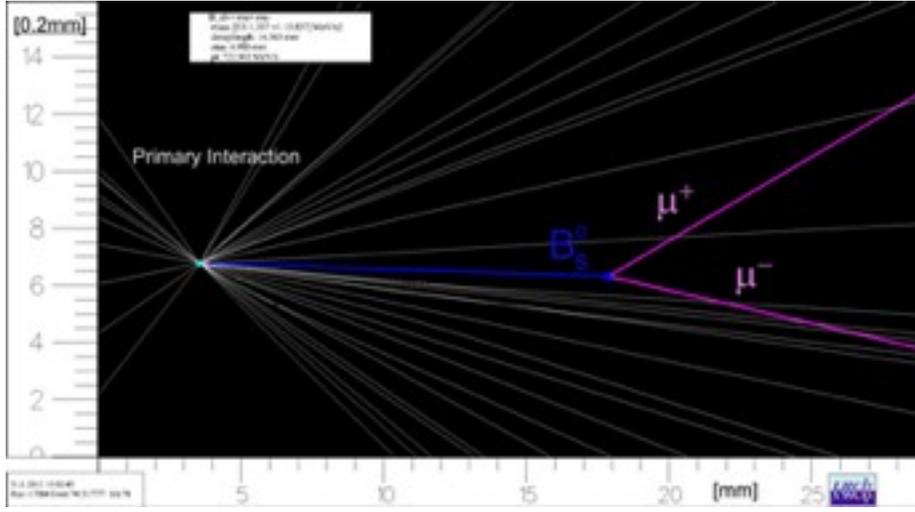
\* results at physical u/d quark masses\*

HPQCD: 1302.2644.

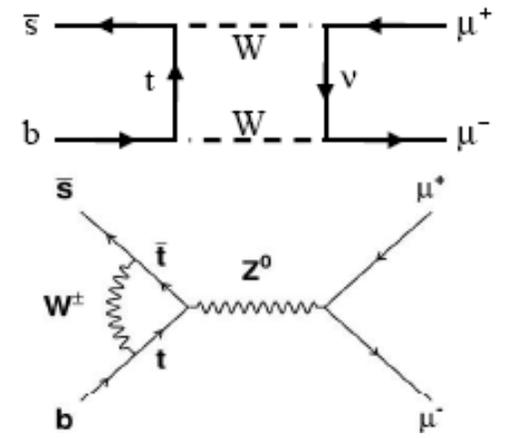
Uses improved NRQCD for b quark and HISQ u/d and s quarks on HISQ 2+1+1 gluon configs

$$f_{B_s} = 224(5) \text{ MeV}$$

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) = 3.47(19) \times 10^{-9}$$

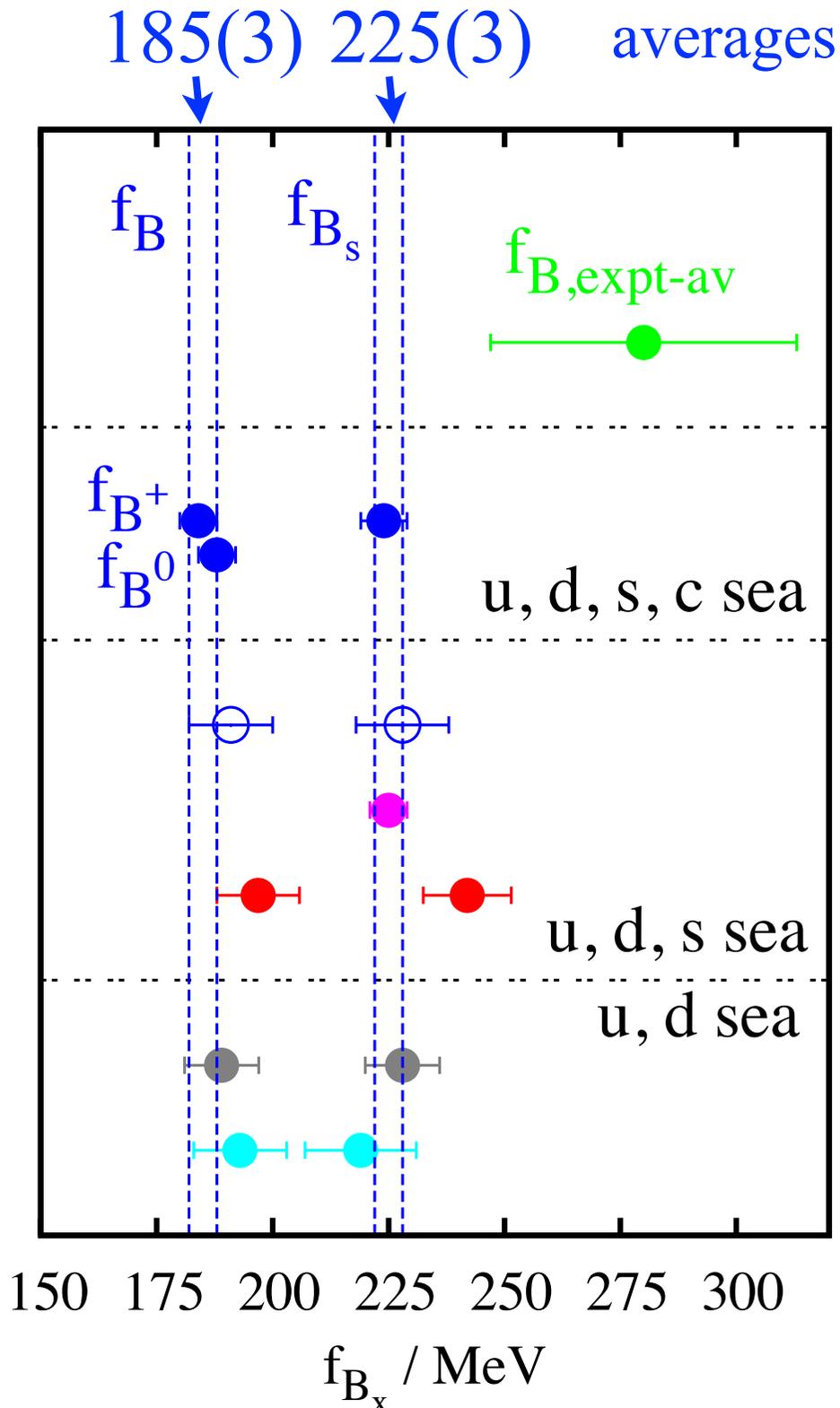


Standard Model processes:



Nov. 2012

# B, B<sub>s</sub> decay constant update 2013



PDG av BR(B- $\rightarrow$  $\tau\nu$ ) + PDG av  $V_{ub}$  (but Belle 1208.4678 = 211(28) MeV)

HPQCD NRQCD 1302.2644 \* results at physical u/d quark masses\*

HPQCD NRQCD 1202.4914  
 HPQCD HISQ 1110.4510  
 FNAL/MILC 1112.3051

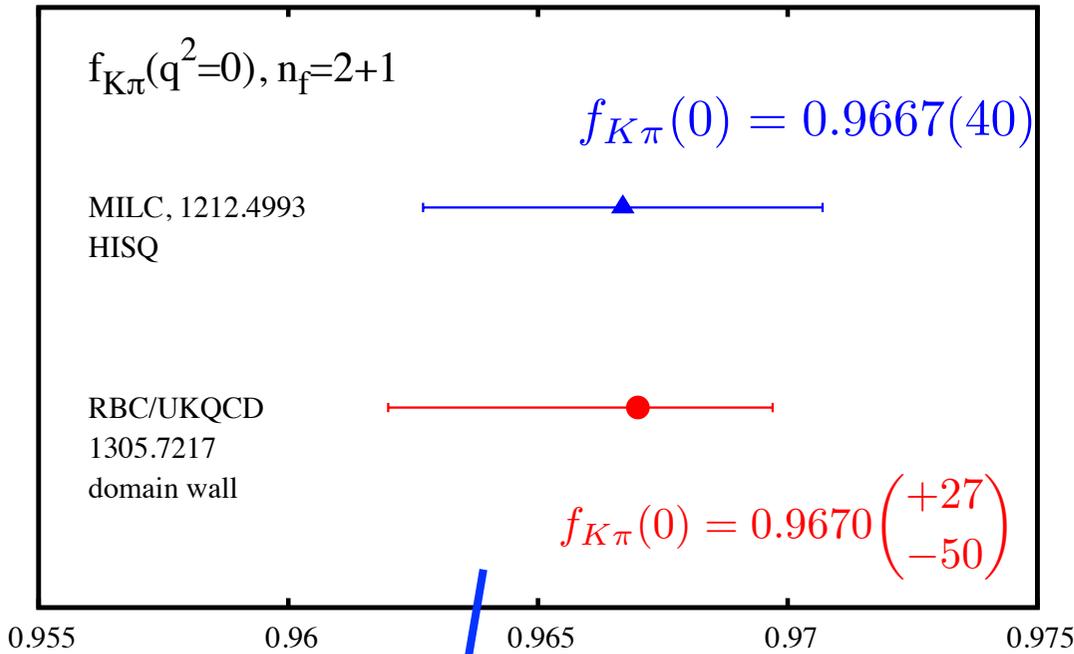
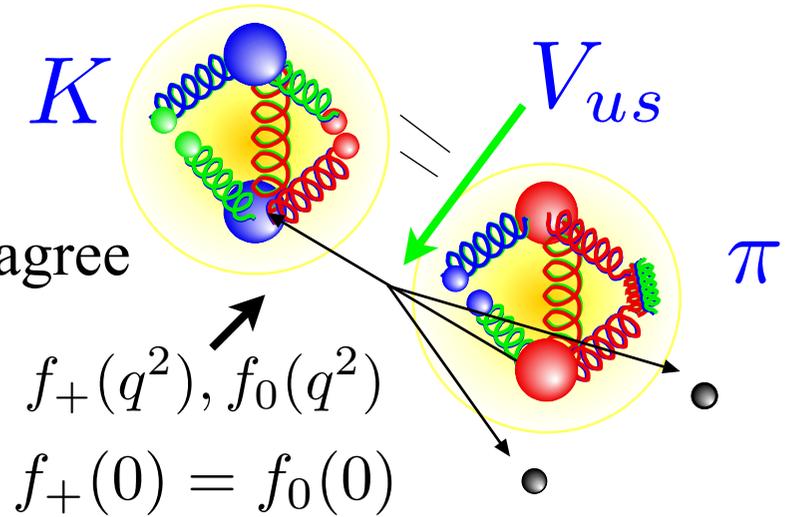
ETMC Lattice2013  
 ALPHA 1210.7932

**NOTE:**  
 $f_{B_s} < f_{D_s}$  now quite clear

# Constraining new physics with lattice QCD: form factors

$$K \rightarrow \pi \ell \nu$$

NEW - now results with full continuum and chiral extrapolation. Different formalisms agree



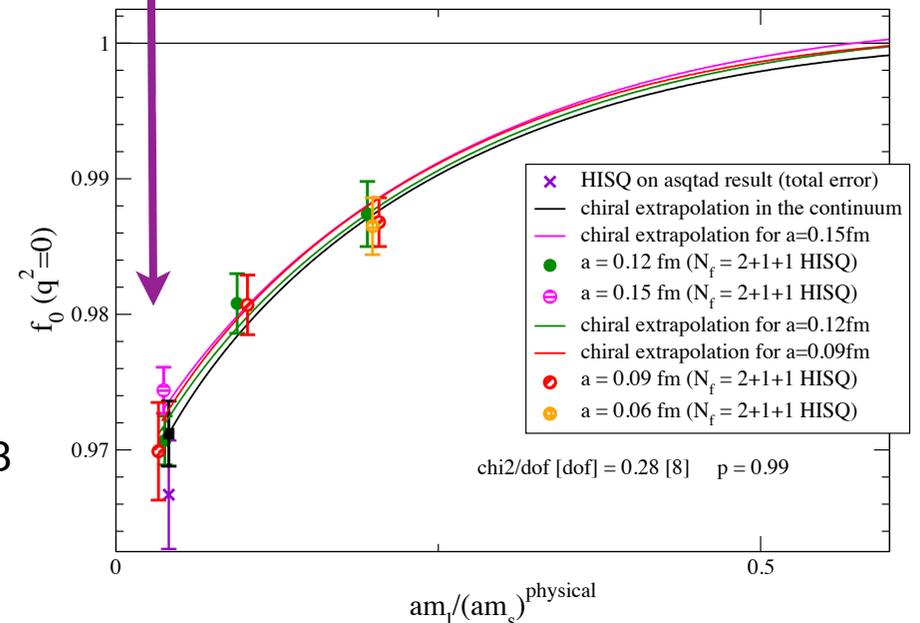
\*MILC in progress: HISQ results with physical u/d quarks - expect ~0.3% error\*

with experiment:

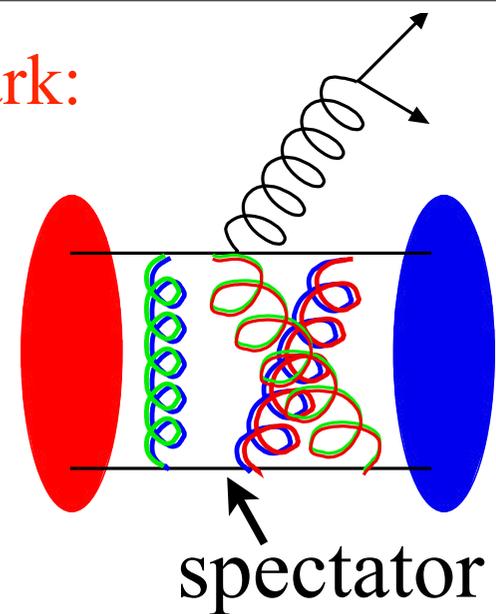
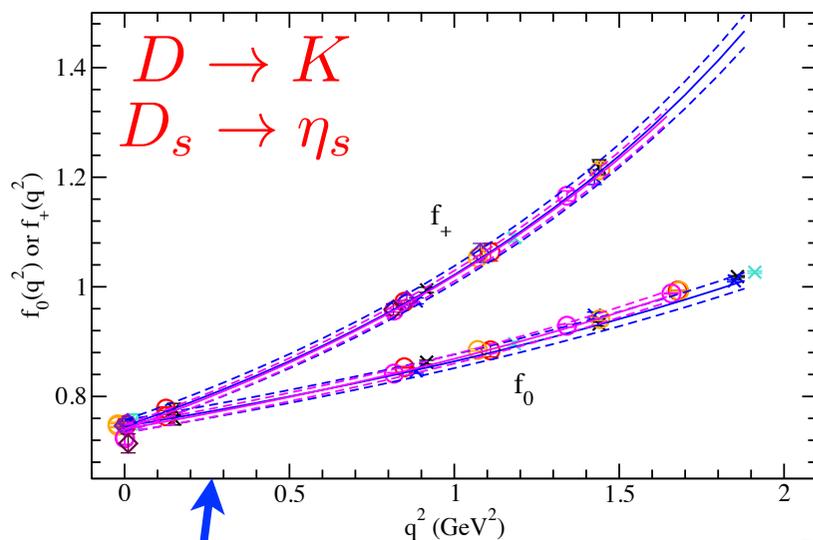
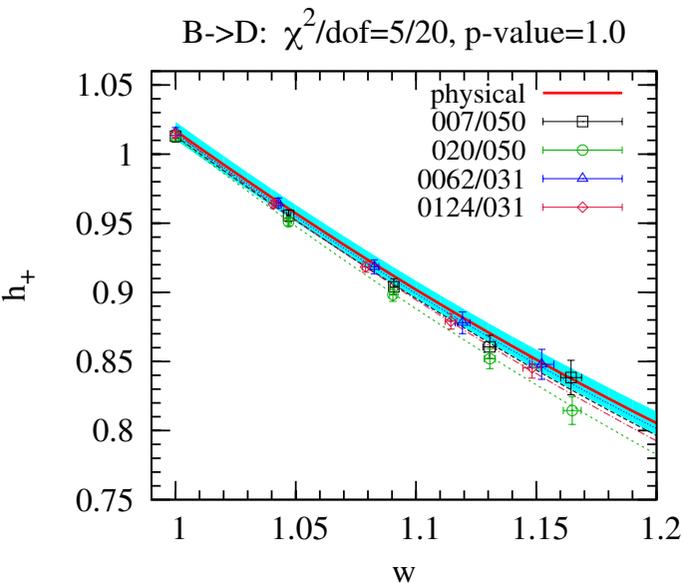
$$|V_{us}| f_{K\pi}(0) = 0.2163(5)$$

1005.2323

$$|V_{us}| = 0.2238(11)$$



# Form factor shapes and dependence on spectator quark:

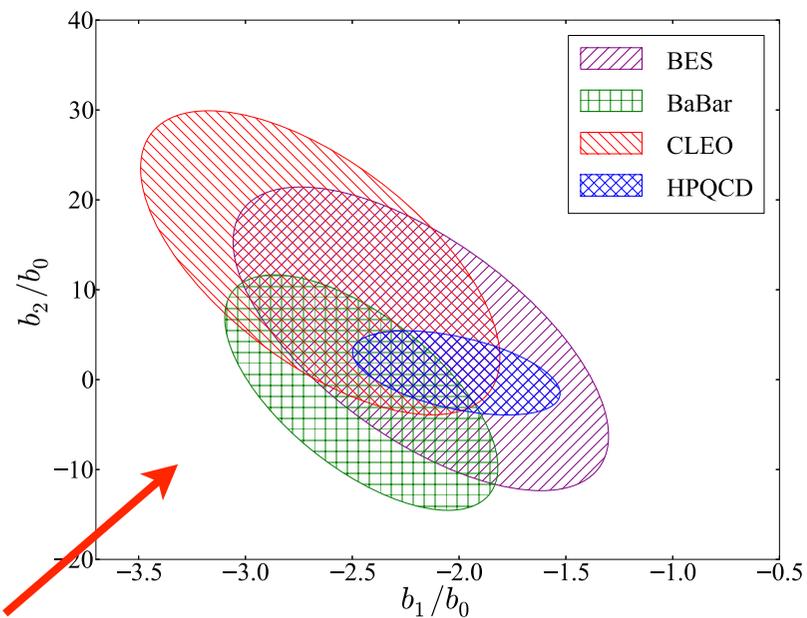


HPQCD:  
1305.1462.

$c \rightarrow s$   
 $b \rightarrow c$

form factors  
same for light or  
strange spectator

compare  $f_+$  shape  
parameters:  
lattice and expt.



$D \rightarrow K l \nu$

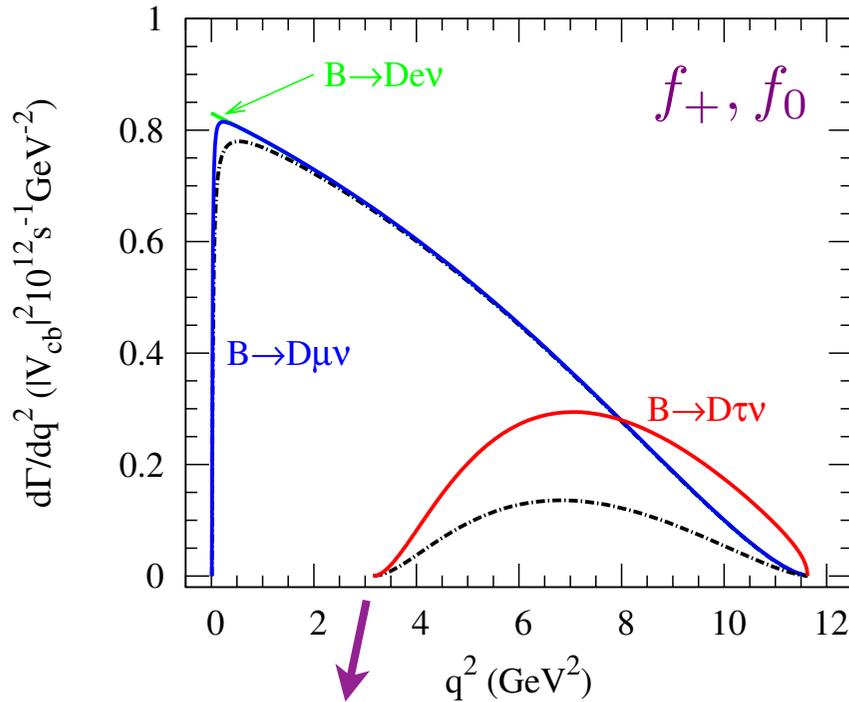
Fermilab/MILC:1202.6346.  
determined ff ratio to use in  
normln of  $B_s \rightarrow \mu^+ \mu^-$

# B semi-leptonic decays and constraints on new physics

B decays with  $\mathcal{T}$  in final state are sensitive to form factors suppressed by lepton masses for  $e, \mu$  so may see new physics.

$$B \rightarrow D\ell\nu$$

Fermilab/MILC:  
1206.4992.



$$\frac{\text{Br}(B \rightarrow D\tau\nu)}{\text{Br}(B \rightarrow D\ell\nu)} = 0.316(14)$$

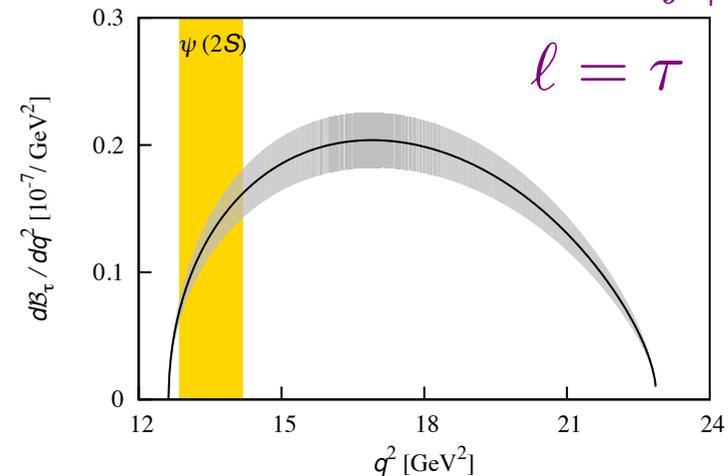
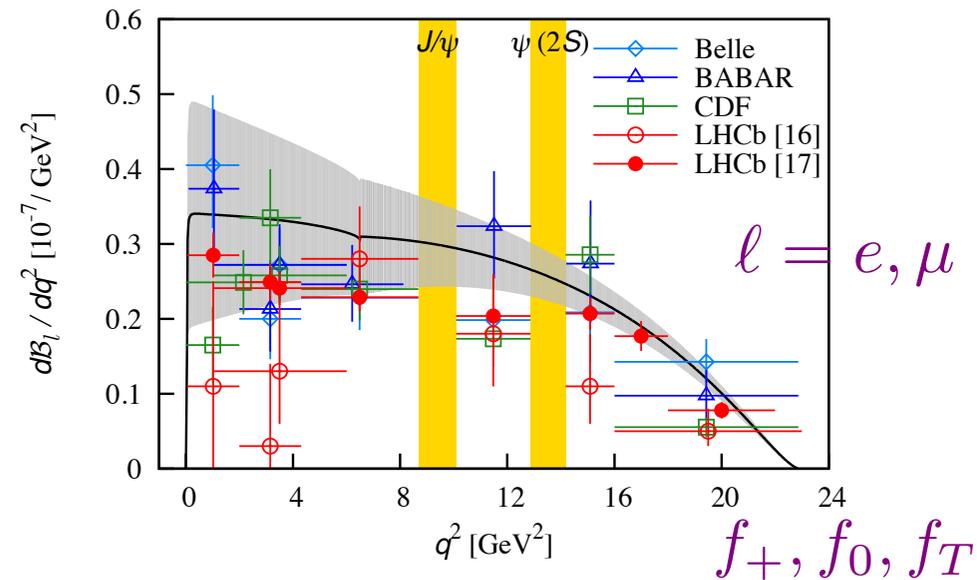
$\ell = e, \mu$

BaBar: 1205.5442;  
1303.0571

$$= 0.440(72)$$

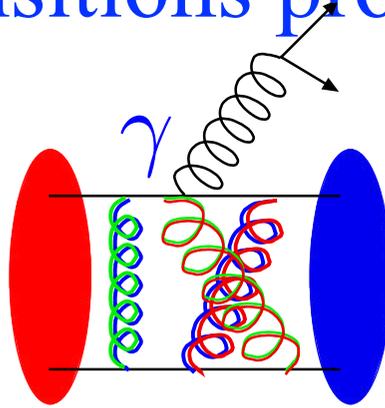
$$B \rightarrow K\ell^+\ell^-$$

HPQCD:  
1306.0434;  
1306.2384.

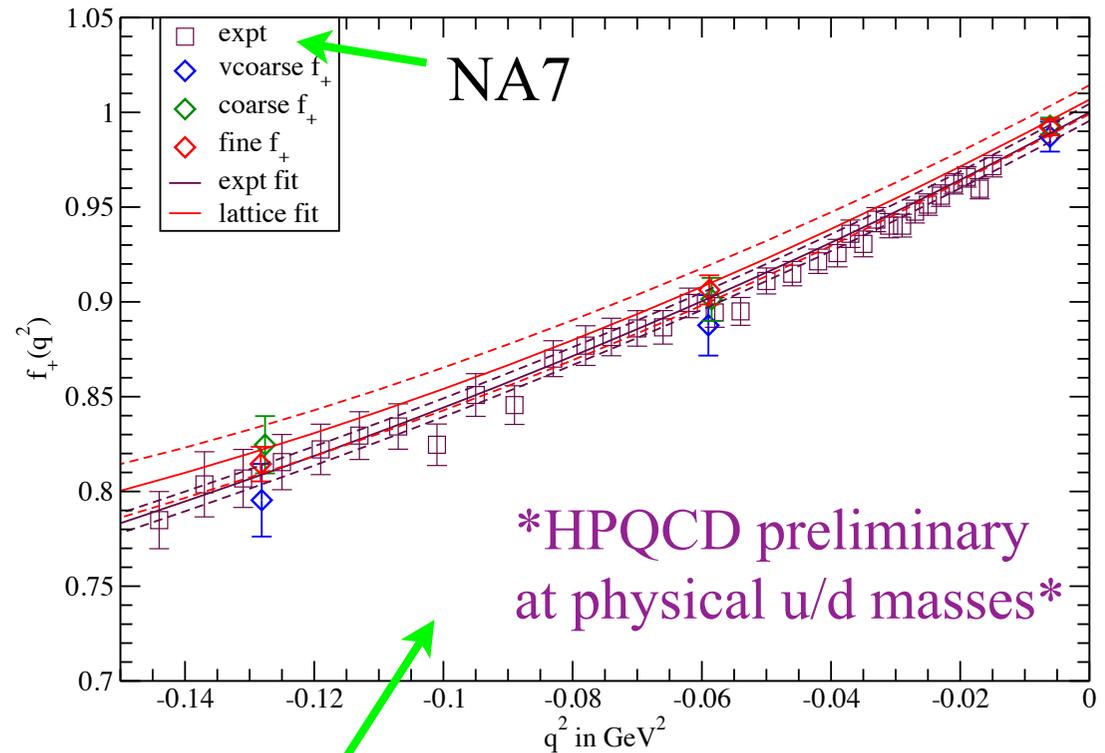
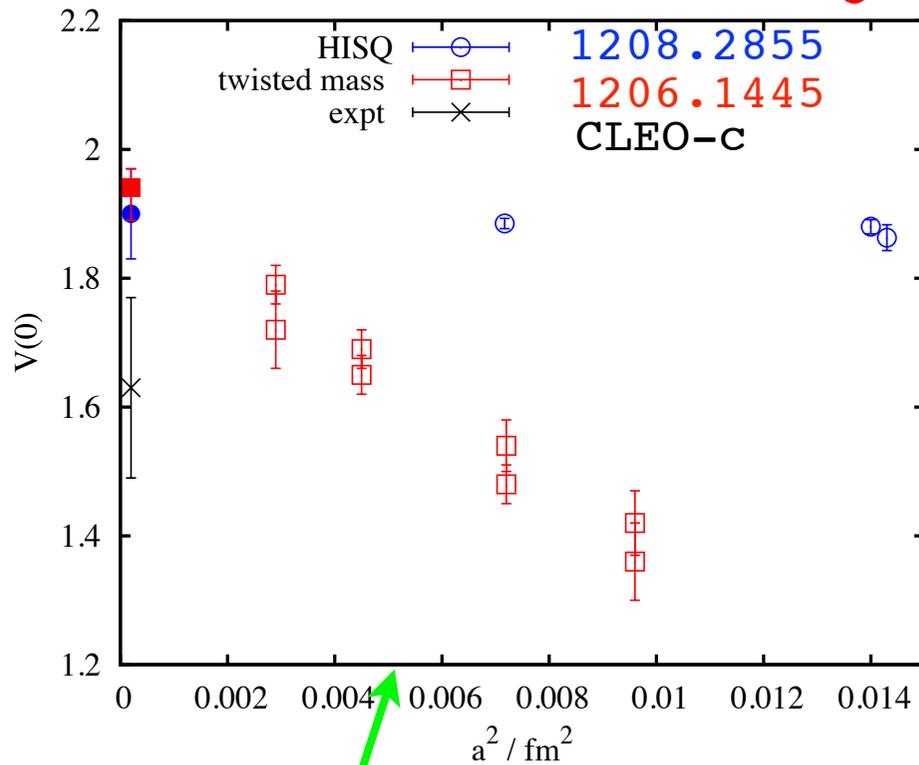


# Electromagnetic transitions provide important tests

$$J/\psi \rightarrow \gamma \eta_c$$



pion electromagnetic form factor



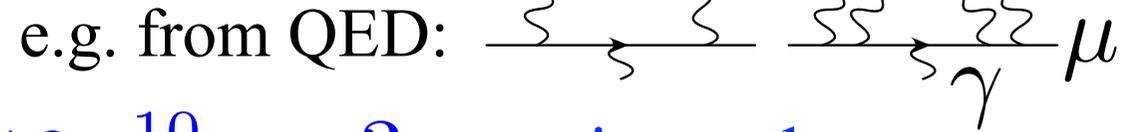
Different quark formalisms agree. Lattice more accurate than experiment.

Test lattice QCD vs direct experiment at low spacelike  $q^2$  ;  $\langle r^2 \rangle$  sensitive to light quark mass  
 ETMC: 0812.4042; JLQCD:0810.2590;  
 Mainz:1306.2916

# Muon anomalous magnetic moment

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

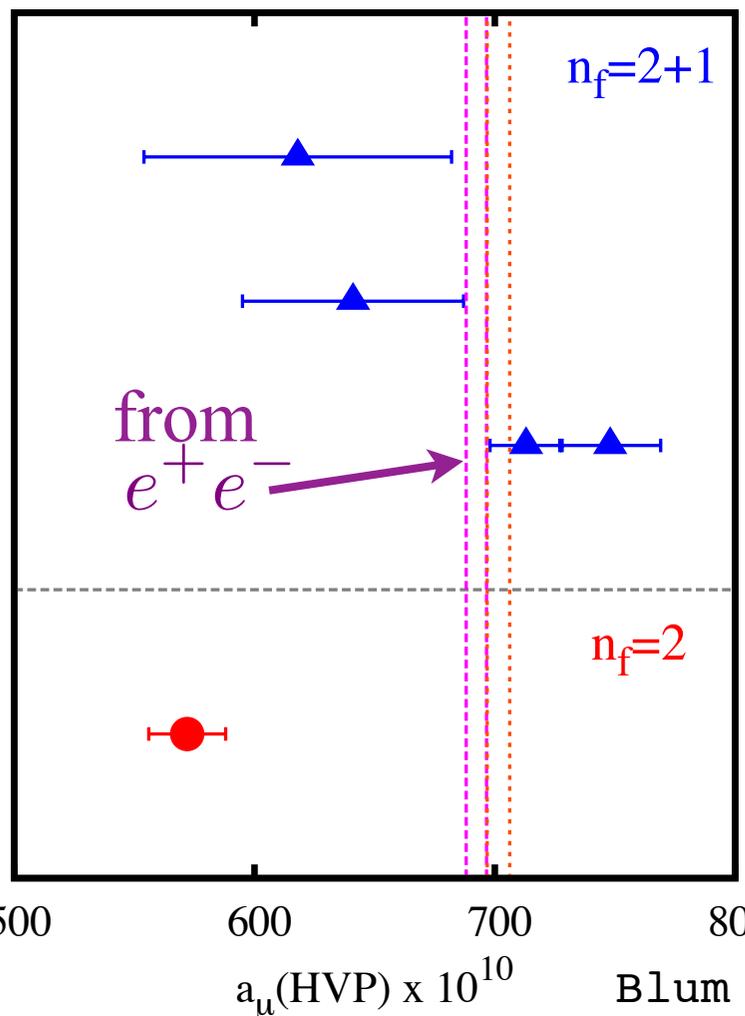
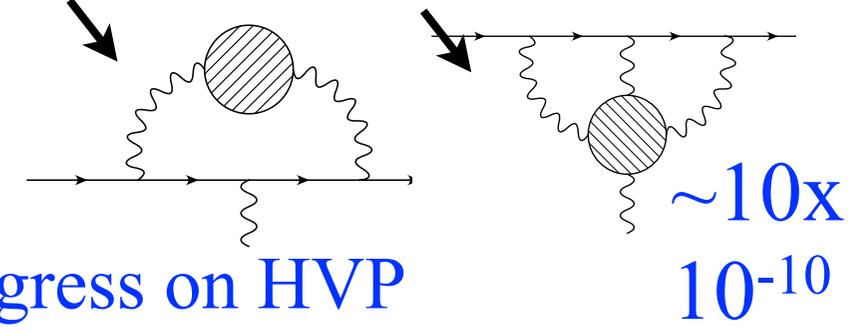
$$\text{anomaly } a_\mu = \frac{g - 2}{2} = \mathcal{O}(10^{-3})$$



$$a_\mu^{\text{expt}} - a_\mu^{\text{SM}} \approx 29(8) \times 10^{-10} = 3\sigma \text{ mismatch}$$

BNL, to be improved at FNAL and J-PARC

QCD contribn from hadronic vac. pol and 'light-by-light'.



Mainz 1112.2894  
clover-quenched s

RBC/UKQCD  
domain wall  
1107.1497

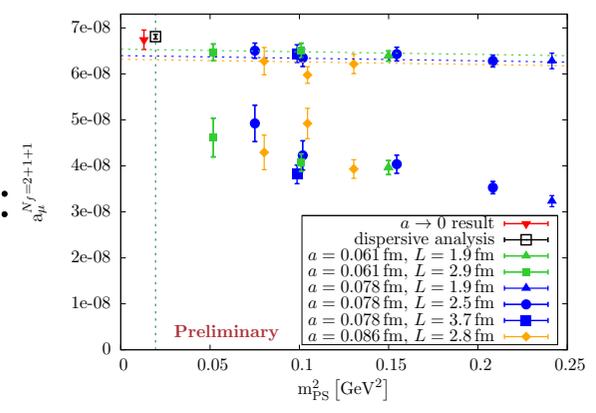
Aubin, Blum  
asqtad  
hep-lat/0608011

ETM 1103.4818  
twisted mass

Progress on HVP  
HLBL needs  
QCD+QED

ETM underway :  
 $n_f=2+1+1$

Blum lattice2012:1301.2607

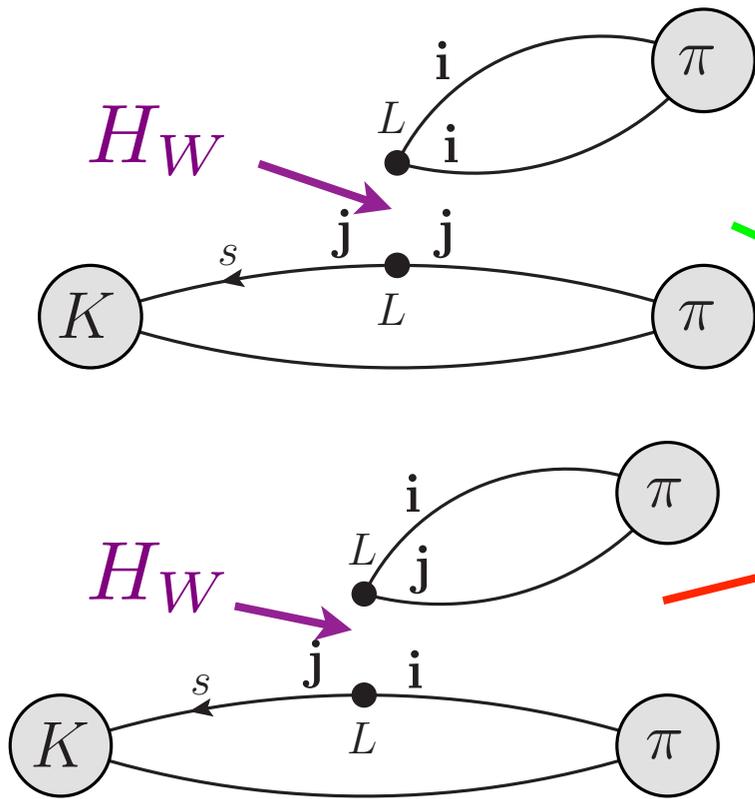


$K \rightarrow \pi\pi$  and the  $\Delta I = 1/2$  rule

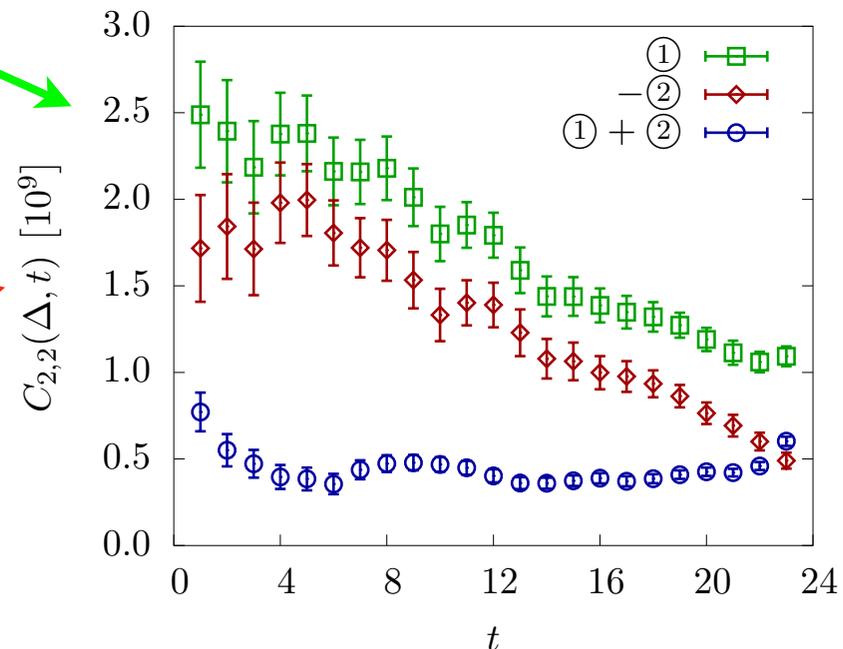
$K \rightarrow \pi\pi_{I=2}$  450 times more likely than  $K \rightarrow \pi\pi_{I=0}$

→ nonperturbative enhancement of matrix elements of 4-quark operators from weak Hamiltonian

I=2 amplitude now calculated directly in lattice QCD; I=0 underway



Find cancellation in  $A_2$  between different color contractions of key operator (same effect will enhance  $A_0$ ).



naive expectn:  $(ME2)=(ME1)/3$

Christ: Lattice2012;  
RBC/UKQCD: 1212.1474; 1111.1699



Ken Wilson

1936-2013

Originator of Lattice QCD

# Conclusion

- Lattice QCD results for gold-plated meson masses, decay constants and form factors provide stringent tests of QCD/ Standard Model.
- Gives QCD parameters and some CKM elements to 1-2% and constrains Beyond the Standard Model physics.

# Future

- sets of ‘2nd generation’ gluon configs now have  $m_{u,d}$  at physical value (so no extrapoln) *or*  $a$  down to 0.05fm (so b quarks are ‘light’) also can include charm in the sea now.
- v. high statistics/large volumes needed for harder calculations (precision baryon physics, flavor singlet /glueball spectroscopy, excited states, nuclear physics) will become available with increased computer power...

Lattice 2013: Mainz, July 29th [www.lattice2013.uni-mainz.de](http://www.lattice2013.uni-mainz.de)