Space-Time, Quantum Mechanics

+

Scattering Amplitudes

Triumph of 20th Century

QM + Relativity

$\downarrow$

Universe is Inevitable

Central Drama of 21st Century

QM + Relativity

↓

Universe (seems) Impossible

★ End of Space-time [Gravity]
Limitations of QM [Cosmology]

★ Why is there a Macroscopic Universe?

NATURALNESS

Why is it big [CC problem]

LHC

with big things in it [hierarchy problem]?

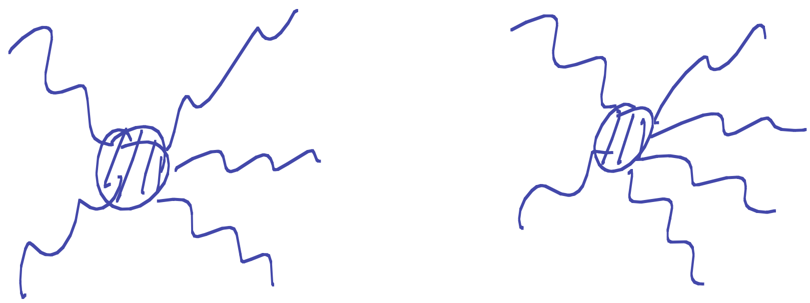
How can Space-Time and

Quantum Mechanics emerge

from more primitive principles?

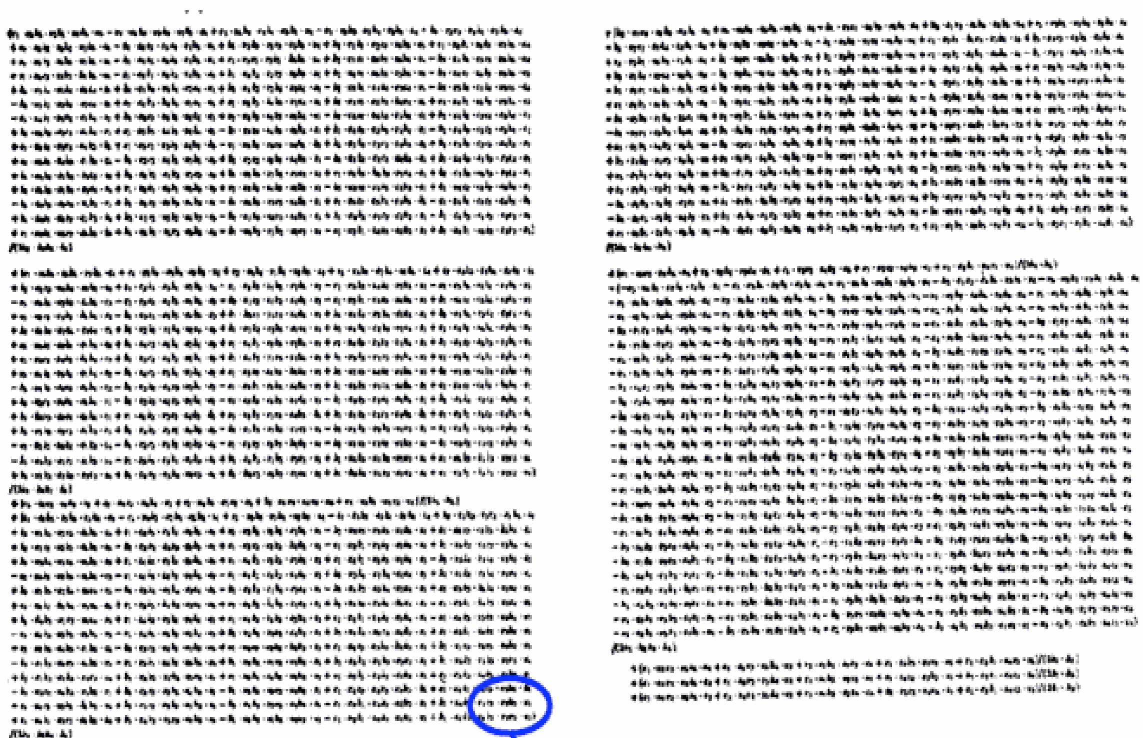
- Sometimes, the most crucial clues are "hiding in plain sight", as funny features of existing theoretical framework.

Feynman Diagrams Get Hard



[Needed For Backgrounds @ LHC]

Result of a brute force calculation:



$k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_1 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5$

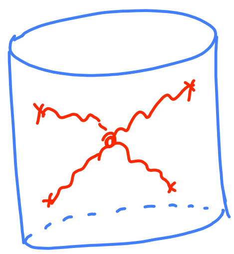
+ 24 more pages ....

$$(1^- 2^+ 3^- 4^+ 5^+ 6^+)$$ [Parke-Taylor]

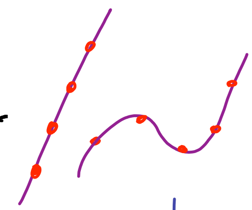
$$\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle} \quad (!)$$

Feynman's way of doing physics makes usual rules of spacetime + QM manifest - but is obviously hiding some extraordinary new structures!

Sitting Under our Noses for 60 yrs

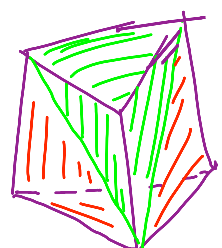


String Theory



Twistor Theory

New Formulation of standard Physics —
emergent spacetime, QM



Algebraic Geometry

Amazing Progress in

Understanding $\Longleftrightarrow$ Computing

Amplitudes in Recent Years

Power of Controlling On-Shell Sing.

Trees + "Integrand" @ Loops:

- Generalized Unitarity [Bern, Dixon, Kosower]

- BCFW recursion [Britto, Cachazo, Feng, Witten]

New "motivic" structures in Full Ampl.

- "Symbology" [Goncharov, Spradlin, Vergu, Volovich]

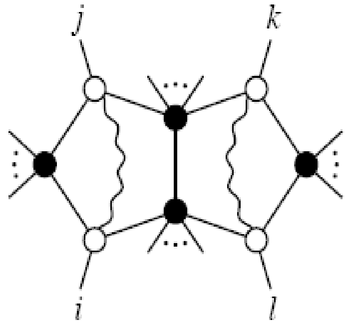
Enormous Progress in Planar $\mathcal{N}=4$ SYM

* Structure of 4,5 pt amps to all loops [Bern, Dixon, Smirnov]
* Dual Conformal Symmetry, Wilson-Loops, Infinite Yangian Symmetry, Momentum-Twistors, ..

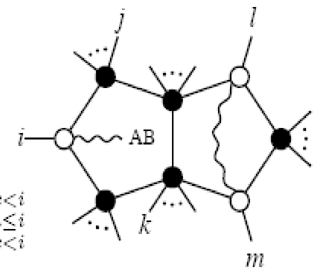
  [Alday, Maldacena; Drummond, Henn, Korchemsky, Sokatchev, Hodges, Caron-Huot, Mason + Skinner, D + H + Plefka ..]
* Grassmannian Structure, All-Loop BCFW for integrand

  [NAH, Cachazo, Cheung + Kaplan, NAH, Bourjaily, C, C-H, Trnka]
* Integrability [Gaiotto, Maldacena, Sever, Vieira, ..]
* "Bootstrap" for full amp [C-H + He, ↑, ..]

$$\mathcal{A}^{2-\text{loop}}_{\text{MHV}} = \frac{1}{2}\sum_{i<j<k<l<i}$$

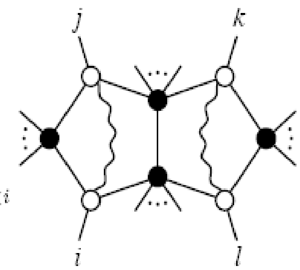


$$\mathcal{A}^{2-\text{loop}}_{\text{NMHV}} = \sum_{\substack{i<j<l<m\leq k<i\\ i<j<k<l<m\leq i\\ i\leq l<m\leq j<k<i}}$$



$$\times\,[i, j, j+1, k, k+1]$$

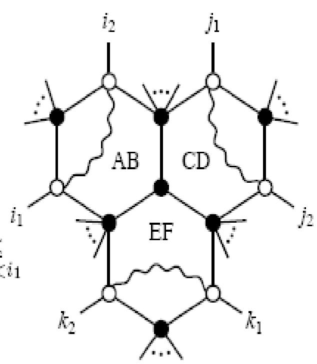
$$+\frac{1}{2}\sum_{i<j<k<l<i}$$



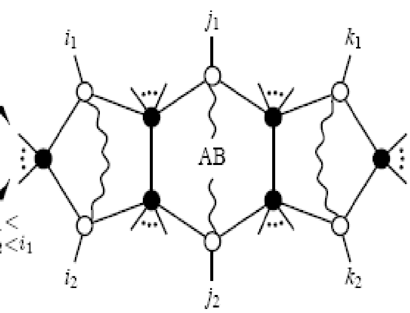
$$\times\left\{\begin{array}{c}\mathcal{A}^{\text{tree}}_{\text{NMHV}}(j,\ldots,k;\, l,\ldots,i)\\ +\mathcal{A}^{\text{tree}}_{\text{NMHV}}(i,\ldots,j)\\ +\mathcal{A}^{\text{tree}}_{\text{NMHV}}(k,\ldots,l)\end{array}\right\}$$

Check:
Local
+
Unitary!

$$\mathcal{A}^{3-\text{loop}}_{\text{MHV}} = \frac{1}{3}\sum_{\substack{i_1\leq i_2<j_1\leq\\ \leq j_2<k_1\leq k_2<i_1}}$$



$$+\frac{1}{2}\sum_{\substack{i_1\leq j_1<k_1<\\ <k_2\leq j_2<i_2<i_1}}$$



[NAH, Bourjaily, Cachazo, Trnka]

$$
\begin{aligned}
&\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{123},0,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},0,\frac{1}{1-u_1},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,0,\frac{1}{1-u_1};1\right)-\\
&\frac{5}{4}\mathcal{G}\left(v_{123},1,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},1;1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},0,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,0;1\right)-\\
&\frac{5}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,1,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1,1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,1,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,\frac{1}{1-u_2},1;1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{231},1,0,\frac{1}{1-u_2};1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},1,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},0;1\right)-\\
&\frac{5}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},0,1;1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,\frac{1}{1-u_3},1;1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{312},1,0,\frac{1}{1-u_3};1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},1,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},0;1\right)-\\
&\frac{5}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},0,1;1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+\\
&\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,1,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3},1;1\right)-\\
&\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1,1;1\right)-\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-\\
&\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-\\
&\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\\
&\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-\\
&\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+\\
&\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+\\
&\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1};1\right)H\left(0;u_1\right)+
\end{aligned}
$$

$$R_6^{(2)}(u_1, u_2, u_3) = \sum_{i=1}^{3} \left(L_4(x_i^+, x_i^-) - \frac{1}{2} \mathrm{Li}_4(1 - 1/u_i) \right)$$
$$- \frac{1}{8} \left(\sum_{i=1}^{3} \mathrm{Li}_2(1 - 1/u_i) \right)^2 + \frac{1}{24} J^4 + \frac{\pi^2}{12} J^2 + \frac{\pi^4}{72}. \quad (3)$$

[Goncharov, Spradlin, Vergu, Volovich]

Big Progress in Gravity

* Magical UV for $\mathcal{N}=8$? [NO PROVEN MAGIC YET]

(Bern, Carrasco, Dixon, Johansson, ..)

* Amazing + mysterious "Color-Kinematics Duality"

$$A^{YM} = \sum_i \frac{c_i N_i}{D_i} \longrightarrow A^{Grav} = \sum_i \frac{\tilde{N}_i N_i}{D_i} \; !$$

(Bern, Carrasco, Johansson)

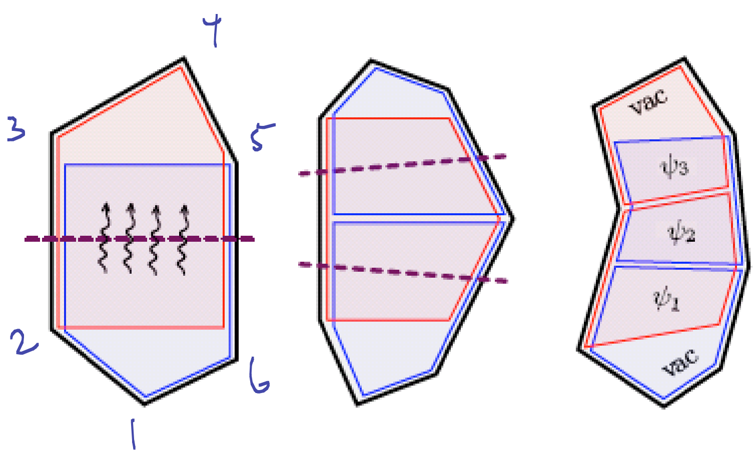
Even by the fast-paced standards of this field, the past year has seen flurry of progress — impossible to even mention all.

Tour-de-Force Results, Unthinkable 5yrs Ago

* Multi-Regge limit of $\mathcal{N}=4$ SYM to 10 Loops, then all loops! [Dixon, Duhr; Pennington]
* 6D SYM diverges @ 6-loops [Bern, Carrasco, Dixon, Douglas, von Hippel, Johansson]
* Surprisingly [?] good behavior of $\mathcal{N}<8$ SUGRA [Bern, Davies, Dennen, Huang]

Breakthrough in $\mathcal{N}=4$ SYM:
using integrability to compute amps
@ finite coupling !!

Basso, Sever, Vieira



"Pentagon Transitions" as building blocks

Remarkable Conceptual Breakthroughs formulating $\mathcal{N}=8$ SUGRA as a novel string theory in Twistor Space!

$$\mathcal{M}_n = \sum_{d=0}^{\infty} \int \frac{\prod_{\mathrm{a}=0}^{d} \mathrm{d}^{4|8}\mathcal{Z}_{\mathrm{a}}}{\mathrm{vol}\,\mathrm{GL}(2;\mathbb{C})}\, \det{}'(\Phi)\det{}'(\tilde{\Phi}) \prod_{i=1}^{n} \mathrm{d}^2\sigma_i\, \delta^2(\lambda_i - \lambda(\sigma_i)) \exp[\![\mu(\sigma_i)\tilde{\lambda}_i]\!]$$

$$\Phi_{ij} = \frac{\langle ij\rangle}{(ij)} \qquad \text{for } i \neq j, \qquad \tilde{\Phi}_{ij} = \frac{[ij]}{(ij)} \qquad \text{for } i \neq j,$$

$$\Phi_{ii} = -\sum_{j\neq i}\left\{\Phi_{ij}\frac{\prod_{k\neq i}(ik)}{\prod_{l\neq j}(jl)}\prod_{\mathrm{a}=0}^{n-d-2}\frac{(jp_{\mathrm{a}})}{(ip_{\mathrm{a}})}\right\}, \qquad \tilde{\Phi}_{ii} = -\sum_{j\neq i}\tilde{\Phi}_{ij}\prod_{\mathrm{a}=0}^{d}\frac{(j\,p_{\mathrm{a}})}{(i\,p_{\mathrm{a}})},$$

[Hodges, Cachazo-Geyer, Cachazo-Skinner, Skinner]

Pioneering Explorations into deep structures in string pert. th:

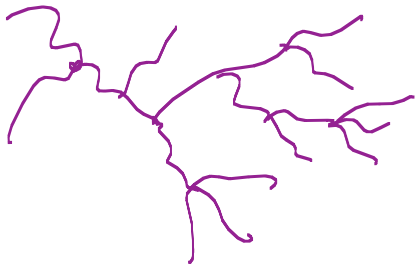
* Tree string amps for all n
  [Mafra, Schlotterer, Stieberger]
* String trees as Mellin Transform of SUGRA (?!)
  [Stieberger, Taylor]
* "Motivic" structure of string trees
  [Schlotterer, Stieberger; Drummond, Ragoucy]

Locality + Unitarity

From Positivity

w/ Jaroslav Trnka

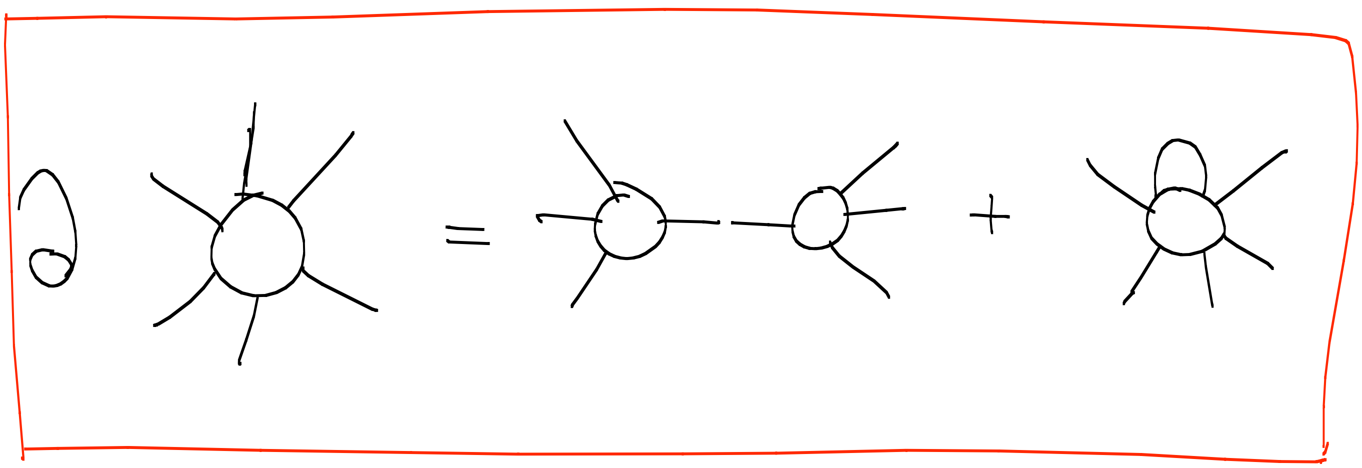
[to appear]

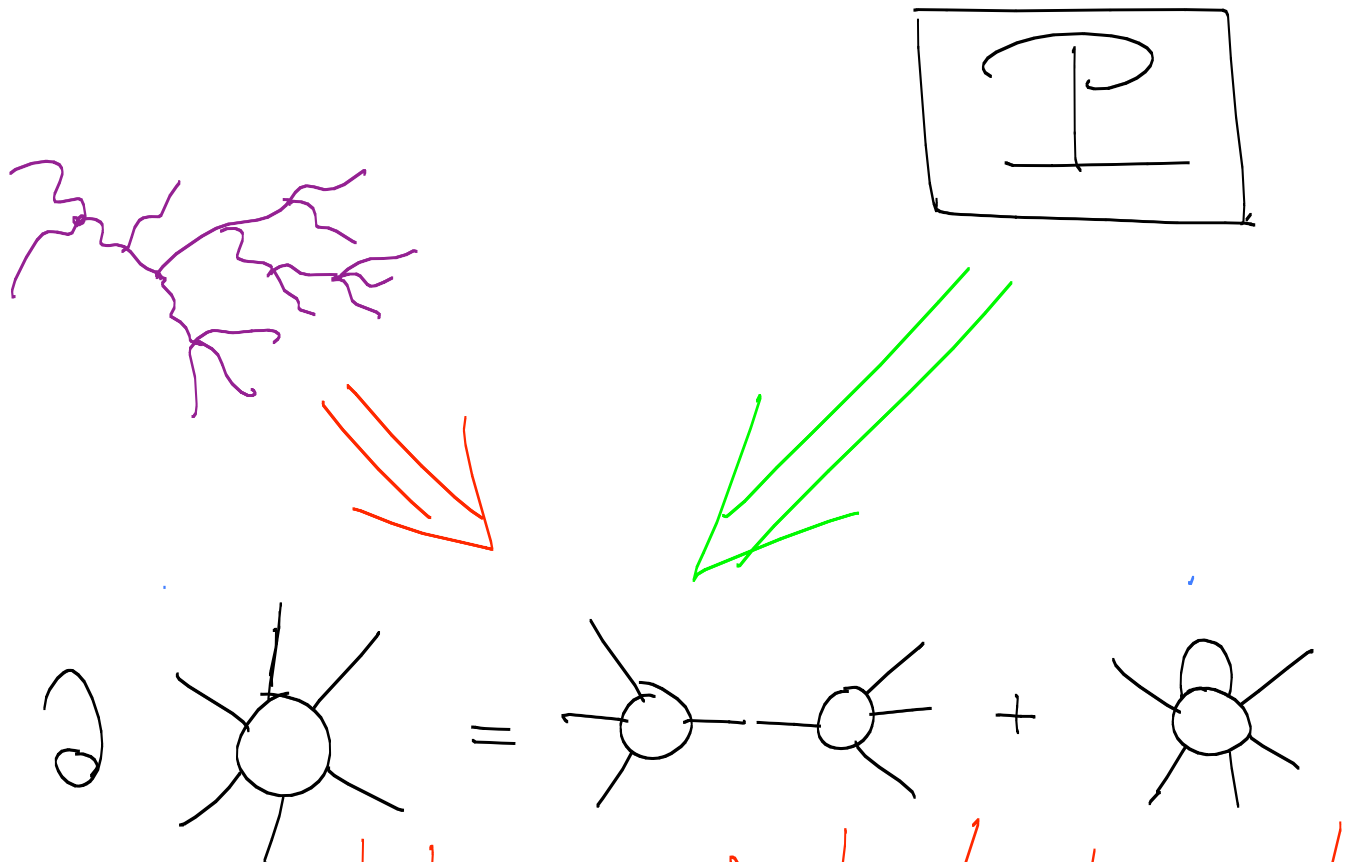


: Locality + Unitarity Manifest

Poles
e.g. $(\sum_i p_i)^2 \to 0$

Factorization/Cut Structure

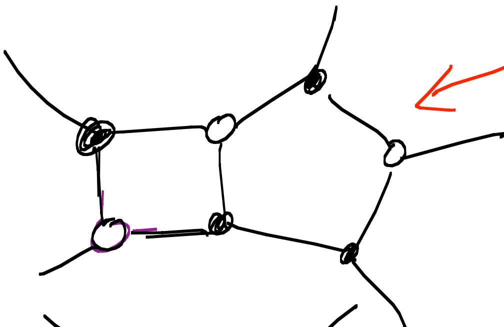


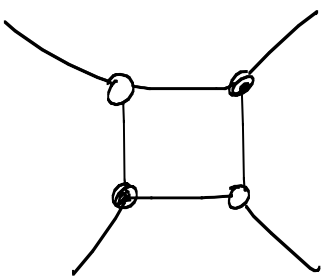


- Locality + Unitarity Emergent
- Infinite "Yangian" Symmetry Manifest

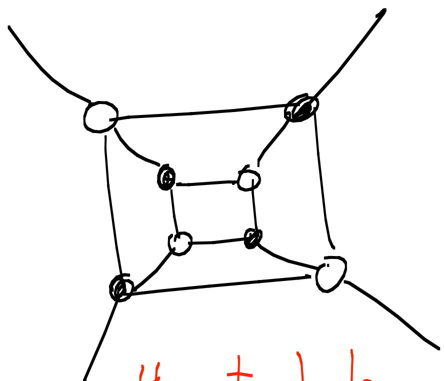
On-Shell Diagrams

All internal lines on-shell





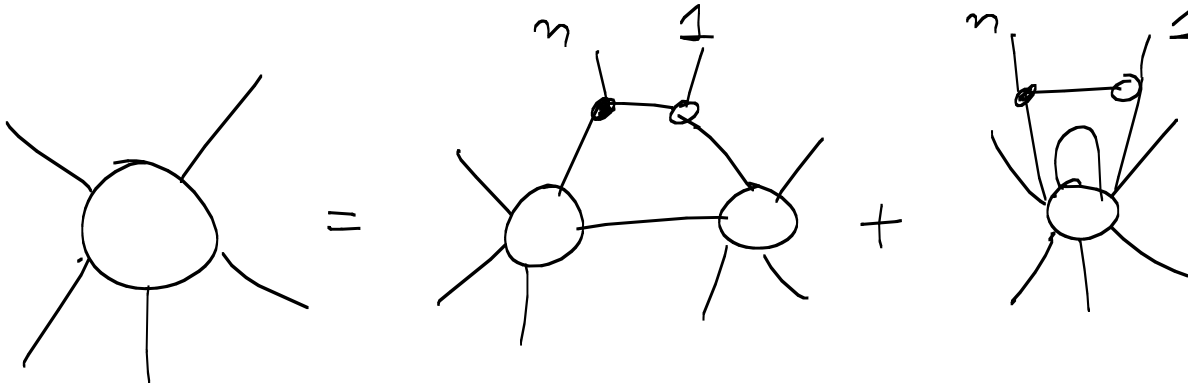
4 pt tree



4-pt 1-loop

NO "VIRTUAL PARTICLES"

Clues in (All-loop) BCFW



Not Local/Unitary Term-By-Term

Yangian Invariant Term-By-Term

Positive Grassmannian

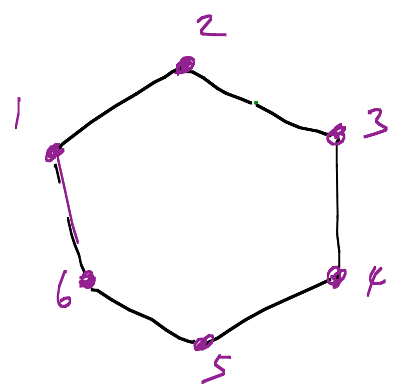
$G(k,n)$: k-planes in n-dimensions

$$C_{\alpha a} = \begin{array}{c} \\ \updownarrow k \end{array} \begin{array}{c} \xleftrightarrow{\quad n \quad} \\ \begin{pmatrix} c_1 & c_2 & \cdots & c_n \end{pmatrix} \end{array} / GL(k)$$

$G^+(k,n)$: all minors $(a_1 \cdots a_k) > 0$

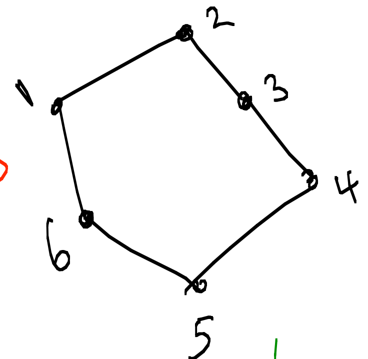
for $a_1 < a_2 \cdots < a_k$

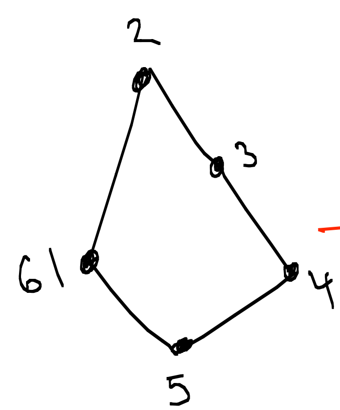
$G^+(3,n)$



$\longrightarrow$ Convex Polygon

Boundaries:

$\longrightarrow$ 

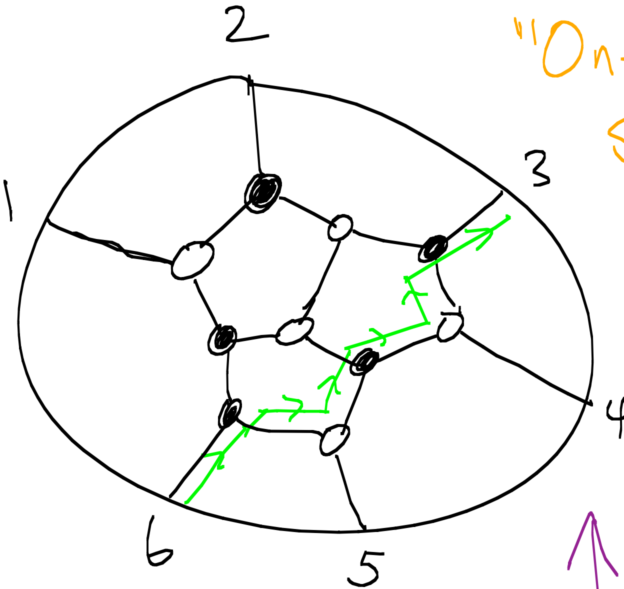
$\longrightarrow$ 

$\longrightarrow$ . . .

$\longrightarrow$ Consecutive linear dependencies $\longrightarrow$ Permutations

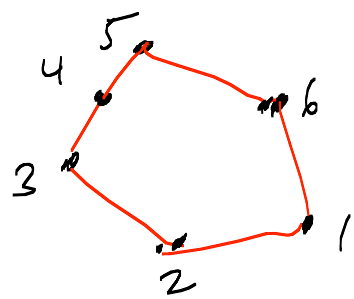
$1 \to 4$
$2 \to 6$
$3 \to 5$
$4 \to 7$
$5 \to 8$
$6 \to 9$

Affine Permutation



"On-shell" Spacetime Picture

Cell of "Positive" Grassmannian

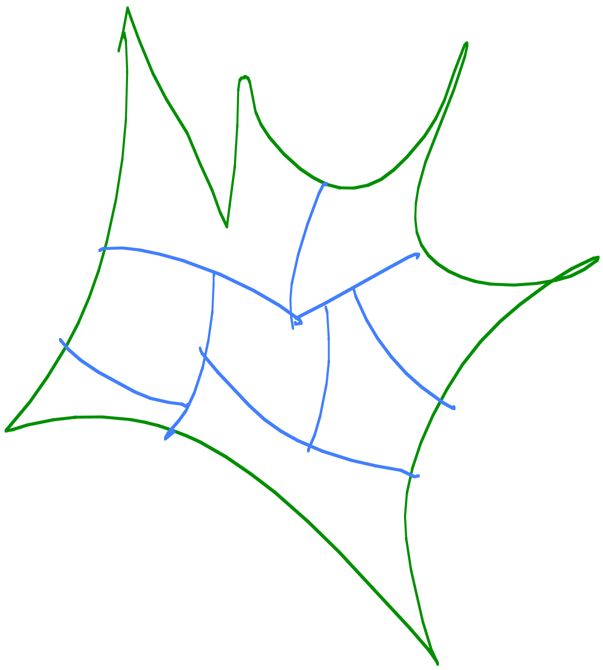


Yangian Invariance

$\Downarrow$

Positive Diffeomorphisms

Vague Fantasy



Amplitude is "the volume" of "some region" in "some space".

Many "triangulations".

Locality + Unitarity Emerge Somehow

Five-Year Path to $\mathbb{P}$

'08 : w/ Kaplan — Extend BCFW

Cachazo, Kaplan

'09 : w/ Cachazo, Cheung + Kaplan — BCFW in Twistor Space

Grassmannian + Leading. Sing.

also Hodges — Polytopes

'10 : w/ Bourjaily, Cachazo, Caron-Huot, Trnka — Recursion for All-loop integrand

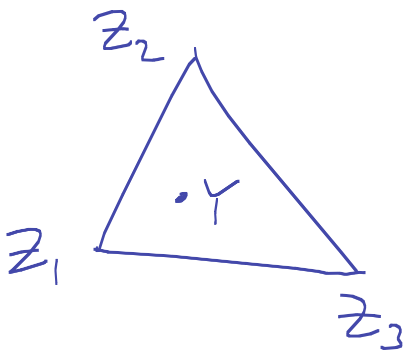
'12 : w/ Bourjaily, Cachazo, Goncharov, Postnikov, Trnka — On-shell diagrams + the Positive Grassmannian

Key New Idea:

External Momentum(-Twistor)

Data is also "Positive"

Triangles $\longrightarrow$ Positive Grassmannian



$$Y^I = c_1 Z_1^I + c_2 Z_2^I + c_3 Z_3^I$$

$$(c_1 c_2 c_3)/GL(1), \quad c_a > 0$$

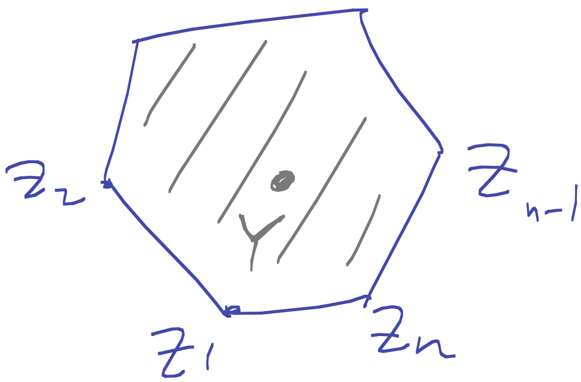
$$\longrightarrow \quad (c_1 \dots c_n)/GL(1) \quad c_a > 0 \quad \text{Simplex}$$

$$\longrightarrow \quad \left(\vec{c}_1 \dots \vec{c}_n\right)/GL(k), \quad (c_{a_1} \dots c_{a_k}) > 0$$

$$a_1 < \dots < a_k$$

Positive Grassmannian

Polygons



$$\langle Z_{a_1} Z_{a_2} Z_{a_3} \rangle > 0$$

$$a_1 < a_2 < a_3$$

key point!

$$Y^I = c_1 Z_1^I + \cdots + c_n Z_n^I$$

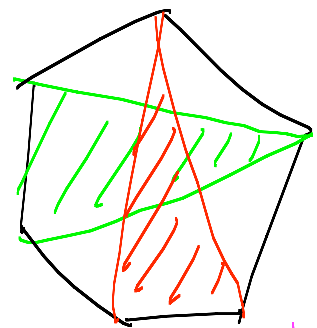
$$(c_1 \cdots c_n) \in G^+(1,n) \longrightarrow \text{Region in } G(1,3)$$

$$(Z_1, \ldots, Z_n) \in G^+(3,n)$$

$$Y^I = c_a Z_a^I$$

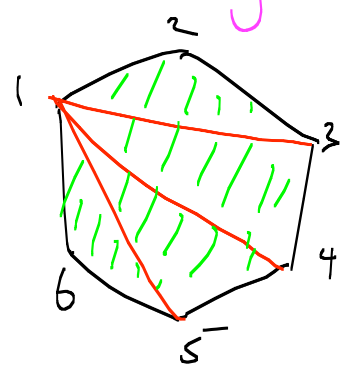
2D $\quad$ $(n-1)$-d

* 2D cells of $G^+(1,n) \longrightarrow$



"Triangles"

* Triangulation, e.g $\sum_i (1\; i\; i+1)$



General Case

$$Y_\alpha^I = C_{\alpha a} Z_a^I$$

$\alpha = 1, .., k$

$I = 1, .., 4+k$

Region in $G(k, k+4)$

$\mathcal{P}_{n,k}$

Familiar Positive Grassmannian

$G^+(k,n)$

External data positive

$G^+(k+4, n)$

- Tree amplitude is "volume" of this region!
- BCFW is (one) "triangulation"

New treatment of SUSY

$$\begin{pmatrix} Z_a \\ \hdashline \eta_a \end{pmatrix} \longrightarrow \begin{pmatrix} Z_a \\ \hdashline \phi_1 \cdot \eta_a \\ \vdots \\ \phi_k \cdot \eta_a \end{pmatrix} = \mathcal{Z}_a$$

$\mathbb{P}^{3|4}$ $\qquad\qquad$ $\mathbb{P}^{3+k}$

External "momentum twistors"

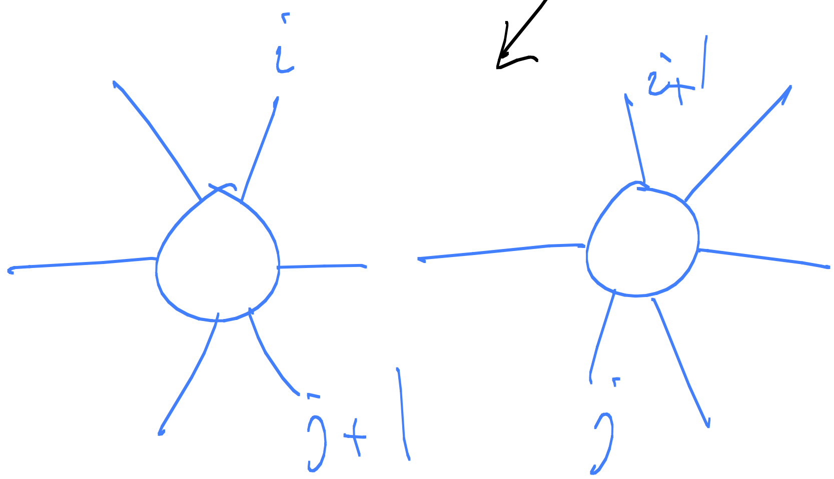
Grassmann Variables For helicity

Super-Geometry $\longrightarrow$ Bosonic Geometry

$$\mathcal{M}_{n,k}[Z_a, \eta_a] = \int d^4\phi_1 \ldots d^4\phi_k \, M[\mathcal{Z}_a]$$

Boundaries of $P_{n,k}$

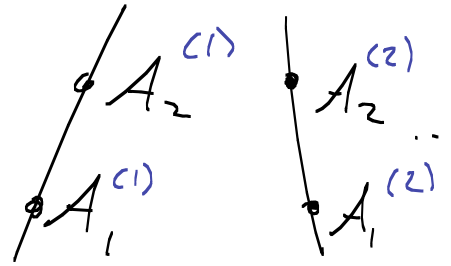
$$\begin{array}{cc} & \begin{array}{cccccccccc} i & i+1 & & & & & j & j+1 & & \end{array} \\ C = & \left[\begin{array}{cccccccccc} * & * & 0 & 0 & 0 & 0 & * & * & 0 & 0 \\ \multicolumn{6}{c}{\boxed{C_L}} & \multicolumn{4}{c}{0} \\ \multicolumn{6}{c}{0} & \multicolumn{4}{c}{\boxed{C_R}} \end{array}\right] \end{array}$$



Unitarity From Positivity

Loops

$k=0$: Lines in $\mathbb{P}^3$



$$A^{I\,(i)}_{\gamma} = Z^{(i)}_{\gamma a} Z^{I+}_{a}$$

$$\left[\begin{array}{c} Z^{(1)} \\ \hline Z^{(2)} \\ \hline \vdots \\ Z^{(L)} \end{array}\right]$$

All minors Positive!

Result of "Hiding Particles"

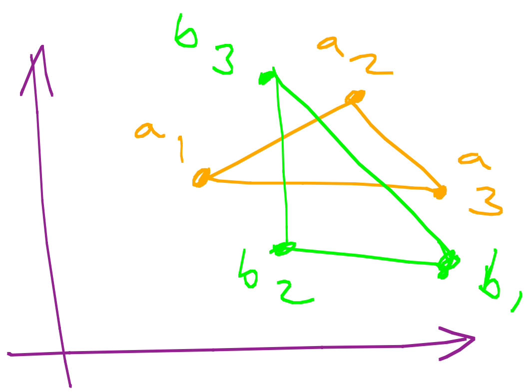
$$
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & \quad 1 \quad 2 \quad \cdots \quad n \end{array} \\
\left[\begin{array}{cccc|c}
1 & 0 & 0 & 0 & \\
0 & 1 & 0 & 0 & Z^{(1)} \\
\cline{5-5}
0 & 0 & 1 & 0 & \\
0 & 0 & 0 & 1 & Z^{(2)}
\end{array}\right]
\end{array}
$$

Simplest Case $n=4$

$$Z_{(i)} = \begin{bmatrix} 1 & x_i & 0 & -z_i \\ 0 & y_i & 1 & w_i \end{bmatrix}$$

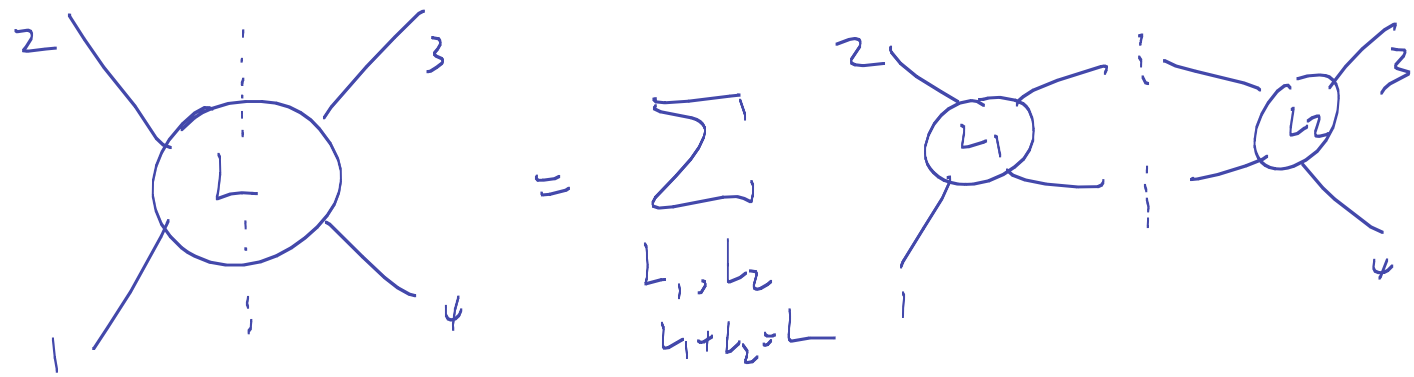
$$\boxed{\vec{a}_i = \begin{pmatrix} x_i^+ \\ y_i^+ \end{pmatrix}, \quad \vec{b}_i = \begin{pmatrix} w_i^+ \\ z_i^+ \end{pmatrix}, \quad (\vec{a}_i - \vec{a}_j)\cdot(\vec{b}_i - \vec{b}_j) < 0}$$

"Triangulate"
= 4-pt to all
loop order!



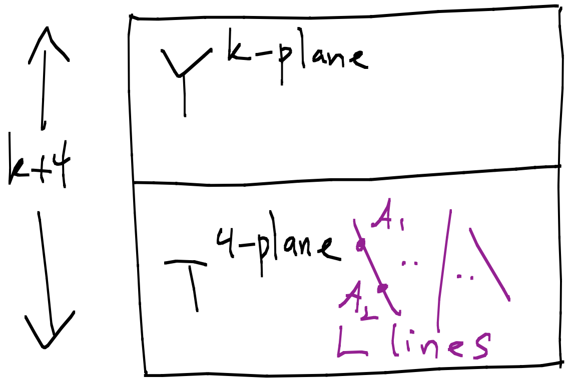
A junior
highschool
geometry
problem

Simple geometric identity from positivity



Again: Unitarity from Positivity

P is for Positive

- 

$$Y_\alpha^I = C_{\alpha a} Z_a^{+I} \qquad \begin{array}{c} \xleftarrow{\;n\;} \\ \begin{pmatrix} C \\ \hline \mathcal{L}^{(1)} \\ \hline \vdots \\ \mathcal{L}^{(L)} \end{pmatrix} \end{array}$$

$$\mathcal{A}_\gamma^{(i)} = \mathcal{L}^{(i)}_{\gamma a} Z_a^{+I}$$

All minors w/ $C > 0$

$\mathcal{P}_{n,k,L}$

- $\Omega_{n,k,L}$: top-form w/ log sing. on $\partial \mathcal{P}_{n,k,L}$

- $$\mathcal{M}_{n,k,L} = \int d^4\phi_1 \cdots d^4\phi_k \, \Omega_{n,k,L} \, \delta^{4k}(Y; Y_0).$$

$$Y_0 = \left(\begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix} \middle| \begin{matrix} 0\,0\,0\,0 \\ \vdots \\ 0\,0\,0\,0 \end{matrix} \right), \quad Z_a = \begin{pmatrix} Z_a \\ \hdashline \phi_1 \cdot \eta_a \\ \vdots \\ \phi_k \cdot \eta_a \end{pmatrix}$$

- All symmetries manifest
- Determining integrand reduced to "triangulating" $\mathcal{P}_{n,k,L}$

[BCFW one triangulation - not best one!]

* This simple mathematical structure gives a complete, autonomous definition of all scattering amplitudes in Planar $\mathcal{N}=4$ SYM, totally free of usual QFT language: no Feynman Diagrams, not even on-shell diagrams, recursion relations etc.

(Momentum) Twistor Space : Kinematics

Grassmannian Positivity : All Dynamics!

[Fascinating hint for important role of positivity in final results — fingerprint appearance of "cluster variables"
Golden, Goncharov, Spradlin, Vergu, Volovich]

We Now Have

A first baby example

of emergent Space-Time + Quantum-Mechanics. Unlike conventional QFT, they don't "fight".

Instead emerge together, from Positivity

This structure is as of yet un-known to the mathematicians — it's very existence is a very big to all algebraic geometers we've talked with.

* Connection between on-shell diagrams + Grassmannian is universal for <u>any</u> theory, combinational properties similar.

Hope: A general reformulation of QFT, with a deeper understanding of emergent S.T. + QM, will emerge by LP 2023!