

Towards the generalization of medium-induced gluon radiation

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Outline

- Motivation
- Jet Quenching
- Medium-induced hard gluon radiation (Phys.Lett.B718,160-168)
 - Hard Gluon Spectrum
 - Interpolating Spectrum
 - Summary
- Extension beyond eikonal approximation (work in progress...)
 - Vertices $q \rightarrow qg$ vs $g \rightarrow gg$
 - Color Structure
 - Summary
- Conclusions

Motivation

- Production of QGP is expected in heavy-ion collisions:
 - Jet Quenching experimentally confirmed:
 - Several observations @ RHIC and LHC
 - Suppression of high- p_T hadrons, strong dijet asymmetry with an almost unchanged azimuthal correlation, jet energy loss with a mild dependency in p_T and jet radius, ...

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 - Most of models claim to describe some or all of jet quenching observables at the same time (Q-Pythia, as an example, 1211.1161)

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Qualitative agreement with data.

But there is space for improvement!

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Include energy conservation by
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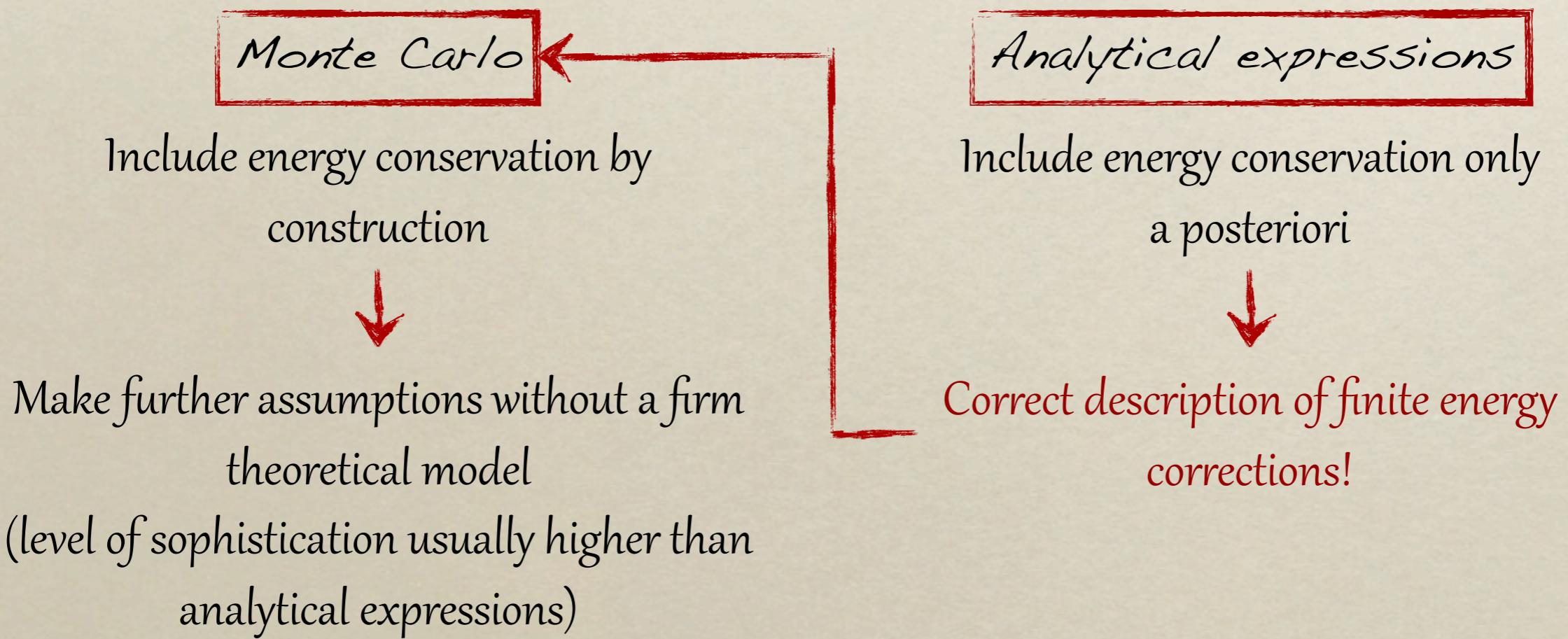
Include energy conservation only a posteriori



Correct description of finite energy corrections!

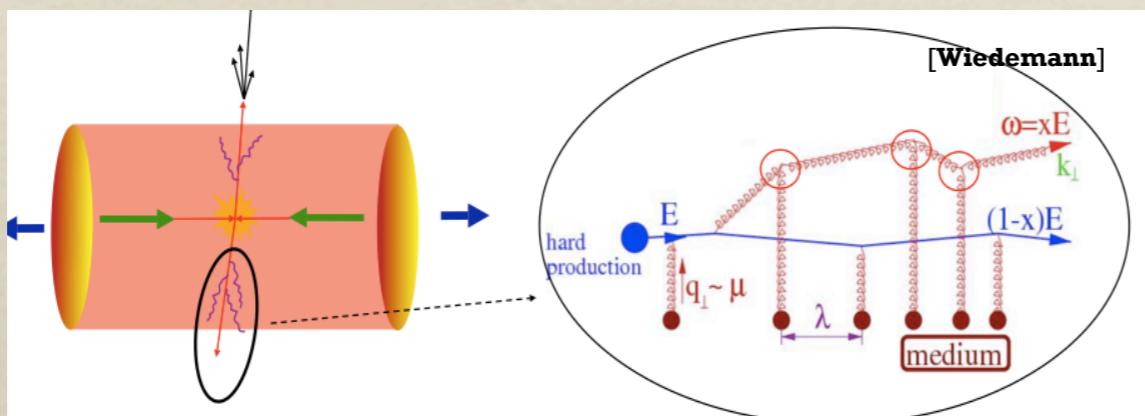
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Jet Quenching

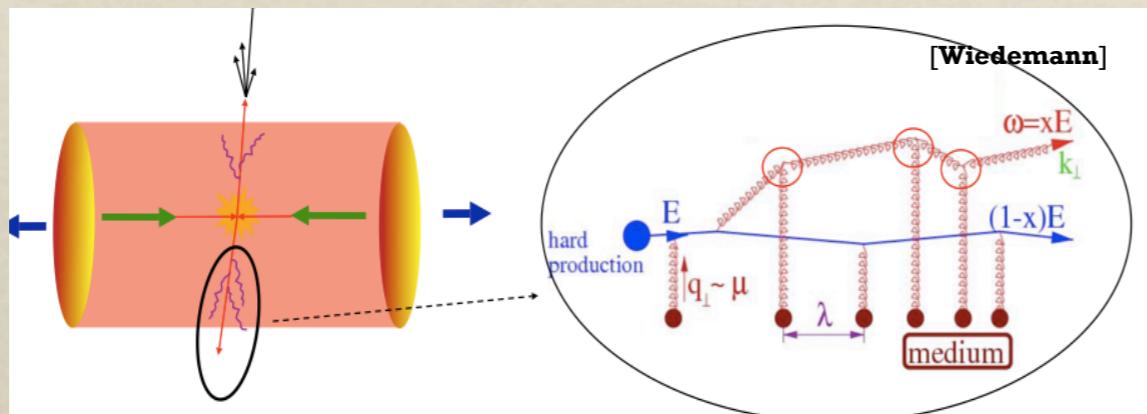
- Jet Quenching models: description in terms of radiative energy loss:



- Usual characteristics:
 - Neglect recoil (elastic energy loss);
 - Work in the limit of soft ($x \rightarrow 0$) and collinear ($k_T \ll \omega$) gluon emissions .
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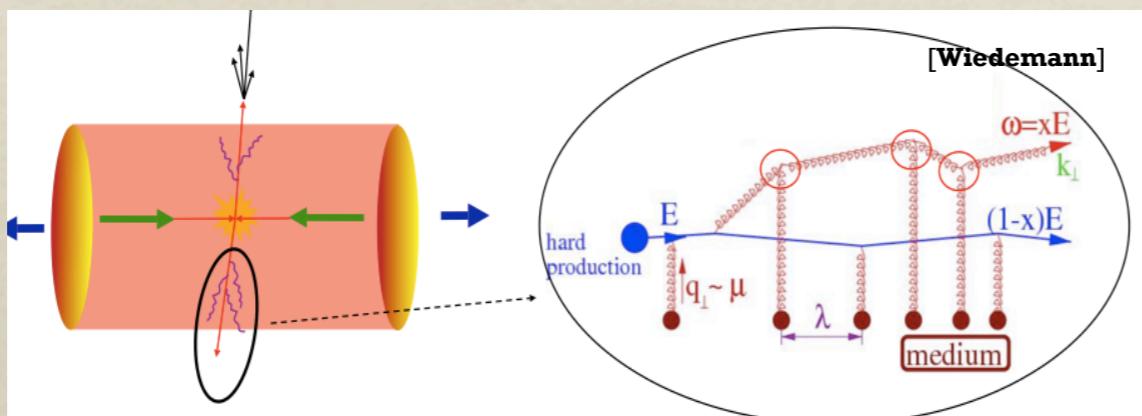
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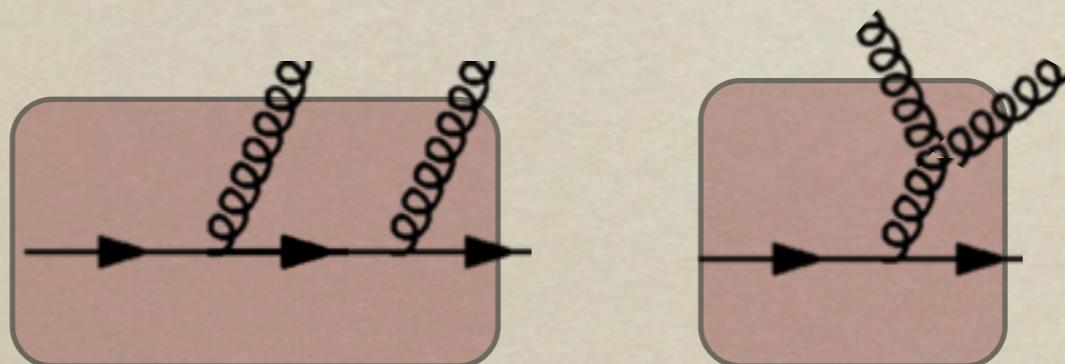
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Phenomenological extension into large x and large angle domain

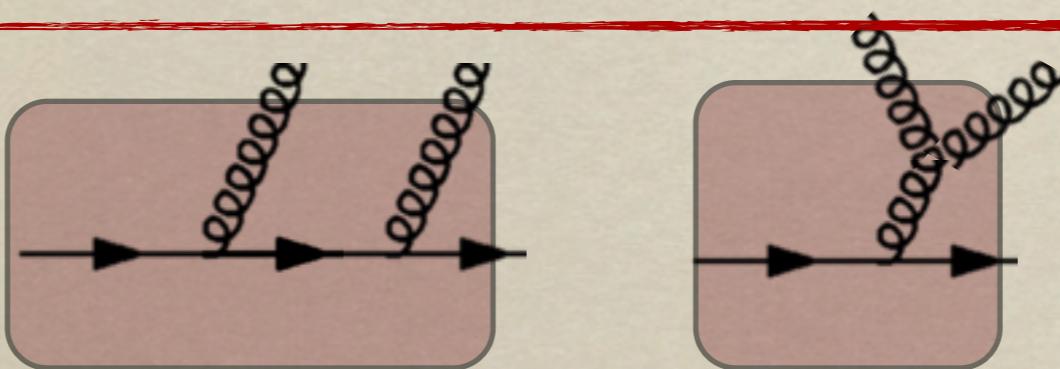
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Work in progress...

Medium-induced hard gluon radiation

Based on: Phys.Lett.B718,160-168

Formalism

- Medium propagation of highly energetic partons:
 - Partons will undergo independent multiple soft scattering with the medium.
 - In the high-energy limit, the re-scattering with the medium conserves the initial energy but results in a rotation of the color field (eikonal approximation).

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- Propagator described by a Wilson line:

$$W(x_{0+}, L_+; \mathbf{x}_\perp) = \mathcal{P} \exp \left\{ ig \int_{x_{0+}}^{L_+} dx_+ A_-(x_+, \mathbf{x}_\perp) \right\}$$

Medium longitudinal boundaries Path-ordering Medium color field Transverse coordinate

Formalism

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↓
Wilson Line

Initial/Final transverse coordinates Path-Integral Longitudinal particle momentum

Formalism

- Medium propagation of less energetic partons:
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Color Rotation

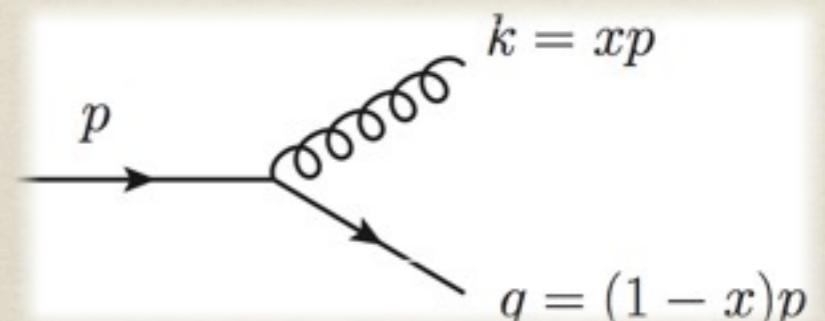
Wilson Line

Brownian Motion

Hard Gluon Spectrum

- Kinematics:

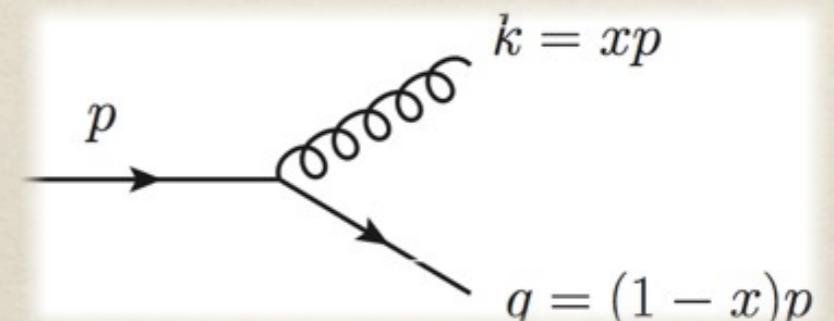
- Initial momenta $p = (p_+, p_-, p_T) = (p_+, 0, 0)$
- Eikonal approximation: $k_T, q_T \ll k_+, q_+$
- Limit $x \rightarrow 1$: $q_+ \ll k_+, p_+$
- Assume scatterings in a frozen color profile (color average).
- Only the softest outgoing particles can undergo Brownian motion.



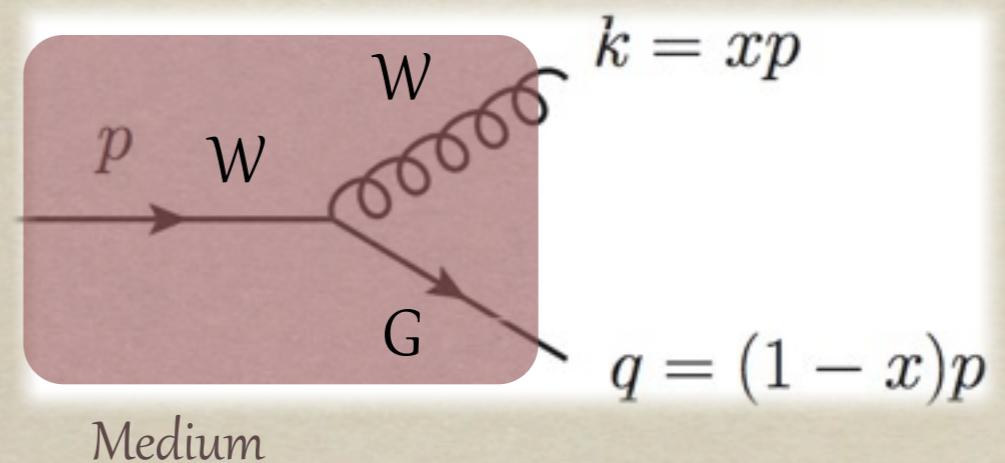
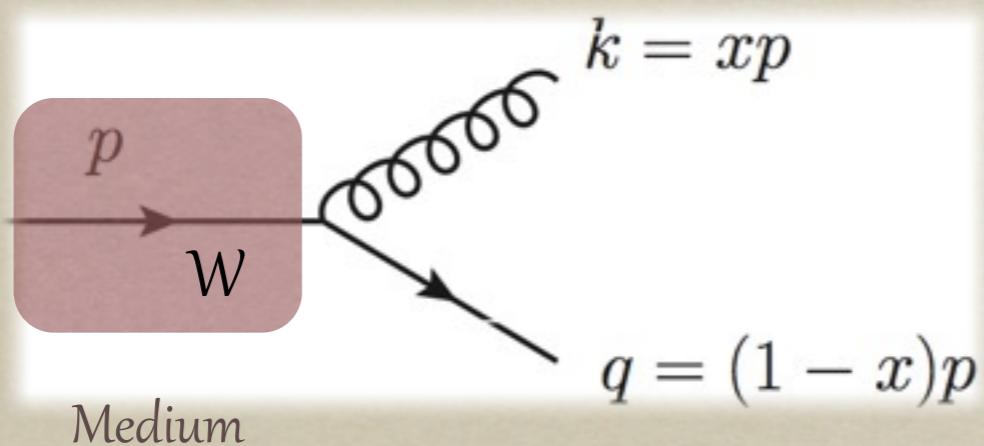
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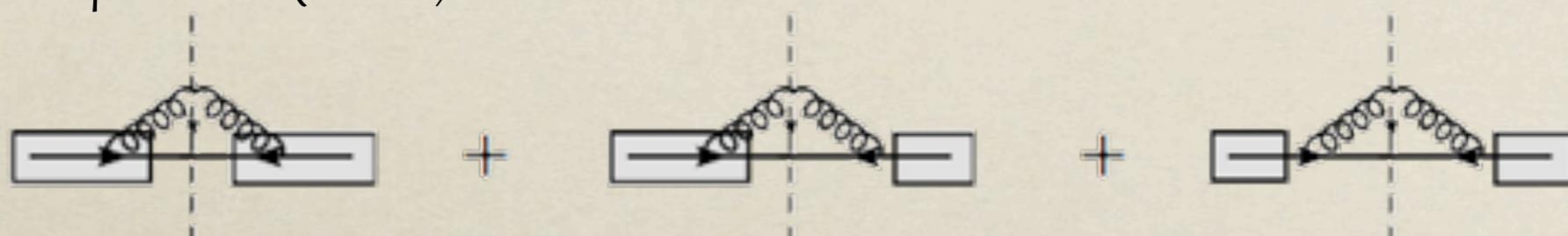


- Diagrams:



Hard Gluon Spectrum

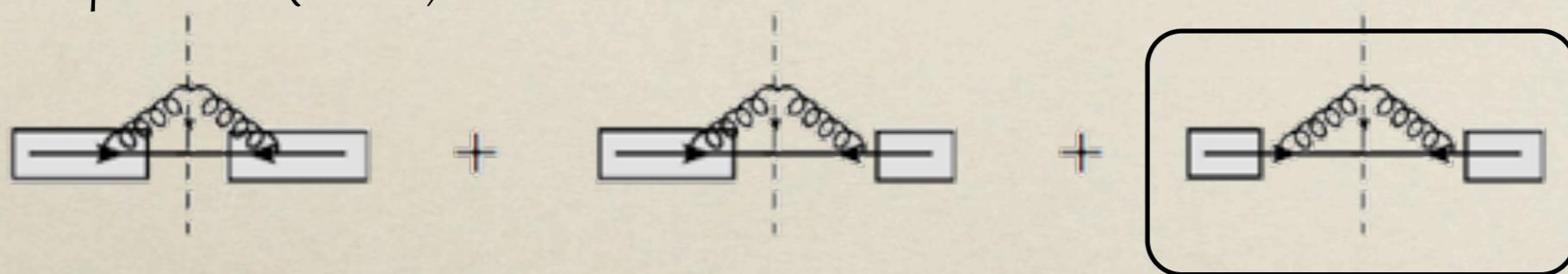
- Total Spectrum ($M M^\dagger$):



$$\left\langle \overline{|M_{tot}|^2} \right\rangle = \left\langle \overline{|M_q|^2} \right\rangle + \left\langle \overline{|M_g|^2} \right\rangle + 2\text{Re} \left\langle \{ \overline{M_g M_q^\dagger} \} \right\rangle$$

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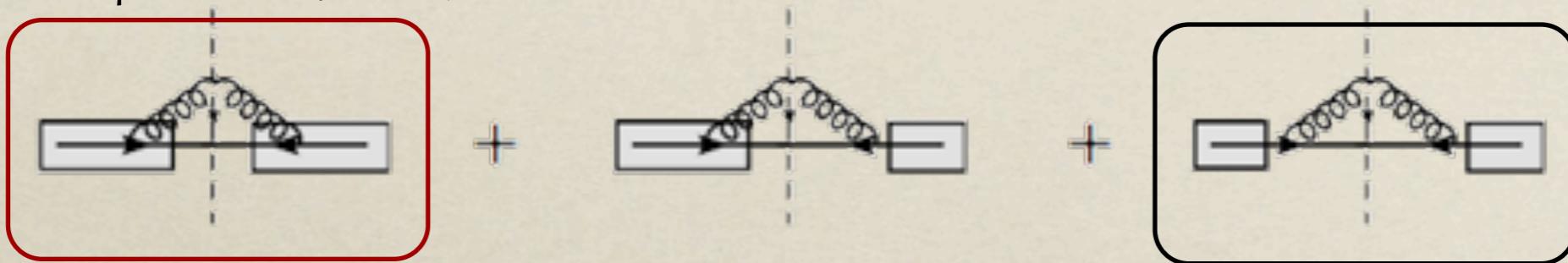
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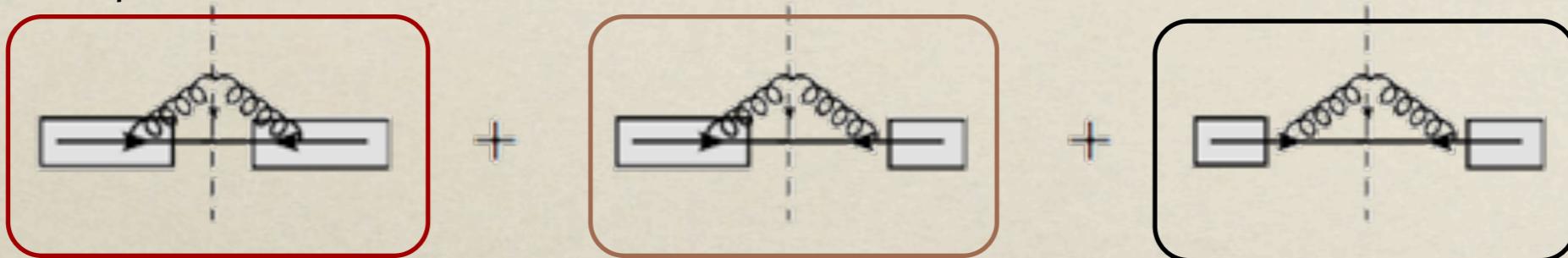
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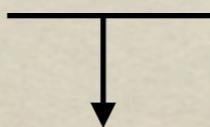
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Vaccum contribution:

$$x \frac{dI}{dxd^2\mathbf{k}_\perp} \Big|_{x \rightarrow 1} \simeq \frac{C_F \alpha_s}{2\pi^2} \frac{1}{\mathbf{k}_\perp^2} = \frac{\alpha_s}{2\pi^2} \frac{1}{\mathbf{k}_\perp^2} P_{g \leftarrow q}(x \rightarrow 1)$$

C_F : Casimir factor (color “strength”)

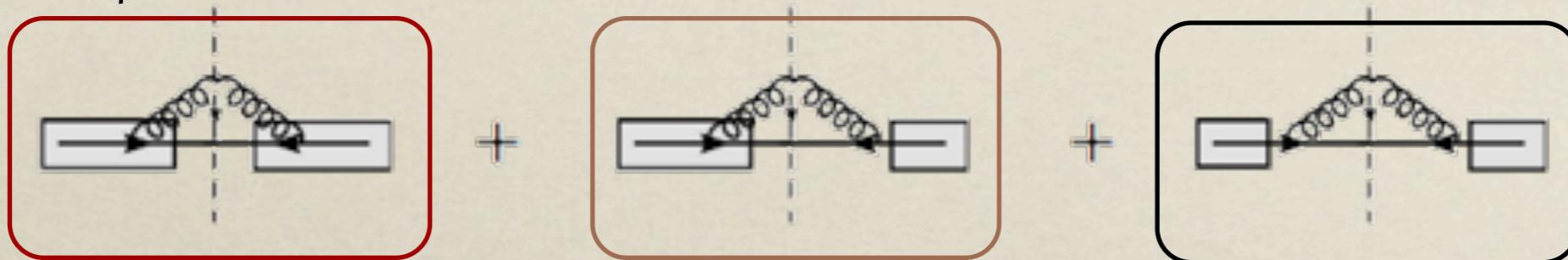
Gluon Bremsstrahlung:

α_s : strong coupling constant

Collinear divergent

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↓ ↓
 Vacuum contribution: Medium contribution

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- Vacuum SF:

$$P_{g \leftarrow q}^{vac}(x) = C_F \left[\frac{1 + (1-x)^2}{x} \right]$$

- Path Integral:

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Medium Information:

$n(\xi)$: density of scattering centers

$\sigma(x_T)$: related to the scattering potential
(usually, static Debye screened potential)

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Hard Gluon Spectrum

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$$k_+ \frac{dI^{med}}{dk_+ d^2 \mathbf{k}_\perp} \Big|_{x \rightarrow 0} = \frac{\alpha_s C_F}{(2\pi)^2 k_+} 2 \operatorname{Re} \left\{ \frac{1}{k_+} \int dy_+ d\bar{y}_+ d\mathbf{x}_\perp e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{-\frac{C_F}{2} \int d\xi n(\xi) \sigma(\mathbf{x}_\perp)} \right. \\ \times \frac{\partial}{\partial \mathbf{y}_\perp} \cdot \frac{\partial}{\partial \mathbf{x}_\perp} \mathcal{K}(y_+, \mathbf{y}_\perp = \mathbf{0}; \bar{y}_+, \mathbf{x}_\perp | k_+) \\ \left. + \int dy_+ d\mathbf{x}_\perp e^{-i\mathbf{x}_\perp \cdot \mathbf{k}_\perp} 2 \frac{\mathbf{k}_\perp}{\mathbf{k}_\perp^2} \cdot \frac{\partial}{\partial \mathbf{y}_\perp} \mathcal{K}(y_+, \mathbf{y}_\perp = \mathbf{0}; L_+, \mathbf{x}_\perp | k_+) \right\}$$

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Full Vacuum Splitting Function

- Medium contribution in the limit $x \rightarrow 0$:

$$k_+ \frac{dI^{med}}{dk_+ d^2 \mathbf{k}_\perp} \Big|_{x \rightarrow 0} = \frac{\alpha_s C_F}{(2\pi)^2 k_+} 2 \text{Re} \left\{ \frac{1}{k_+} \int dy_+ d\bar{y}_+ d\mathbf{x}_\perp e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} e^{-\frac{C_F}{2} \int d\xi n(\xi) \sigma(\mathbf{x}_\perp)} \right.$$

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Vacuum Splitting Function in the limit $x \rightarrow 0$

Hard Gluon Spectrum

- Medium contribution in the limit $x \rightarrow 1$:

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Quark longitudinal momentum

- Medium contribution in the limit $x \rightarrow 0$:

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Gluon longitudinal momentum

Hard Gluon Spectrum

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Related to the final quark

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Related to the final gluon

How to interpolate between the two limits?

Interpolating Spectrum

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 - Only one transverse coordinate: $q_T \sim k_T$
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 - $\xrightarrow{x p_+ = k_+, x \rightarrow 0}$
 - $\xrightarrow{(1-x)p_+ = q_+, x \rightarrow 1}$
 - Single inclusive spectrum of the “radiated parton” (quark or gluon):

$$E_+ \frac{dI^{med}}{dE_+ d\mathbf{q}_\perp^2} = \frac{\alpha_s C_F}{4\pi} x(1-x) P_{g \leftarrow q}(x)(I_1 + I_2)$$

$$\begin{aligned} I_1 &= \frac{1}{E_+^2} \text{Re} \int dy_+ d\bar{y}_+ d\mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} e^{-\frac{1}{2} \int_{\bar{y}_+}^{L_+} d\xi n(\xi) \sigma(\mathbf{x}_\perp)} \\ &\times \frac{\partial}{\partial \mathbf{x}_\perp} \cdot \frac{\partial}{\partial \mathbf{y}_\perp} \mathcal{K}(y_+, \mathbf{y}_\perp = \mathbf{0}_\perp; \bar{y}_+ \mathbf{x}_\perp | E_+) \end{aligned}$$

Loss of parton identification...
 Total energy loss instead of soft or
 hard gluon radiation spectrum!

$$I_2 = \frac{2}{E_+} \frac{\mathbf{q}_\perp}{\mathbf{q}_\perp^2} \cdot \int dy_+ d\mathbf{x}_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{x}_\perp} \frac{\partial}{\partial \mathbf{y}_\perp} \mathcal{K}(y_+, \mathbf{y}_\perp = \mathbf{0}_\perp; L_+, \mathbf{x}_\perp | E_+)$$

Interpolating Spectrum

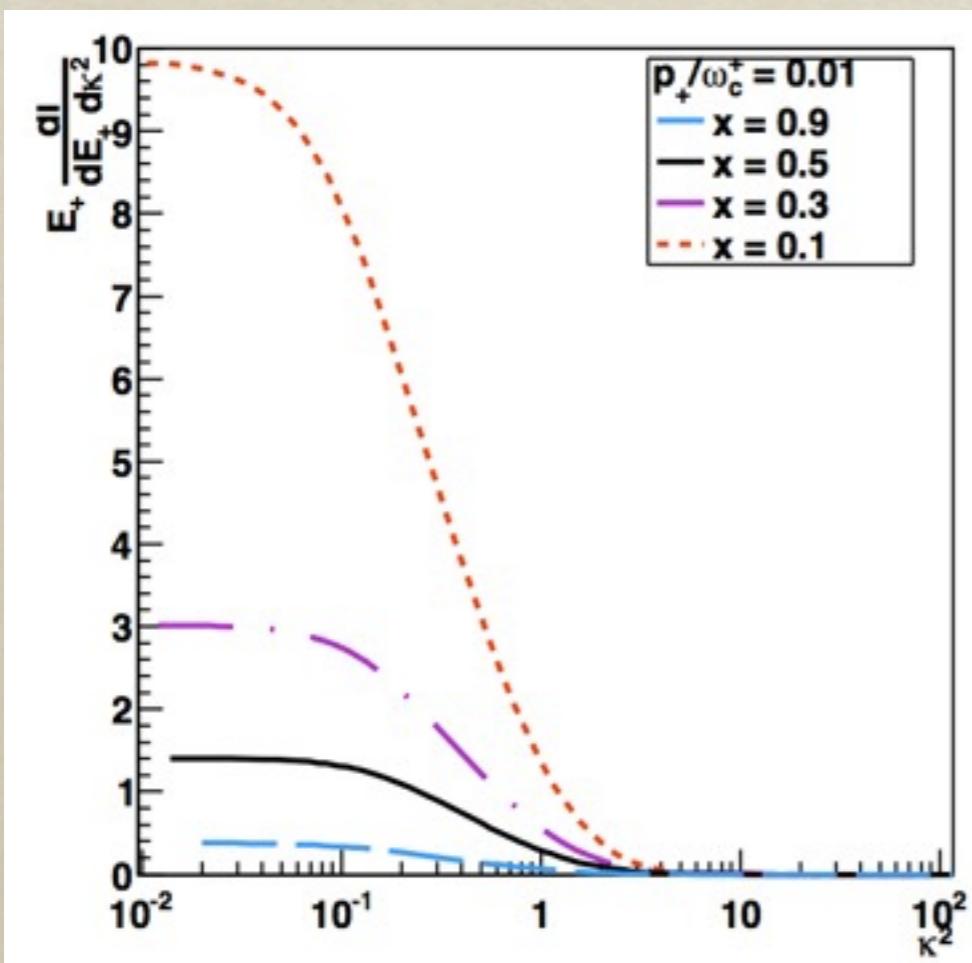
- Numerical results in multiple soft scattering approximation (medium spectrum):

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$$\frac{p_+}{\omega_c^+} = \frac{2p_+}{\hat{q}_A L_+^2} \quad \kappa^2 = \frac{q_\perp^2}{\hat{q}_A L_+}$$

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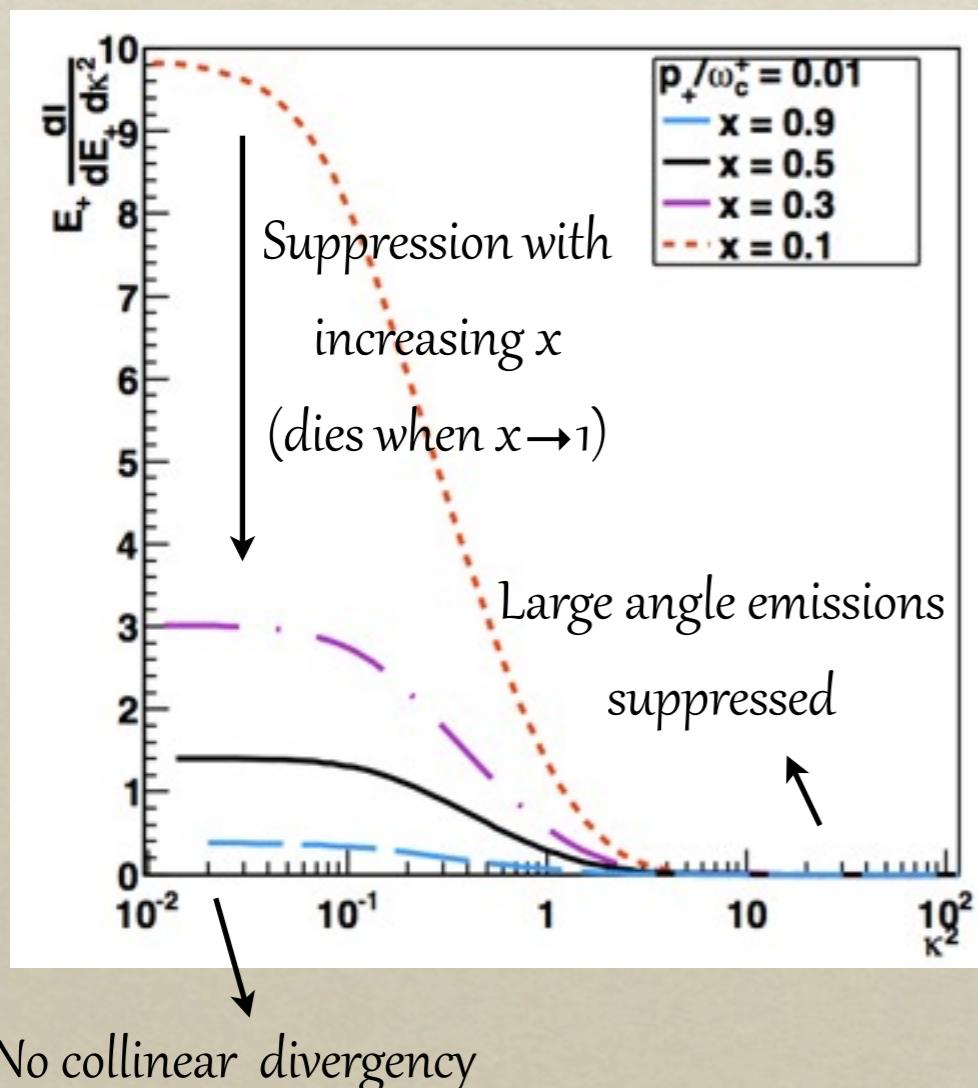
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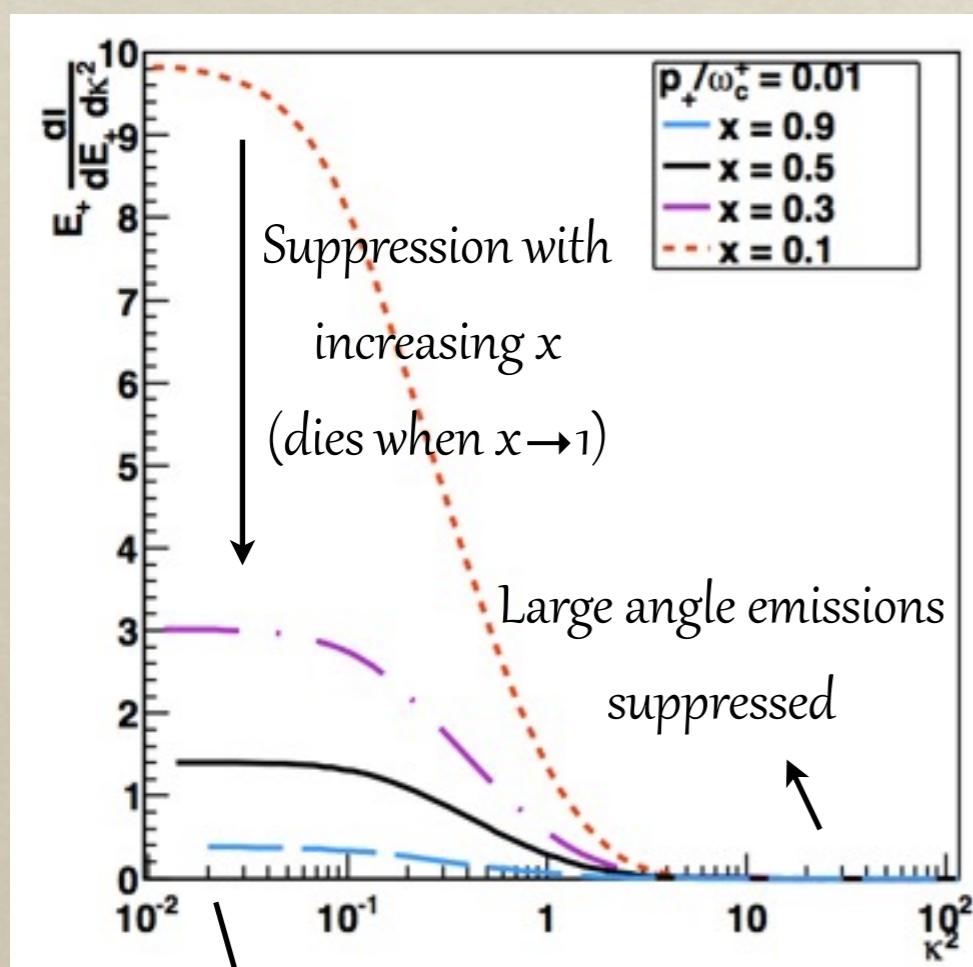
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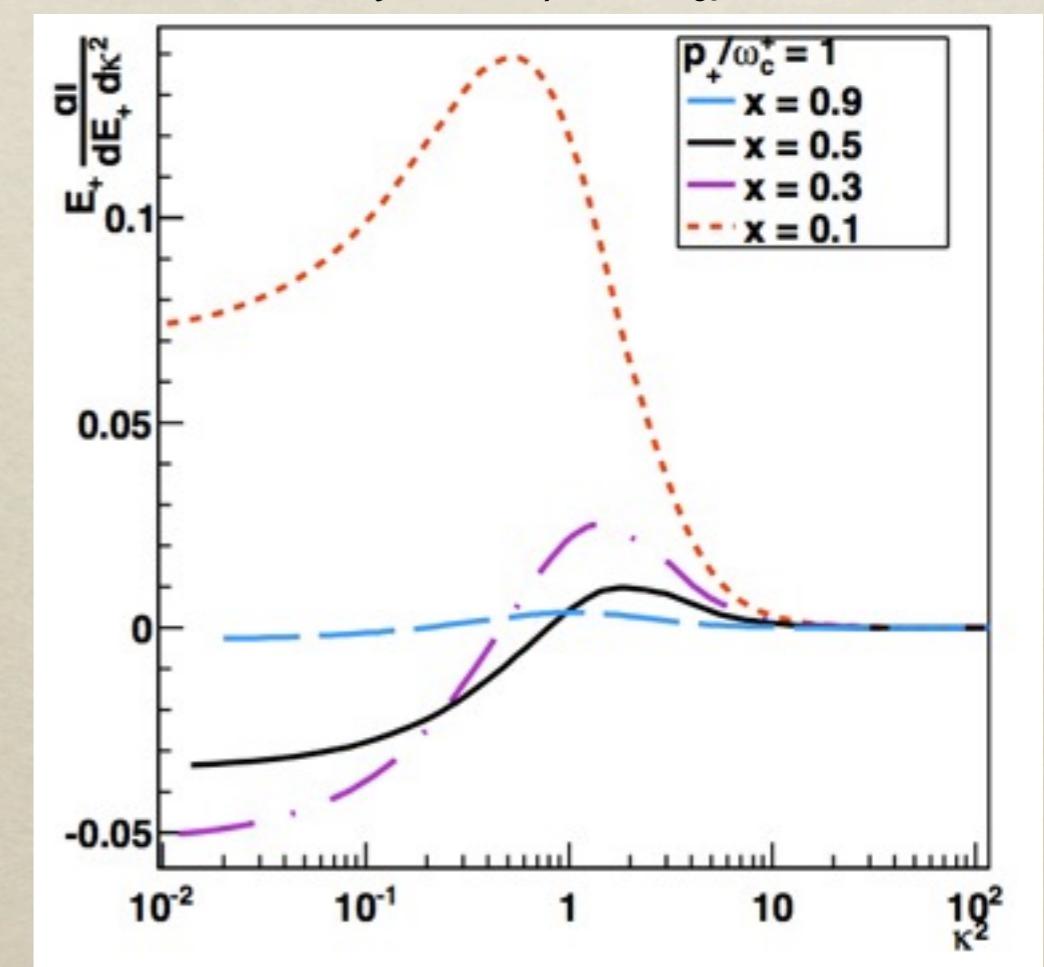
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Decrease
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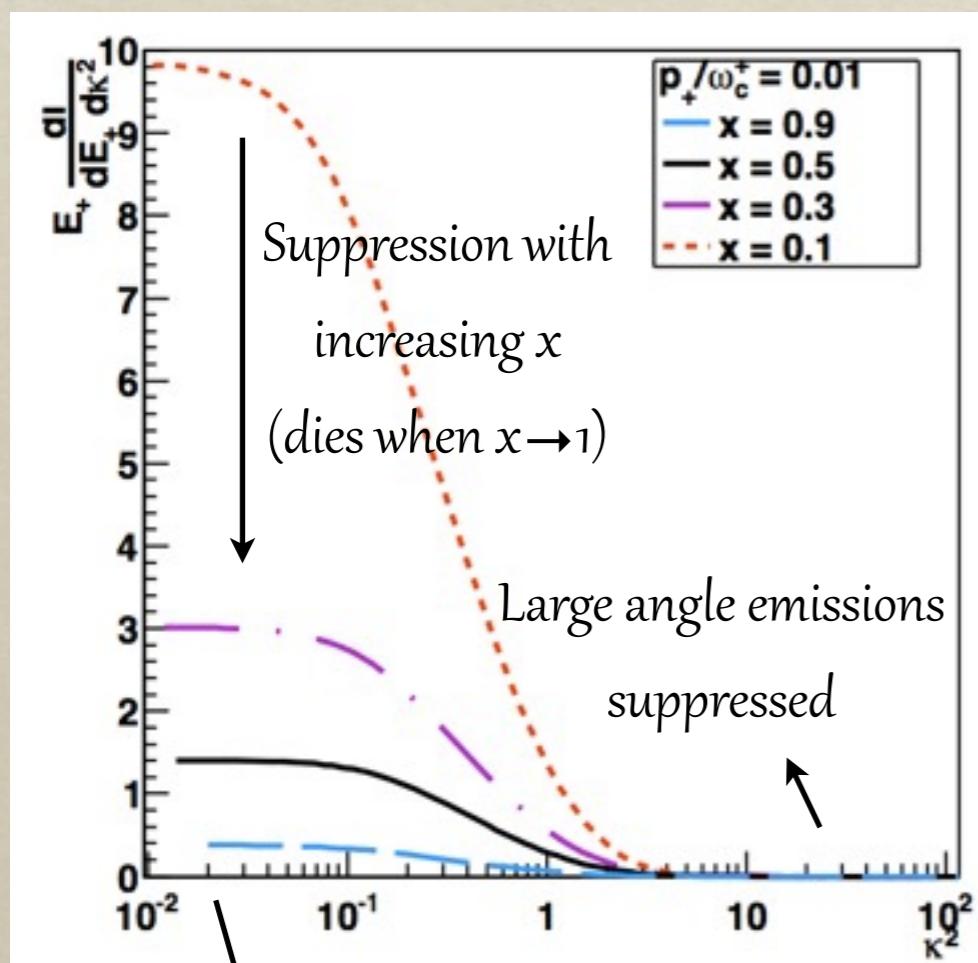
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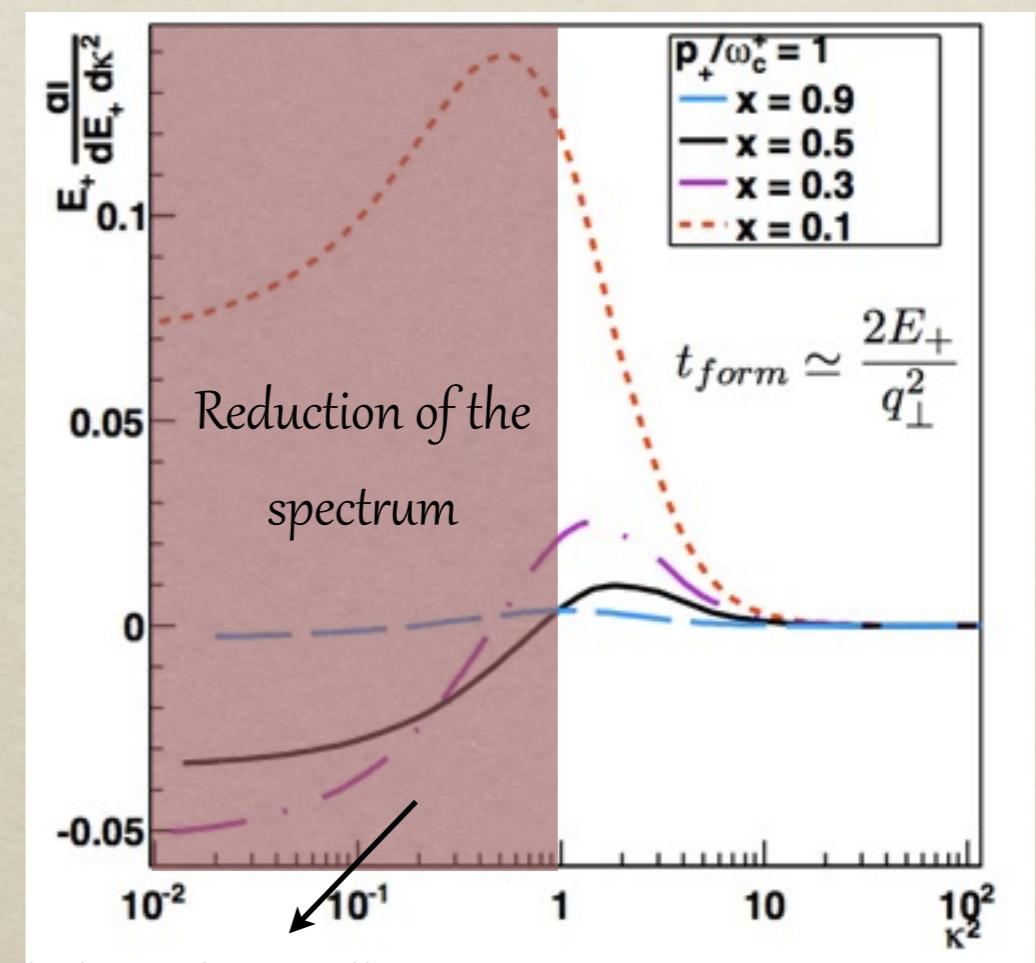
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No collinear divergency

Decrease
of ω_c^+

If $t_{form} \gg L_+$, the whole medium will
act as a single scattering center



LPM effect in QCD

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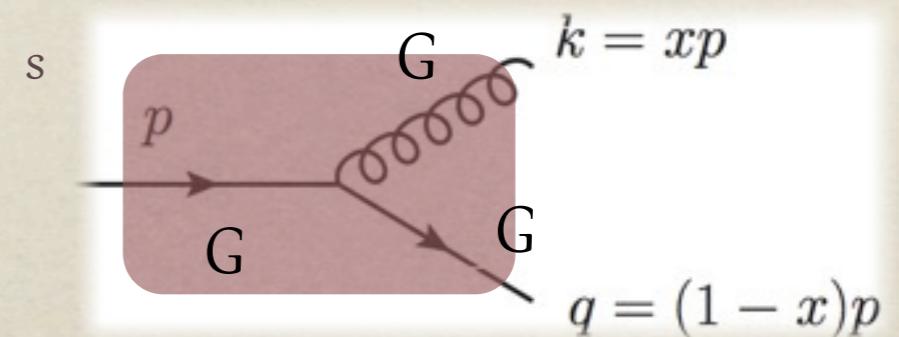
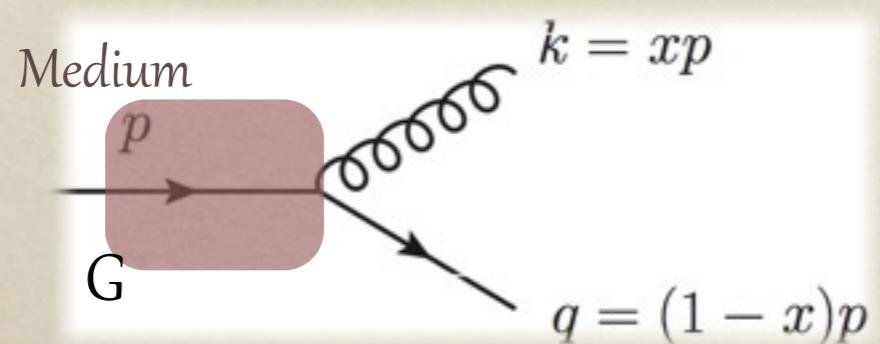
Done in the next section!

Extension beyond eikonal approximation

Article in preparation...

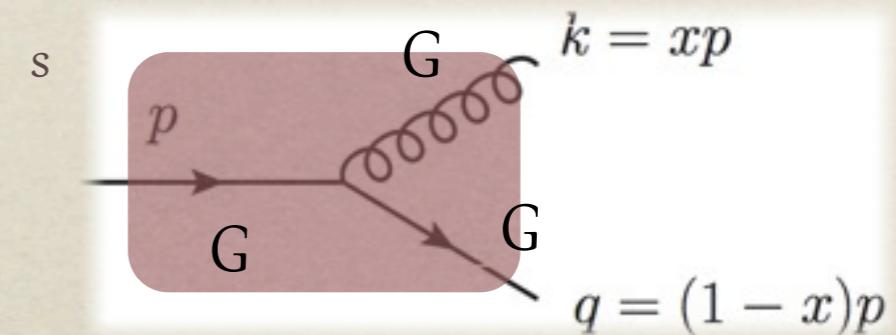
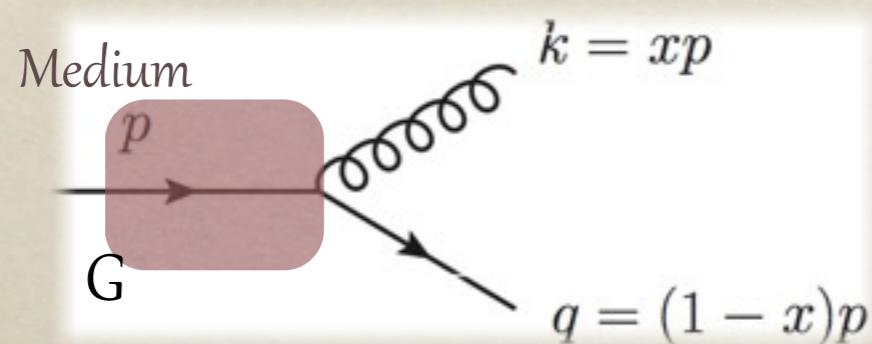
Vertices $q \rightarrow qg$ vs $g \rightarrow gg$

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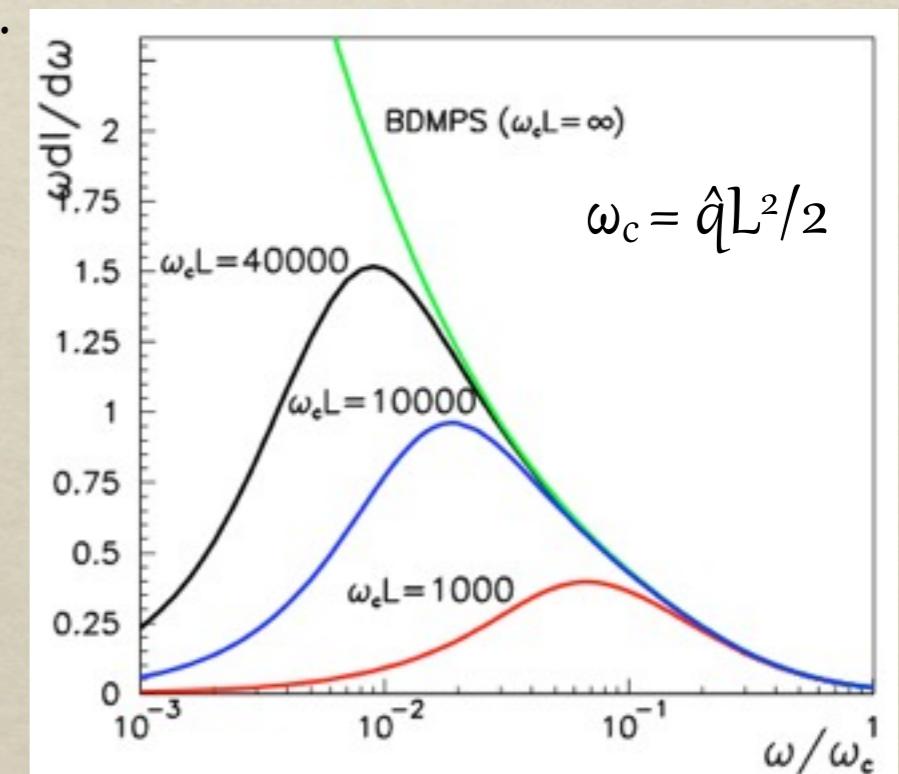
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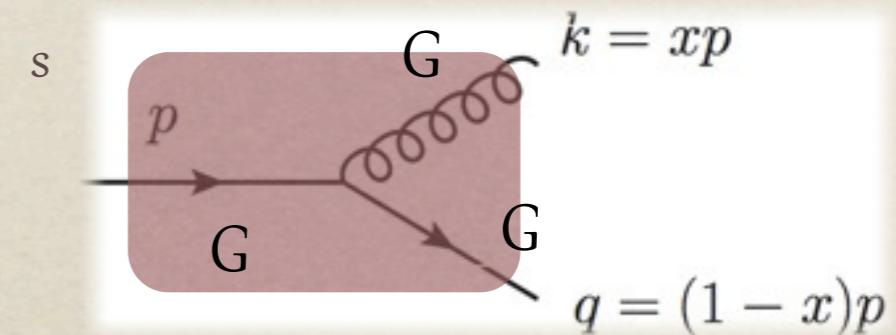
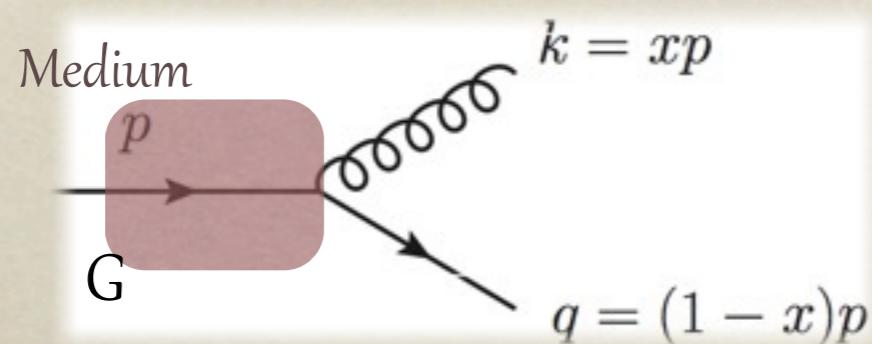
$$t_{\text{form}} \ll L \Rightarrow \omega \ll \omega_c$$

arXiv:0712.3443



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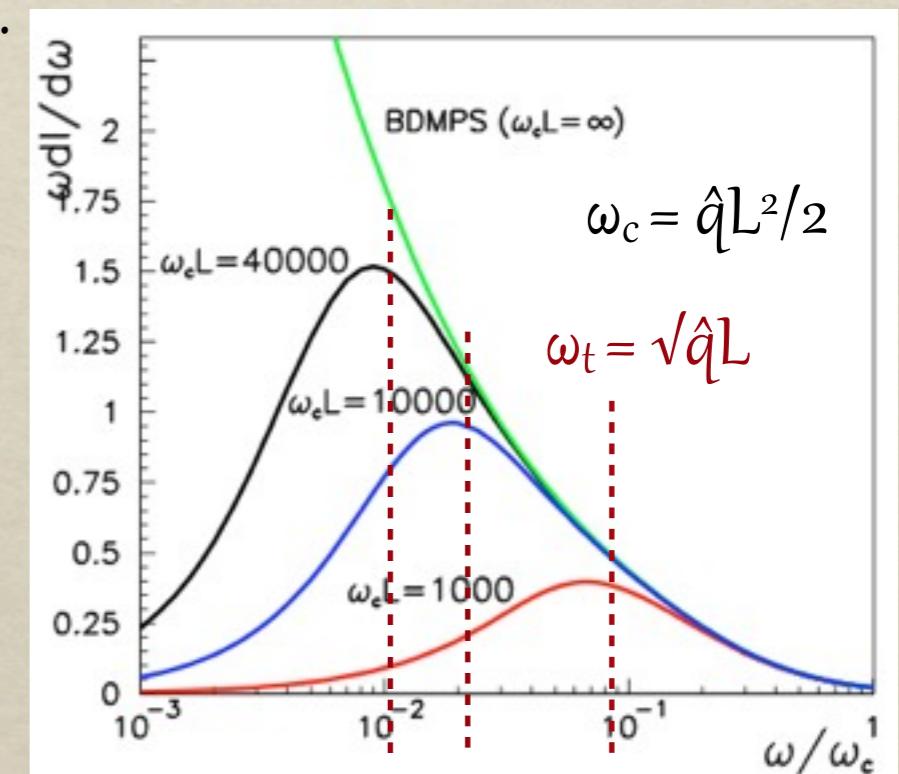
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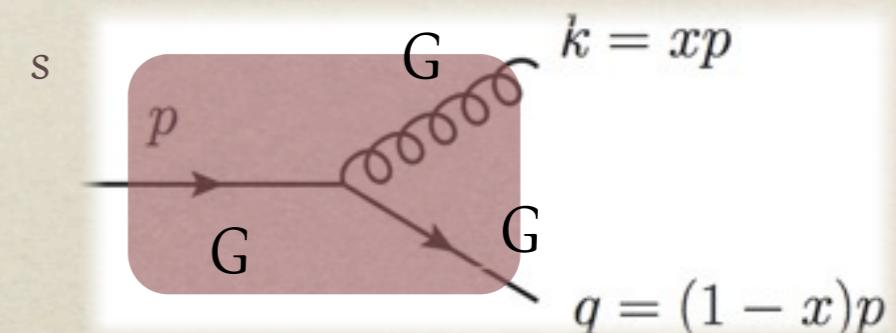
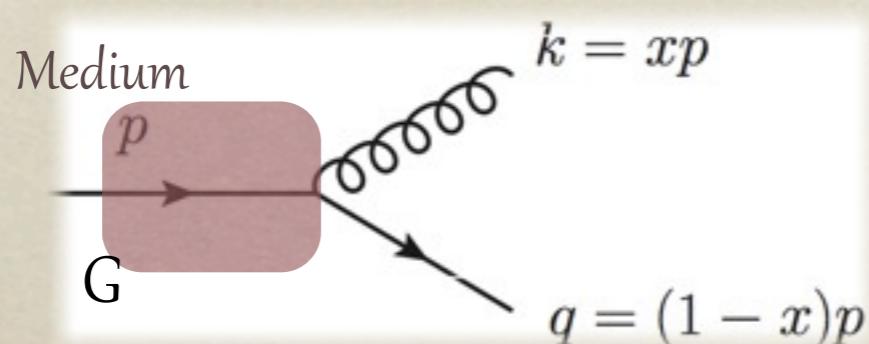
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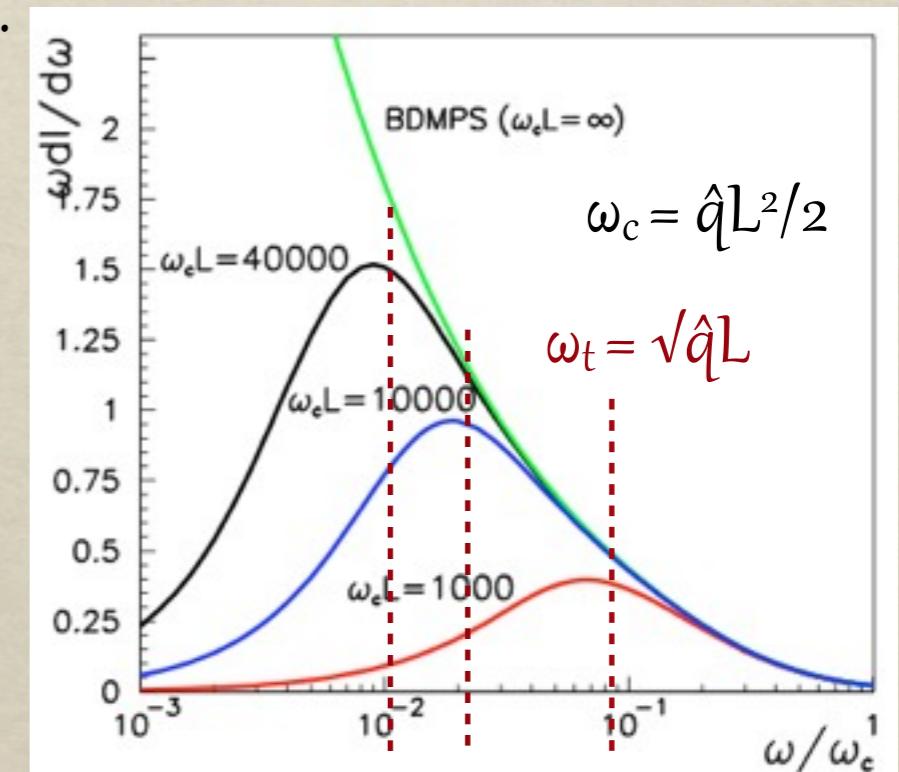
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Satisfied for:

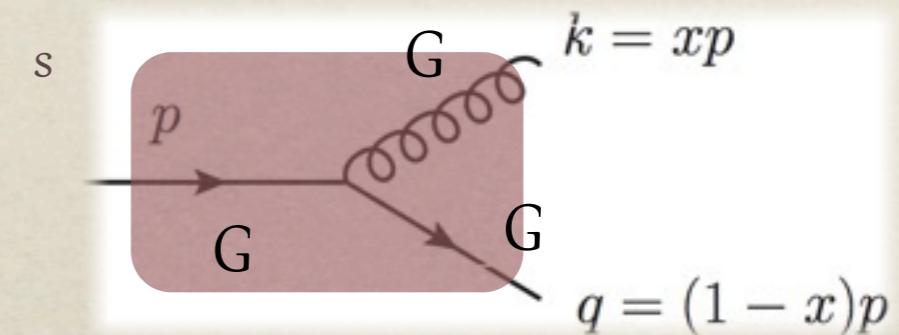
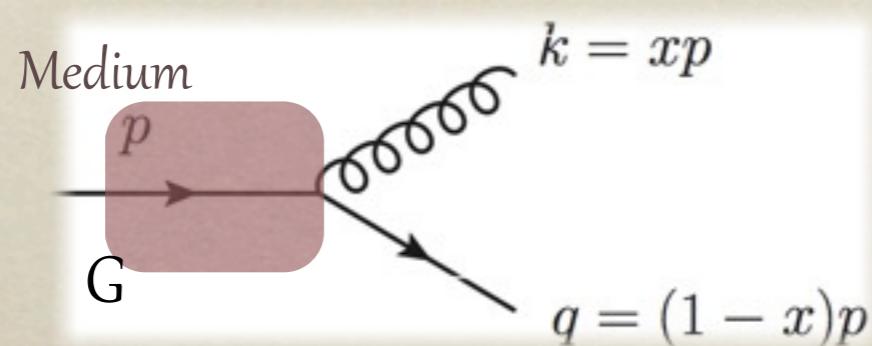
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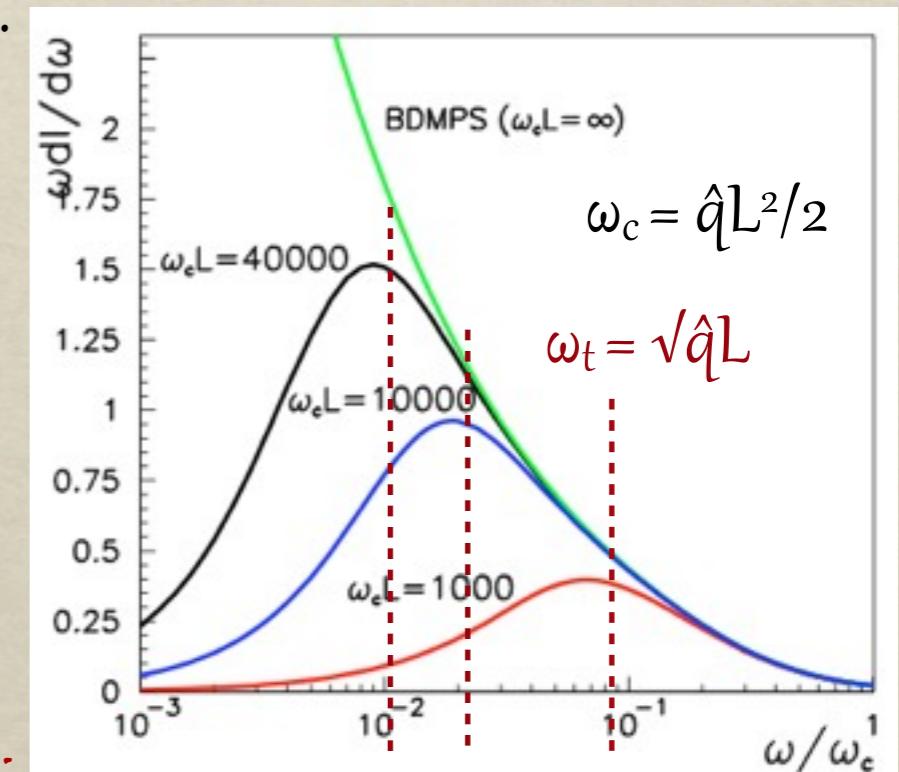
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Nevertheless...

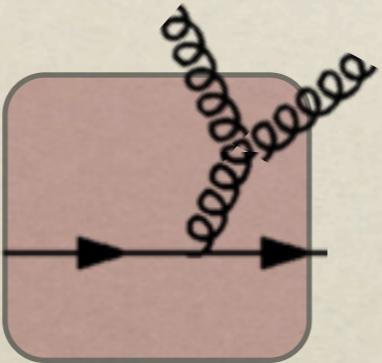
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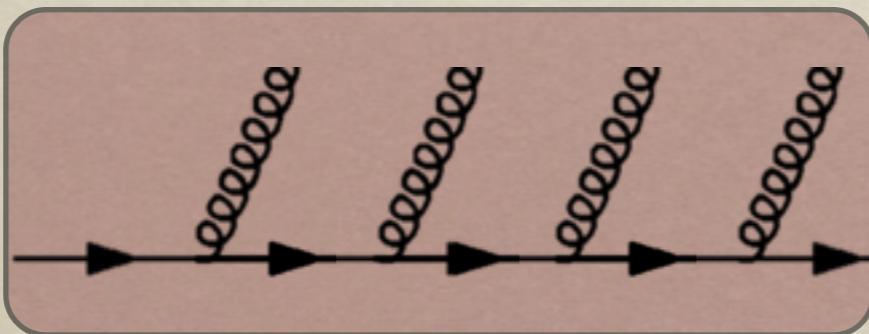
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- Problem with the approximation $t_{\text{form}} \ll L$?

- Diagram:



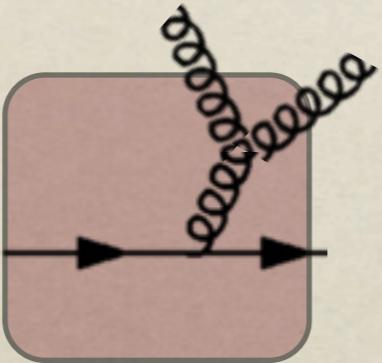
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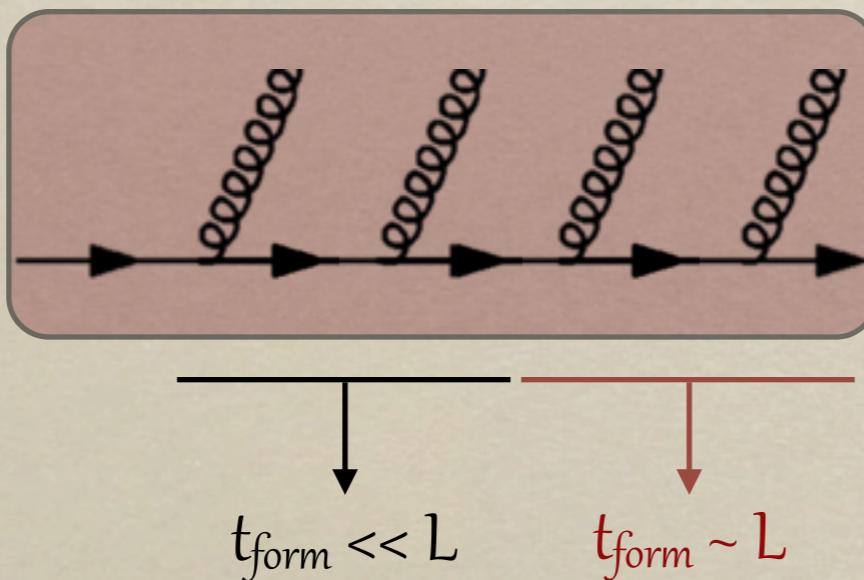
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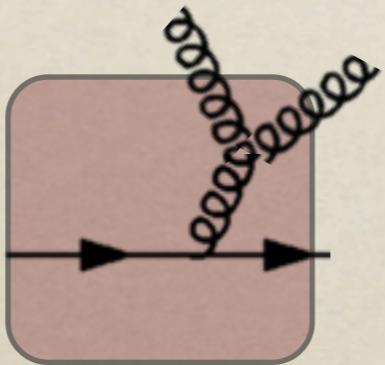
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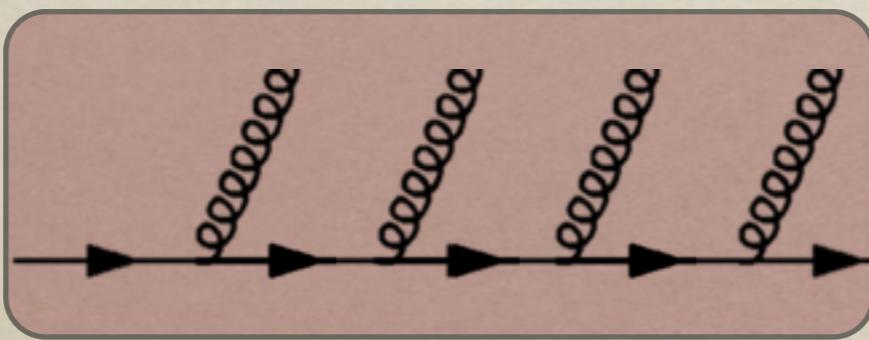
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$$t_{\text{form}} \ll L$$

$$t_{\text{form}} \sim L$$

MCs need this correction!

Vertices $q \rightarrow qg$ vs $g \rightarrow gg$

- Interesting to study $q \rightarrow qg$:
 - Not constraining to $t_{\text{form}} \ll L$;
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 - $S_{g \leftarrow g}(x) = S_{g \leftarrow g}(1-x)$ but not the $S_{g \leftarrow q}(x)$;
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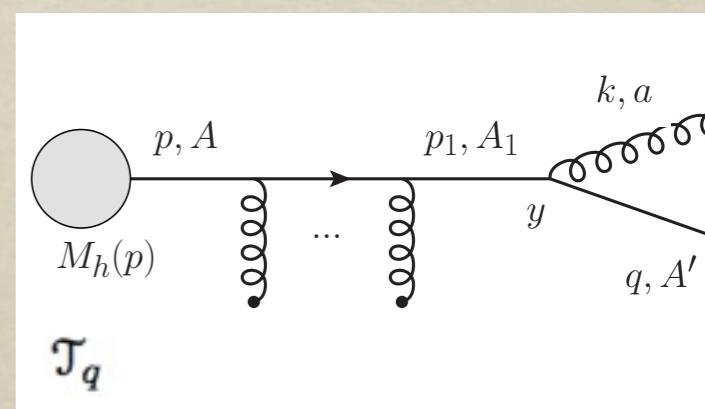
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 - Matrix-elements:
 - T_g is the same as in the hard limit case, but...

General case

$$\mathcal{T}_q = -g T_{BA_1}^a \frac{1}{4(k \cdot q)} \int d\mathbf{x}_\perp e^{-i\mathbf{x}_\perp \cdot (\mathbf{k}_\perp + \mathbf{q}_\perp)} G_{A_1 A}(x_{0+}, \mathbf{0}_\perp; L_+, \mathbf{x}_\perp | p_+) \\ \times \bar{u}(q) \epsilon_k^*(k + q) \gamma_+ \gamma_- M_h(p_+),$$

Hard Limit

$$\mathcal{T}_q = -g T_{A'A_1}^a \frac{(q + k)_+}{2(q \cdot k)} W_{A_1 A}(x_{0+}, L_+; \mathbf{0}_\perp) \bar{u}(q) \epsilon^* \gamma_- M_h(k + q).$$



Vertices $q\rightarrow qq$ vs $g\rightarrow gg$

- Total Spectrum ($M M^\dagger$):

$$\langle \overline{|M_{tot}|^2} \rangle = \langle \overline{|M_q|^2} \rangle + \langle \overline{|M_g|^2} \rangle + 2\text{Re} \langle \{ \overline{M_g M_q^\dagger} \} \rangle$$

- Dirac structure:

$$\langle \overline{|M_q|^2} \rangle \propto \frac{2g^2 x(1-x)}{[(1-x)\mathbf{k}_\perp - z\mathbf{q}_\perp]^2} P_{g \leftarrow q}(x)$$

$$\langle \overline{M_g M_q^\dagger} \rangle \propto \frac{1}{2(k \cdot q)p_+} \left\{ \mathbf{k}_\perp^2 \frac{(1-x)(2-x)}{x^2} + \mathbf{q}_\perp^2 \frac{1}{1-x} - \mathbf{q}_\perp \cdot \mathbf{k}_\perp \frac{3-x}{x} \right\}$$

$$\langle \overline{|M_g|^2} \rangle \propto \frac{1}{p_+^2} \frac{[(1-x)\mathbf{k}_\perp - x\mathbf{q}_\perp]^2 + (1-x)^2 \mathbf{k}_\perp^2}{2x^2(1-x)}$$

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$$\langle \overline{M_g M_q^\dagger} \rangle \propto \frac{1}{2(k \cdot q)p_+} \left\{ \mathbf{k}_\perp^2 \frac{(1-x)(2-x)}{x^2} + \mathbf{q}_\perp^2 \frac{1}{1-x} - \mathbf{q}_\perp \cdot \mathbf{k}_\perp \frac{3-x}{x} \right\}$$

$$\langle \overline{|M_g|^2} \rangle \propto \frac{1}{p_+^2} \frac{[(1-x)\mathbf{k}_\perp - x\mathbf{q}_\perp]^2 + (1-x)^2 \mathbf{k}_\perp^2}{2x^2(1-x)}$$

Only term with factorization of the
Altarelli-Parisi



Extra-term with respect to
 $g \rightarrow gg$ case

Vertices $q \rightarrow qg$ vs $g \rightarrow gg$

- Total Spectrum ($M M^\dagger$):

$$\langle \overline{|M_{tot}|^2} \rangle = \langle \overline{|M_q|^2} \rangle + \langle \overline{|M_g|^2} \rangle + 2\text{Re} \langle \{ \overline{M_g M_q^\dagger} \} \rangle$$

- Dirac structure:

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Only term with factorization of the
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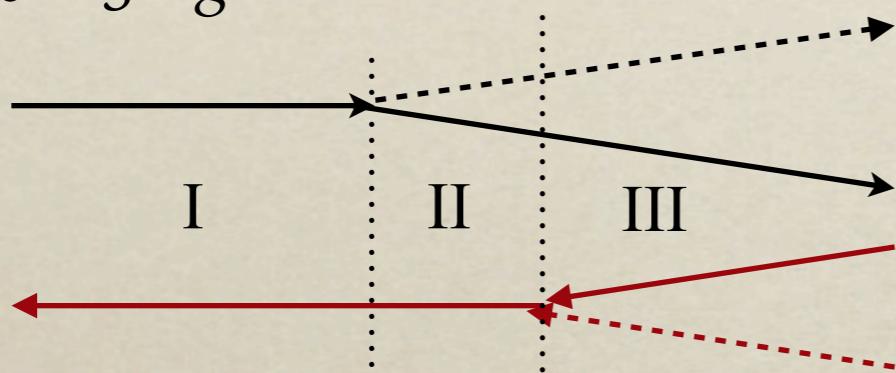
Extra-term with respect to
 $g \rightarrow gg$ case

When $k_T = -q_T$, we recover the results from the hard gluon spectrum!

Color Structure

- Color Structure in $\langle |M_g|^2 \rangle$:

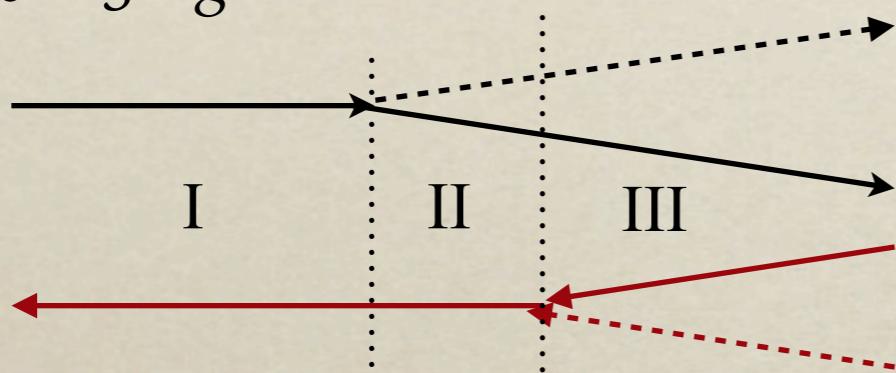
- 3 regions:



Color Structure

- Color Structure in $\langle |M_g|^2 \rangle$:

- 3 regions:



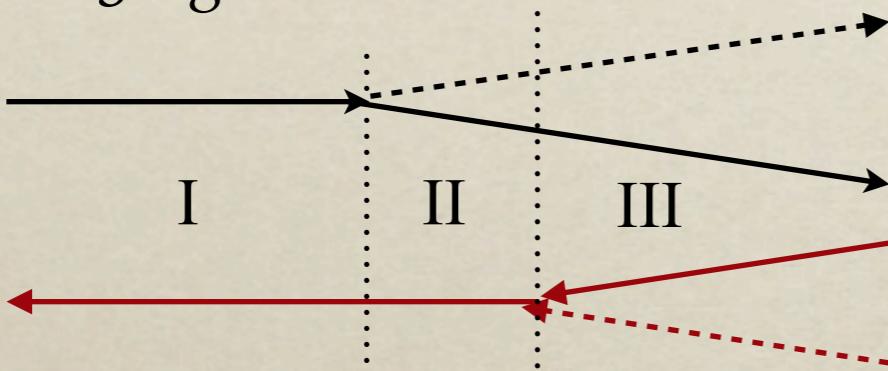
- No assumption on the t_{form} (presence of region II), but large N_c limit:

$$\text{wavy line} = \longleftrightarrow$$

Color Structure

- Color Structure in $\langle |M_g|^2 \rangle$:

- 3 regions:



- No assumption on the t_{form} (presence of region II), but large N_c limit:

$$\text{wavy line} = \text{double line}$$

$$\left\langle [W_q W_g^\dagger]_{ij} [W_g W_q^\dagger]_{kl} \right\rangle \propto \text{Tr} \langle W_q W_g^\dagger \rangle \text{Tr} \langle W_g W_q^\dagger \rangle \delta_{ij} \delta_{kl}$$

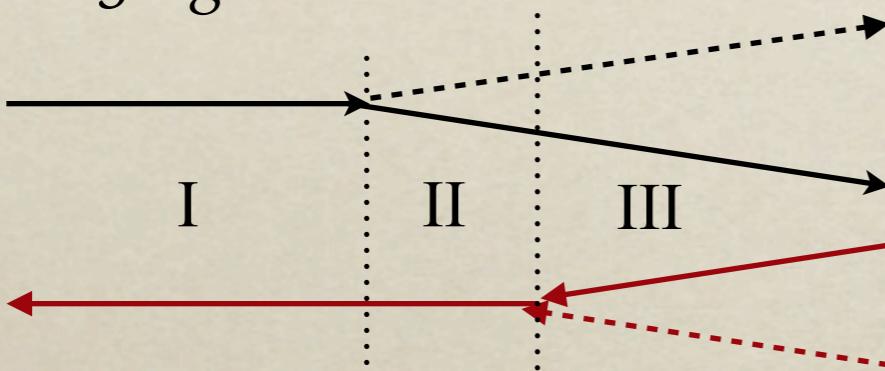
II

$$\begin{array}{c} \text{double line} \\ \text{with a loop} \end{array} = \begin{array}{c} \text{double line} \\ \text{with a loop} \end{array} + \mathcal{O}(N^{-1})$$

Color Structure

- Color Structure in $\langle |M_g|^2 \rangle$:

- 3 regions:



- No assumption on the t_{form} (presence of region II), but large N_c limit:

$$\text{wavy line} = \longleftrightarrow$$

$$\left\langle \left[W_q W_g^\dagger \right]_{ij} \left[W_g W_q^\dagger \right]_{kl} \right\rangle \propto \text{Tr} \left\langle W_q W_g^\dagger \right\rangle \text{Tr} \left\langle W_g W_q^\dagger \right\rangle \delta_{ij} \delta_{kl}$$

II

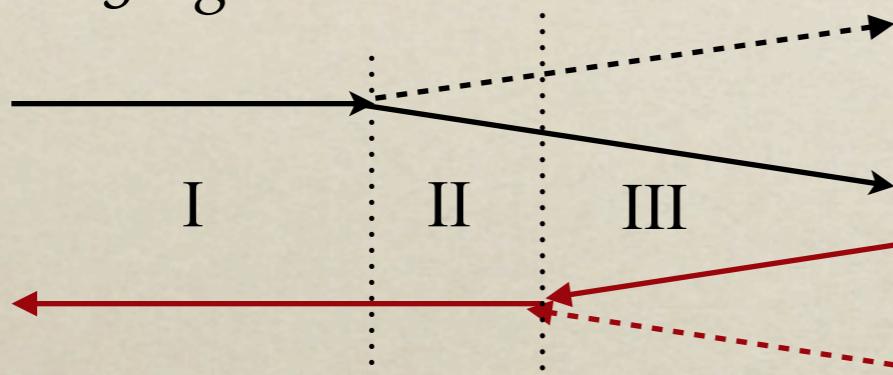
Close region III

$$\longleftrightarrow = \longleftrightarrow + \mathcal{O}(N^{-1})$$

Color Structure

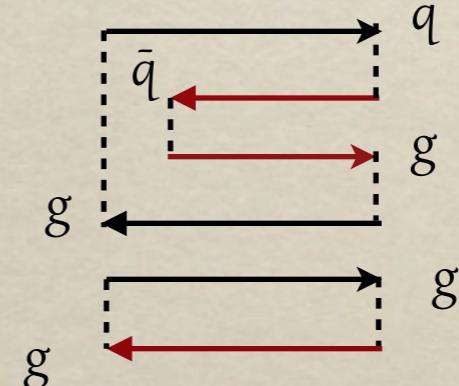
- Color Structure in $\langle |M_g|^2 \rangle$:

- 3 regions:



- No assumption on the t_{form} (presence of region II), but large N_c limit:

III $\text{Tr} \langle W_g W_{\bar{g}}^\dagger \rangle \text{Tr} \langle W_q W_g^\dagger W_{\bar{g}} W_{\bar{q}}^\dagger \rangle$



=

Color Structure

- Quadrupole:
 - Expand the Wilson lines in an infinitesimal internal:

$$W(\mathbf{x}_\perp)_{ij} = V(\mathbf{x}_\perp)_{i\alpha} \left\{ \delta_{\alpha j} \left(1 - \frac{C_F}{2} B(0) \right) - iT_{\alpha j}^a A^a(\mathbf{x}_\perp) \right\} \quad B(\mathbf{x}_\perp - \mathbf{y}_\perp) \propto \langle A^a(\mathbf{x}_\perp) A^a(\mathbf{y}_\perp) \rangle$$

→ =

- Result:

Color Structure

- Quadrupole:
 - Expand the Wilson lines in an infinitesimal internal:

$$W(\mathbf{x}_\perp)_{ij} = V(\mathbf{x}_\perp)_{i\alpha} \left\{ \delta_{\alpha j} \left(1 - \frac{C_F}{2} B(0) \right) - iT_{\alpha j}^a A^a(\mathbf{x}_\perp) \right\} \quad B(\mathbf{x}_\perp - \mathbf{y}_\perp) \propto \langle A^a(\mathbf{x}_\perp) A^a(\mathbf{y}_\perp) \rangle$$

$$\longrightarrow = \longrightarrow$$

- Result:

Color Structure

- Quadrupole:

- Expand the Wilson lines in an infinitesimal internal:

$$W(\mathbf{x}_\perp)_{ij} = V(\mathbf{x}_\perp)_{i\alpha} \left[\delta_{\alpha j} \left(1 - \frac{C_F}{2} B(0) \right) - iT_{\alpha j}^a A^a(\mathbf{x}_\perp) \right]$$

$$B(\mathbf{x}_\perp - \mathbf{y}_\perp) \propto \langle A^a(\mathbf{x}_\perp) A^a(\mathbf{y}_\perp) \rangle$$

$$\longrightarrow = \longrightarrow \rightarrow$$

- Result:

Color Structure

- Quadrupole:

- Expand the Wilson lines in an infinitesimal internal:

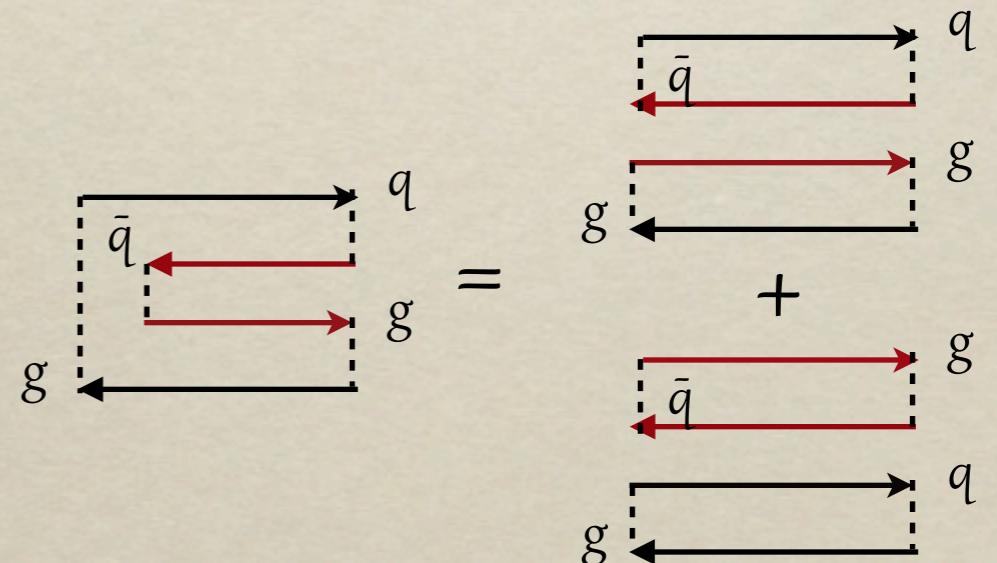
$$W(\mathbf{x}_\perp)_{ij} = V(\mathbf{x}_\perp)_{i\alpha} \left[\delta_{\alpha j} \left(1 - \frac{C_F}{2} B(0) \right) - iT_{\alpha j}^a A^a(\mathbf{x}_\perp) \right]$$

$B(\mathbf{x}_\perp - \mathbf{y}_\perp) \propto \langle A^a(\mathbf{x}_\perp) A^a(\mathbf{y}_\perp) \rangle$

$\longrightarrow = \longrightarrow \rightarrow$

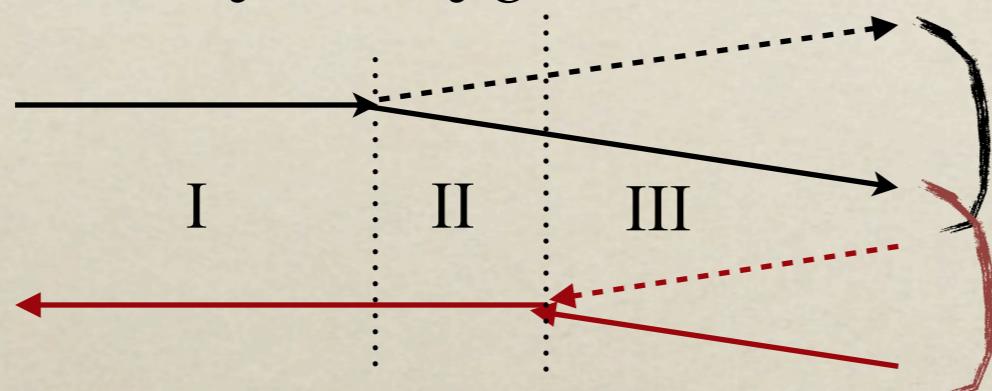
- Result:

$$\begin{aligned} & \text{Tr} \langle W(\mathbf{x}_{1\perp}) W^\dagger(\mathbf{x}_{2\perp}) W(\mathbf{x}_{3\perp}) W^\dagger(\mathbf{x}_{4\perp}) \rangle \\ & \propto \text{Tr} \langle W(\mathbf{x}_{1\perp}) W^\dagger(\mathbf{x}_{2\perp}) \rangle \text{Tr} \langle W(\mathbf{x}_{3\perp}) W^\dagger(\mathbf{x}_{4\perp}) \rangle \\ & - \text{Tr} \langle W(\mathbf{x}_{1\perp}) W^\dagger(\mathbf{x}_{4\perp}) \rangle \text{Tr} \langle W(\mathbf{x}_{3\perp}) W^\dagger(\mathbf{x}_{2\perp}) \rangle \end{aligned}$$

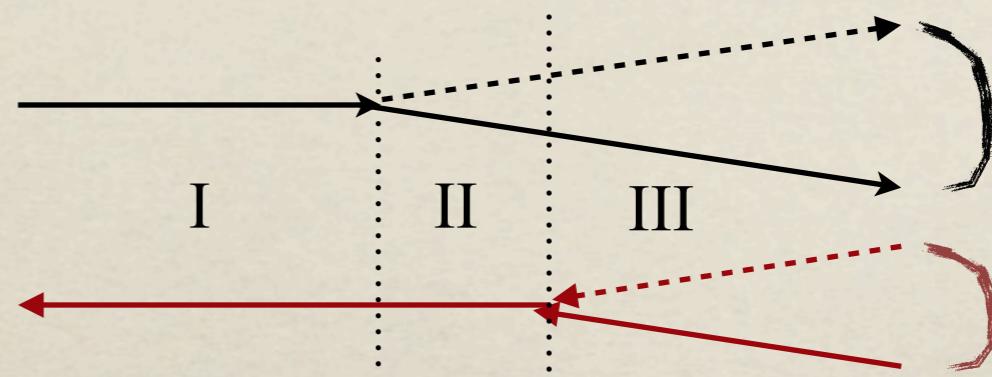


Color Structure

- Color final configurations:



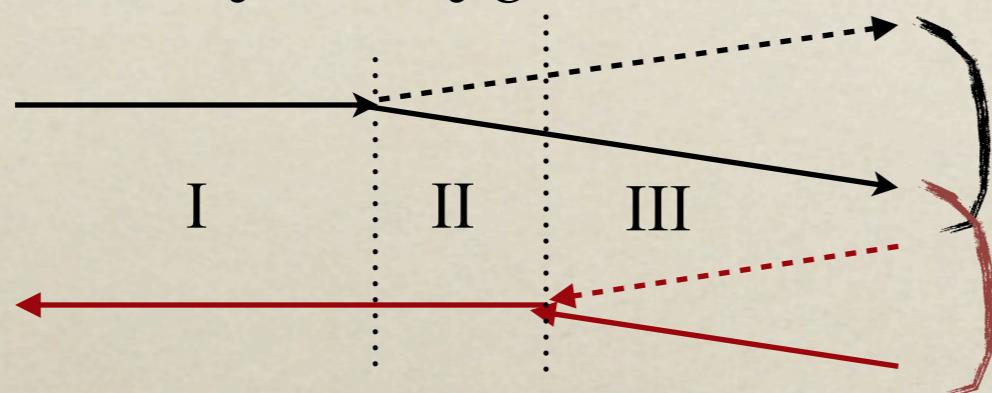
Quark and gluon factorize



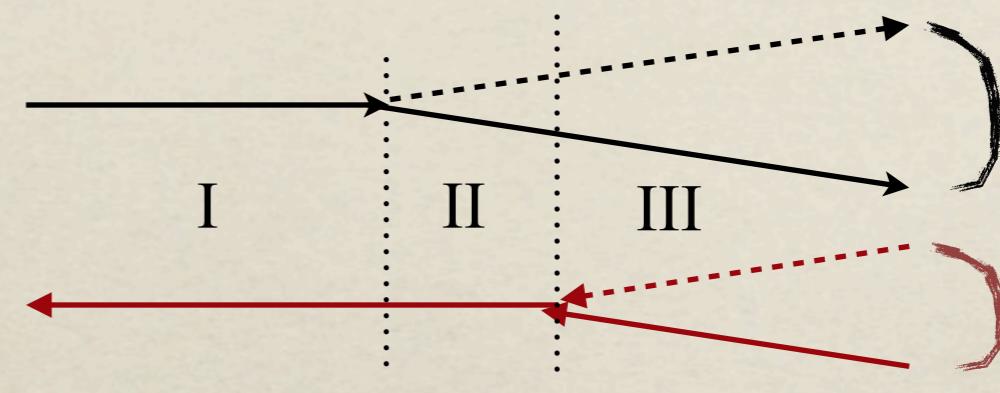
Correlation between final partons

Color Structure

- Color final configurations:

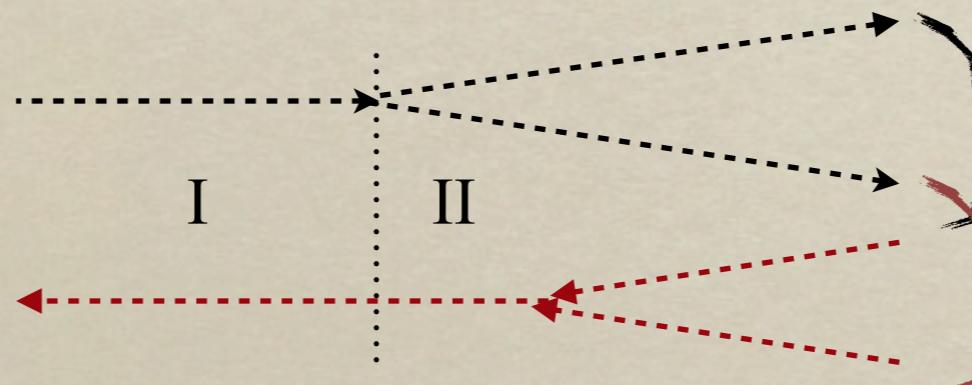


Quark and gluon factorize



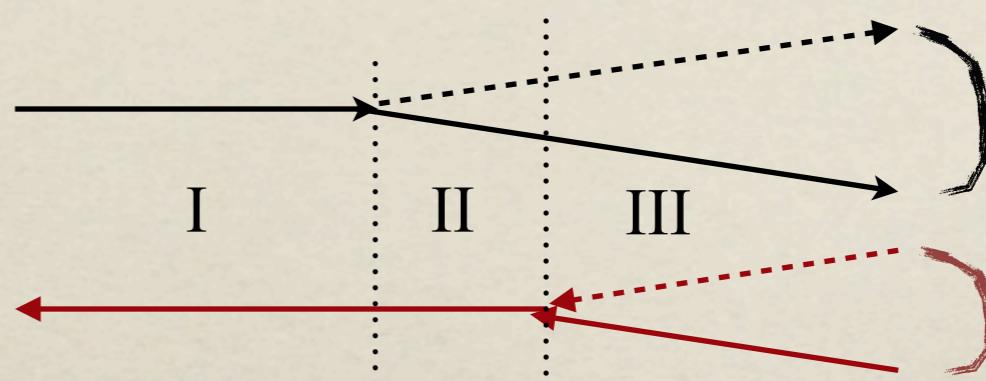
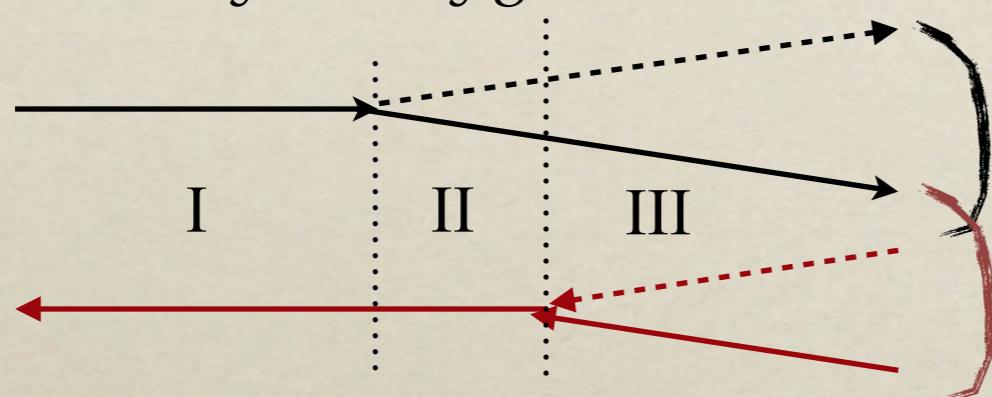
Correlation between final partons

- In the $g \rightarrow gg$ case, only one configuration (final particles are indistinguishable):



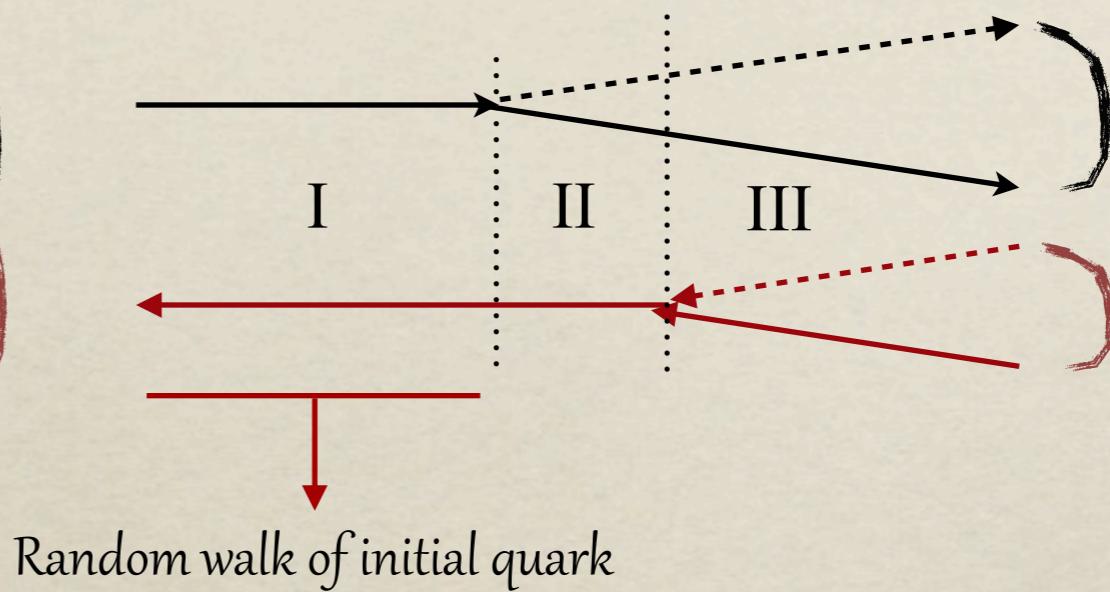
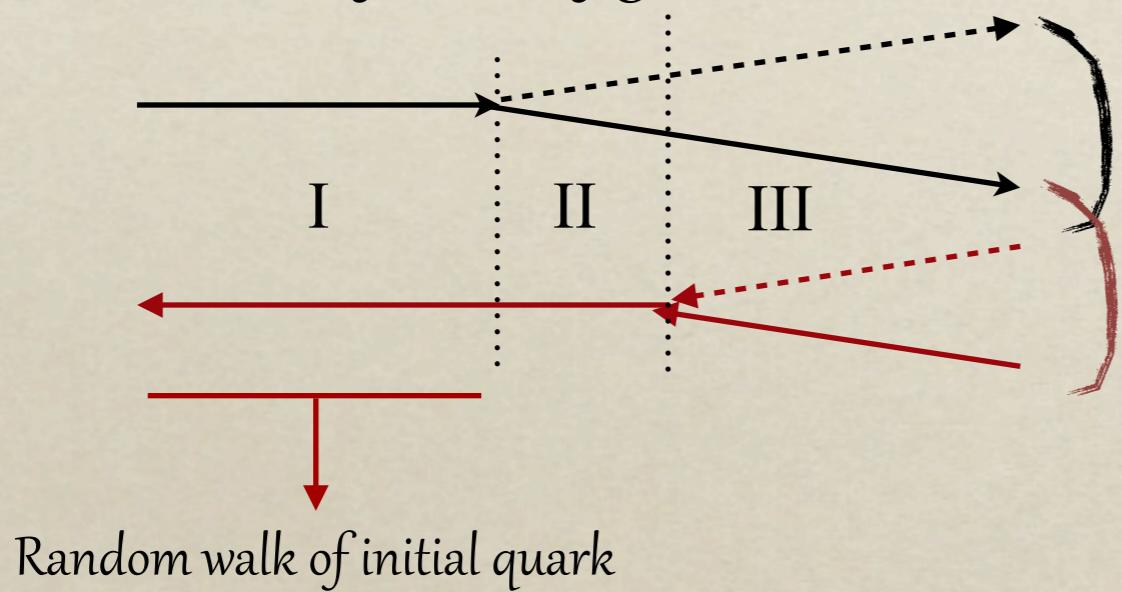
Color Structure

- Color final configurations:



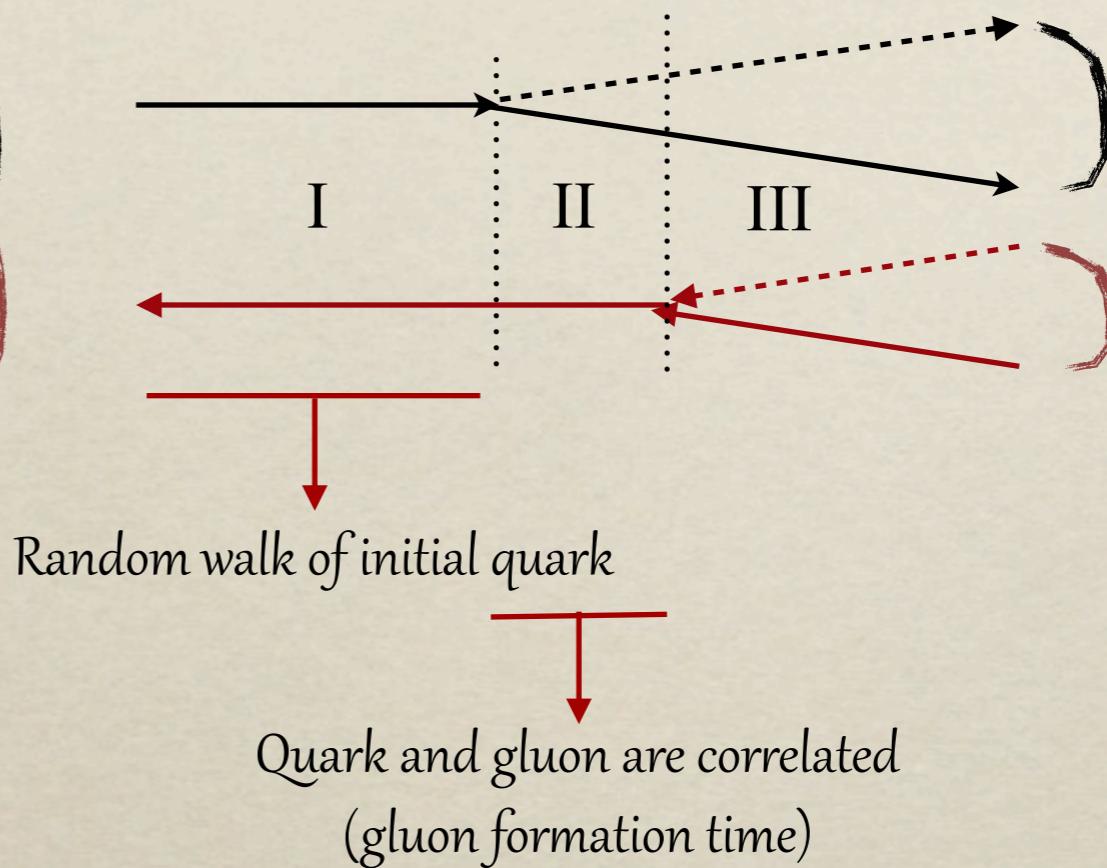
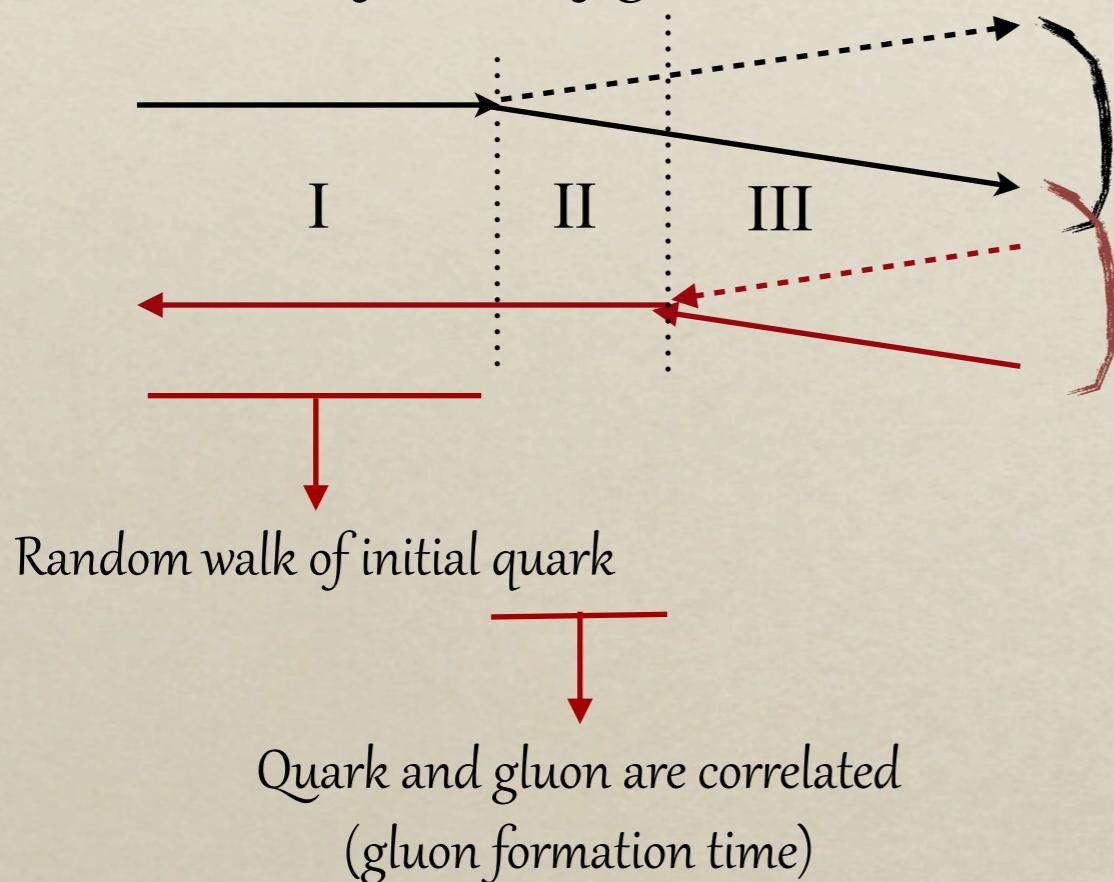
Color Structure

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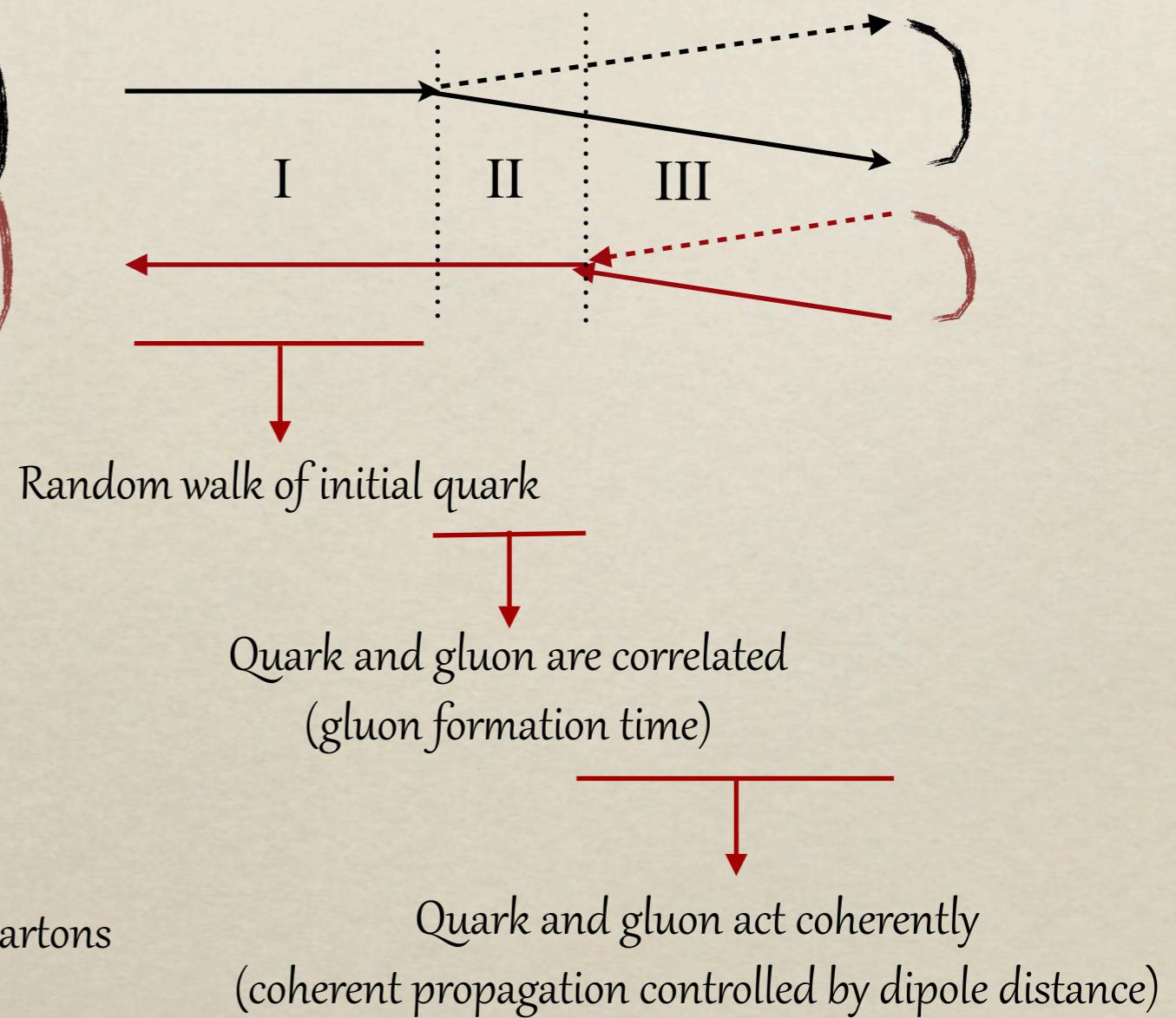
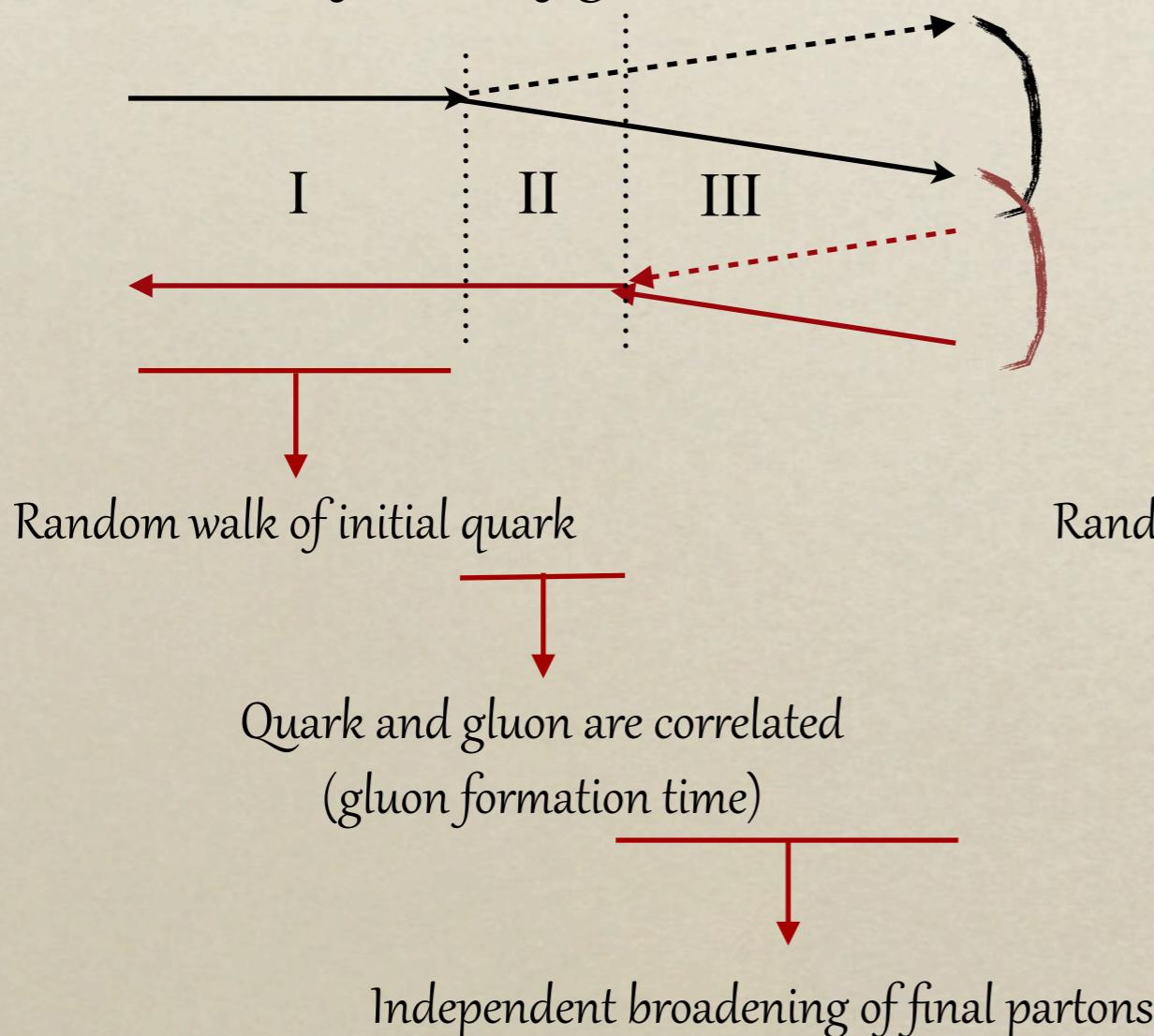
Color Structure

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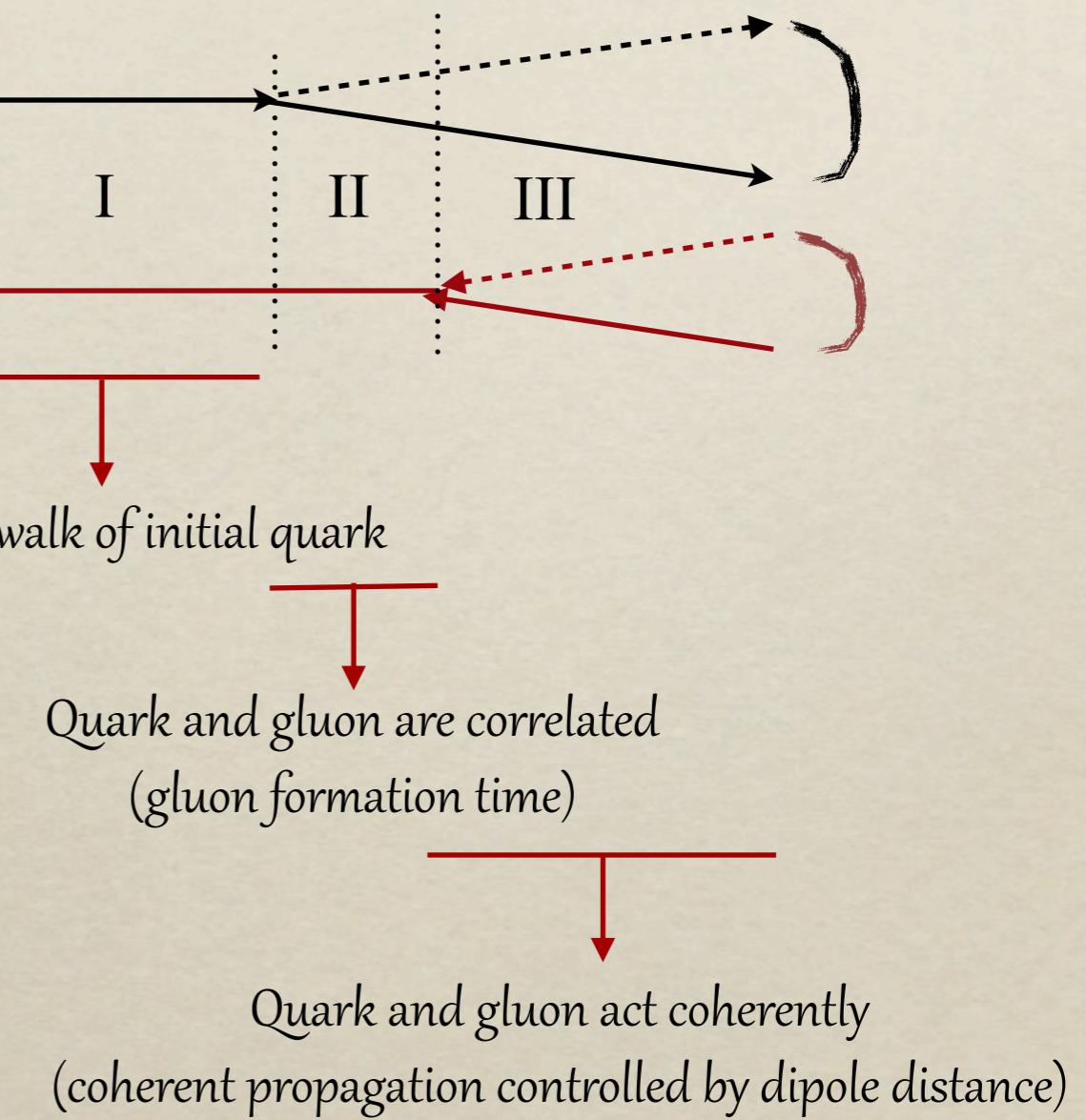
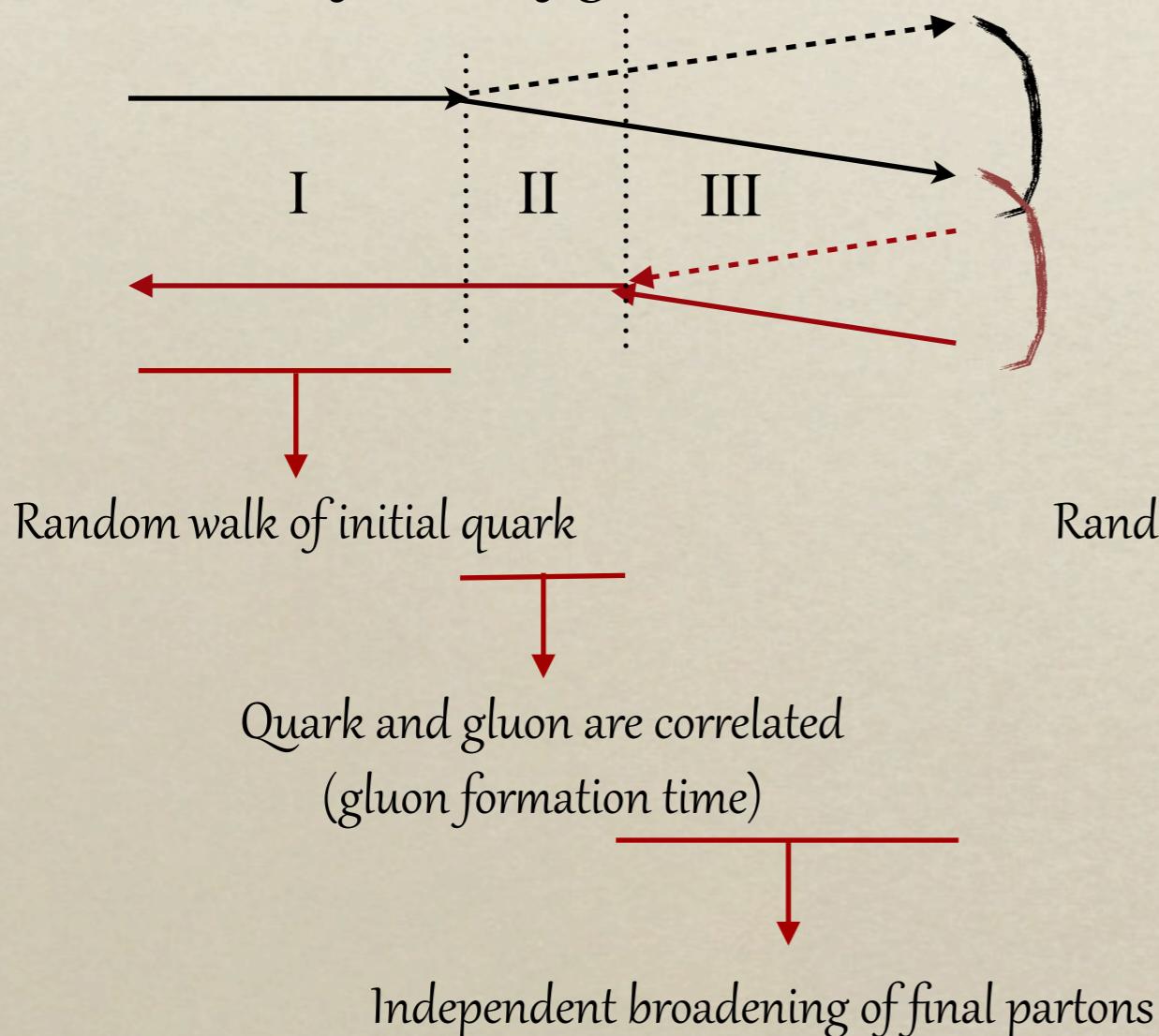
Color Structure

- Color final configurations:



Color Structure

- Color final configurations:



Summary (II)

- Extension beyond eikonal approximation:
 - Able to extend previous work by allowing all particles undergo Brownian motion in the transverse plane;
 - Results in the large N_c limit, but with no constrains on the formation time;
 - Able to solve the quadrupole in this limit;
 - Dirac and color structure with more information since there is no symmetry between the final partons;
 - Interference term included in calculations;
 - Found a “generalization” of a non-eikonal in-medium antenna.

Conclusions/Prospects

- Study of Jet Quenching is important since it provides a way of probing matter created in heavy-ion collisions.
- Further understanding the mechanisms of interaction and propagation with the medium:
 - Already several efforts up to now:
 - Medium-induced hard gluon radiation, Medium-induced gluon branching (small t_{form}), (Massive) Medium antenna, SCFT...

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Thanks!

Backup Slides

Interpolating Spectrum

- Multiple Soft Scattering Approximation:

$$n(\xi)\sigma(\mathbf{r}_\perp) \simeq \frac{1}{2}\hat{q}(\xi)\mathbf{r}_\perp^2 + \mathcal{O}(\ln r^2)$$

- Static medium: $\hat{q} = \mu^2/\lambda$
- Allow us to find analytical expressions:

$$\begin{aligned}\mathcal{K}(y_+, \mathbf{y}_\perp = \mathbf{0}_\perp; \bar{y}_+, \mathbf{x}_\perp | q_+) &= \mathcal{K}_{osc}(y_+, \mathbf{y}_\perp = \mathbf{0}_\perp; \bar{y}_+, \mathbf{x}_\perp | q_+) \\ &= \frac{A_1}{\pi i} \exp [i A_1 B_1 (\mathbf{x}_\perp^2 + \mathbf{y}_\perp^2) - 2i A_1 \mathbf{x}_\perp \cdot \mathbf{y}_\perp]\end{aligned}$$

$$A_1 = \frac{q_+ \Omega}{2 \sin [\Omega(\bar{y}_+ - y_+)]} , \quad B_1 = \cos [\Omega(\bar{y}_+ - y_+)] , \quad \Omega = \frac{1-i}{2} \sqrt{\frac{\hat{q}_F}{q_+}}$$