Probabilistic picture for Jet evolution in Heavy-Ion Collisions

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In collaboration with

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arXiv: 1209.4585 [hep-ph] JHEP 1301 (2013) 143
arXiv: 1301.6102 [hep-ph]

work in progress...

OUTLINE

- Motivation: in-medium jet modification at the LHC
- Probabilistic picture for in-medium jet evolution:

factorization of multiple-branchings:

1 - Incoherent branchings: Time scale separation, $t_{\rm br} \ll L$

resum. large $\alpha_s L$

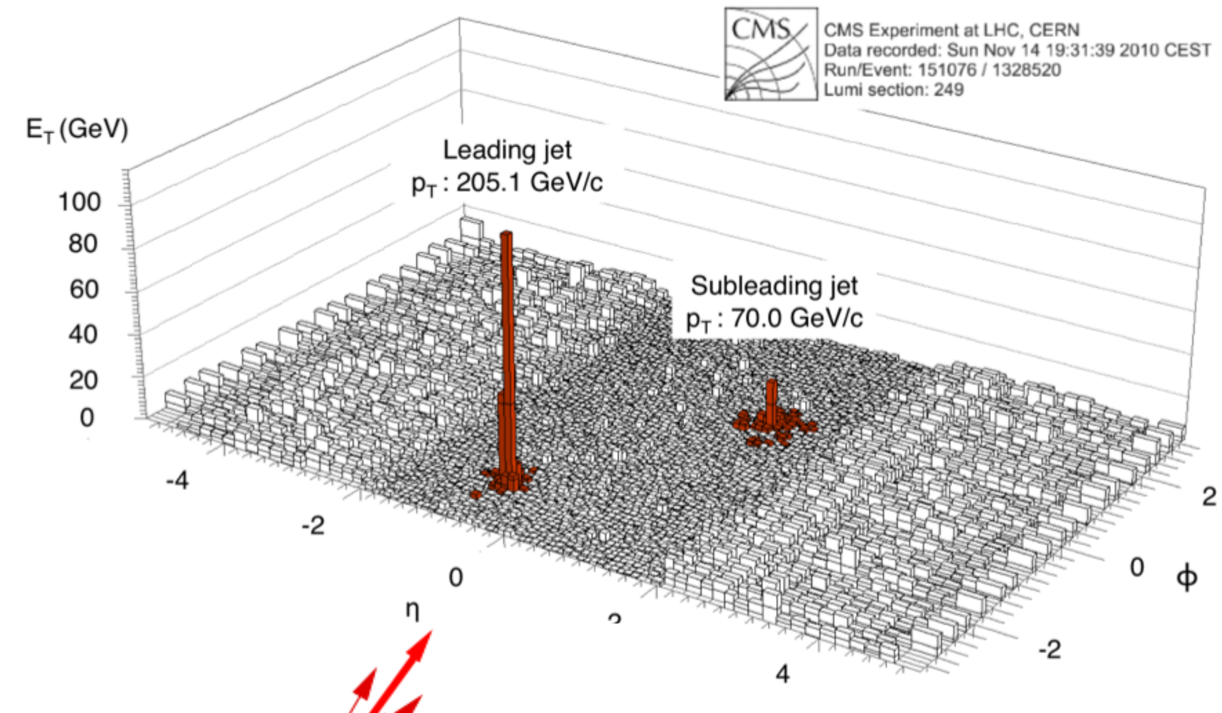
2 - Coherent branchings: resum. Double Logs $\alpha_s \log^2\left(\hat{q}L/m_D^2\right)$

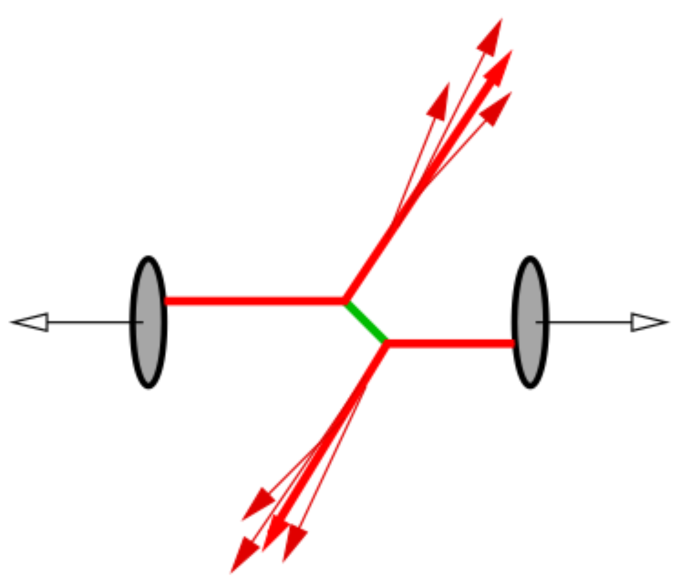
in a renormalization of the quenching parameter $\hat{q}$

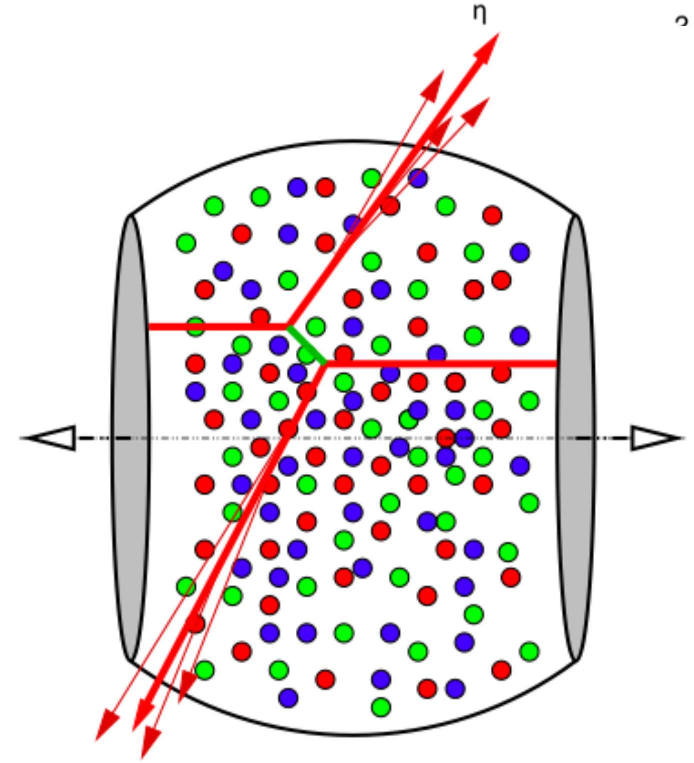
Jets in HIC at the LHC

- JET QUENCHING :

a tool to probe the Quark-Gluon-Plasma and QCD dynamics at high parton density



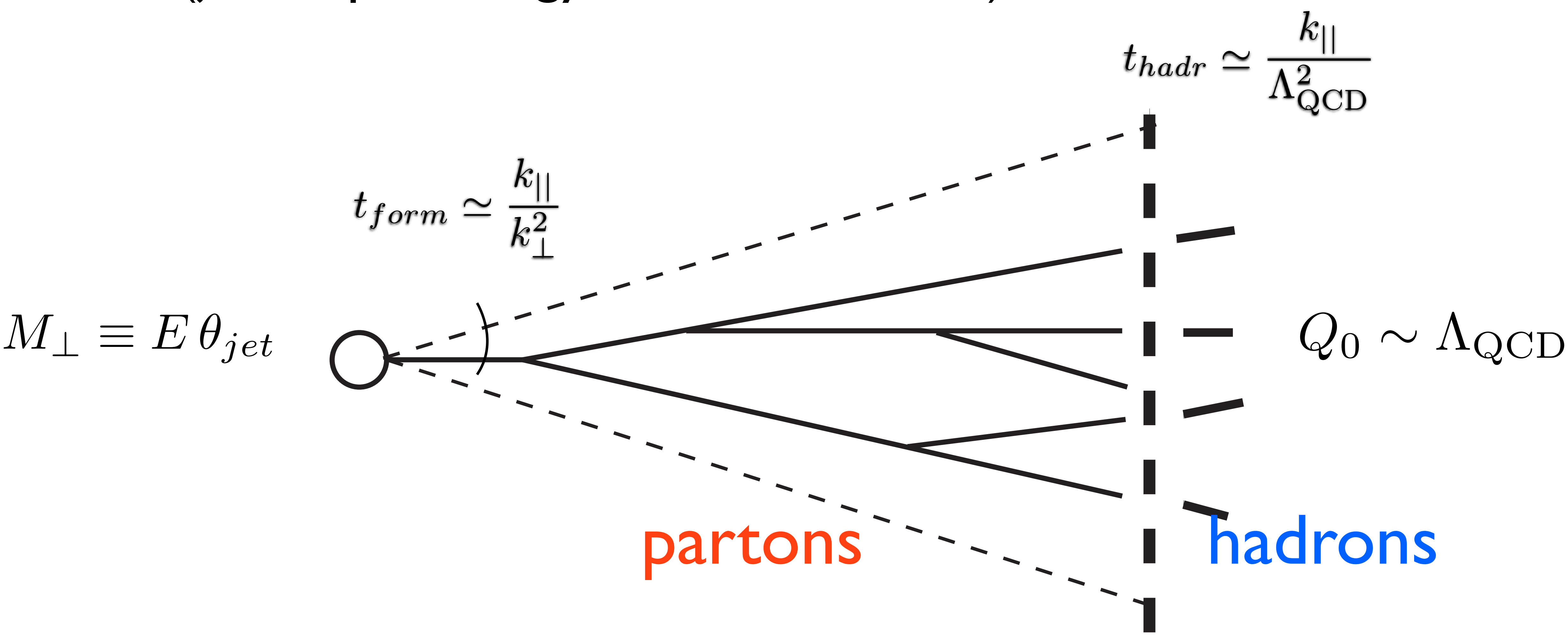




- in-medium jet modification: departures from p-p baseline

JETS IN VACUUM

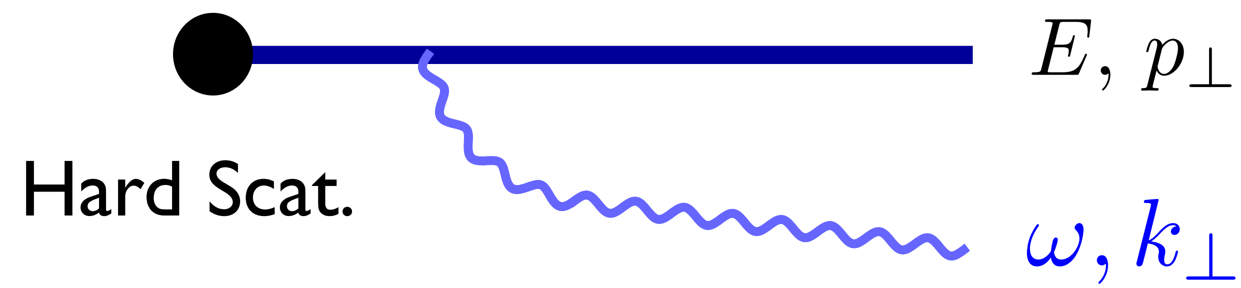
- Originally a hard parton (quark/gluon) which fragments into many partons with virtuality down to a non-perturbative scale where it hadronizes
- LPHD: Hadronization does not affect inclusive observables (jet shape, energy distribution etc..)



JETS IN VACUUM

- The differential branching probability

$$dP \simeq \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d^2 k_\perp}{k_\perp^2}$$



  - soft and collinear singularities

$$Q_0 < k_\perp < M_\perp$$

- Phase-space enhancement (Double Logs)

$$\boxed{\alpha_s \to \alpha_s \ln^2 \frac{M_\perp}{Q_0}}$$

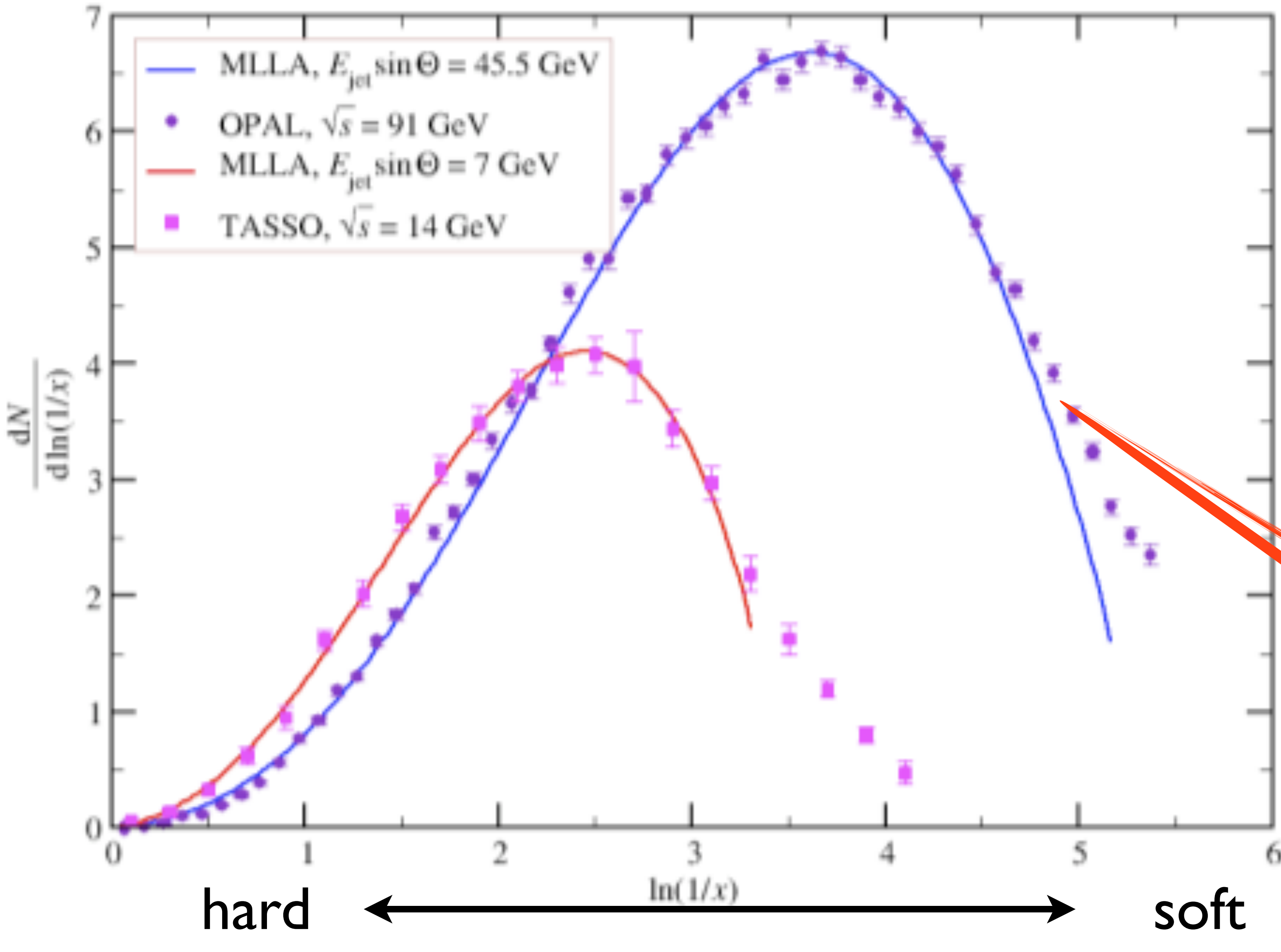
- Multiple branchings are not independent and obeys Angular Ordering (for inclusive observables): Due to color coherence (interferences) large-angle gluon emissions are strongly suppressed. AO ordering along the parton cascade :

$$\theta_{jet} > \theta_1 > \ldots > \theta_n$$

[Bassetto, Mueller, Ciafaloni, Marchesini, Dokshitzer, Khoze, Troyan, Fadin, Lipatov, 80's]

JETS IN VACUUM

Fragmentation function



$$D(x) \equiv x\frac{dN}{dx}$$

$$x \equiv E_h/E_{jet}$$

AO limits phase space for soft emissions!

TASSO Collaboration, Z. Phys. C 47 (1990) 187

OPAL Collaboration, Phys. Lett. B 247 (1990) 617

IN-MEDIUM JET EVOLUTION

- What is the space-time structure of in-medium jets?

- probabilistic picture?

  resummation scheme?

  ordering variable?

MEDIUM-INDUCED GLUON RADIATION

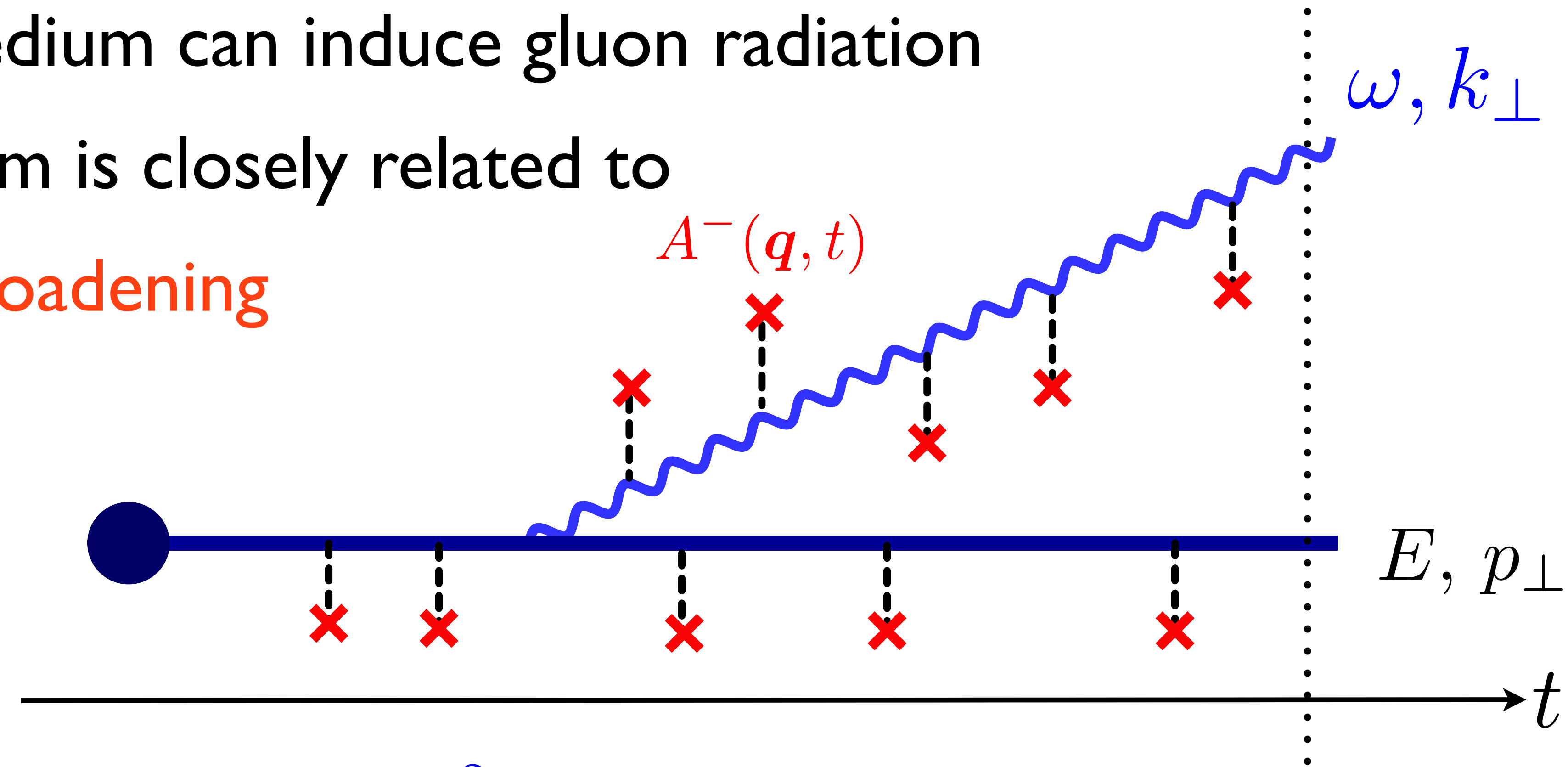
Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996) Arnold, Moore, Yaffe (2001)

- Scatterings with the medium can induce gluon radiation
- The radiation mechanism is closely related to

transverse momentum broadening

$$\Delta k_\perp^2 \simeq \hat{q}\, \Delta t$$

- where the quenching parameter



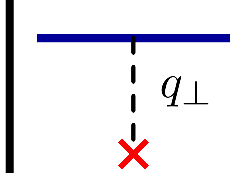
$$\hat{q} \equiv \int_{\boldsymbol{q}} \boldsymbol{q}^2\, \mathcal{C}(\boldsymbol{q}) \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

LC time $t \equiv x^+$

L

is related to the collision rate in a thermal bath

P. Aurenche, F. Gelis and H. Zaraket, (2002)

$$\mathcal{C}(\boldsymbol{q}, t) = 4\pi\alpha_s\, C_R\, n(t)\, \gamma(\boldsymbol{q}) \equiv \left| \vphantom{\frac{a}{b}} \right.$$  $$\left. \vphantom{\frac{a}{b}} \right|^2$$

where $\gamma(\boldsymbol{q}) = \dfrac{g^2}{\boldsymbol{q}^2(\boldsymbol{q}^2 + m_D^2)}$

Independent scatterings: Gaussian distribution for the background field $A^-(\boldsymbol{q}, t)$

$$\left\langle A_a^-(\boldsymbol{q}, t) A_b^{*-}(\boldsymbol{q}', t') \right\rangle = \delta_{ab}\, n(t)\, \delta(t - t')(2\pi)^2 \delta^{(2)}(\boldsymbol{q} - \boldsymbol{q}')\, \gamma(\boldsymbol{q}) \,,$$

MEDIUM-INDUCED GLUON RADIATION

- How does it happen? After a certain number of scatterings coherence between the parent quark and gluon fluctuation is broken and the gluon is formed (decoherence is faster for soft gluons)

$$t_f \equiv \frac{\omega}{\langle q_\perp^2 \rangle} \simeq \frac{\omega}{\hat{q}\, t_f} \quad \Rightarrow \quad t_f = t_{\rm br} \equiv \sqrt{\frac{\omega}{\hat{q}}}$$

- The BDMPS spectrum $$\omega \frac{dN}{d\omega} = \frac{\alpha_s C_R}{\pi} \sqrt{\frac{2\omega_c}{\omega}} \propto \alpha_s \frac{L}{t_{\rm br}}$$

with $\omega_c = \frac{1}{2}\hat{q}\, L^2$ is the maximum frequency at which the medium acts fully coherently on the (maximum suppression). Typically, $\omega_c \simeq 50\, GeV$

- Soft gluon emissions $\omega \ll \omega_c$

  ➡ Short branching times $t_{\rm br} \ll L$ and large phase-space:

When $\alpha_s \frac{L}{t_{\rm br}} \gtrsim 1$ Multiple branchings are no longer negligible

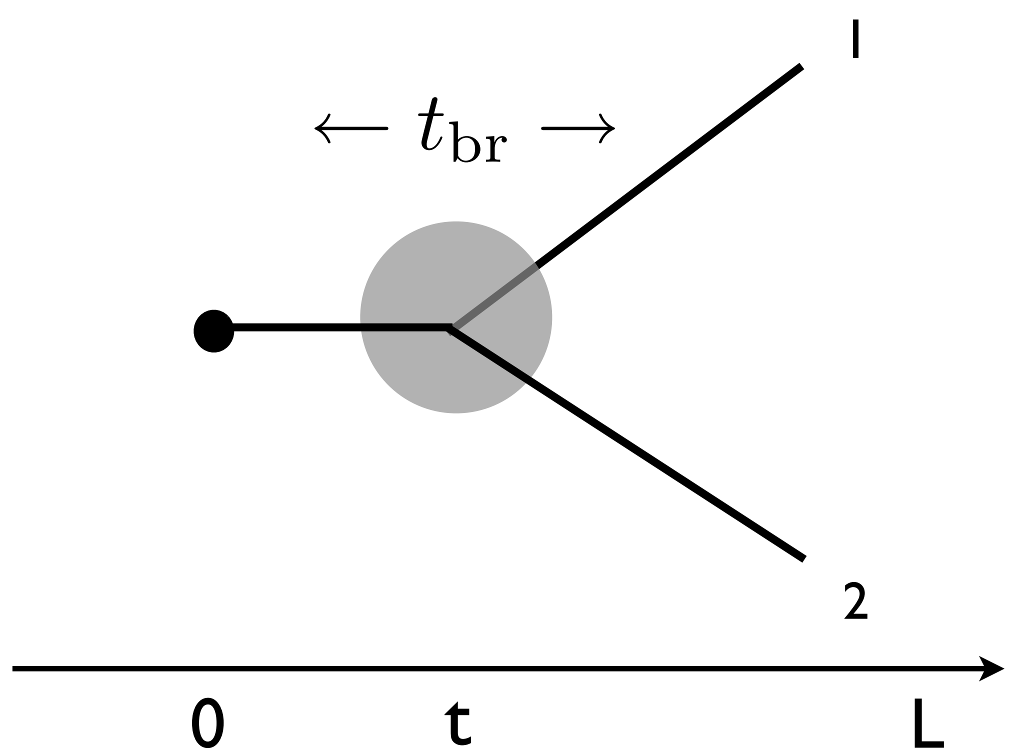
BUILDING IN-MEDIUM JET EVOLUTION:
Some necessary steps

➡ Going beyond the eikonal (soft gluon) approximation

➡ Fully differential in momentum space

➡ Factorization of multiple branchings in the decoherence regime

DECOHERENCE OF MULTI-GLUON EMISSIONS



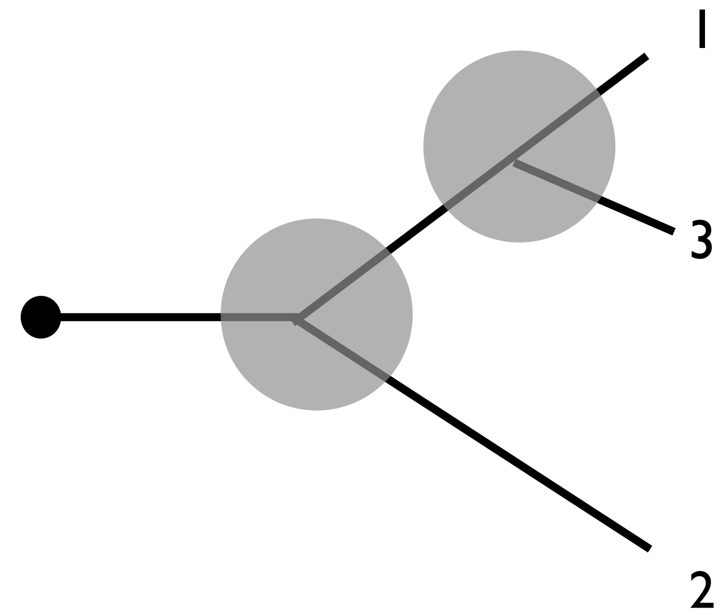
- The branching can occur anywhere along the medium with a constant rate
- Time scale separation: compared to the time scale of the jet evolution in the medium L the branching process is quasi-local $t_{\rm br} \ll L$
- Off-spring gluons are independent after they are formed as they are separated over a distance that is larger then the in-medium correlation length

[See F. Dominguez's talk]

DECOHERENCE OF MULTI-GLUON EMISSIONS

incoherent emissions

- For large media two subsequent emissions are independent and therefore factorize

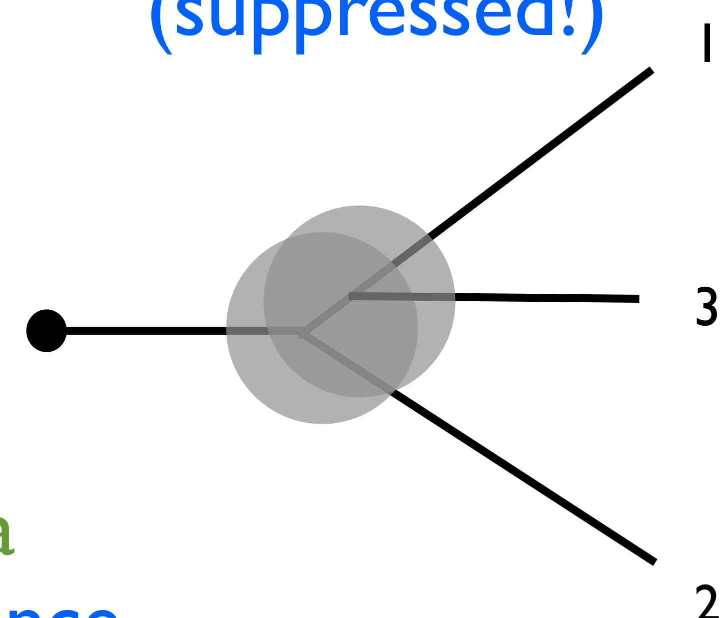


$$\propto \left(\alpha_s \frac{L}{t_{\rm br}}\right)^2$$

coherent emissions
(suppressed!)

- Interferences are suppressed by a factor

$$t_{\rm br}/L \ll 1$$

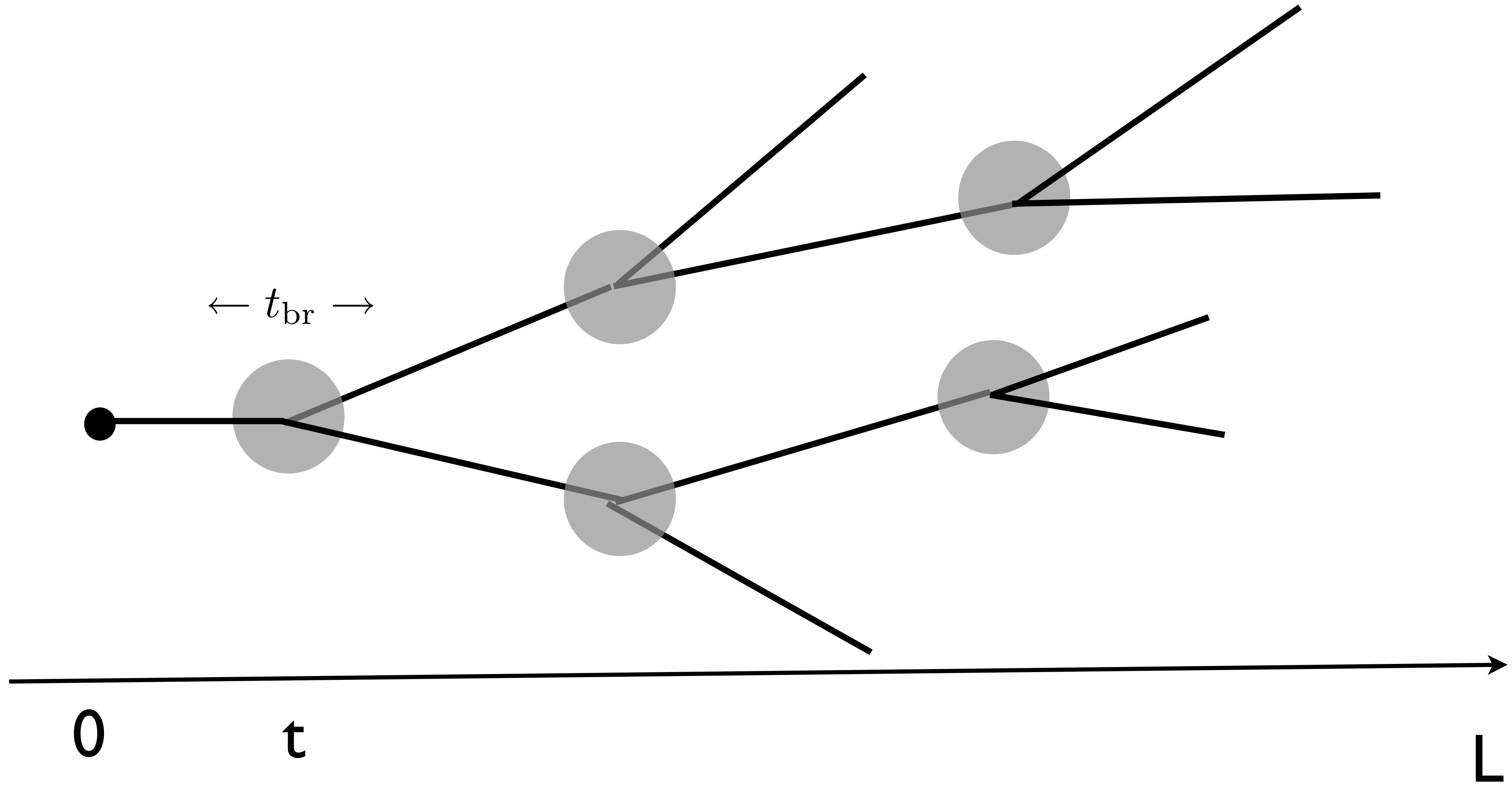


$$\propto \alpha_s \left(\alpha_s \frac{L}{t_{\rm br}}\right)$$

Note that this is not the case in a vacuum shower where color coherence is responsible for Angular-Ordering

Y. M.-T, K. Tywoniuk, C.A. Salgado (2010-2012)
J. Casalderray-Solana, E. Iancu (2011)

DECOHERENCE OF MULTI-GLUON EMISSIONS



Successive branchings are then independent and quasi-local.

Time-scale separation: $t_{\rm br} \ll t \sim L$

Markovian Process

$\Rightarrow$ Probabilistic Scheme $\qquad \sigma = \sum_n a_n \left(\alpha_s \frac{L}{t_{\rm br}} \right)^n$

Building blocks of medium-induced cascade

I - The rate of elastic scatterings   C

II - The rate of inelastic scatterings   K

I - Rate for inelastic scatterings

The rate of elastic scatterings reads

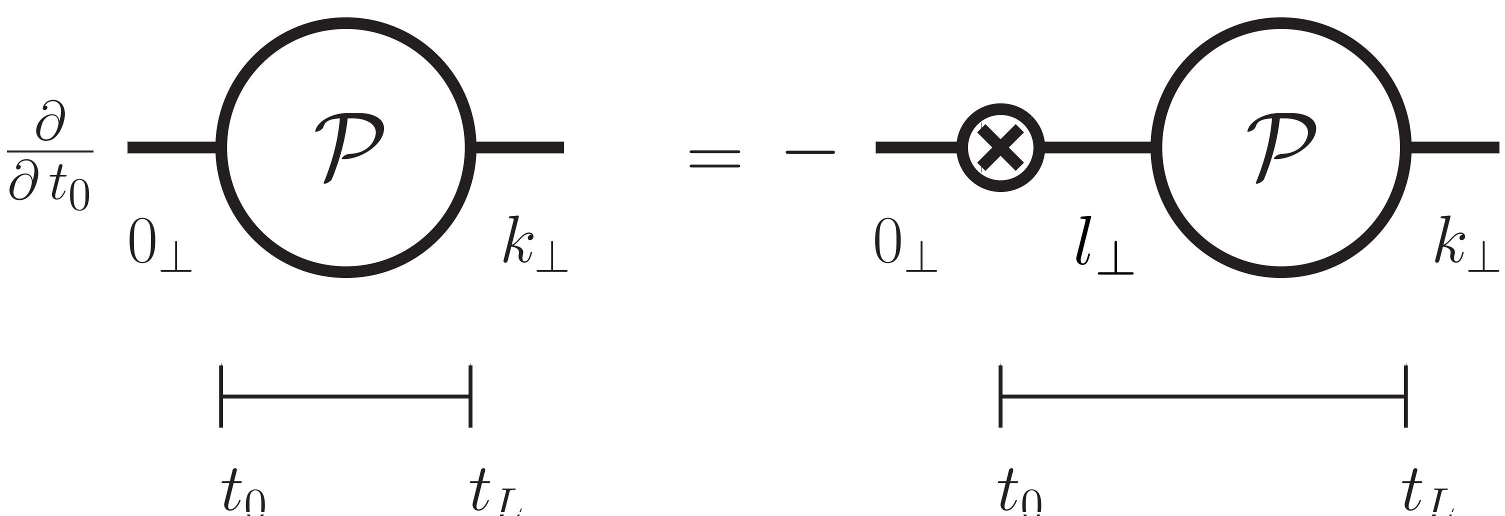
$$\mathcal{C}(\boldsymbol{l},t) = 4\pi\alpha_s C_A n(t)\left[\gamma(\boldsymbol{l}) - \delta^{(2)}(\boldsymbol{l})\int \mathrm{d}^2\boldsymbol{q}\,\gamma(\boldsymbol{q})\right]$$

- when there are no branchings partons scatter off the color charges of the medium and acquire a transverse momentum $k_\perp$ after a time

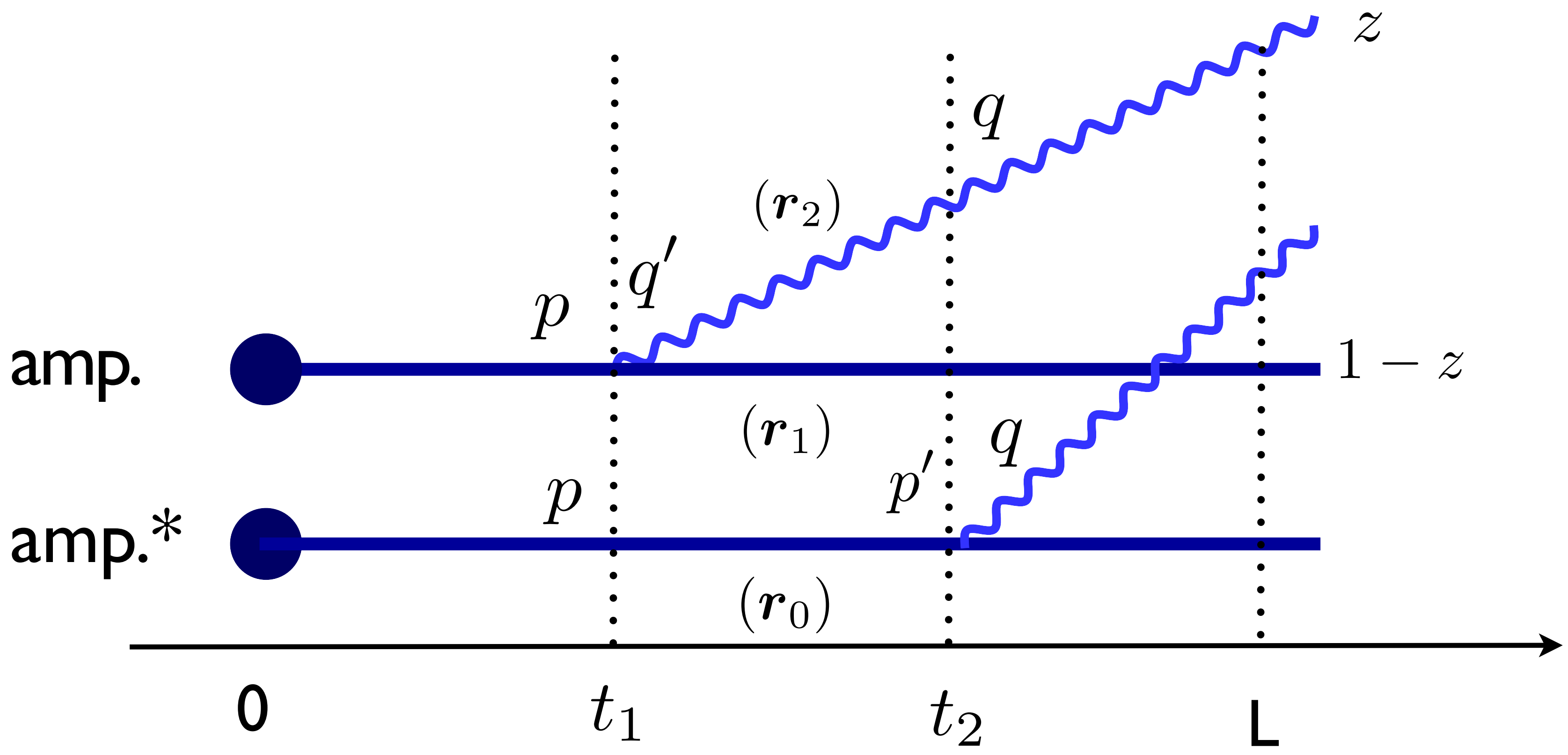
$\Delta t = t_L - t_0$ with a probability $\mathcal{P}$

- The broadening a probability obeys the evolution equation

$$\boxed{\frac{\partial}{\partial t_0}\mathcal{P}(\boldsymbol{k};t_L,t_0) = -\int \frac{\mathrm{d}^2\boldsymbol{l}}{(2\pi)^2}\,\mathcal{C}(\boldsymbol{l},t_0)\,\mathcal{P}(\boldsymbol{k}-l;t_L,t_0)\,,}$$



I - Rate for inelastic scatterings



the dipole cross-section is related to the collision rate

$$\sigma(\boldsymbol{r}) = \int_{\boldsymbol{q}} C(\boldsymbol{q})\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$$

- The 3-point function correlator accounts for instantaneous multiple scatterings of a 3 parton syst. It solves the Dyson-like equation

$$S^{(3)}(t_2,t_1) = S_0^{(3)}(t_2,t_1) + \int_{t_1}^{t_2} dt'\, S_0^{(3)}(t_2,t')\,\sigma_3(t')\,S^{(3)}(t',t_1)$$

- It is related to the expectation value of 3 wilson lines at time-dependent transverse coordinates (Brownian motion in T-space)

$$S^{(3)} \sim \langle\, \mathrm{tr}\, T^a\, U_F(\boldsymbol{r}_1)\, T^b\, U_F^\dagger(\boldsymbol{r}_0) U_{ab}(\boldsymbol{r}_2)\rangle_{\rm med}$$

I - Rate for inelastic scatterings

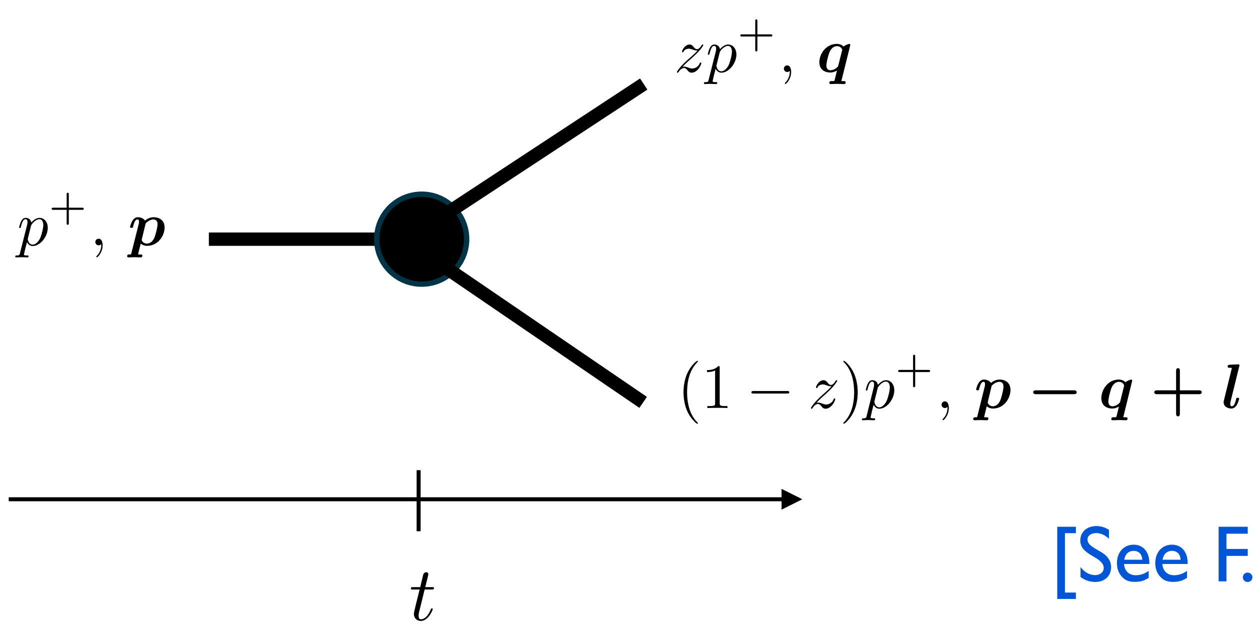
- We work in the approximation of small branching times: $\Delta t \equiv t_2 - t_1 \sim t_{\rm br} \ll t_1, t_2$

Hence, one can neglect the difference Δt everywhere except in the 3-point function,

$$\int_0^L dt_1 \int_{t_1}^L dt_2 \approx \int_0^L dt \int_0^\infty d\Delta t$$

Therefore, independent branchings can be described by the quasi-local branching rate K and t is the ordering variable

$$\mathcal{K}(\boldsymbol{Q}, \boldsymbol{l}, z, p^+; t) \equiv \frac{P_{gg}(z)}{[z(1-z)p^+]^2} \, \mathrm{Re} \int_0^\infty \mathrm{d}\Delta t \int_{\boldsymbol{P}} (\boldsymbol{P}\cdot\boldsymbol{Q}) \, S^{(3)}(\boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{l}, z, p^+; t+\Delta t, t)$$



[See F. Dominguez's talk]

Differential gluon distribution

The inclusive distribution of gluons with momentum k inside a parton with momentum p is defined as (with $x \equiv k^+/p^+$):

$$k^+ \frac{\mathrm{d}\ N}{\mathrm{d}k^+ \mathrm{d}^2\boldsymbol{k}} (k^+, \boldsymbol{k}, p^+, \boldsymbol{p}; t_L, t_0) \equiv D(x, \boldsymbol{k} - x\boldsymbol{p}, p^+; t_L, t_0)\,,$$

Rotational invariance implies the dependence on the reduced variable

$$\boldsymbol{k} - x\boldsymbol{p} \equiv \omega(\boldsymbol{\theta}_{\boldsymbol{k}} - \boldsymbol{\theta}_{\boldsymbol{p}}) = \omega\boldsymbol{\theta}_{\boldsymbol{k}p}$$

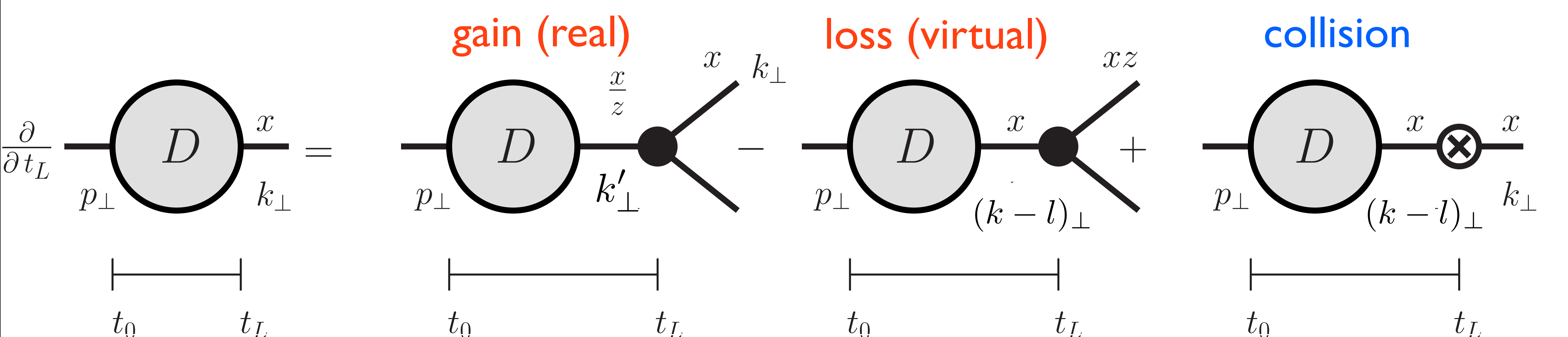
Differential gluon distribution

Given the branching and elastic rates K(t) and C(t) respectively, with t being the ordering variable, it is then straightforward to write the evolution equation for D

$$\frac{\partial}{\partial t_L} D(x,\boldsymbol{k},t_L) = \alpha_s \int_0^1 \mathrm{d}z \int_{\boldsymbol{Q},\boldsymbol{l}} \left[2\mathcal{K}\left(\boldsymbol{Q},\boldsymbol{l},z,\frac{x}{z}p_0^+,t_L\right) D\left(\frac{x}{z},(\boldsymbol{k}-\boldsymbol{Q}-z\boldsymbol{l})/z,t_L\right) \right.$$
$$\left. -\mathcal{K}\left(\boldsymbol{Q},\boldsymbol{l},z,xp_0^+,t_L\right) D\left(x,\boldsymbol{k}-\boldsymbol{l},t_L\right)\right] - \int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l},t_L) D\left(x,\boldsymbol{k}-l,t_L\right) .$$

[Integrating over kt we recover the rate equation: Baier, Mueller, Schiff, Son (2001) Jeon , Moore (2003),]

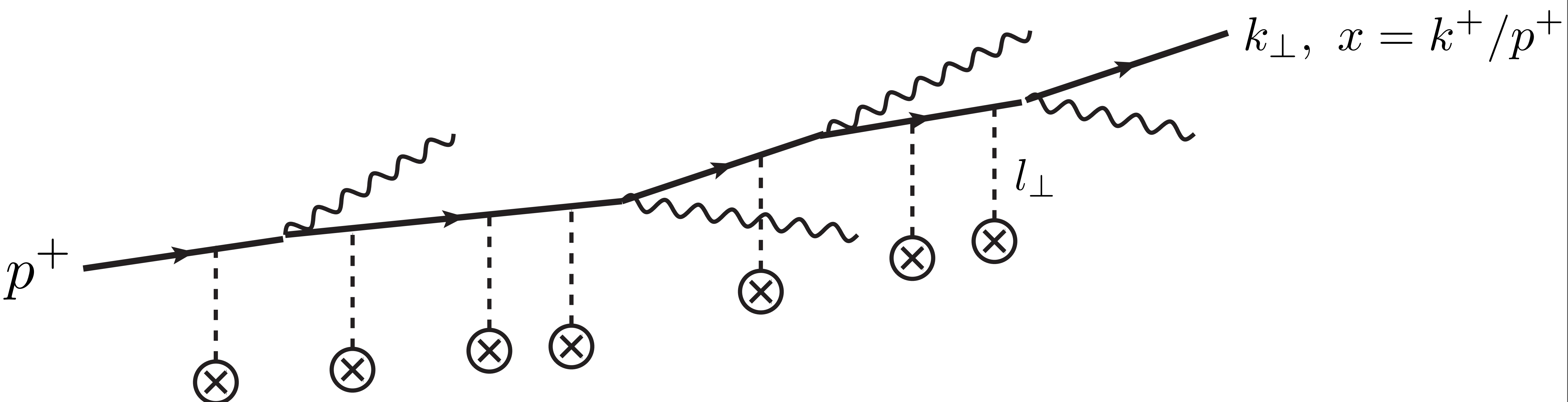
[See J.-P. Blaizot's talk]



Renormalization of the quenching parameter

Diffusion approximation

Let us consider a highly energetic particle passing through the medium : x ~ 1 . The broadening acquired during a single scattering or a branching is small compared to the total broadening. This allows us to expand the distribution D for small transverse momentum exchange $l_\perp \ll k_\perp$



$$D\left(x, \boldsymbol{k}-\boldsymbol{l}\right) = D\left(x, \boldsymbol{k}\right) - \boldsymbol{l} \cdot \frac{\partial}{\partial \boldsymbol{k}} D\left(x, \boldsymbol{k}\right) + \frac{1}{2!} l^i l^j \frac{\partial}{\partial k_i} \frac{\partial}{\partial k_j} D\left(x, \boldsymbol{k}\right) + \cdots$$

Hence, the elastic term, where the quenching parameter appears naturally as a diffusion coefficient, yields

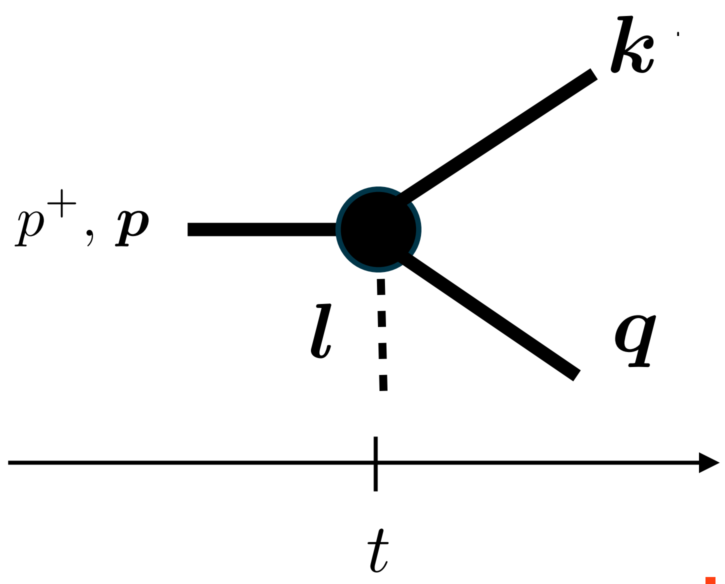
$$\int \frac{\mathrm{d}^2 \boldsymbol{l}}{(2\pi)^2} \mathcal{C}(\boldsymbol{l}, t_L)\, D\left(x, \boldsymbol{k}-\boldsymbol{l}, t_L\right) \approx \frac{1}{4} \hat{q}_0(t_L) \left(\frac{\partial}{\partial \boldsymbol{k}}\right)^2 D\left(x, \boldsymbol{k}, t_L\right) \, .$$

Renormalization of the quenching parameter

In the diffusion approximation the equation for D reduces to

$$\frac{\partial}{\partial t_L} D(x,\boldsymbol{k},t_L) = \alpha_s \int_0^1 \mathrm{d}z \left[2\mathcal{K}\left(z, \frac{x}{z}p^+, t_L\right) D\left(\frac{x}{z}, \frac{\boldsymbol{k}}{z}, t_L\right) - \mathcal{K}\left(z, xp^+, t_L\right) D\left(x, \boldsymbol{k}, t_L\right) \right]$$

$$- \frac{1}{4}\left[\hat{q}_0(t_L) + \hat{q}_1(t_L)\right] \left(\frac{\partial}{\partial \boldsymbol{k}}\right)^2 D\left(x, \boldsymbol{k}, t_L\right) \,.$$



Diffusion coefficient

Inelastic correction (radiation) can be absorbed in a redefinition of $\hat{q}(\boldsymbol{k}^2)$

elastic quenching parameter

$$\hat{q}_0(t) \equiv \int_{\boldsymbol{q}} \boldsymbol{q}^2\, \mathcal{C}(\boldsymbol{q}, t)$$

$$\hat{q}_1(t, \boldsymbol{k}^2) \equiv 2\alpha_s \int \mathrm{d}z \int_{\boldsymbol{q},\boldsymbol{l}}^{\boldsymbol{k}^2} \left[(\boldsymbol{q}+\boldsymbol{l})^2 - \boldsymbol{l}^2\right] \mathcal{K}\left(\boldsymbol{q}, \boldsymbol{l}, z, p^+, t\right)$$

Renormalization of the quenching parameter

We find a Double-Log (DL) enhancement in the radiative correction

$z \sim 1 \text{ and } \boldsymbol{q}^2 \gg k_{\rm br}^2 = \sqrt{\omega \hat{q}_0} \equiv \hat{q}\, t_{\rm br}$

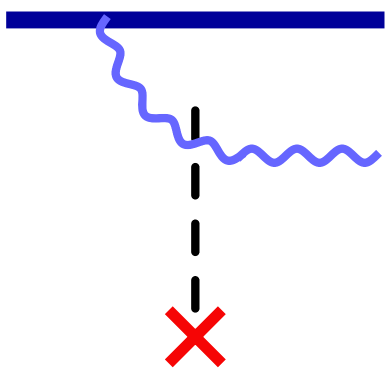
$$\hat{q}_1(t, \boldsymbol{k}^2) \approx \frac{\alpha_s C_A}{\pi} \int_{\hat{q}_0 \lambda^2}^{\boldsymbol{k}^4/\hat{q}_0} \frac{d\omega}{\omega} \int_{k_{\rm br}^2}^{\boldsymbol{k}^2} \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \hat{q}_0(t)$$

In agreement with a recent result on radiative corrections to pt-broadening. A. H. Mueller, B. Wu, T. Liou arXiv: 1304.7677

The double logs correspond to gluons that are formed before the medium resolves the system «gluon-emitter» (no LPM suppression)

Ordering in formation time

$$\frac{\omega}{k_\perp^2} \ll \frac{\omega}{q_\perp^2} \ll t_{\rm br} \equiv \sqrt{\frac{\omega}{\hat{q}_0}}$$



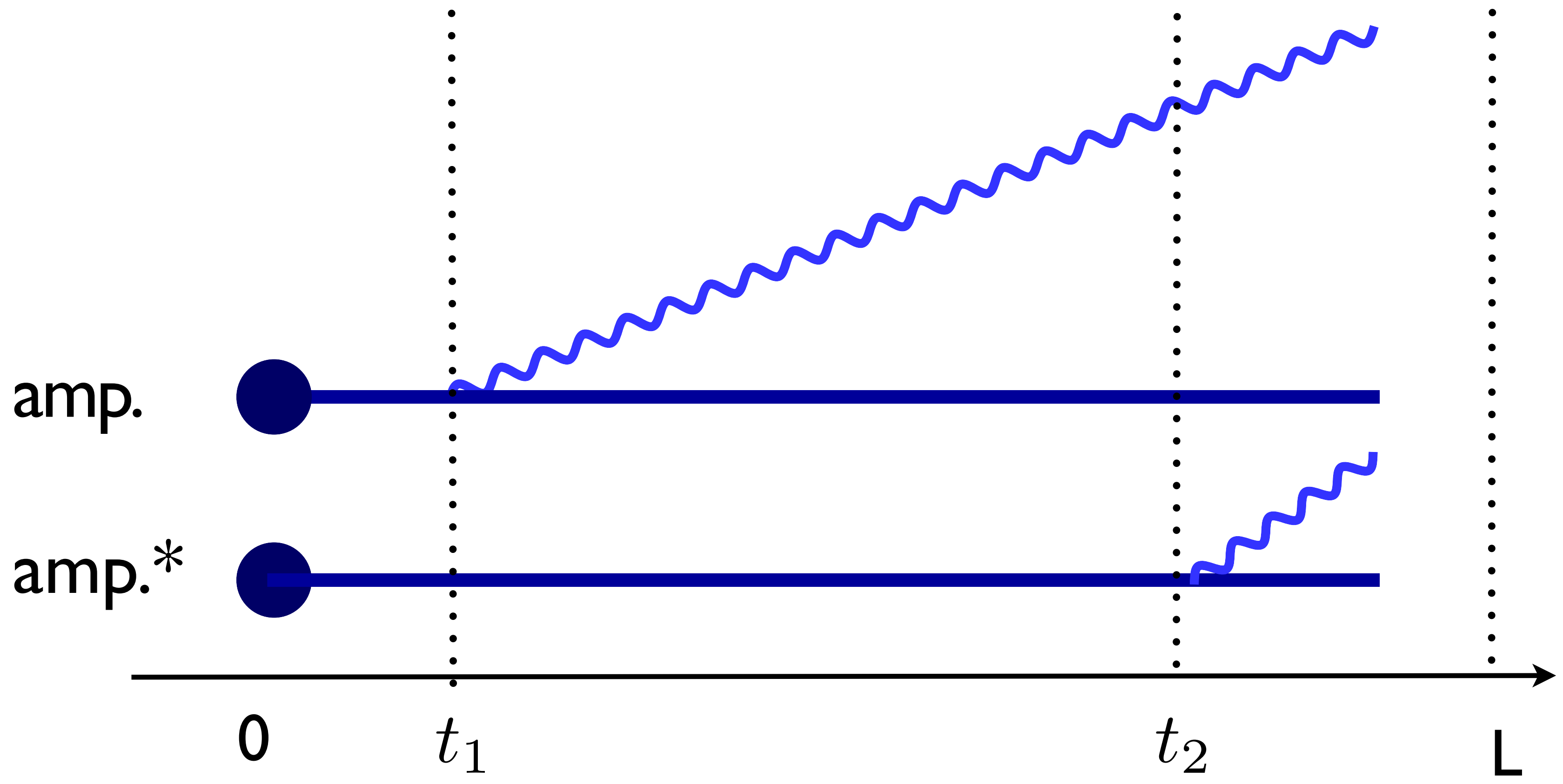
DL: a single (relatively) hard scattering

$$\hat{q}(t, \boldsymbol{k}^2) \approx \hat{q}_1(t, \boldsymbol{k}^2) + \hat{q}_0(t) \equiv \hat{q}_0(t) \left[1 + \frac{\alpha_s C_A}{2\pi} \log^2\left(\frac{\boldsymbol{k}^2}{m_D^2}\right)\right]$$

Renormalization of the quenching parameter

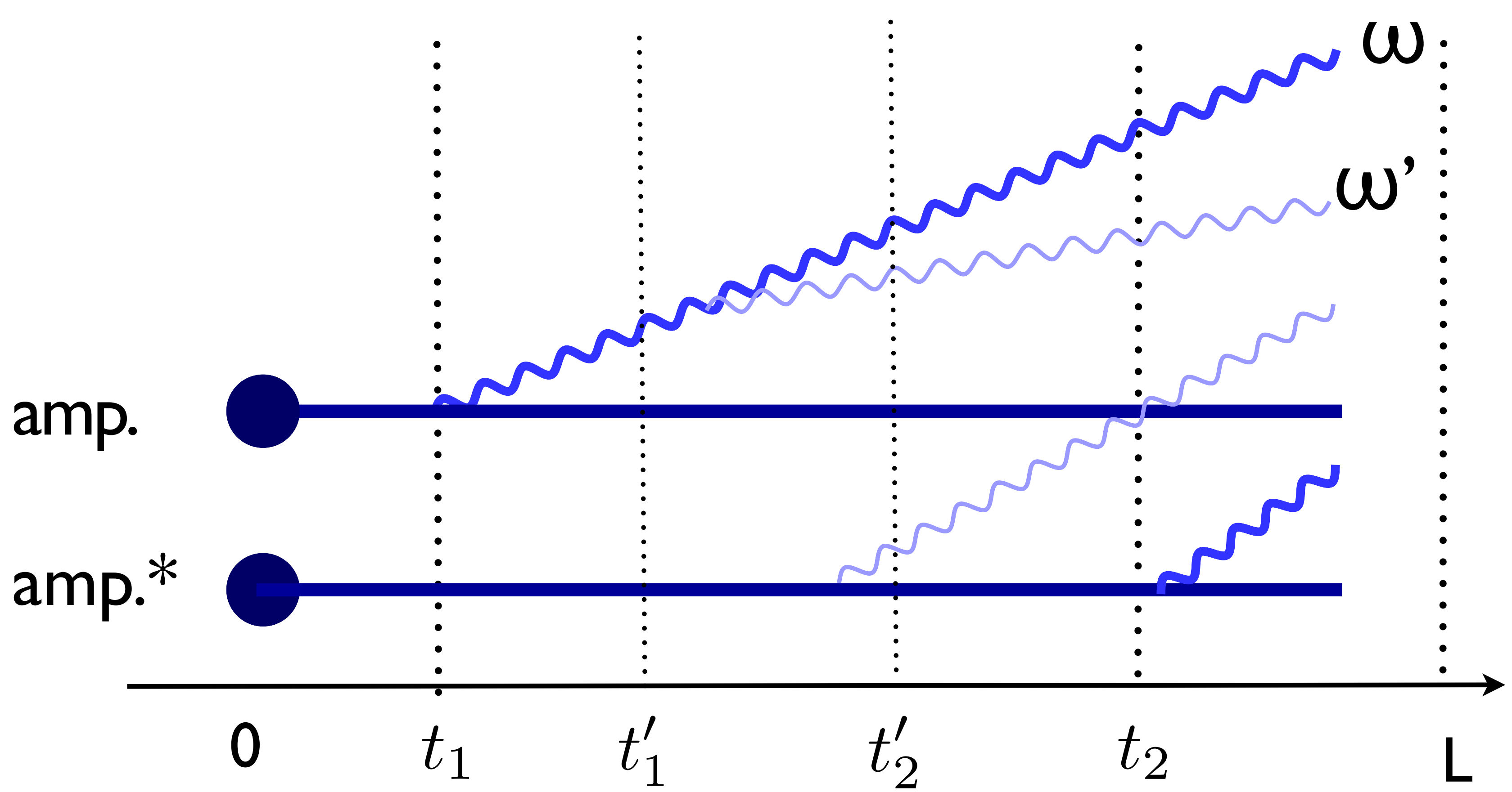
To proof that the DL's can be fully absorbed in a renormalization of the quenching parameter we shall compute the radiative correction to the 3-point function, i.e., to the radiation rate K.

$$\mathcal{K}[\hat{q}_0] \to \mathcal{K}[\hat{q}_0 + \hat{q}_1]$$



Renormalization of the quenching parameter

On top of the branching described by K we allow the radiation of an additional, softer, gluon ω' << ω < E, which is integrated out.



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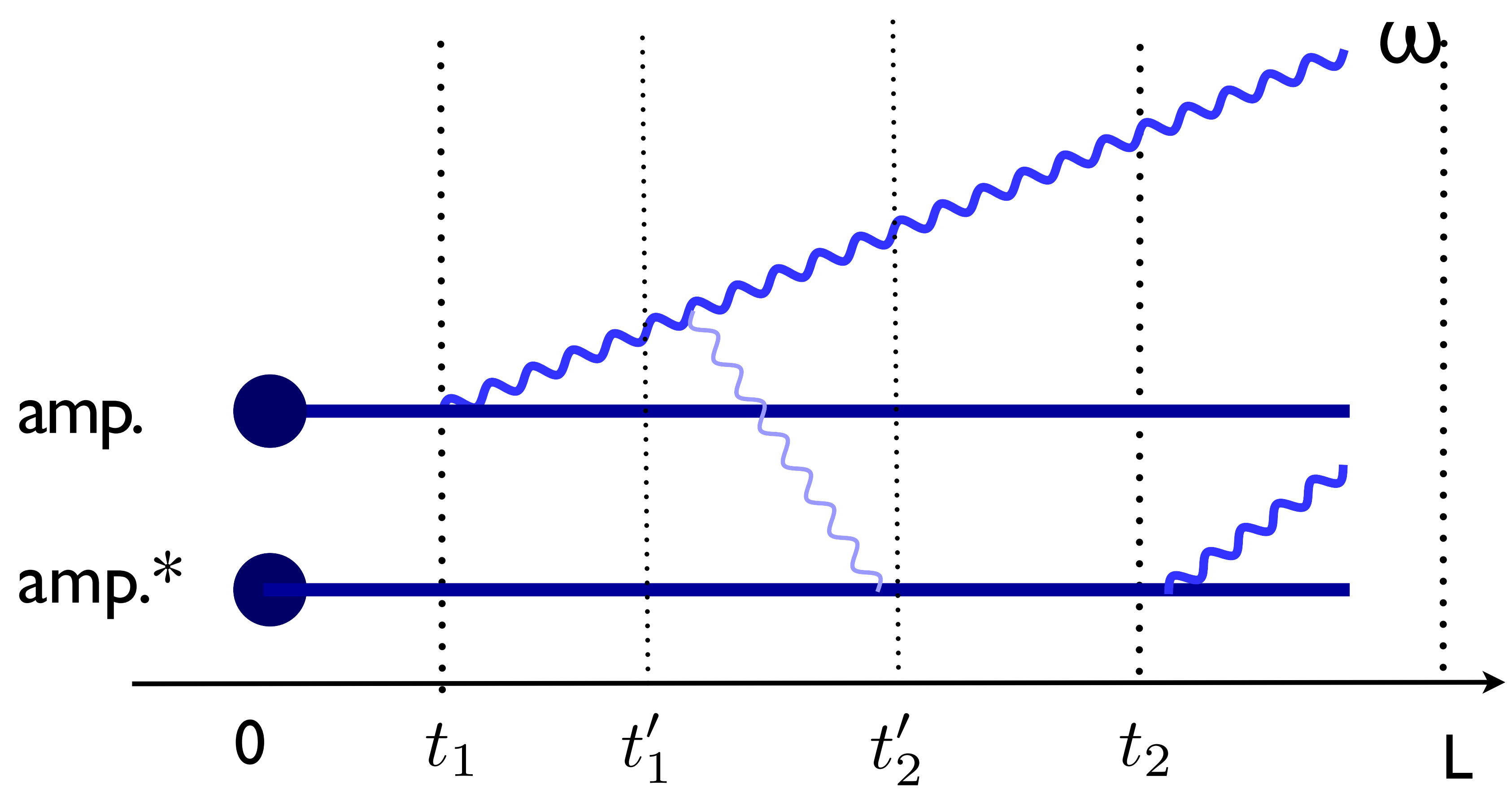
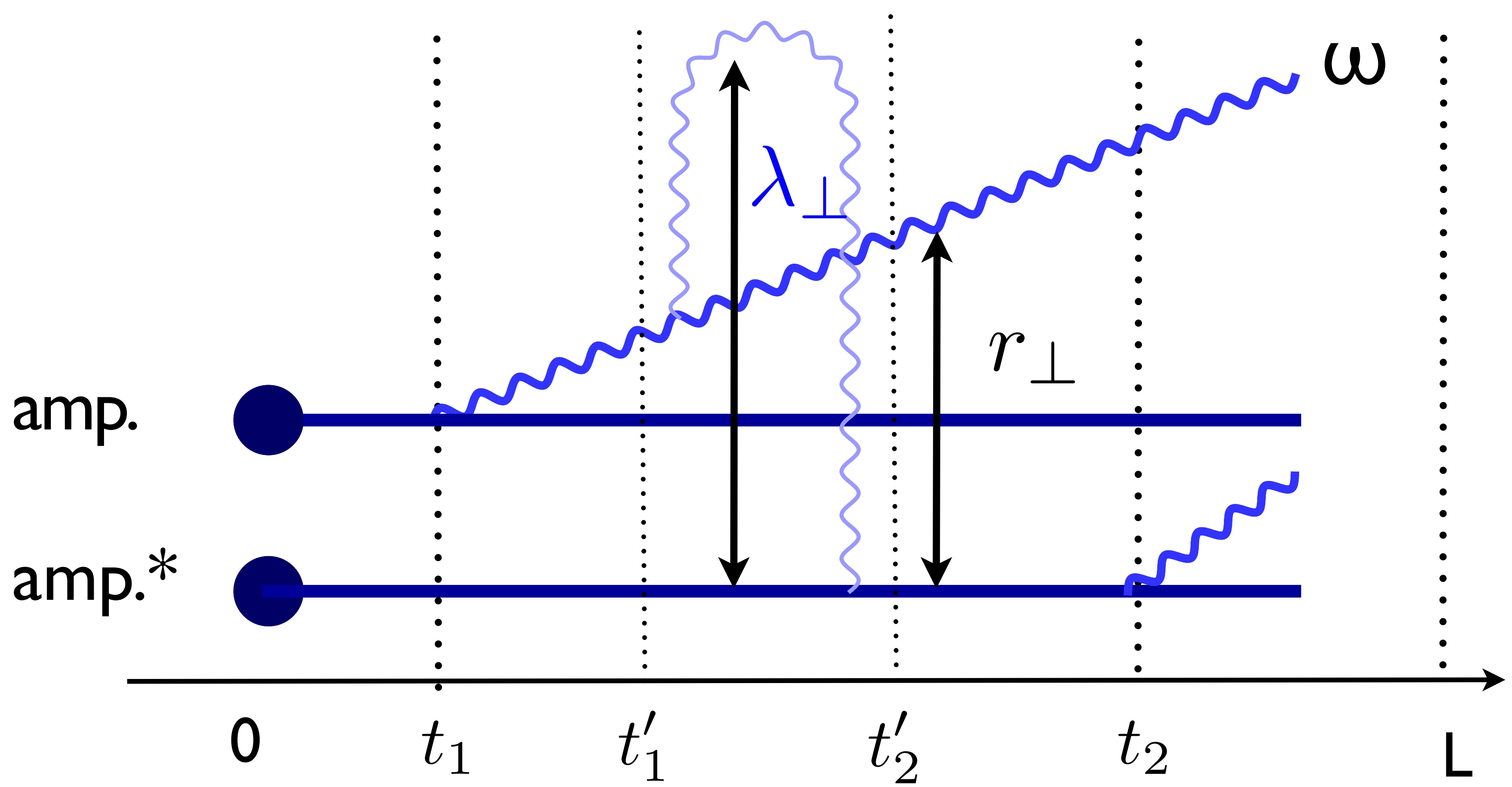


Renormalization of the quenching parameter

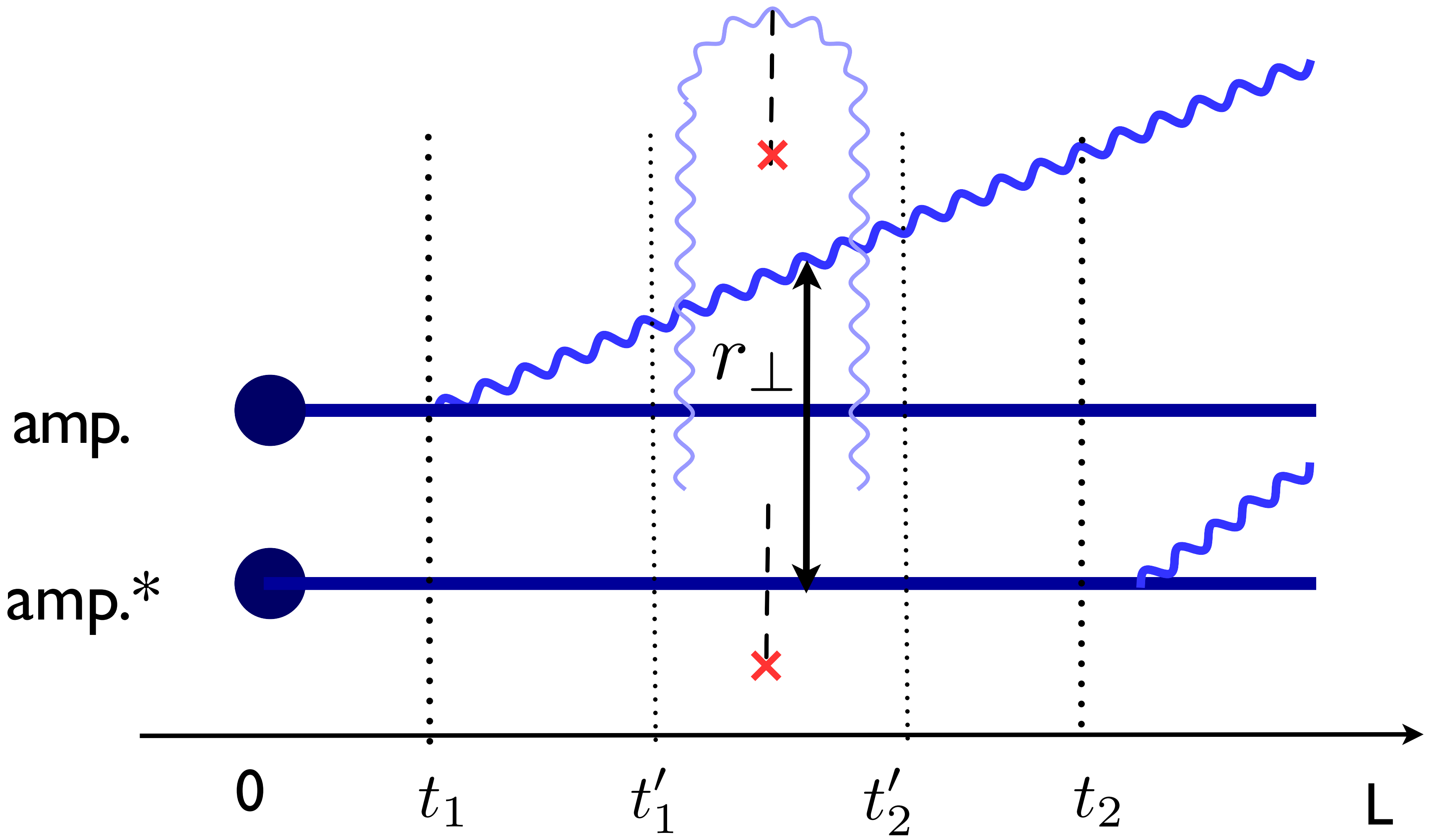
DL region: The transverse wave length of the gluon' is typically larger than the gluon-quark-antiquark system and fluctuates over a much smaller time than the branching time of gluon

$$\lambda_\perp \gg r_\perp \;\; \text{or} \;\; q_\perp \ll r_\perp^{-1} \qquad \text{and} \qquad \Delta t' \ll \Delta t \simeq t_{\rm br}(\omega)$$



The fluctuation is instantaneous for gluon ω and can occur with constant rate along the branching time Δt

Renormalization of the quenching parameter



+ Topologies

$$\equiv \int_0^{t_2-t_1} d\Delta t' \; \hat{q}_0 \; r_\perp^2 \; \frac{\alpha_s C_A}{2\pi} \log^2\left(\frac{1}{r_\perp^2 m_D^2}\right) \equiv \int_0^{t_2-t_1} d\Delta t' \; \delta\sigma_3(r_\perp)$$

We obtain a correction to the radiation rate

$$\mathcal{K}(t_2,t_1) = \mathcal{K}_0(t_2,t_1) + \int_{t_1}^{t_2} dt'\, \mathcal{K}_0(t_2,t')\; [\sigma_3(t') + \delta\sigma_3(t')]\; \mathcal{K}(t',t_1)$$

Renormalization of the quenching parameter

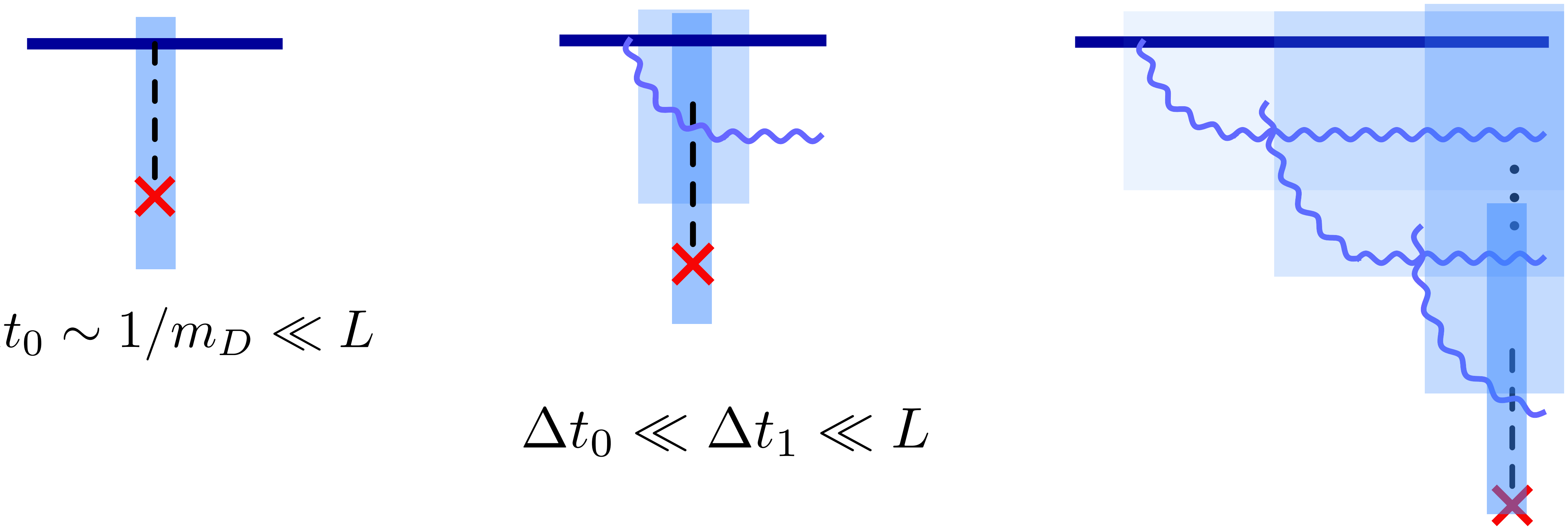
➡ The DL's are resummed assuming strong ordering in formation time (or energy) and transverse mom. of overlapping successive gluon emissions (coherent branchings!)

$$\boxed{\frac{\partial\, \hat{q}(\Delta t, \boldsymbol{k}^2)}{\partial \log(\Delta t/\Delta t_0)} = \int_{\hat{q}\Delta t}^{\boldsymbol{k}^2} \bar{\alpha}_s(\boldsymbol{q}^2)\, \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\, \hat{q}(\Delta t, \boldsymbol{q}^2)}$$

$$\hat{q}_0 \Delta t_0 \equiv Q_0^2$$

$$\hat{q}_0 \Delta t_{\max} \equiv \boldsymbol{k}^2$$

$\hat{q}_0$ $\hat{q}_1$ $\hat{q}_n$



$$\Delta t_0 \sim 1/m_D \ll L$$

$$\Delta t_0 \ll \Delta t_1 \ll L$$

$$\Delta t_0 \ll \Delta t_1 \ll \ldots \Delta t_n \ll L$$

$$\boxed{\hat{q}(\boldsymbol{k}) \sim \hat{q}_0 \left(\frac{\boldsymbol{k}^2}{m_D^2}\right)^{\sqrt{\frac{4\alpha_s N c}{\pi}}}}$$

with $\boldsymbol{k}^2 \sim \hat{q} L$

[In preparation]

Radiative Energy Loss

As a consequence, the DL's not only affects the pt-broadening but also the radiative energy loss expectation:

$$\Delta E \equiv \int d\omega\, \omega\, dN/d\omega$$

Typically the transport coefficient runs up to the scale $\boldsymbol{k}^2 \sim \hat{q}_0 L$

$$\Delta E \simeq \alpha_s \hat{q}_0 L^2 \;\rightarrow\; \Delta E \simeq \alpha_s \hat{q}_0 L^2 \left[1 + \frac{\alpha_s C_A}{2\pi} \log^2\left(\hat{q}_0 L/m_D^2\right)\right]$$

When the logs become large (asymptotic behavior)

$$\boxed{\rightarrow\; \Delta E \simeq \frac{\alpha_s \hat{q}_0 L^2}{4\sqrt{\pi}\, \bar{\alpha}_s^{3/4} \log^{3/2}(\hat{q}_0 L/m_D^2)} \left(\frac{\hat{q}_0 L}{m_D^2}\right)^{\sqrt{\frac{4\alpha_s C_A}{\pi}}}}$$

Radiative Energy Loss

Path-length dependence of mean energy-loss:

I- weak coupling (BDMPS) $\Delta E \sim L^2$

II- strong coupling (ADS/CFT) $\Delta E \sim L^3$

[F. Dominguez et al (2008) C. Marquet (2009)]

III- weak coupling (BDMPS+Dlogs) $\Delta E \sim L^{2+\gamma}$

where $0 < \gamma \equiv \sqrt{4\alpha_s N_c/\pi} < 1$

Larger quenching parameter ⇒ Larger energy loss

SUMMARY

✓ In the limit of a dense medium, parton branchings decohere due to rapid color randomization except for strongly collimated partons (unresolved by the medium)

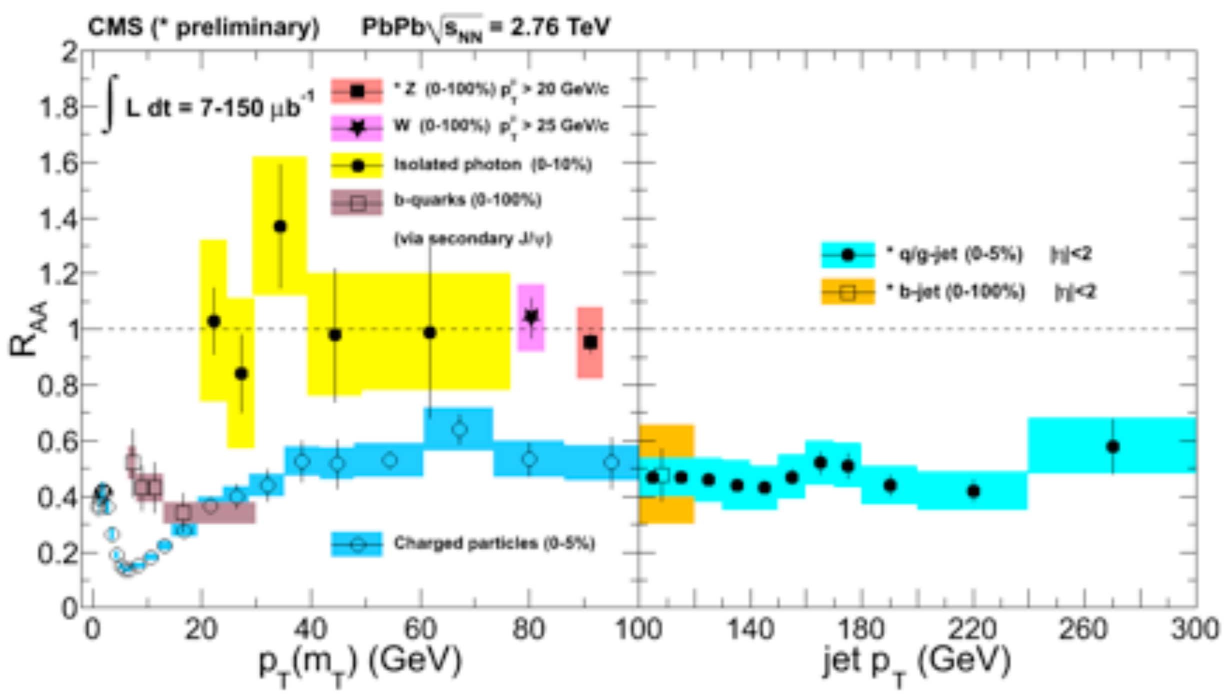
✓ In the decoherent limit: factorization of multiple gluon emissions

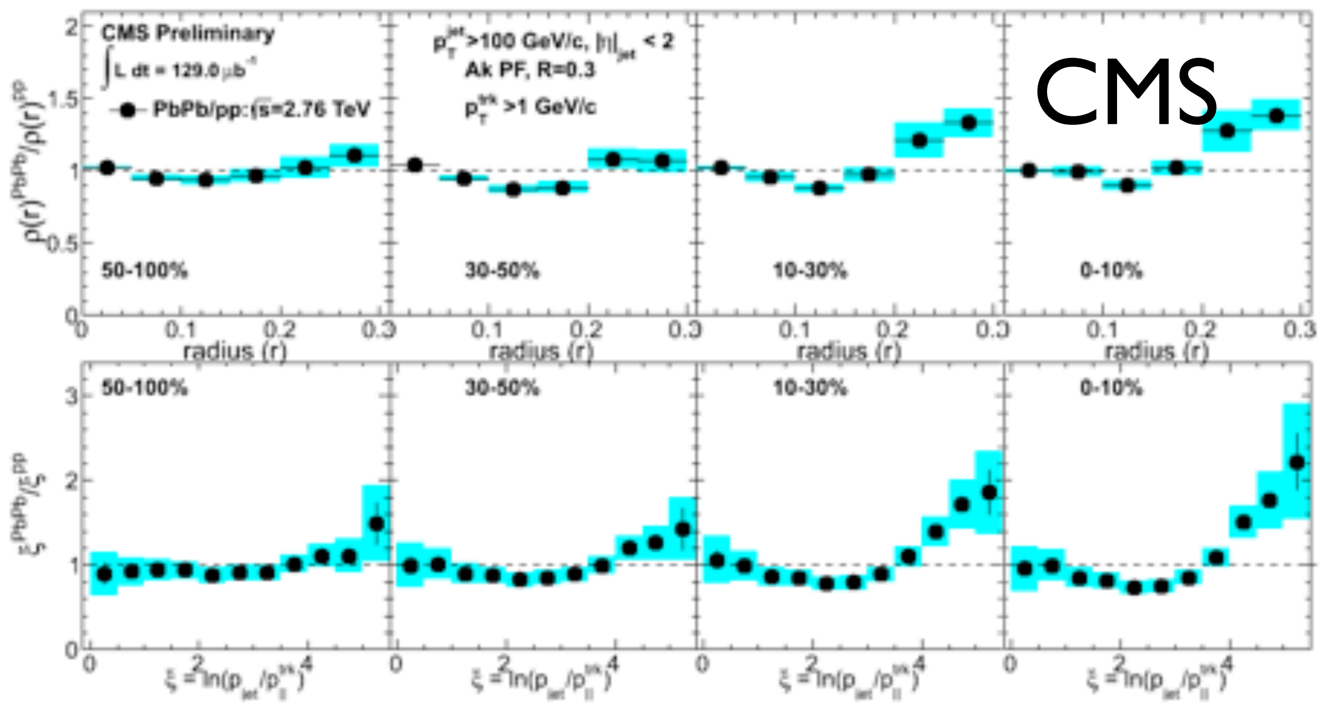
➡ Probabilistic picture ⇒ Monte-Carlo

Implementation

✓ Coherent radiations with formation times much shorter then the branching time lead to potentially large Double Log enhancement that can be resummed and absorbed in a renormalization of the quenching parameter

Jets in HIC at the LHC





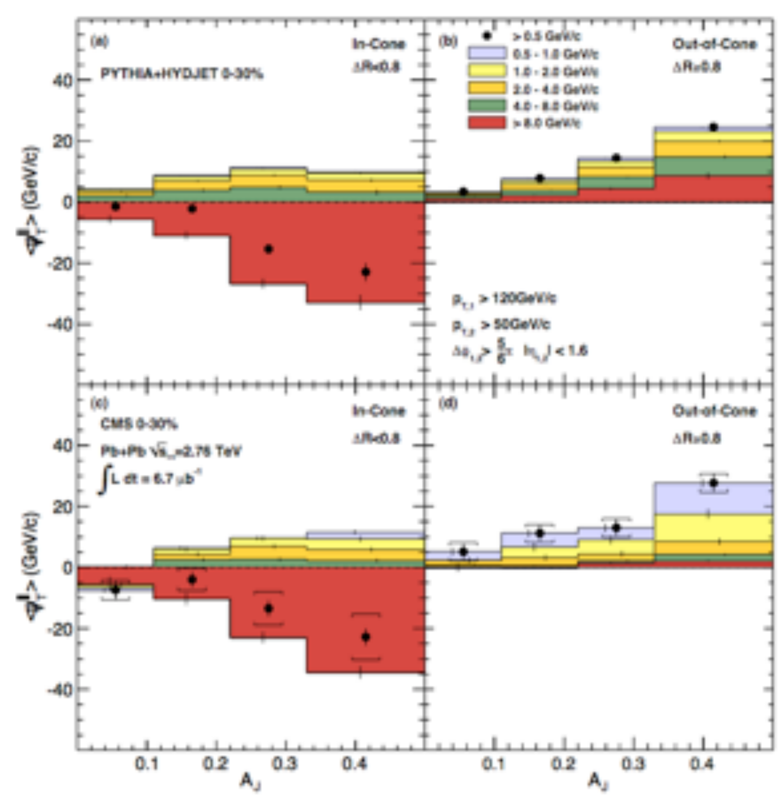
$$R_{AA} \equiv \frac{1}{N_{coll}} \frac{dN^{jet}_{PbPb}}{dN^{jet}_{pp}}(p_T)$$

Fragmentation fct.

$$\frac{D_{PbPb}}{D_{pp}}\left(\xi = \ln\frac{p_h}{p_{jet}}\right)$$

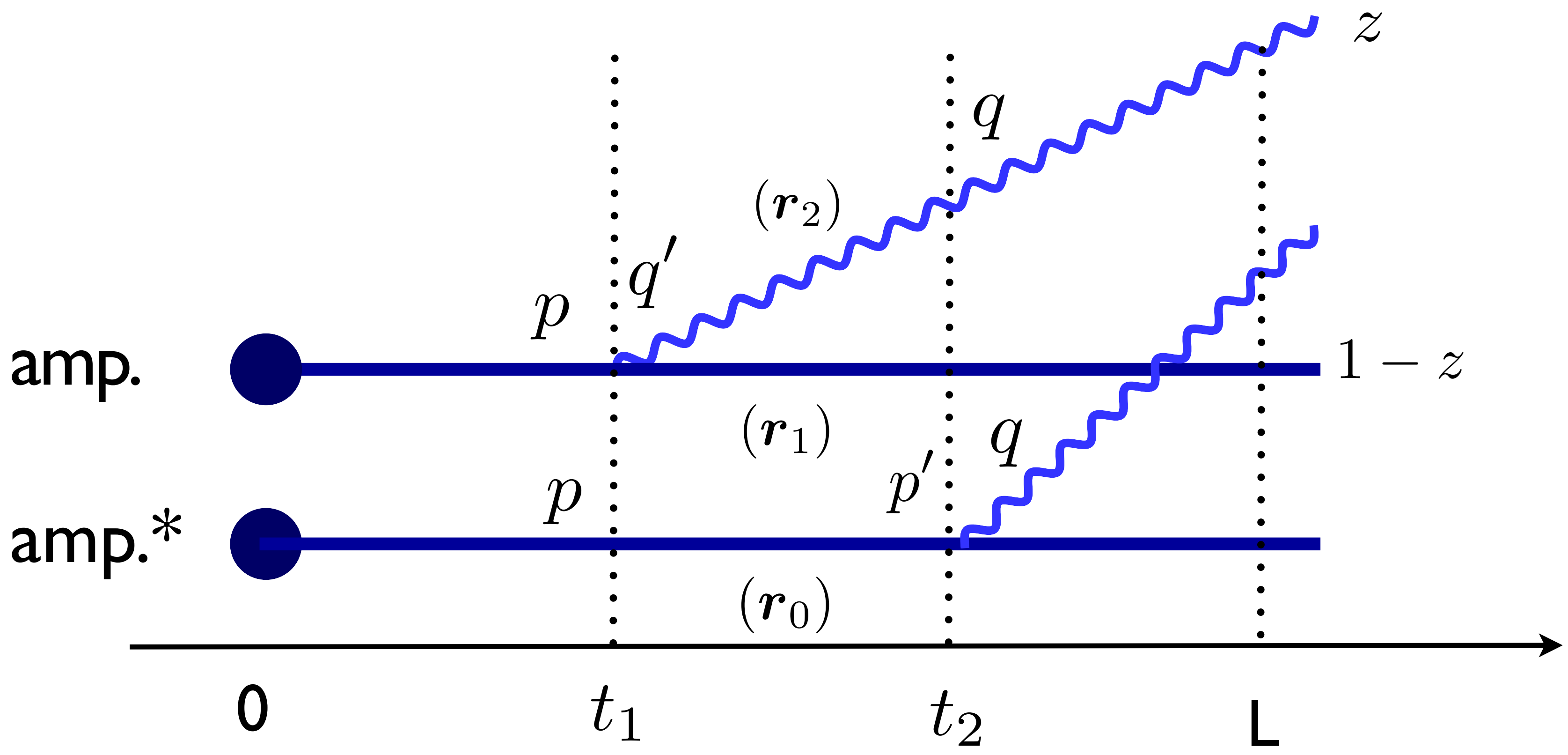
dijet asymmetry

$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$



(I) Significant dijet energy asymmetry
(II) Soft particles at large angles
(III) medium-modified fragmentation

I - Rate for elastic scatterings



the dipole cross-section is related to the collision rate

$$\sigma(\boldsymbol{r}) = \int_{\boldsymbol{q}} C(\boldsymbol{q})\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}$$

$$S^{(3)}(\boldsymbol{P},\boldsymbol{Q},\boldsymbol{l},z,p^+;t_2,t_1) = \int \mathrm{d}\boldsymbol{u}_1\mathrm{d}\boldsymbol{u}_2\mathrm{d}\boldsymbol{v}\; e^{i\boldsymbol{u}_1\cdot\boldsymbol{P}-i\boldsymbol{u}_2\cdot\boldsymbol{Q}+i\boldsymbol{v}\cdot\boldsymbol{l}}$$
$$\times \int_{\boldsymbol{u}_1}^{\boldsymbol{u}_2} \mathcal{D}\boldsymbol{u}\, \exp\left\{ \frac{iz(1-z)p^+}{2}\int_{t_1}^{t_2} \mathrm{d}t\; \dot{\boldsymbol{u}}^2 - \frac{N_c}{4}\int_{t_1}^{t_2} \mathrm{d}t\, n(t)\left[\sigma(\boldsymbol{u}) + \sigma(\boldsymbol{v}-z\boldsymbol{u}) + \sigma(\boldsymbol{v}+(1-z)\boldsymbol{u})\right]\right\}$$

Transverse momenta generated in the splitting (in the amp. and comlex. conj.)

$\boldsymbol{Q} \equiv \boldsymbol{q} - z\boldsymbol{p}'$ $\quad \boldsymbol{P} \equiv \boldsymbol{q}' - z\boldsymbol{p}$ are conjugate to the dipole size $\boldsymbol{u} \equiv \boldsymbol{r}_2 - \boldsymbol{r}_1$

Transverse momentum acquired by collisions conjugate to the diff. of centers of mass

$$\boldsymbol{p}' - \boldsymbol{p} \equiv \boldsymbol{l}$$

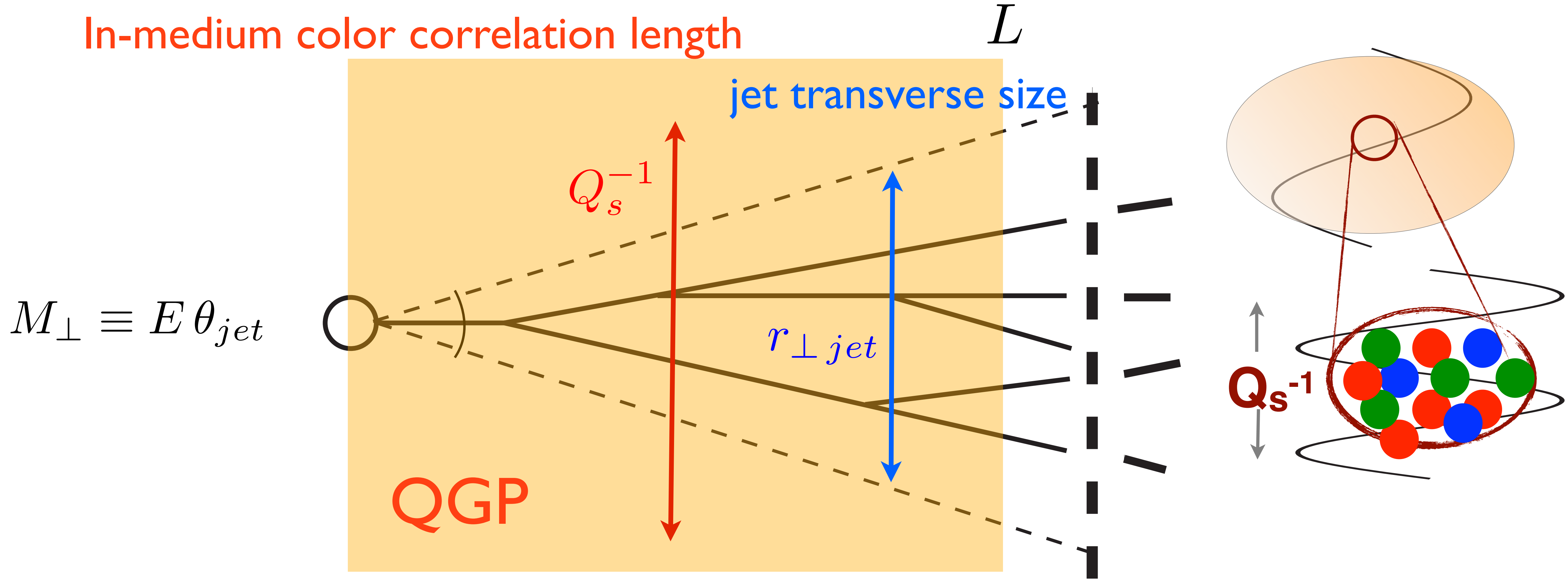
$$\boldsymbol{v} \equiv z\boldsymbol{r}_2 + (1-z)\boldsymbol{r}_1 - \boldsymbol{r}_0$$

MULTISCALE PROBLEM

$$M_\perp \equiv E\,\theta_{jet}$$
$$Q_0 \sim \Lambda_{\rm QCD}$$

+

$$Q_s \equiv \sqrt{\hat{q}L} \equiv m_D\,\sqrt{N_{\rm scat}}$$
$$r_{\perp\,jet}^{-1} \equiv (\theta_{jet}L)^{-1}$$



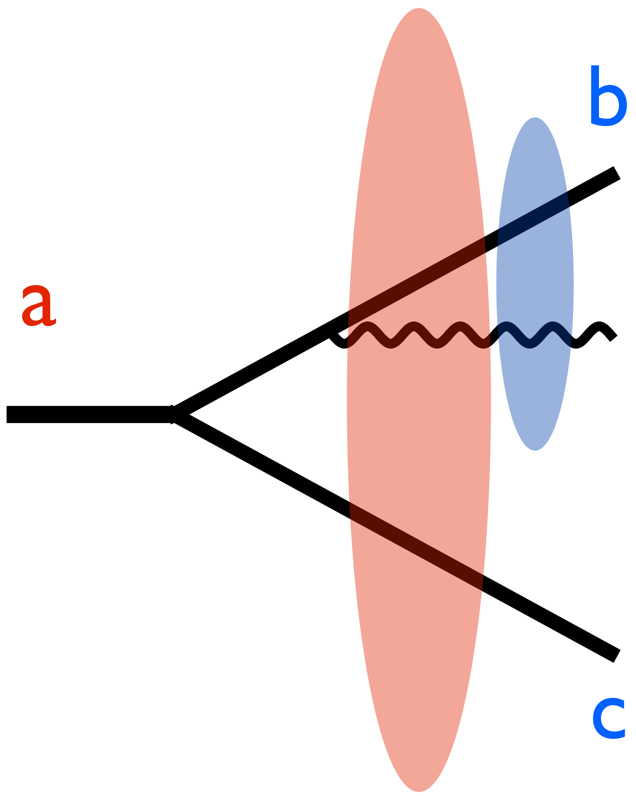
Color transparency for $r_\perp < Q_s^{-1}$ or $\theta_{jet} < \theta_c \sim \dfrac{1}{\sqrt{\hat{q}L^3}}$

Decoherence $r_\perp > Q_s^{-1}$

Y. M.-T, K. Tywoniuk, C. A. Salgado (2010-2012)
J. Casalderray-Solana, E. Iancu (2011)

COLOR COHERENCE IN A FEW WORDS

Consider the radiation of a gluon off a system of two color charges a and b.



large angle gluon radiation does not resolve the inner structure of the emitting system

Incoherent emissions at small angles

$$\omega \frac{dN_{\mathrm{a}}}{d\omega d^2 k_\perp} \propto \frac{\alpha_s C_{\mathrm{b}}}{k_\perp^2} + (\mathrm{b} \to \mathrm{c}) \qquad \theta \ll \theta_{bc} \quad (k_\perp \ll \omega\theta_{bc})$$

large angle emission by the total charge (destructive interferences)

$$\omega \frac{dN_{\mathrm{a}}}{d\omega d^2 k_\perp} \propto \frac{\alpha_s C_{\mathrm{a}}}{k_\perp^2} \qquad \theta \gg \theta_{bc} \quad (k_\perp \gg \omega\theta_{bc})$$

Energy flow: democratic branching

Integrating over transverse momenta, the contribution to the classical broadening vanishes

$$\int_{\boldsymbol{l}} \mathcal{C}(\boldsymbol{l}, t_L) = 0$$

We obtain the simplified equation J.-P. Blaizot, E. Iancu, Y. M.-T., arXiv: 1301.6102 [hep-ph]

$$\frac{\partial}{\partial \tau} D(x,\tau) = \int \mathrm{d}z \, \hat{\mathcal{K}}(z) \left[\sqrt{\frac{z}{x}} D\left(\frac{x}{z}, \tau\right) - \frac{z}{\sqrt{x}} D(x,\tau) \right],$$

Similar eq. postulated: R. Baier, A. H. Mueller, D. Schiff, D. T. Son (2001) S. Jeon, G. D. Moore(2003)

Toy Model: Keeping the singular part at z=0 and z=1

$$\mathcal{K} = P(z) \sqrt{\frac{\hat{q}_{eff}}{z(1-z)E}} \approx \sqrt{\frac{\hat{q}}{E}} \frac{1}{z^{3/2}(1-z)^{3/2}}$$

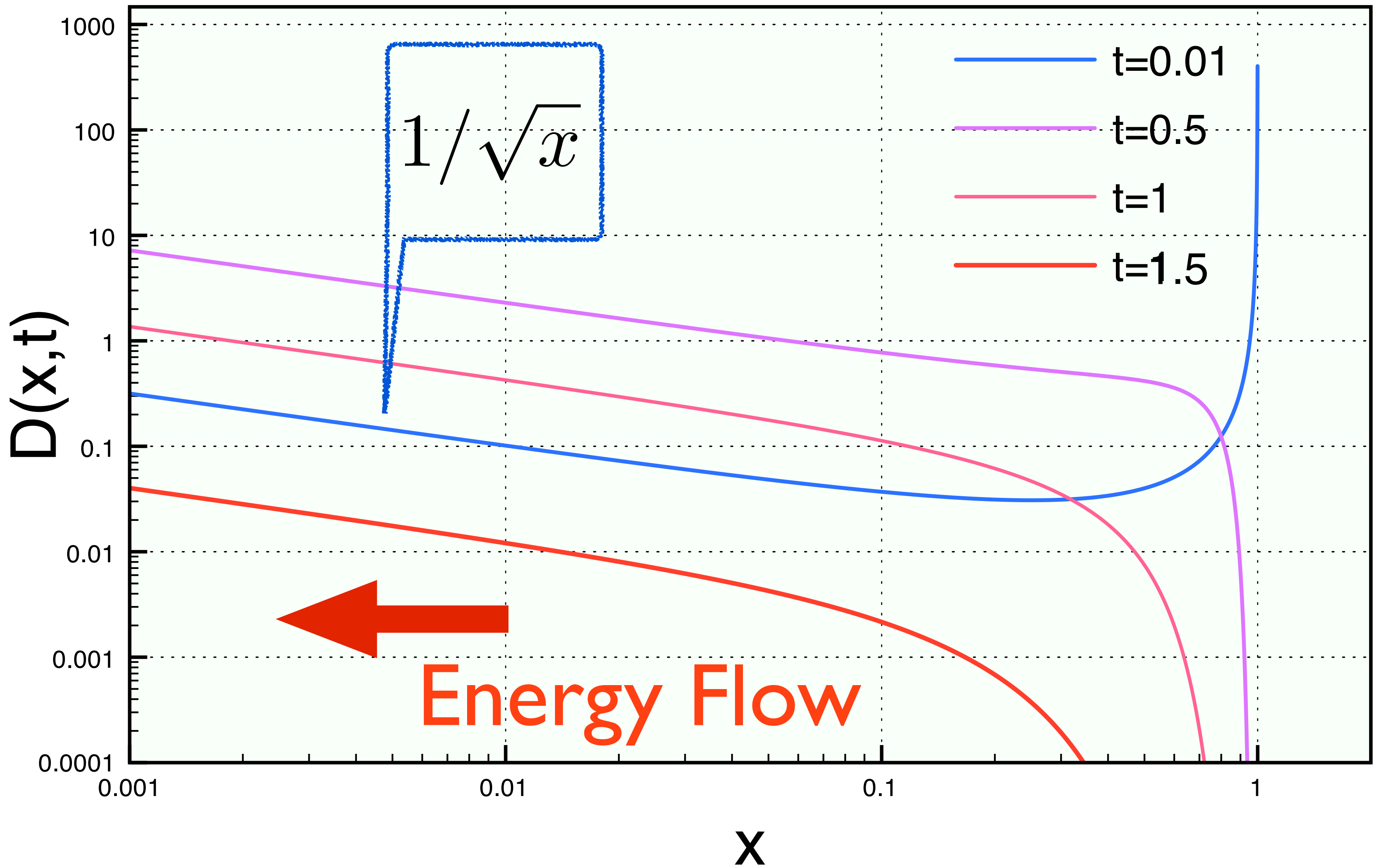
The exact solution for D(x,E,L) reads

$$D(x) = \frac{\bar{\alpha}}{(1-x)^{3/2}} \sqrt{\frac{\hat{q}L^2}{Ex}} \exp\left[-\pi \frac{\bar{\alpha}^2 \hat{q} L^2}{(1-x)E} \right]$$

Energy flow: democratic branching

Initial condition: $D_0(x) = \delta(1-x)$

$$t = \bar{\alpha}\sqrt{\frac{\hat{q}L^2}{E}}$$



scaling spectrum

$x \ll 1$

$$D(x) \sim \frac{t}{\sqrt{x}}\, e^{-\pi t^2}$$

Partons disappear in the medium when

$$E < \bar{\alpha}_s^2\, \hat{q}L^2$$

Energy flows uniformly from hard to soft modes without accumulation ➡ indication of wave turbulence

Energy in the spectrum $\int_0^1 dx\, D(x) = e^{-\pi t^2} < 1$ ➡ indication of a condensate at x=0

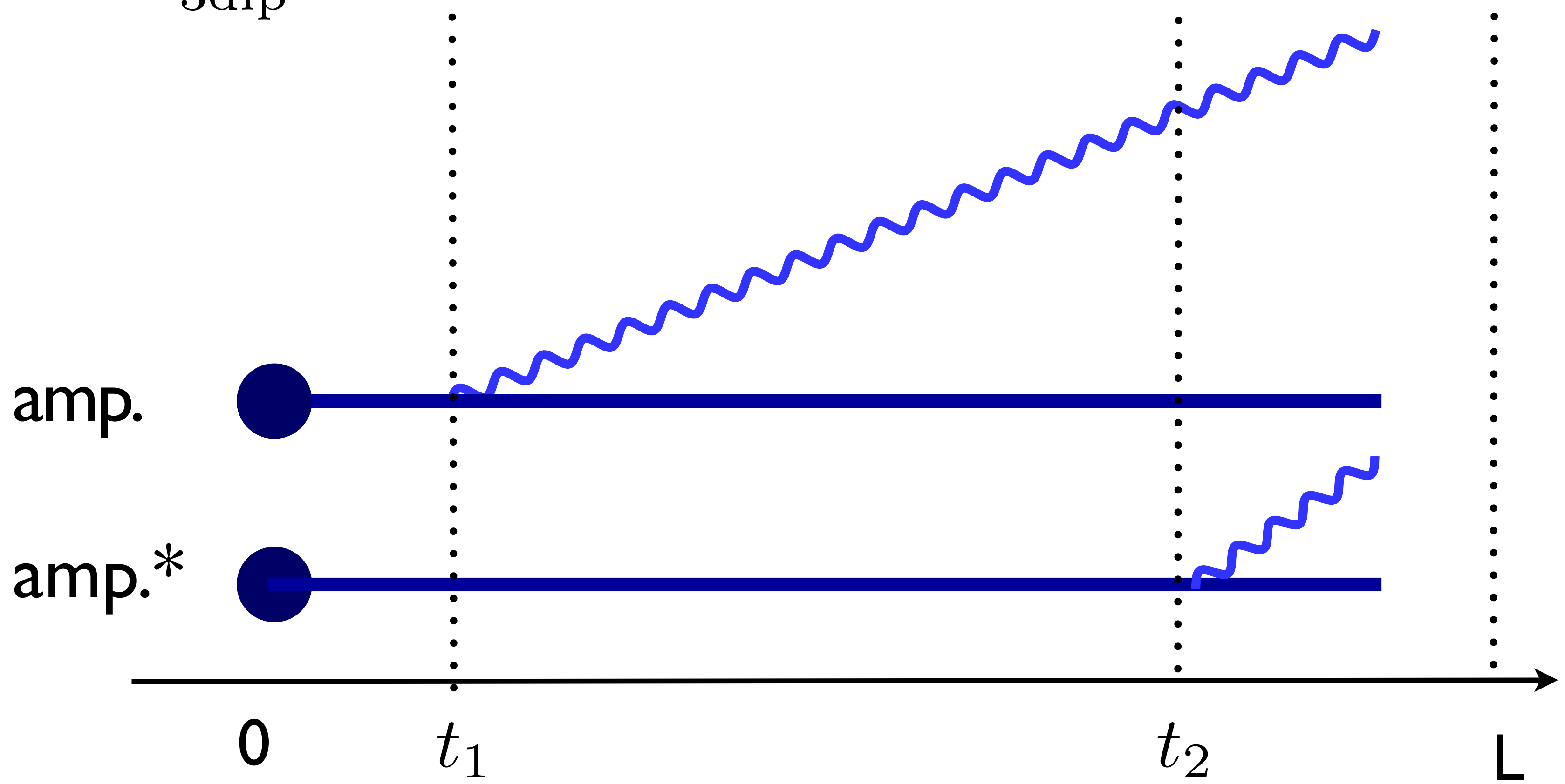
Renormalization of the quenching parameter

Remember: The radiation rate obeys a Dyson-like equation

$$\mathcal{K}(t_2,t_1) = \mathcal{K}_0(t_2,t_1) + \int_{t_1}^{t_2} dt'\, \mathcal{K}_0(t_2,t')\,\sigma_3(t')\,\mathcal{K}(t',t_1)$$

where the instantaneous interaction with the medium is encoded in the 3-dipole cross-section

$$\sigma_3(t',r_\perp) \sim \sum_{\text{3dip}} \hat{q}\; r_\perp^2 \qquad \mathcal{K} \sim \langle\, \mathrm{tr}\, T^a\, U_F(r_q)\, T^b\, U_F^\dagger(r_{\bar{q}}) U_{ab}(r_g) \rangle_{\text{med}}$$

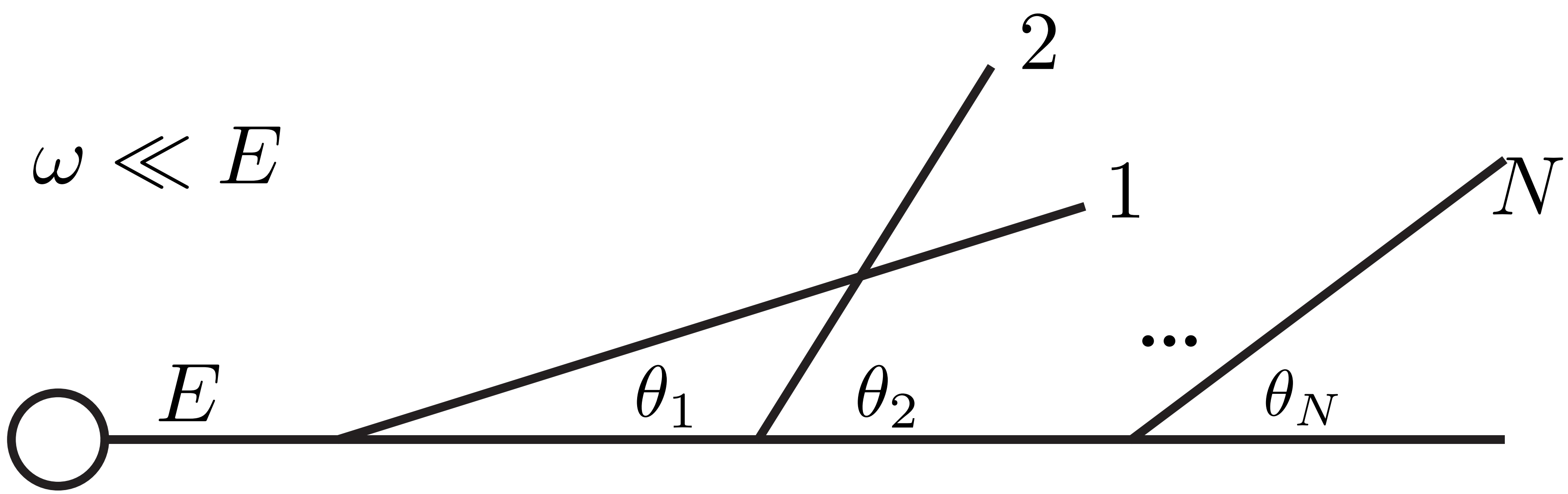


FACTORIZATION OF BRANCHINGS IN VACUUM

Ladder diagrams (no interferences) resum mass singularities:
Strong ordering in k_T (DGLAP)

$$\frac{d}{d\ln M_\perp} D_A^B(x, M_\perp) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} P_A^C(z)\, D_c^B(x/z, M_\perp)$$

In the soft regime $\omega \ll E$



$$M_\perp \gg k_{\perp 1} \gg k_{\perp 2} \gg \ldots \gg k_{\perp N}$$

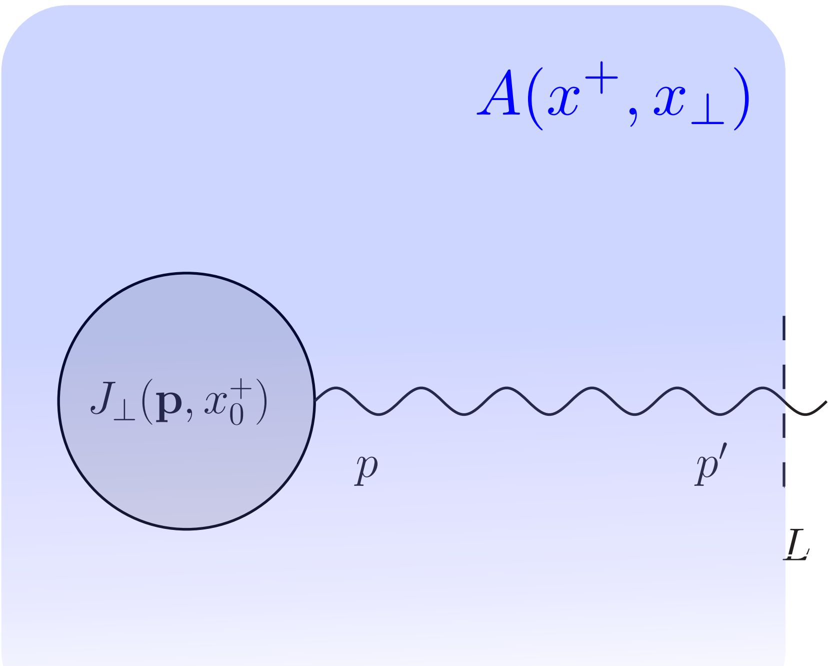
Radiation suppressed at $\theta_2 > \theta_1$ because of coherence phenomena: Interference of 1 with 2 at large angles

$k_{2\perp} \ll k_{1\perp}$ $\qquad \theta_1 < \theta_2 \ll \frac{\omega_1}{\omega_2}\theta_1$ $\qquad$ k_T ordering fails!

BUILDING IN-MEDIUM JET EVOLUTION:

parton shower in classical background field $A(x^+, x_\perp)$

0th order (no-splitting) and 1st order (1-splitting)



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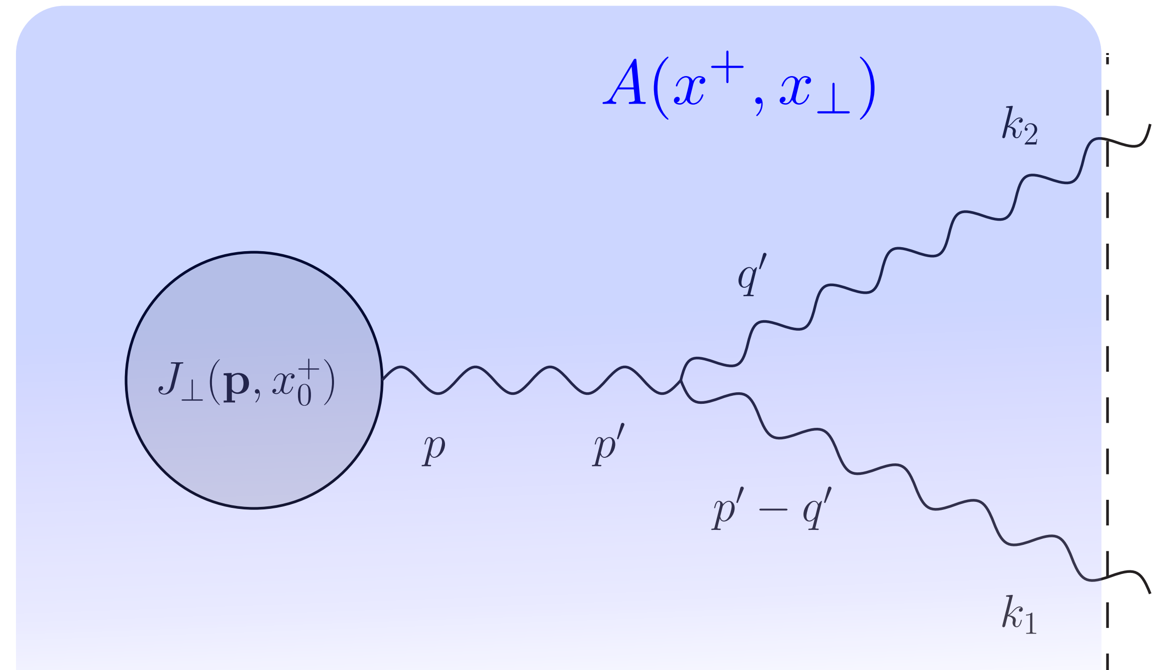


- mixed representation

  $(p_\perp, p^+, x^+ \equiv t)$

- eikonal propagation: $p^+ \gg k_\perp \sim p_\perp$

- Propagators: Brownian motion in transverse plan

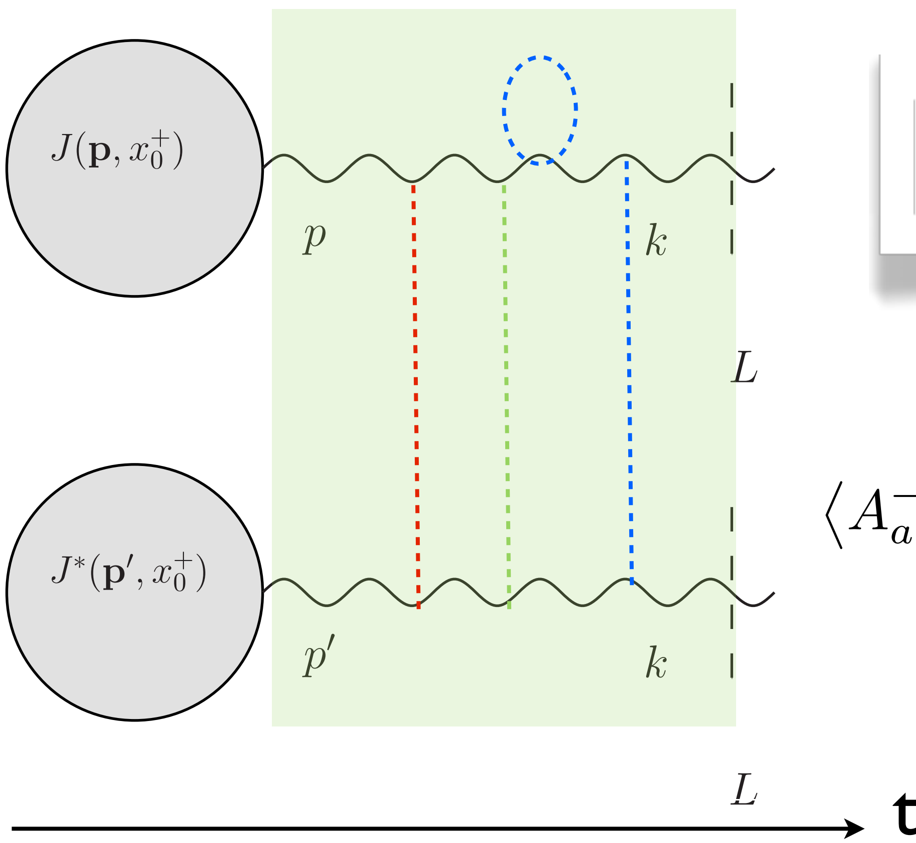
$$\mathcal{G}_{ac}(X, Y; k^+) = \int \mathcal{D}\boldsymbol{r}_\perp \, \mathrm{e}^{i \frac{k^+}{2} \int_{y^+}^{x^+} d\xi \, \dot{\boldsymbol{r}}_\perp^2(\xi)} \, \tilde{U}_{ac}(x^+, y^+; \boldsymbol{r}_\perp)$$

- For instance the 0th order amplitude reads

$$\mathcal{M}_{0,\lambda}(\mathbf{k}) = e^{ik^- L^+} \int \frac{d\boldsymbol{p}}{(2\pi^2)} \, \mathcal{G}(\mathbf{k}, L^+; \mathbf{p}, x_0^+) \, \boldsymbol{\epsilon}_\lambda \cdot \boldsymbol{J}(\mathbf{p}, x_0^+)$$

BUILDING IN-MEDIUM JET EVOLUTION:

0th order (no-splitting)



$$\mathcal{M}_{0,\lambda}(\mathbf{k}) = e^{ik^- L^+} \int \frac{d\boldsymbol{p}}{(2\pi^2)}\, \mathcal{G}(\mathbf{k}, L^+; \mathbf{p}, x_0^+)\, \boldsymbol{\epsilon}_\lambda \cdot \boldsymbol{J}(\mathbf{p}, x_0^+)$$

- Medium average (Gaussian white noise)

$$\langle A_a^-(\boldsymbol{q}, t) A_b^{*-}(\boldsymbol{q}', t')\rangle = \delta_{ab} n(t) \delta(t-t')(2\pi)^2 \delta^{(2)}(\boldsymbol{q}-\boldsymbol{q}')\, \gamma(\boldsymbol{q})\,,$$

- 2-point function correlator

$$S^{(2)} \equiv \langle \mathcal{G}\mathcal{G}^\dagger \rangle$$

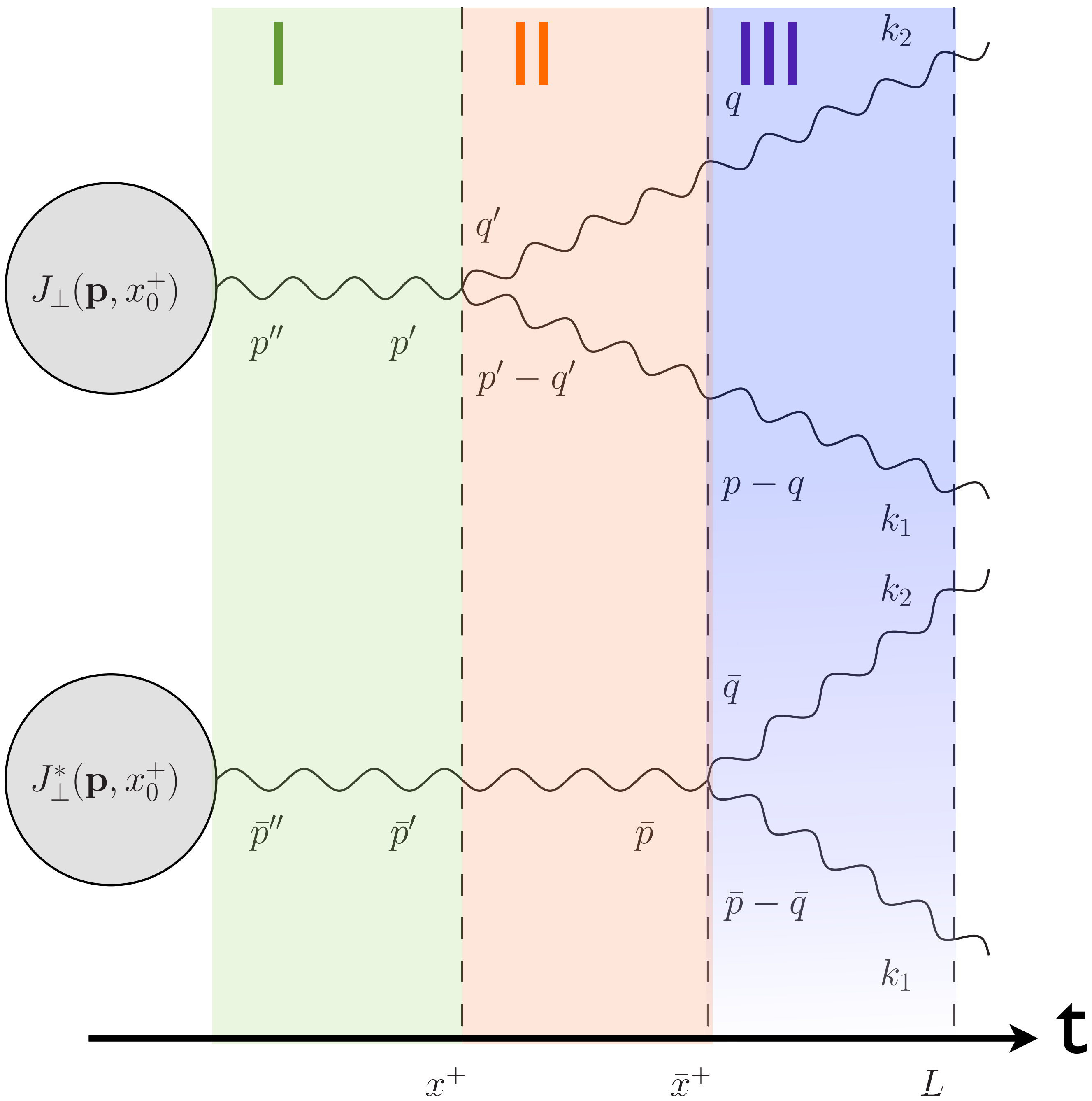
$$\delta^{ab} \langle \mathcal{G}^{aa'}(\mathbf{k}, L^+; \mathbf{p}, x_0^+) \mathcal{G}^{\dagger b'b}(\mathbf{p}', x_0^+; \mathbf{k}, L^+)\rangle = \delta^{a'b'} (2\pi)^2 \delta^{(2)}(\boldsymbol{p}-\boldsymbol{p}')\ \mathcal{P}(\boldsymbol{k}-\boldsymbol{p}, L^+ - x_0^+)$$

- Prob. for kt broadening $\quad \mathcal{P}(\boldsymbol{k}, \xi) = \dfrac{4\pi}{\hat{q}\,\xi} e^{-\frac{\boldsymbol{k}^2}{\hat{q}\xi}}$

$$\frac{d\sigma_0}{d\Omega_k} = \int \frac{d\boldsymbol{p}}{(2\pi^2)}\, \mathcal{P}(\boldsymbol{k}, L^+; \boldsymbol{p}, x_0^+)\ \boldsymbol{J}^2(\mathbf{p}, x_0^+)$$

BUILDING IN-MEDIUM JET EVOLUTION:

1th order (1-splitting)



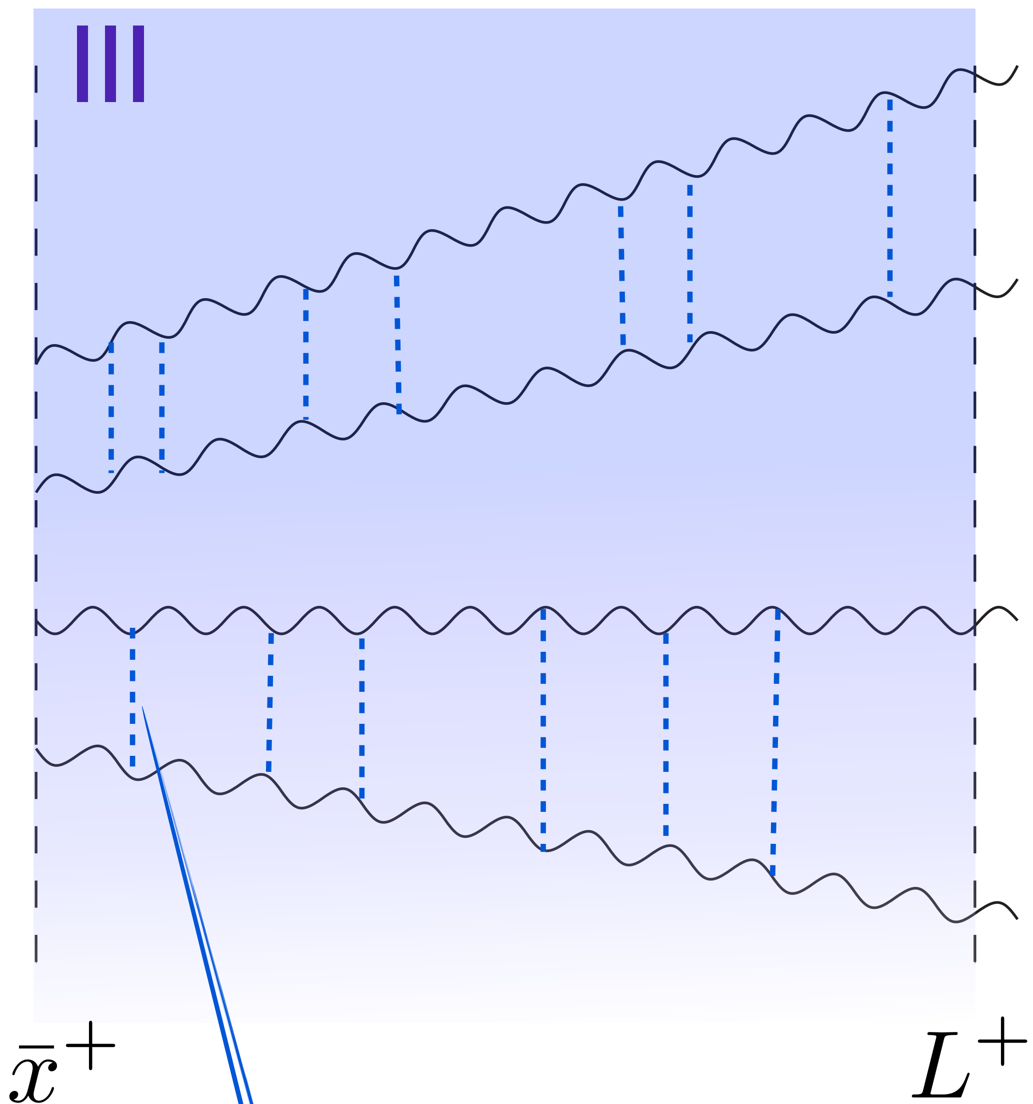
$$S^{(2)} \equiv \langle \mathcal{G}_0 \mathcal{G}_0^\dagger \rangle$$

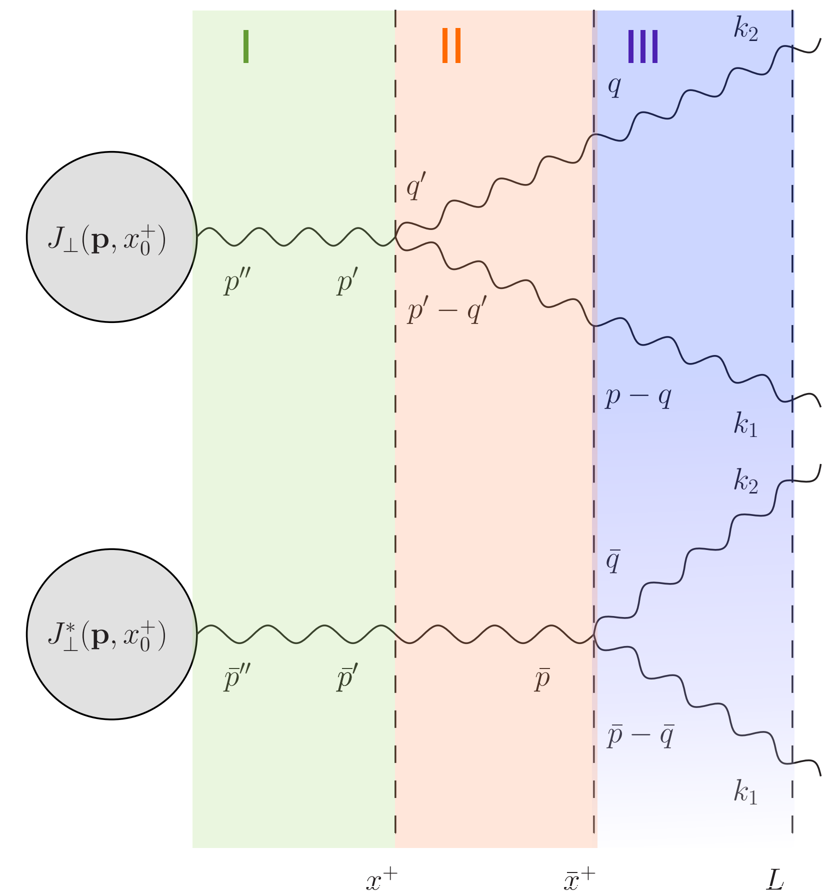
$$S^{(3)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_0^\dagger \rangle$$

$$S^{(4)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_1^\dagger \mathcal{G}_2^\dagger \rangle$$

BUILDING IN-MEDIUM JET EVOLUTION:

Factorization of the 4-point function





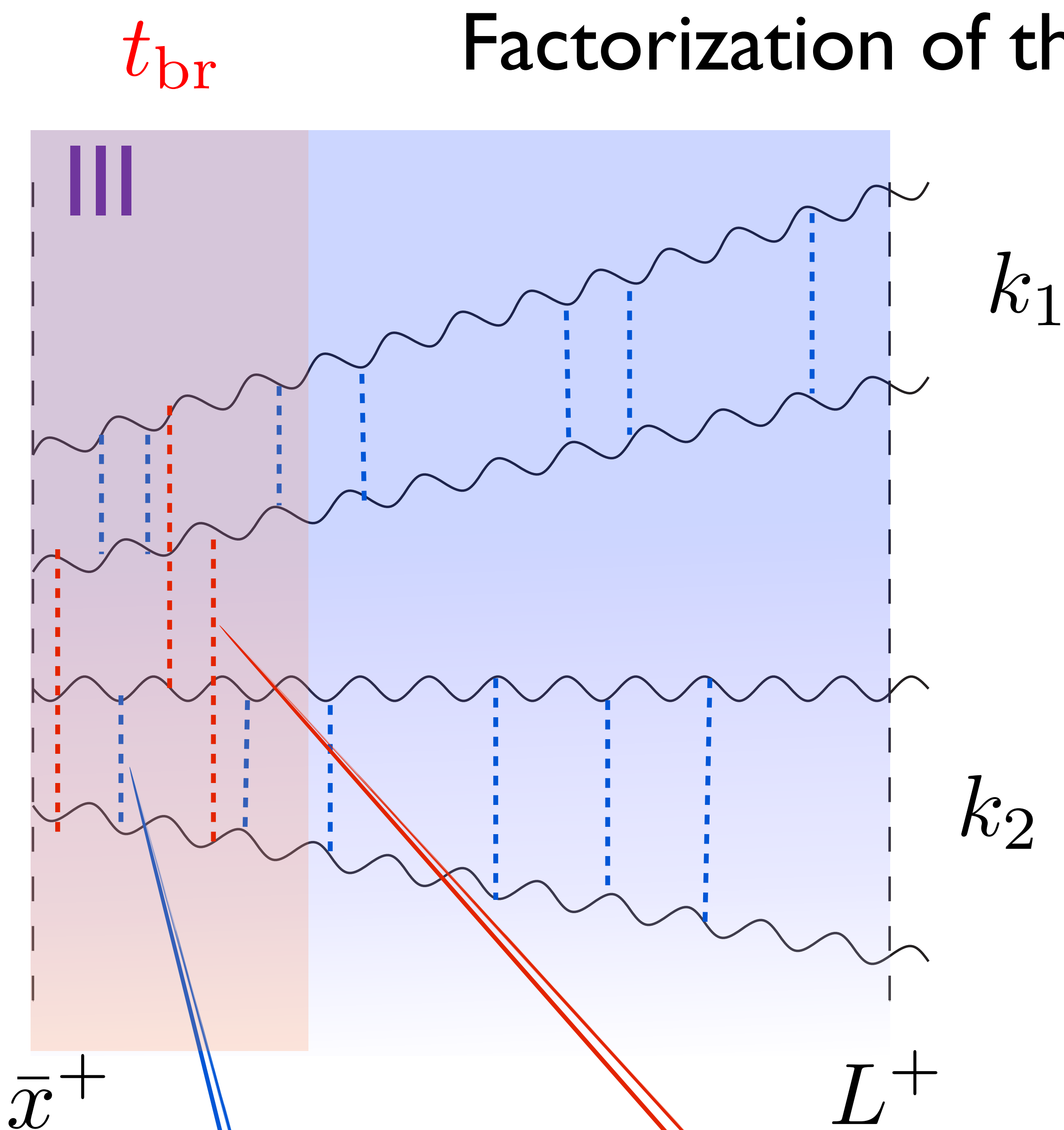
Color decorrelation (decoherence)

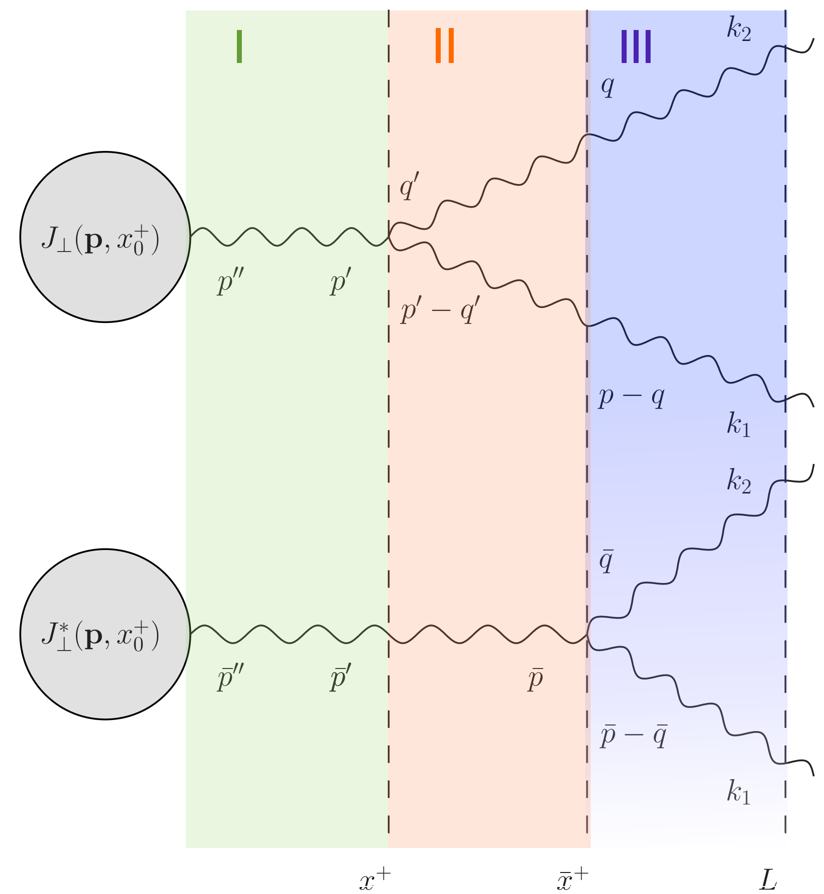
$$
\begin{aligned}
S^{(4)} &\equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_1^\dagger \mathcal{G}_2^\dagger \rangle \\
&\approx \boxed{\langle \mathcal{G}_1 \mathcal{G}_1^\dagger \rangle \langle \mathcal{G}_2 \mathcal{G}_2^\dagger \rangle} + \mathcal{O}\left(\frac{t_{\mathrm{br}}}{L}\right)
\end{aligned}
$$

gluons are decorrelated after they are produced

BUILDING IN-MEDIUM JET EVOLUTION:

Factorization of the 4-point function





Color decorrelation (decoherence)

$$\propto \hat{q} \int_0^{t_{\rm br}} d\xi \, (\boldsymbol{r}_{1\bar{1}} \cdot \boldsymbol{r}_{2\bar{2}}) \sim \frac{\hat{q}\, t_{\rm br}}{Q_s^2}$$

$$Q_s^2 = \hat{q} L$$

$$S^{(4)} \equiv \langle \mathcal{G}_1 \mathcal{G}_2 \mathcal{G}_1^\dagger \mathcal{G}_2^\dagger \rangle$$

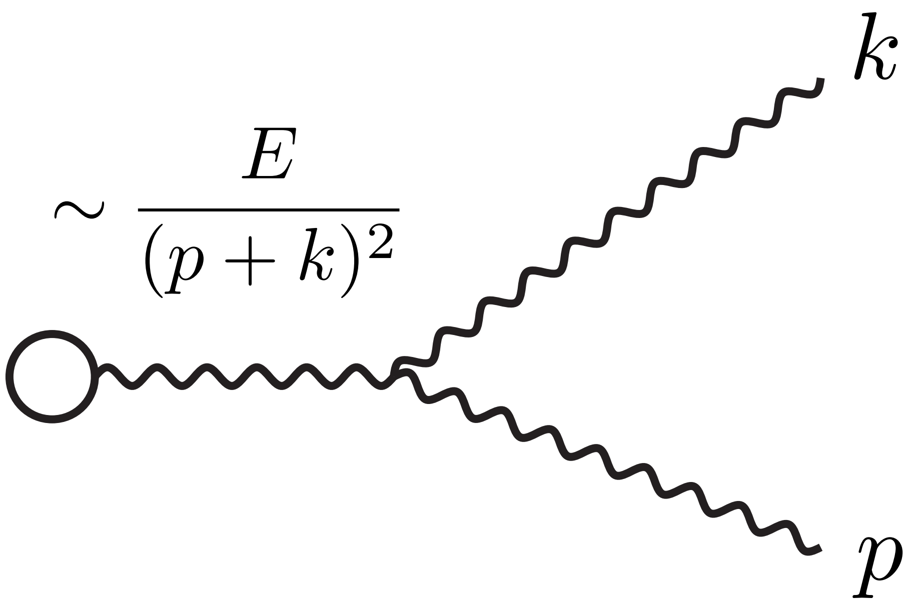
$$\approx \boxed{\langle \mathcal{G}_1 \mathcal{G}_1^\dagger \rangle \langle \mathcal{G}_2 \mathcal{G}_2^\dagger \rangle} + \boxed{\mathcal{O}\left(\frac{t_{\rm br}}{L}\right)}$$

gluons are decorrelated after they are produced

FACTORIZATION OF BRANCHINGS IN VACUUM

$$M_\perp \equiv E\,\theta_{jet}$$

$$\sim \frac{E}{(p+k)^2}$$



$$k_\perp > Q_0 \qquad z = \omega/E$$

the diff-branching probability

$$dP = \frac{\alpha_s C_R}{\pi}\,P(z)\,dz\,\frac{d^2 k_\perp}{k_\perp^2}$$

soft and collinear divergences

phase-space enhancement

$$\boxed{\alpha_s \to \alpha_s \ln^2 \frac{M_\perp}{Q_0}}$$

A highly virtual parton branches typically over a time (formation time)

$$t_f \equiv \frac{E}{(p+k)^2} \sim \frac{E}{2p\cdot k} \sim \frac{\omega}{k_\perp^2}$$

For arbitrary number of parton branchings the logarithmic regions are accounted for via strong ordering of formation times

$$t_{fN} \gg \ldots \gg t_{f2} \gg t_{f1}$$