

# A novel method to simulate the critical current degradation of Nb<sub>3</sub>Sn PIT strands under transverse compression

Tiening Wang

Tufts University

# Outline

- Introduction
- Methodology
  - Finite element analysis
  - Calculation based on a scaling law
- Results and discussion
- Conclusion

# Introduction

Understanding critical current degradation of Nb<sub>3</sub>Sn superconducting strands under **transverse (Lorentz) compression** is necessary for large magnet application

Due to the experiment cost of full size cable, experiments on sub-sized cable or single strand are conducted but generally experiments are challenging due to the small diameter of the strands (~ 1 mm).

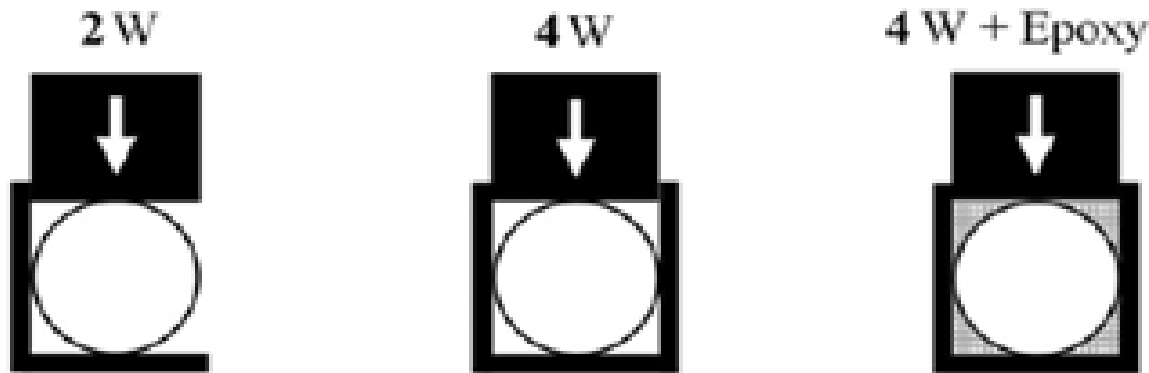
We developed a method to estimate the critical current behavior under transverse load based on Finite Element Analysis (FEA) and an available tensile scaling law.

A successful method would allow a platform for the optimization of the strand design and for the prediction of the behavior of a cable without costly experiments.

# Methodology

## Two steps to calculate the critical current ( $I_c$ ) degradation

- FEA is based on the experiments at University of Geneva.



- Calculate  $I_c$  degradation based on
  - 1) Strain of the  $Nb_3Sn$  filaments from FEA
  - 2) An available scaling law.
- Comparison between the calculated results and the experiment results.

# Methodology

## Finite element analysis

FEA procedure in ANSYS®

1. Solid geometry modeling
2. Meshing (Finite element model)
3. Boundary conditions and Loading
4. Material properties
5. Solution
6. Post-Processing

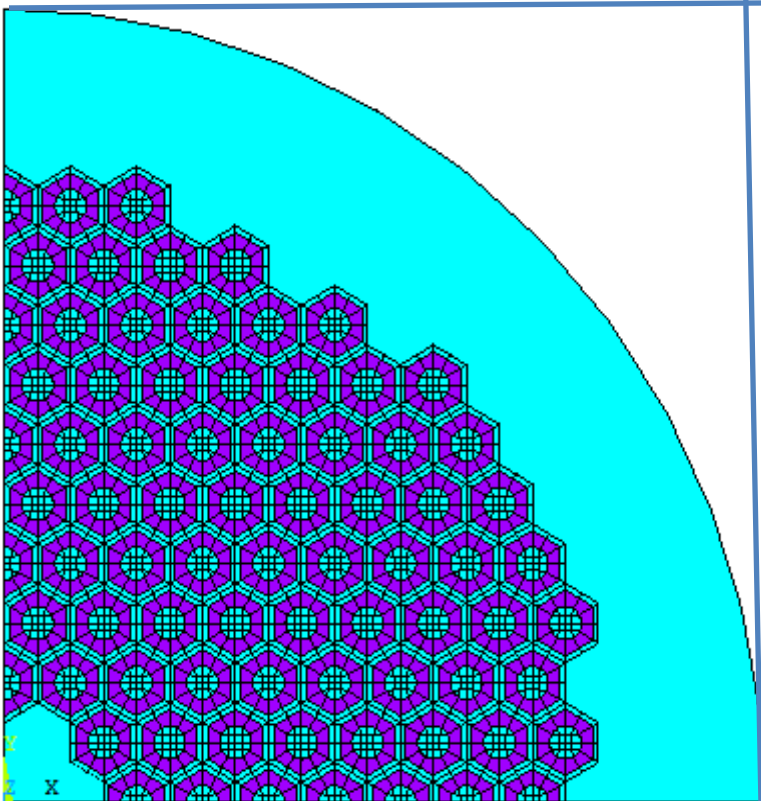
# Solid geometry modeling:

Quarter model:

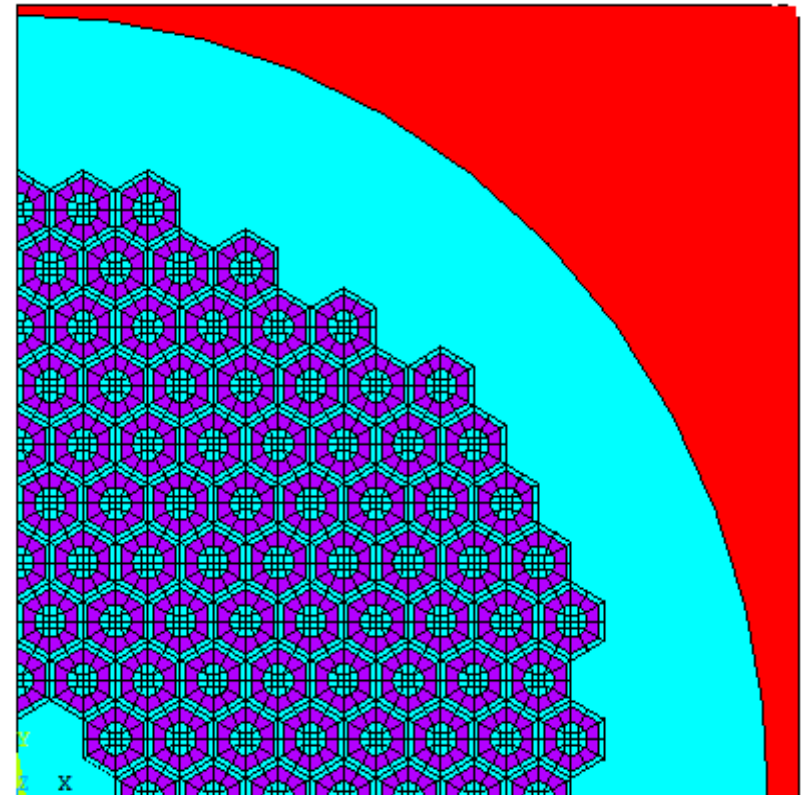
Follow “bottom-up” procedure: Key points -> Lines -> Areas

Materials: Copper(bronze),  $\text{Nb}_3\text{Sn}$ , and Epoxy

PIT288 with 4W condition



PIT288 with 4W+Epoxy condition

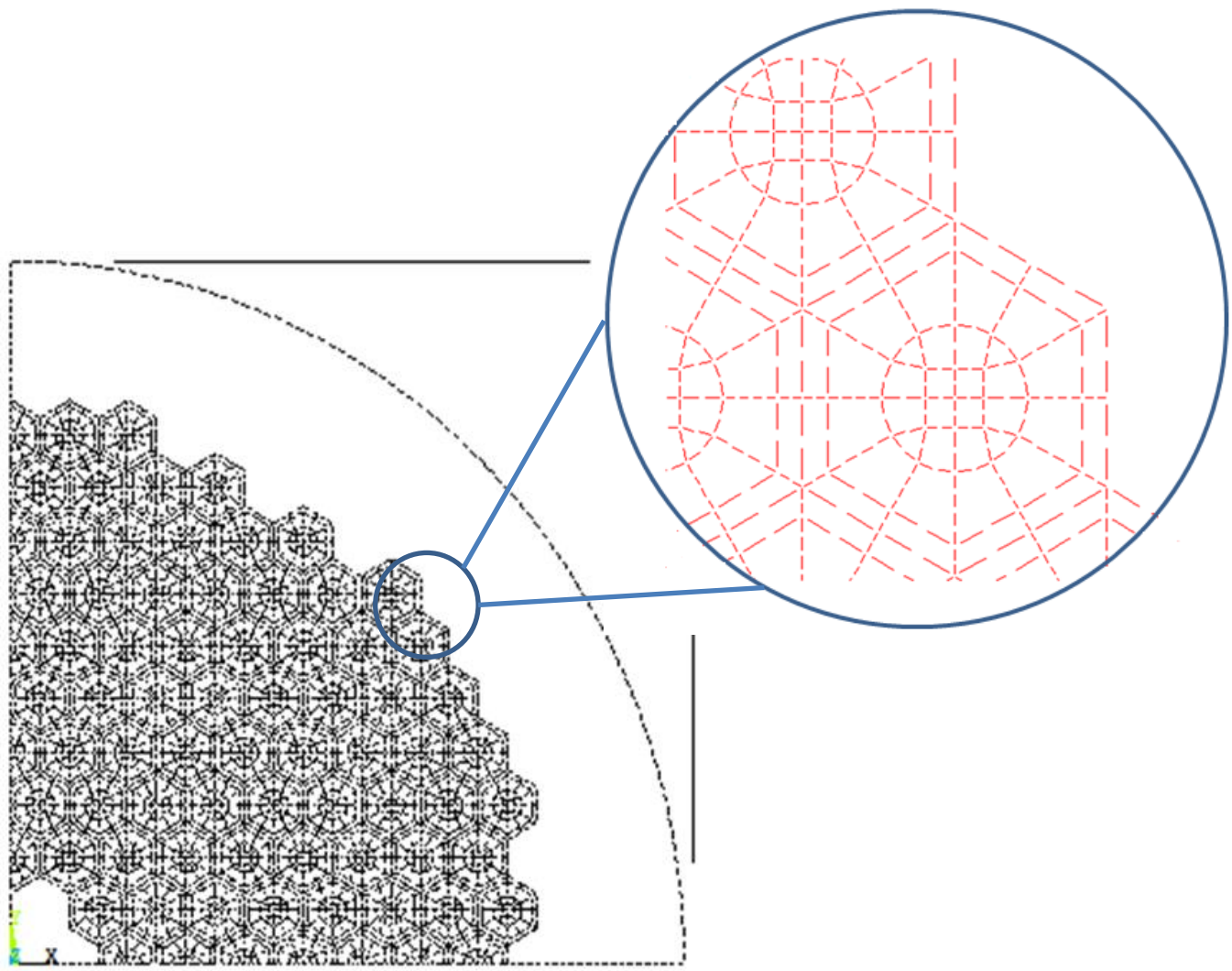


# Meshing

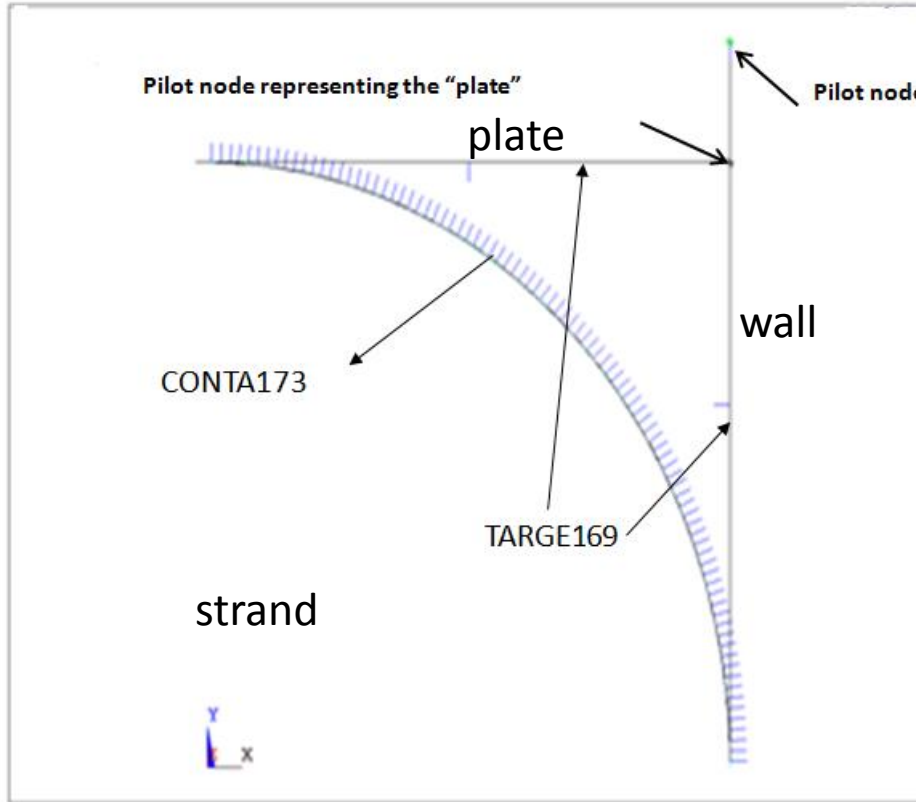
## Strand mesh

Element type: PLANE183 with plane strain option

Division on the lines to control the size and shape of elements.



**Contact pairs** include contact element CONTA172 and Target element TARGE169



Two contact pairs are used for the contact between:  
1 "plate and strand"  
2 "wall and strand"

Plate and wall are modeled as rigid bodies (no deformation) with TARGE169.

Pilot nodes are generated by an option of TARGE169. Pilot motion represents the motion of whole body.

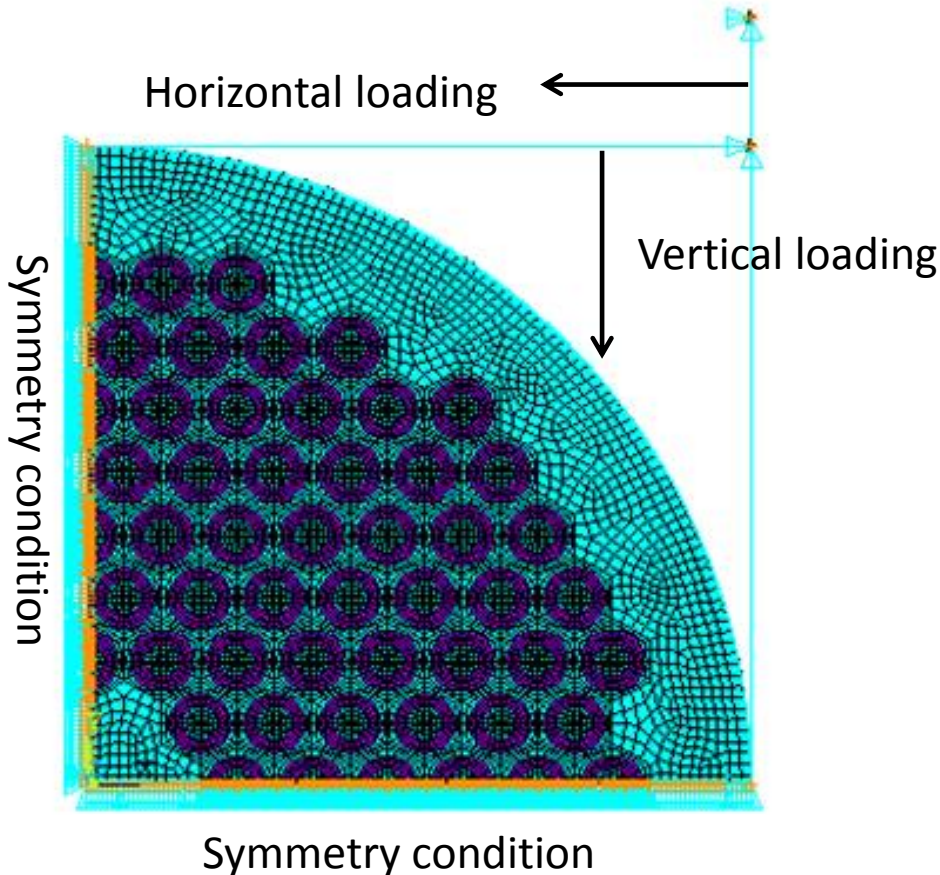
The load and boundary conditions are applied on the wall and the plate via pilot nodes.



# Boundary conditions and loading

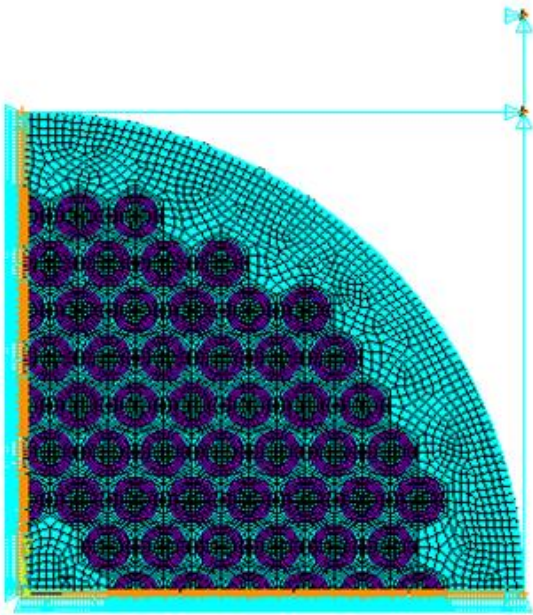
Boundary conditions (except symmetry conditions) and loads.

Loading direction	Plate: x-direction	Plate: y-direction	Wall: x-direction	Wall: y-direction
Vertical loading	Fixed	4wall: -0.07 mm 4wall+epoxy: -120 N/mm	Fixed	Unknown
Horizontal loading	Unknown	Fixed	4wall: -0.07 mm 4wall+epoxy: -120 N/mm	Fixed



Because of the twisting of the filaments, configurations of strand cross section are different at different axial positions.

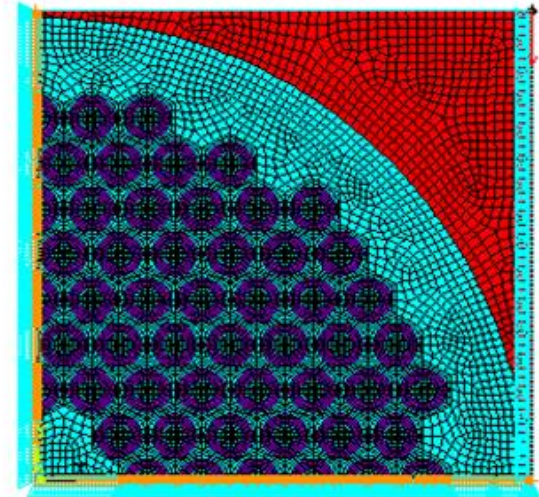
We apply the loads in vertical and horizontal directions to take into consideration twisting.



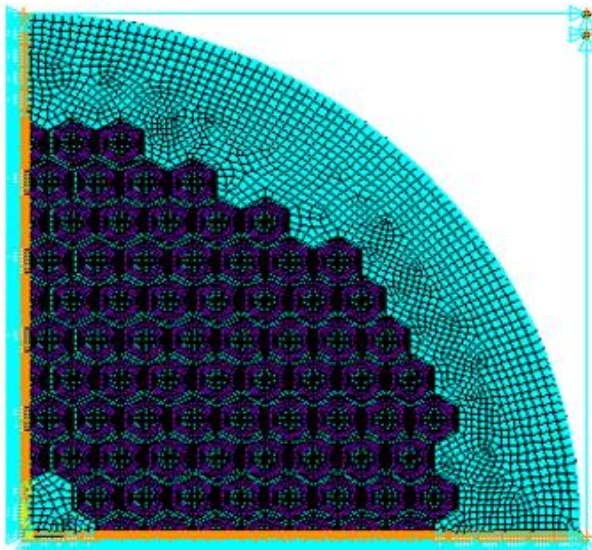
PIT192  
4W

MAT NUM  
U  
ROT  
F

PIT192  
4W+Epoxy



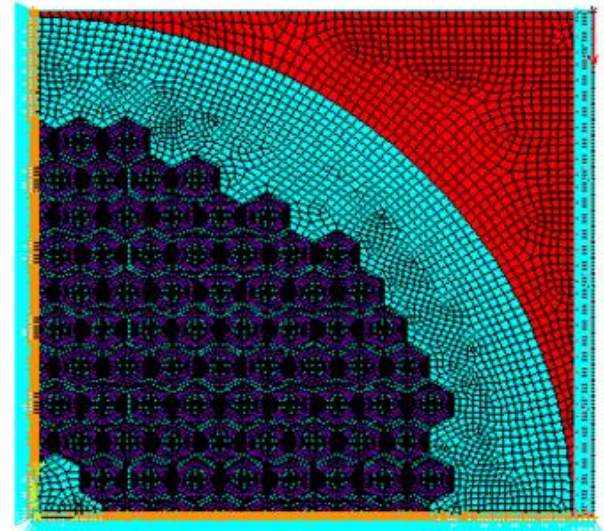
Horizontal loading ←  
Vertical loading ↓



PIT288  
4W

ELEMENTS  
MAT NUM  
U  
ROT  
F

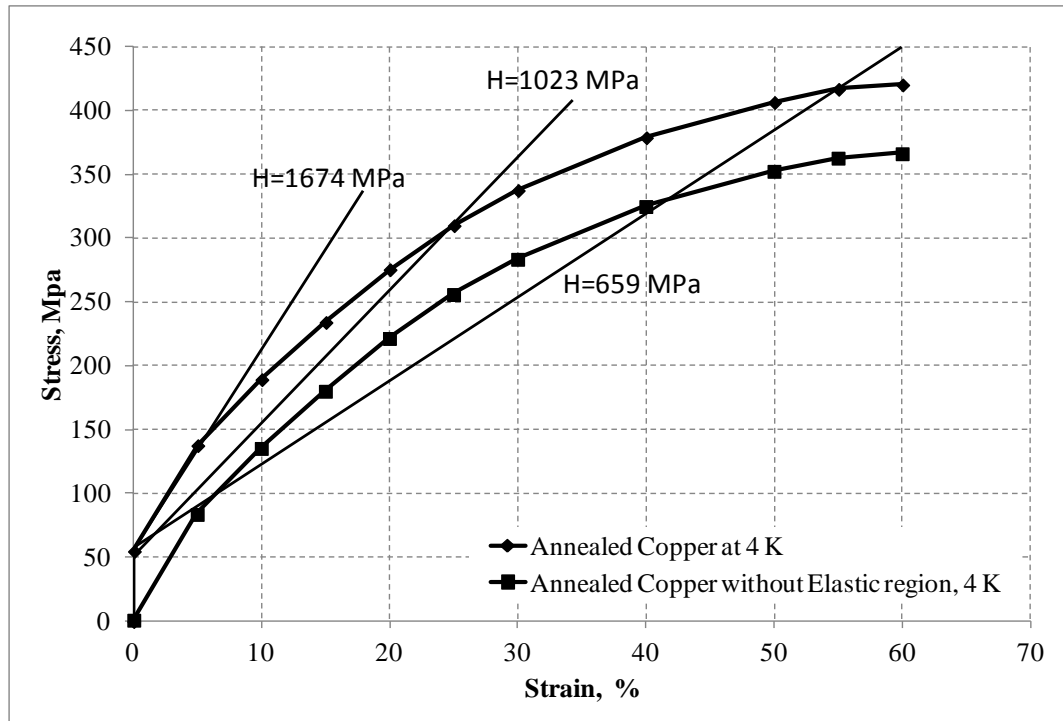
PIT288  
4W+Epoxy



# Material properties

## 1. Copper/Bronze

Bi-linear form, parameters: Young's modulus, Yield stress, and Plastic tangent modulus (H).



Copper behaves as a plastic material after cool-down. In the simulation, we ignore the elastic region of the stress-strain curve by setting yield stress = 1 MPa.

Good agreement on displacement curves as function of applied force for available experimental data [19] was obtained for H=1300 MPa [20].

2. Nb<sub>3</sub>Sn: Elastic material: E = 50~100 GPa in literatures at 4 K.

3. Epoxy: Elastic material: E is estimated as 9 GPa at 4 K.

# Solution

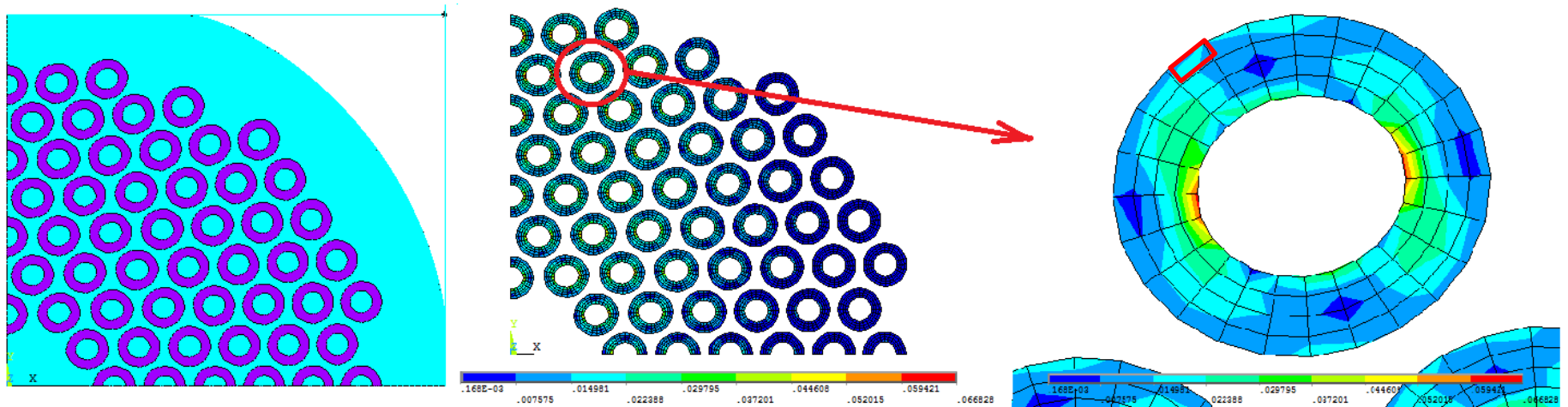
The simulation is a non-linear problem ( including contact behavior, large deformation and elasto-plastic material property).

The load is applied by multiple load steps to avoid convergence problem.

# Post-processing

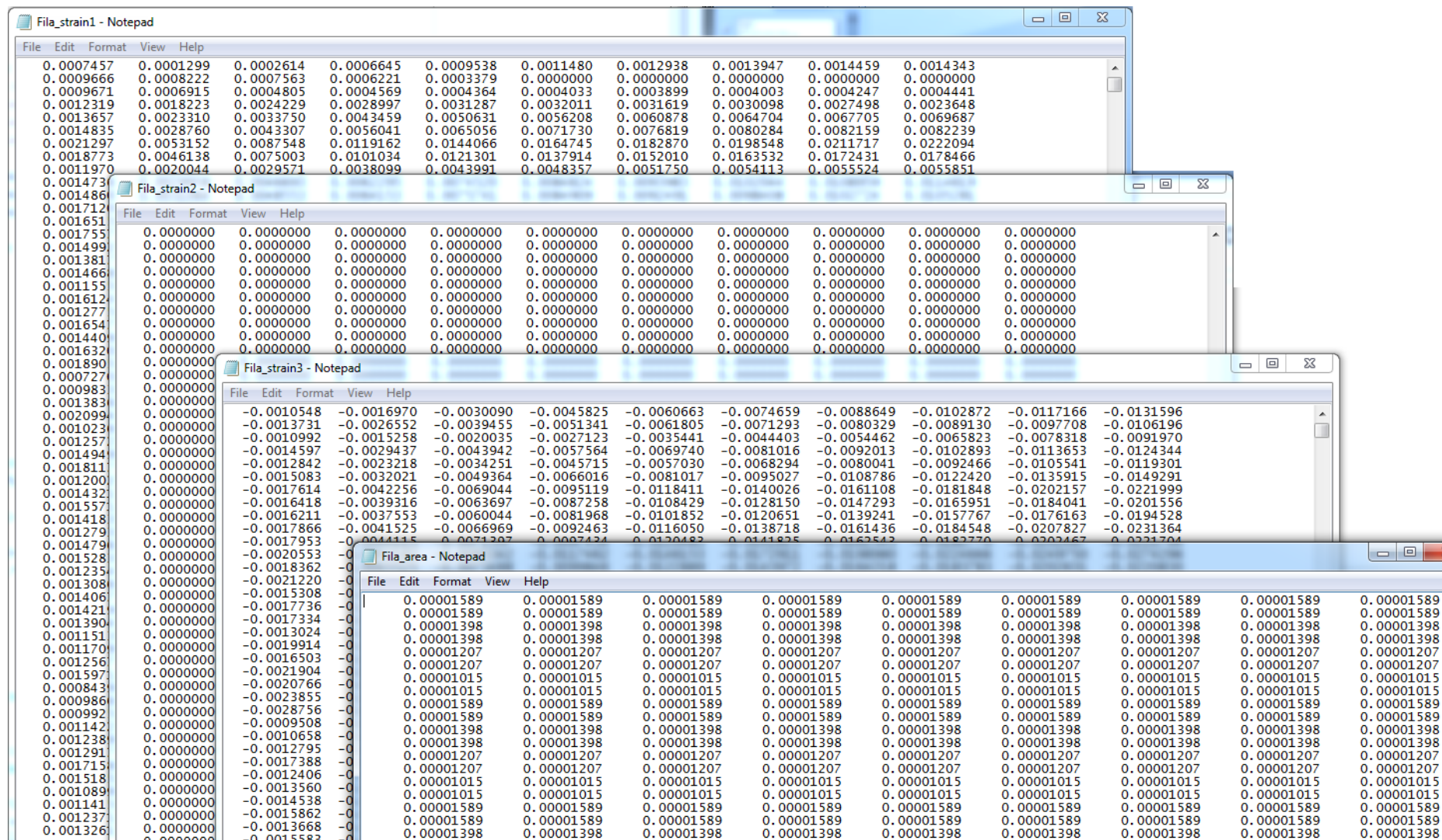
We output

1. 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> principle strains of all Nb<sub>3</sub>Sn filament elements.
2. Filament area of all Nb<sub>3</sub>Sn filament elements.
3. Compression force.



# Output files

Load step: 1 → 10



# Methodology

## Calculation based on a scaling law

A scaling law developed by B. Bordini et al [17] is used to calculate the critical current degradation. The calculation procedure is listed as follow:

Description of step	Relevant equations
1. Calculate the first invariant of strain tensor and the second invariant of deviator strain tensor, $I_1$ and $J_2$ , with the principle strains.	$I_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ $J_2 = \frac{1}{6} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2]$
2. Calculate the strain function $s(\varepsilon)$ with $I_1$ and $J_2$ assuming the effective Poisson's ratio of the PIT strand is 0.36	$s(\varepsilon) = \frac{e^{-C_1 \frac{J_2+3}{J_2+1} J_2} + e^{-C_1 \frac{I_1^2+3}{I_1^2+1} I_1^2}}{2}$
3. Calculate the critical temperature $T_c(\varepsilon)$ as a function of strain function.	$T_c(\varepsilon) = T_c(0) s(\varepsilon)^{\frac{1}{w}}$ <p><math>t = T/T_c(\varepsilon)</math>, T is the experiment temperature</p>
4. Calculate the second critical magnetic field $B_{c2}(T, \varepsilon)$ and the normalized magnetic field $b$ .	$B_{c2}(T, \varepsilon) = B_{c2}(0,0) s(\varepsilon) (1 - t^v)$ <p><math>b = B/B_{c2}(0,0)</math>, B is the background field</p>
5. Calculate the pinning force $F_p(T, \varepsilon)$ and $F_p(T, 0)$ .	$F_p(T, \varepsilon) = J_c(T, \varepsilon) \times B = Cg(s(\varepsilon))h(t)b^p(1 - b)^q$ <p><math>g(s(\varepsilon)) = s(\varepsilon)</math>; <math>h(t) = (1 - t^2)(1 - t^{1.52})</math></p>
6. Calculate the critical current degradation of Nb <sub>3</sub> Sn element $i$ , $R_i$ .	$R_i = \frac{I_c(T, \varepsilon)}{I_c(T, 0)} = \frac{F_p(T, \varepsilon)}{F_p(T, 0)}$
7. Calculate the critical current degradation of the strand.	$\frac{I_c}{I_{c0}} = \frac{\sum_{i=1}^N A_i R_i}{\sum_{i=1}^N A_i}$

```

T=4.2;    % Operation temperature
B=19;    % Magnetic field in the experiment

%Read raw data
importfile('F_D.txt');    % Compression force
importfile('Fila_area.txt');    % Filament element area
importfile('Fila_strain1.txt');    % 1st principle strain
importfile('Fila_strain2.txt');    % 2nd principle strain
importfile('Fila_strain3.txt');    % 3rd principle strain
Fila_strain1=Fila_strain1*100;
Fila_strain2=Fila_strain2*100;
Fila_strain3=Fila_strain3*100;

% Define the constants
w=3;    v=1.5;    p=0.5;    q=1.91;
% PIT data
Tc0=17;    Bc200=30.97;    C1=0.735;

% Square of the 1st invariant
I1_sqr=(Fila_strain1+Fila_strain2+Fila_strain3).^2;
% The 2nd invariant
J2=1/6.*((Fila_strain1-Fila_strain2).^2+(Fila_strain2-Fila_strain3).^2+(Fila_strain3-Fila_strain1).^2);
A1=exp(-1.*J2.*C1.*(J2+3)./(J2+1));
A2=exp(-1.*I1_sqr.*C1.*(I1_sqr+3)./(I1_sqr+1));

% S function
s=1/2.*(A1+A2);

% t
Tce=Tc0.*s.^(1/w);    t=T./Tce;    % Reduced temperature

% b
Bc2=Bc200.*s.*(1-t.^v);    b=B./Bc2;    % Reduced magnetic field

% Fp (pinning force)=B*Jc=B*(g(strain0)*h(t)*f(b))
Fp=B.*s.*(1-t.^2).*(1-t.^1.52).*b.^p.*(1-b).^q;

% Calculate the pinning force with 0 strain (s=1) at 4.2 K and magnetic field of B.
t0=T./(Tc0.*1);    Bc20=Bc200.*1.*(1-t0.^v);
b0=B./Bc20;    Fp0=B.*1.*(1-t0.^2).*(1-t0.^1.52).*b0.^p.*(1-b0).^q;

% Ri of each Nb3Sn element
Ratio=Fp./Fp0;

```

Calculation step  
 1 → 6

```

% Set the ratio to 0 if the corresponding Bc2 < B(background field)
n=size(Ratio,1);
m=size(Ratio,2);
i=1;
j=1;
] for i=1:n;
]   for j=1:m;
   if Bc2(i,j)<B;
       Ratio(i,j)=0;
   end
   end
- end
- end

```

```

% Calculate the Ic degradation of the strand
Ic_degra_whole=(sum(Fila_area.*Ratio)./sum(Fila_area))';

```

```

%Reset the strain data
Fila_strain1=Fila_strain1/100;
Fila_strain2=Fila_strain2/100;
Fila_strain3=Fila_strain3/100;

```

Final result, step 7

We use the minimum value of  $R_i$  from vertical loading and from horizontal loading to estimate **the smallest critical current**.

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Ratio_combined=min(Ratio_h,Ratio_v);
Ic_combined=(sum(Fila_area.*Ratio_combined)./sum(Fila_area))';

```

# Results and discussion

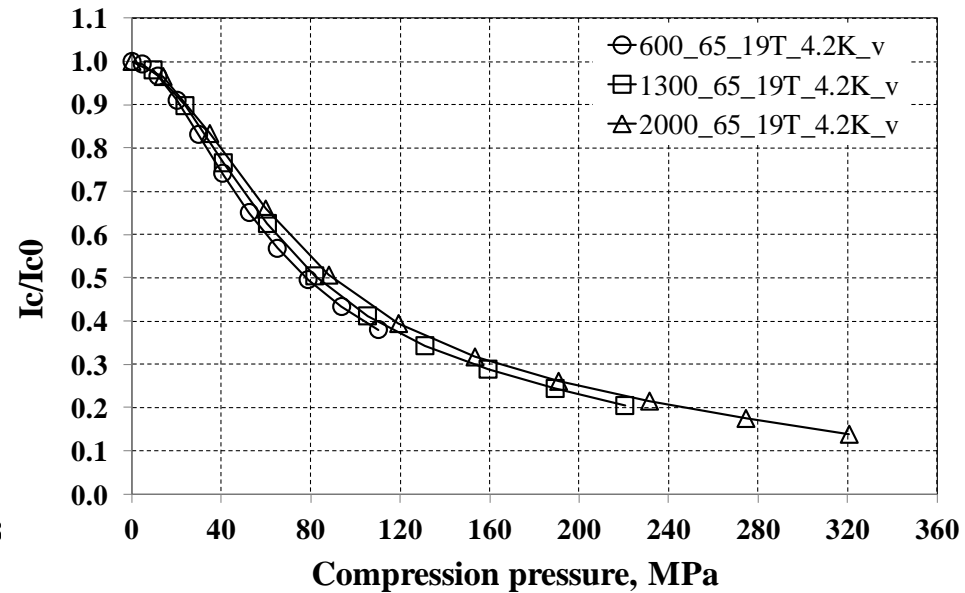
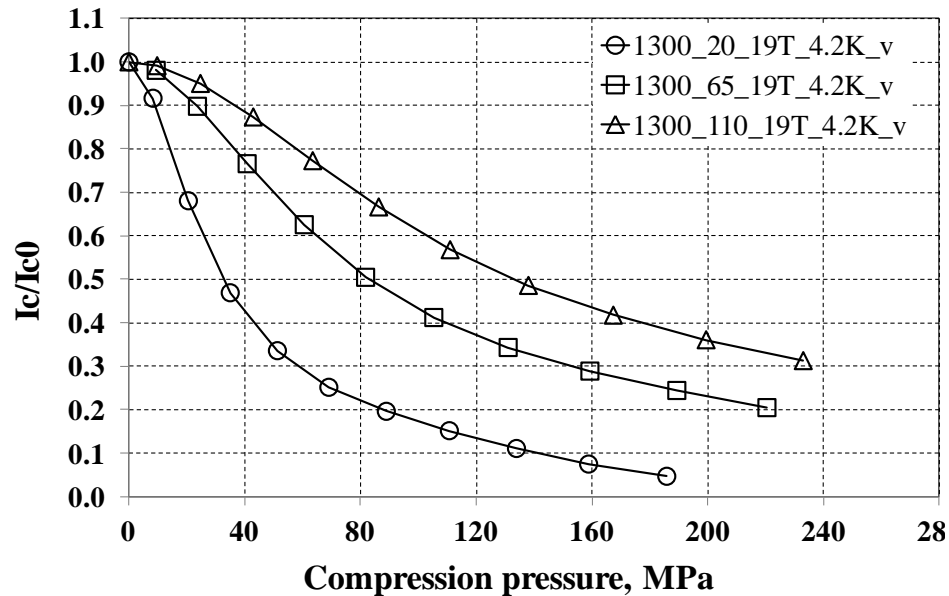
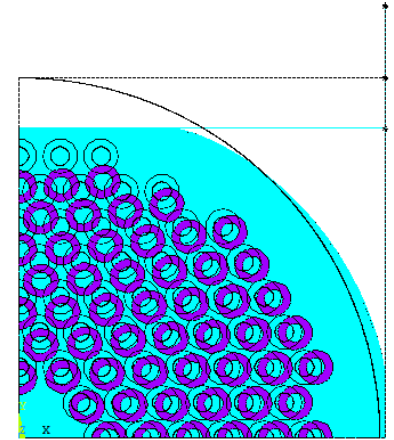
Parametric study of the effects of:

- Tangent modulus of Copper (bronze), range from 600 to 2000 MPa;
- Young's modulus of Nb<sub>3</sub>Sn, range from 20 to 110 GPa;

Based on PIT192\_4W simulation

X-axis: Compression force per unit axial length / Radius of the quarter FE model.

Y-axis:  $I_c/I_{c0}$  of the whole strand

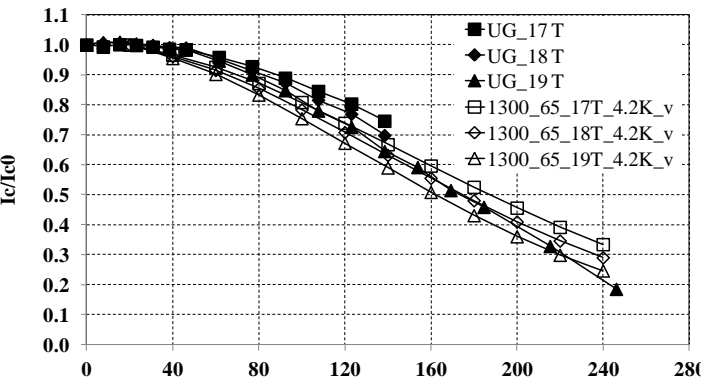


Conclusion:  $I_c/I_{c0}$  is sensitive to Young's modulus of Nb<sub>3</sub>Sn.

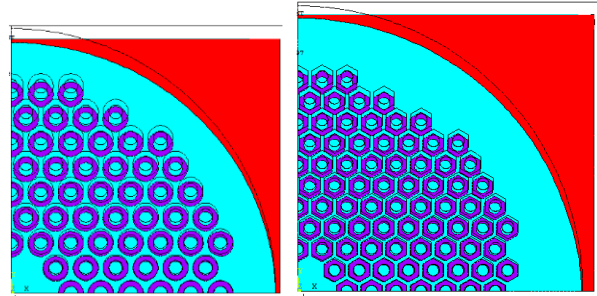
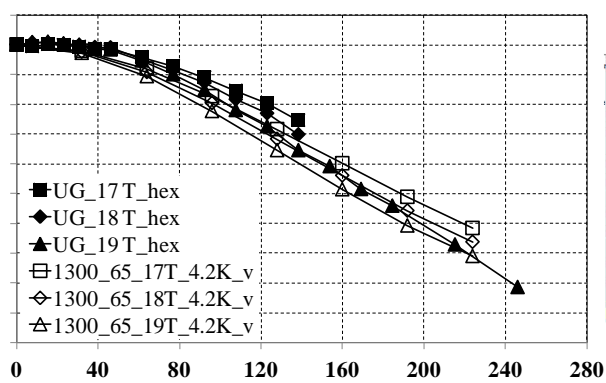


# PIT192 and PIT288 under 4W+Epoxy condition:

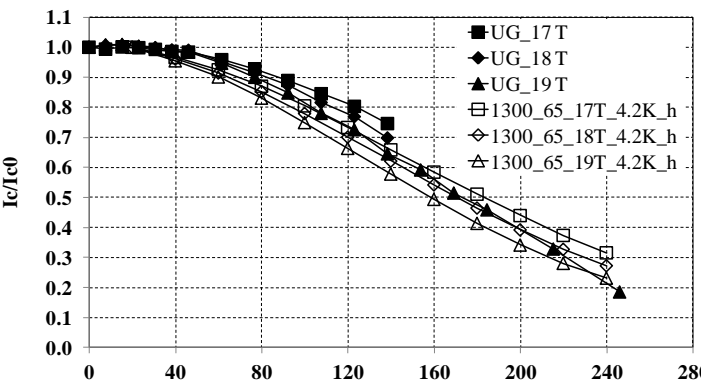
PIT192\_4W+epoxy\_vertical



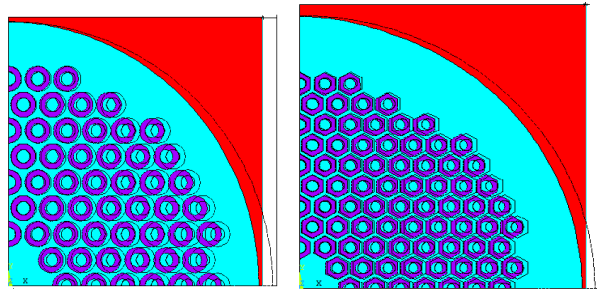
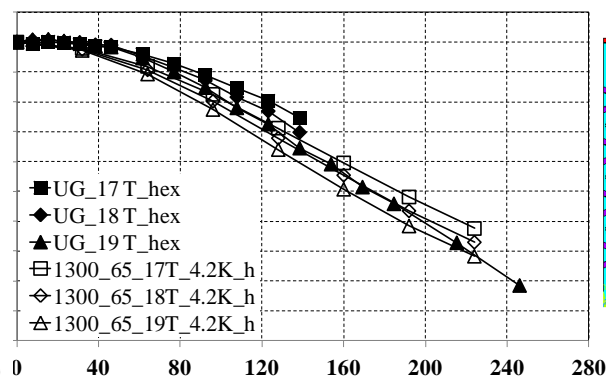
PIT288\_4W+epoxy\_vertical



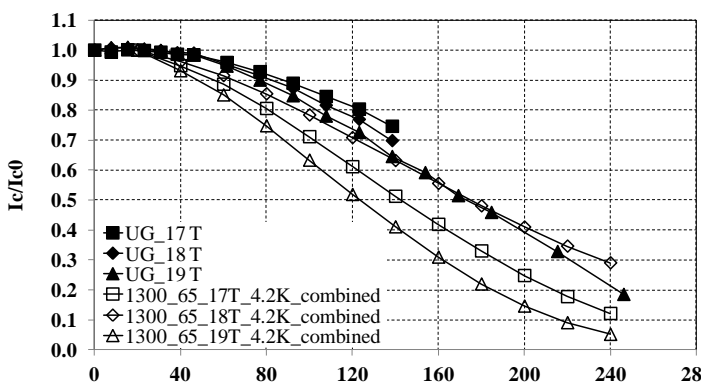
PIT192\_4W+epoxy\_horizontal



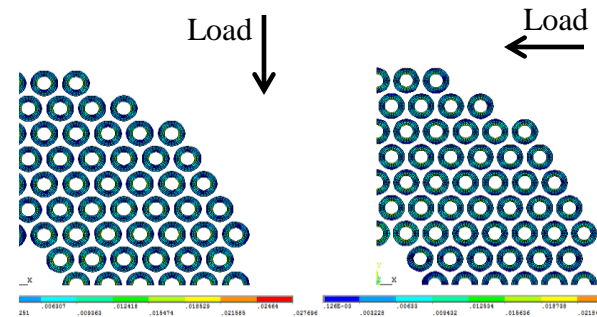
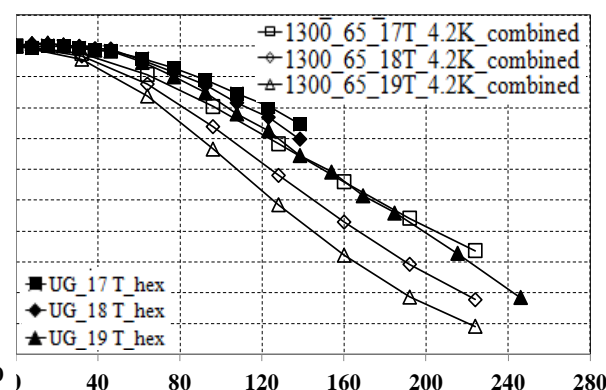
Compression pressure, MPa



PIT192\_4W+epoxy\_combined



PIT288\_4W+epoxy\_combined



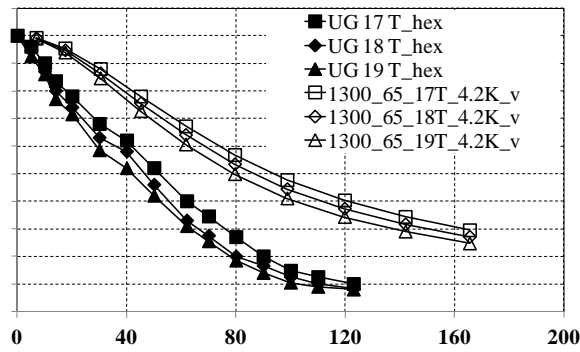
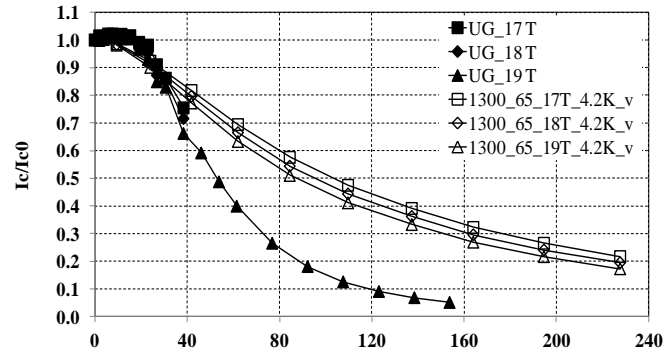
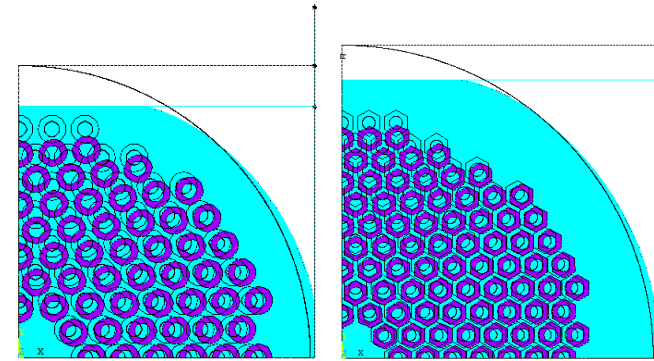
Compression pressure, MPa

Compression pressure, MPa

# PIT192 and PIT288 under 4W condition:

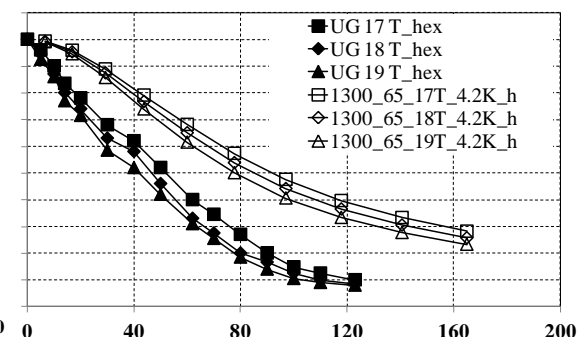
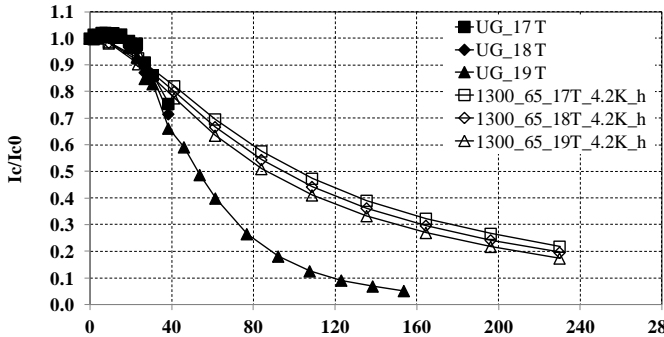
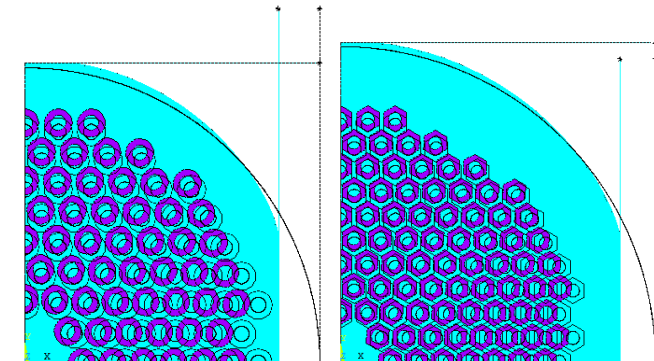
PIT192\_4W\_vertical

PIT288\_4W\_vertical



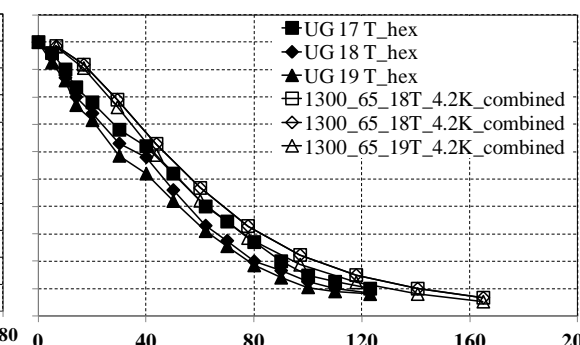
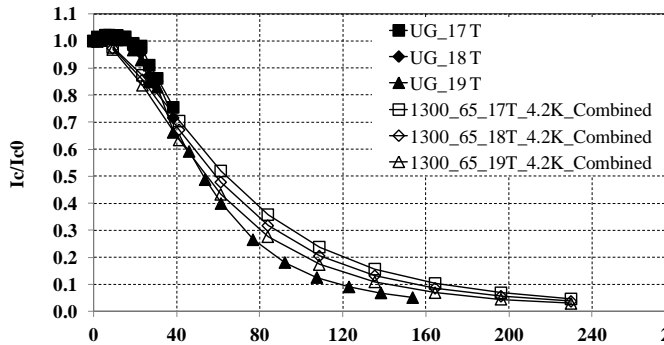
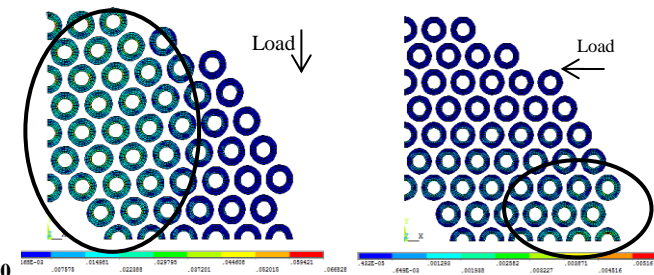
PIT192\_4W\_horizontal

PIT288\_4W\_horizontall



PIT192\_4W\_combined

PIT288\_4W\_combined



Compression pressure, MPa

Compression Pressure, MPa

# Summary

PIT192 and PIT288 Ic degradation Calculation vs Experiment		
	4W	4W+Epoxy
Vertical	<<	✓
Horizontal	<<	✓
Combined	✓ (<)	✓ (>)

- Nb<sub>3</sub>Sn Young's modulus of 65 GPa seems appropriate for this work. The same value was used successfully in previous studies [10].
- 4W+Epoxy results of vertical/horizontal loading agreed with the experiment very well. The combined result overestimates the Ic degradation.
- 4W results of vertical/horizontal loading showed smaller degradation than the experiment. The combined results (largest strain configuration) is also smaller than the experiment. This may be caused by the uncertainties of the real loading condition in the 4W experiment (strand is not supported as well as in 4W+epoxy. This might cause more filament damages, lower Ic.)
- Limitation of methodology: residual compressive strain of the filaments and current sharing between the filaments are not considered.

# Conclusion

- Ic degradation is sensitive to Young's modulus of Nb<sub>3</sub>Sn. Transverse stiffness of the strand is determined by the plastic tangent modulus of copper.
  - More accurate Young's modulus of Nb<sub>3</sub>Sn measurements at 4 K are necessary in the future.
  - More accurate measurements for annealed copper under transverse load to study the compression displacement as a function of compression force are necessary.
- Preliminary results obtained by the new method are compared with experiments.
  - Good agreement is obtained when strand is positioned in the holder filled with epoxy.
  - Simulation showed smaller degradation in 4W cases.
- The Nb<sub>3</sub>Sn material property used in this work is consistent with that in a previous Internal-tin triplet strand calculation, which indicates that this method may be used for different types of strands.
- This method can serve as a platform for optimization of the strand as geometry parameters are easily adjustable in model (sub-element spacing, shape of the sub-element and the size of the strand)
- This method may be used for critical current calculation of higher stages of cable.
- More experiments are needed to verify the use of this methodology

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# Thank you

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Phillip Mallon   Nathaniel Allen   Daniel Catanzano

Thank you