Losses Scaling Law for Superconducting Filaments Magnetized by an Elliptical Field.

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Chats
Applied Superconductivity
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NASA's Distributed Propulsion Aircraft

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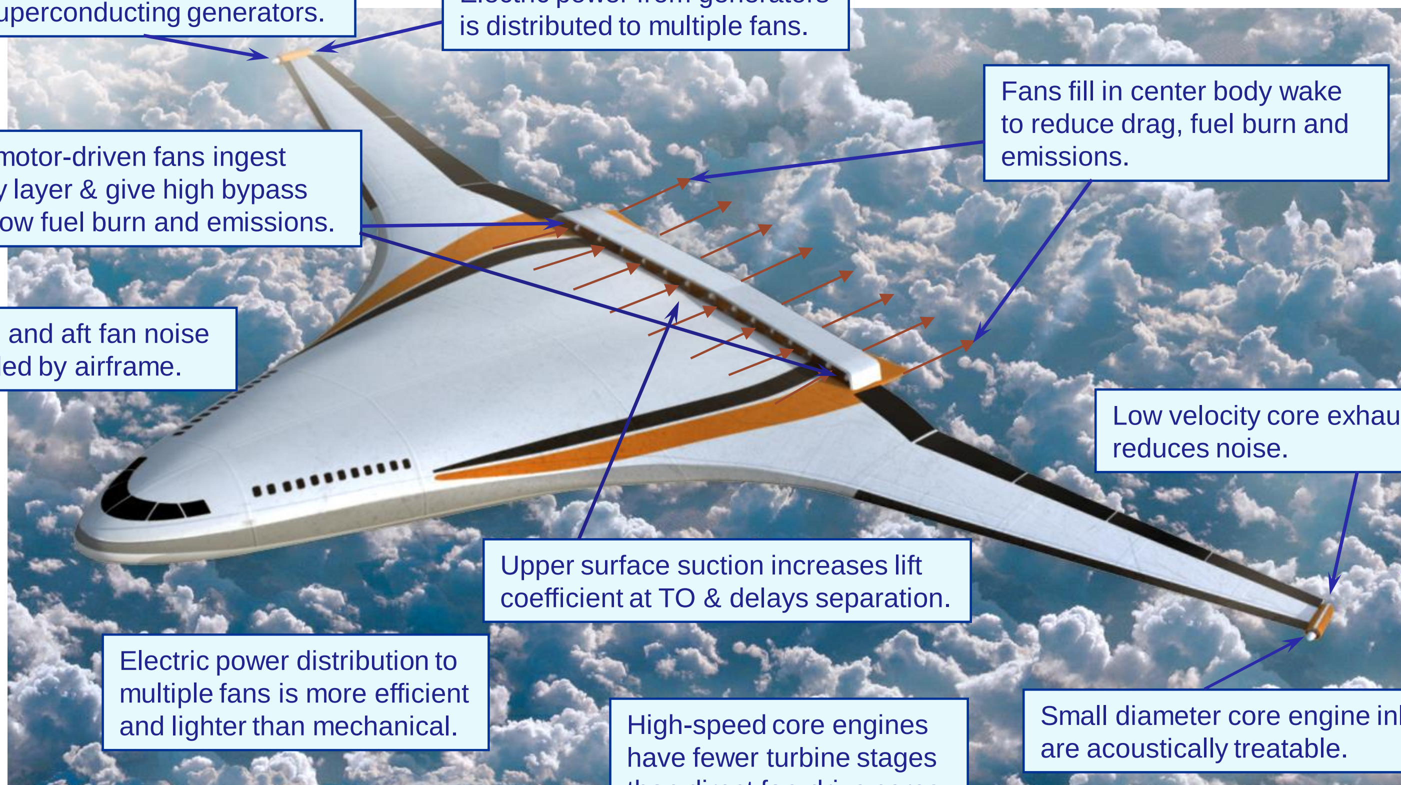
National Aeronautics and Space Administration

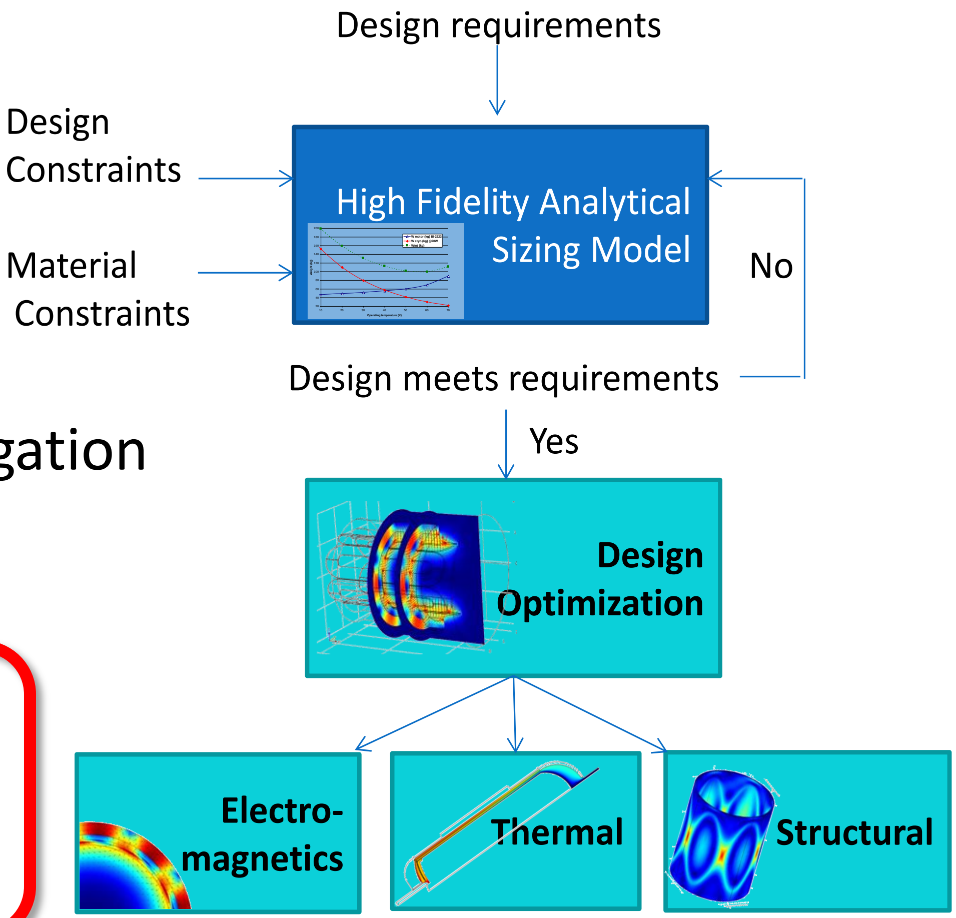
BENEFITS



THE TURBOELECTRIC APPROACH CONTRIBUTES TO EVERY CORNER OF THE SFW TRADE SPACE

Project Objectives

- Develop a **high fidelity** sizing tool for **fully superconducting rotating machines**.
  - Accurate 3D geometry represented
    - Electromagnetics, mechanical and thermal
  - Portable code in Python and C



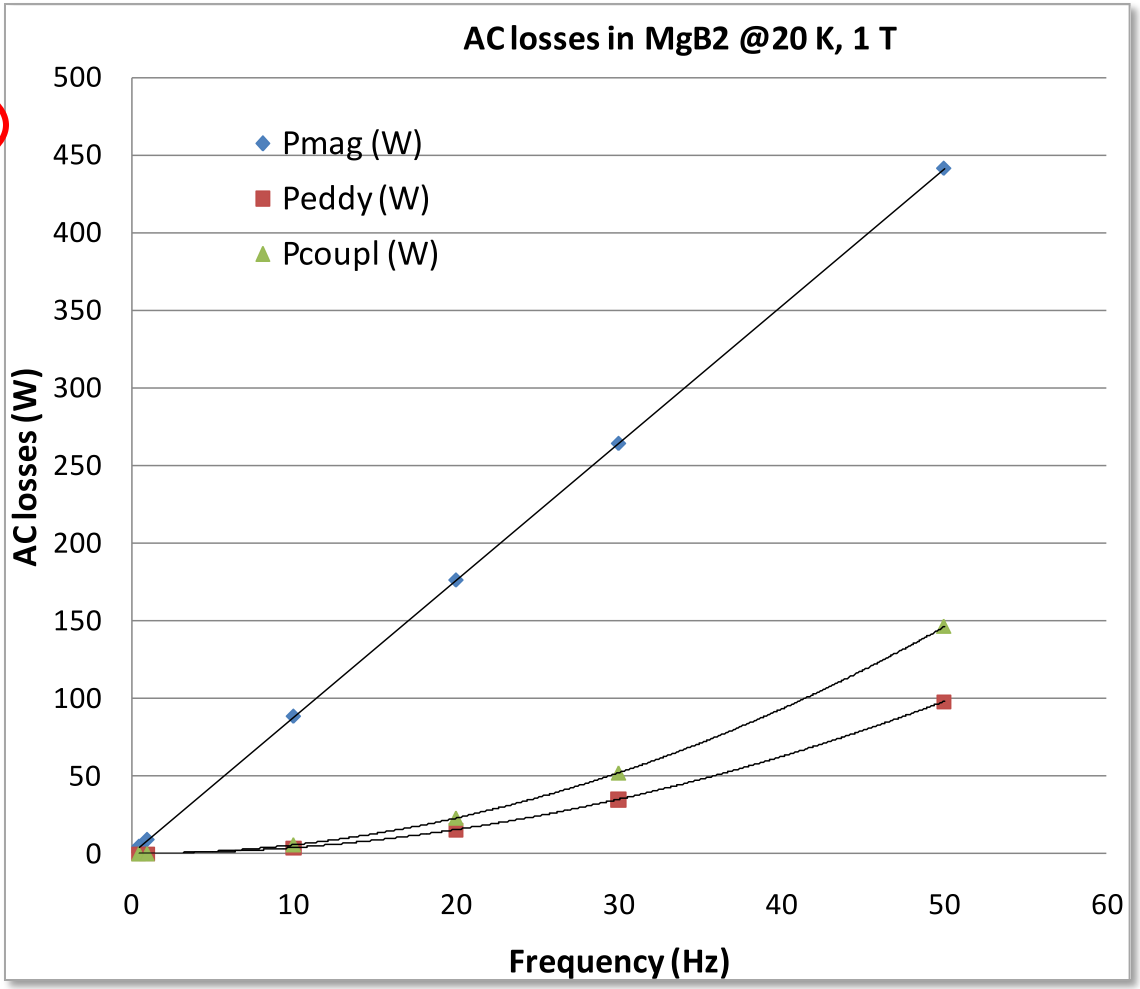
- Develop model for quench propagation
  - Address detection and protection

- Develop **new model** for AC losses for superconducting stators
  - Based on FEA simulations

- Validate AC losses model **experimentally**

Frequency Dependence of AC Losses

- 3 sources of AC losses in superconductors:
  - Magnetization losses
    $P_{mag} \propto B, f, J_c$
  - Coupling losses
    $P_{coupl} \propto B^2, f^2$
  - Eddy current losses
    $P_{Eddy} \propto B^2, f^2$



- **Magnetization losses are dominating at low frequency**

Magnetization (Hysteresis) Losses Calculation

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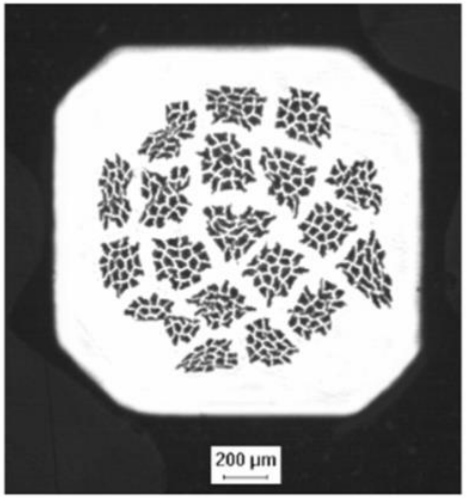
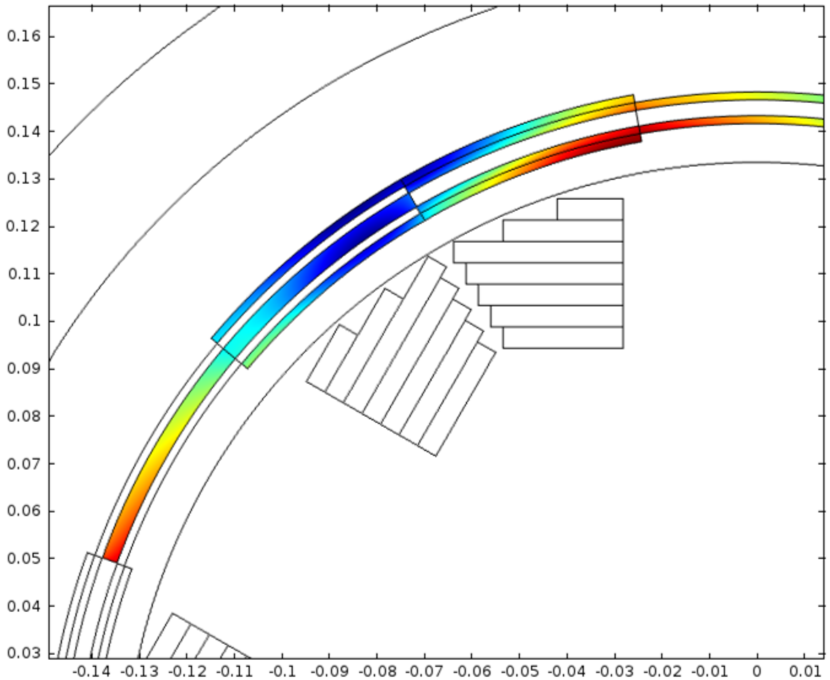
- Assumes AC field (alternating)
  - Limited to self field
- Transport current and applied field are in phase



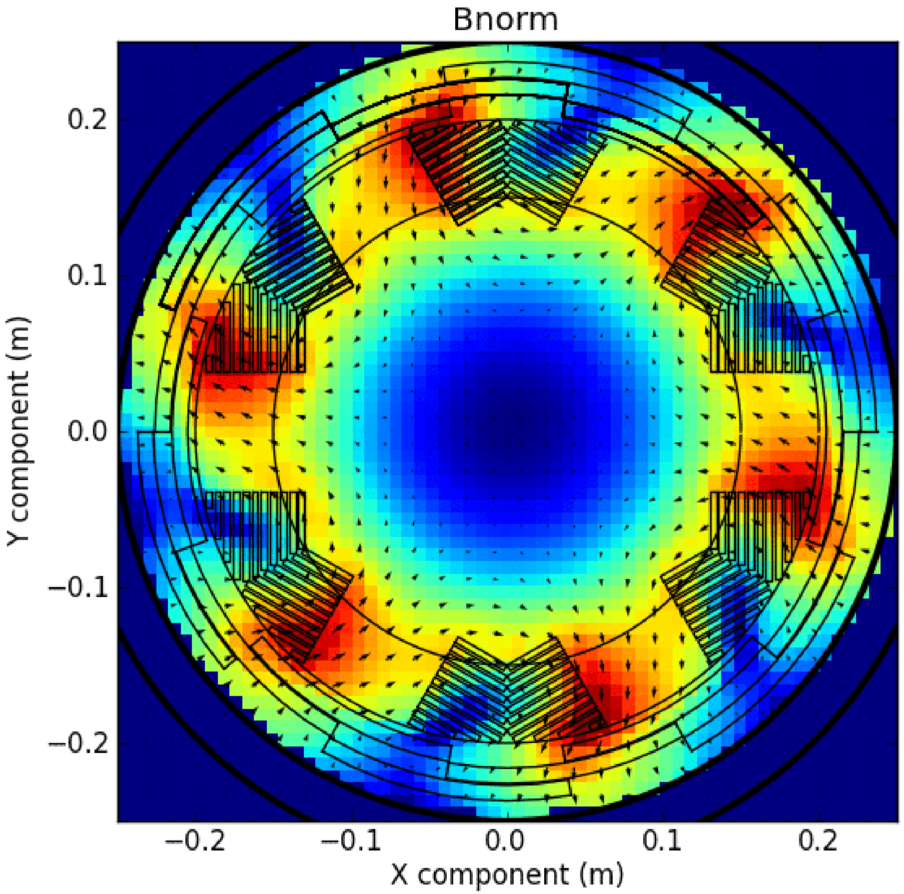
$$P_m = \left(\frac{8}{3\pi} J_c \lambda_s d B_0 \right)\left(1 + F^2\right) \quad [W/m^3]$$

J_c: filament critical current density
λ_s: fraction of superconductor
d: filament size
B_0: flux density variation
F: I/Ic

Stator Peak Field at Full Load



- Flux density contribution from both rotor and stator



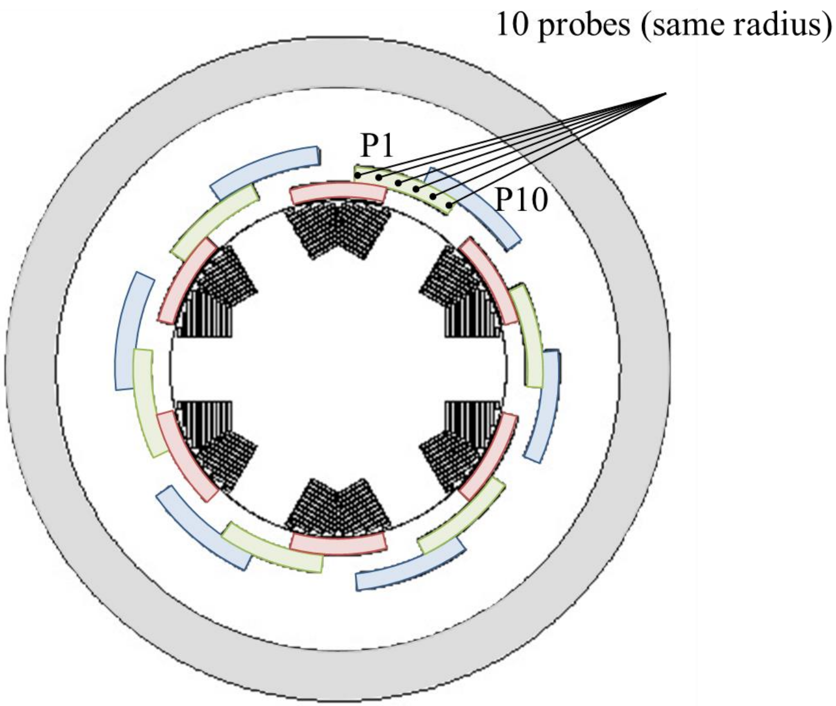
- Non uniform flux density distribution

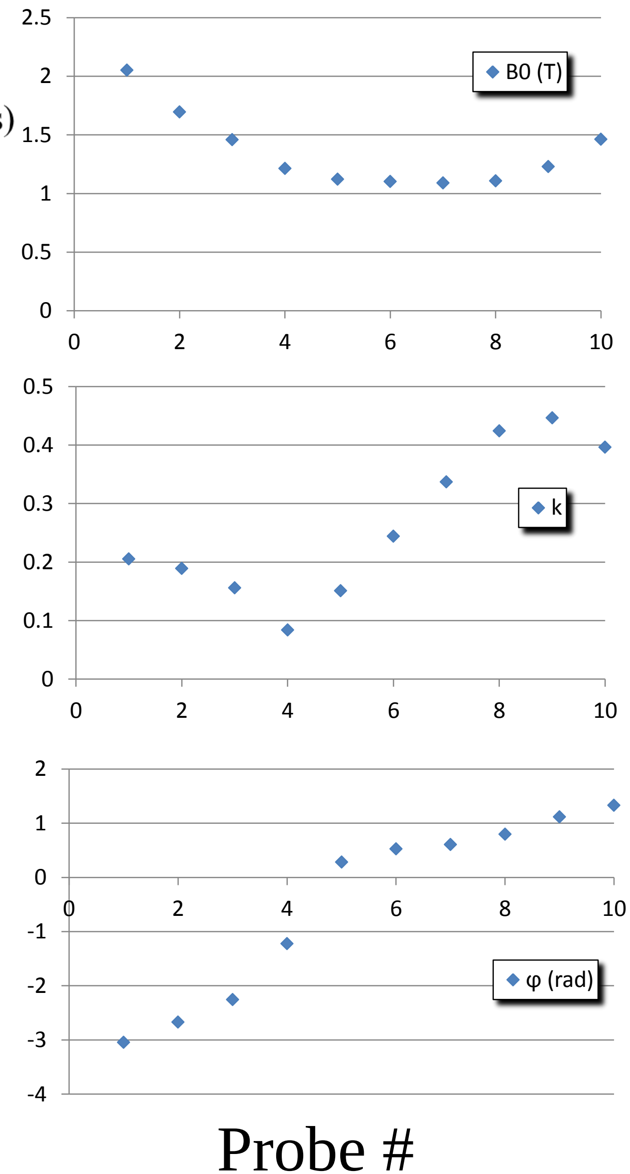
Introduction (1/3)

- Magnetization losses in stator wires of rotating machines:
  - Stator: **Rotating** field generated by temporal variation of currents inside the windings.
  - Rotor: **Rotating** and **Alternating** field produced by rotation of the spatial distribution of the currents.

$\rightarrow$ Local field = rotating field + alternating field

Introduction (2/3)





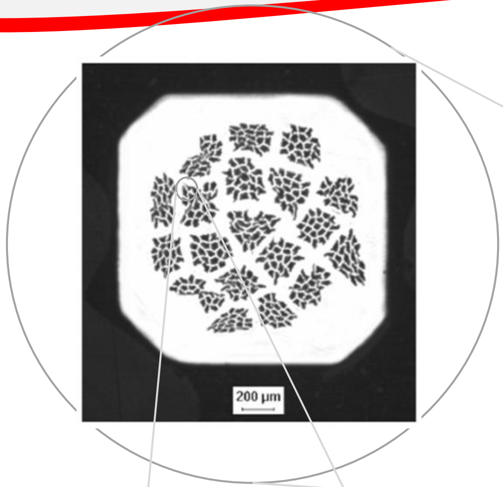
Peak field

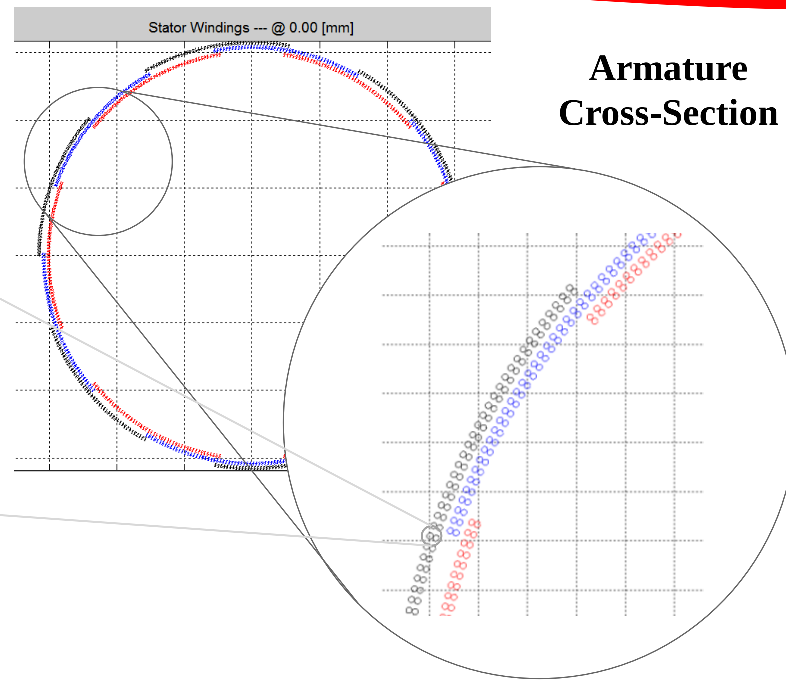
Rotating/pulsating

Phase angle rotating-pulsating

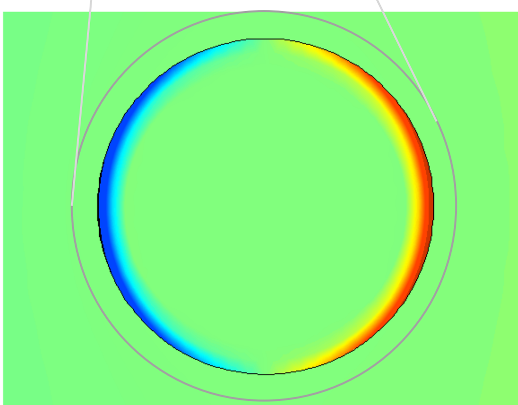
In 1 pole, each stator conductor is in a unique magnetic configuration

Introduction (3/3)





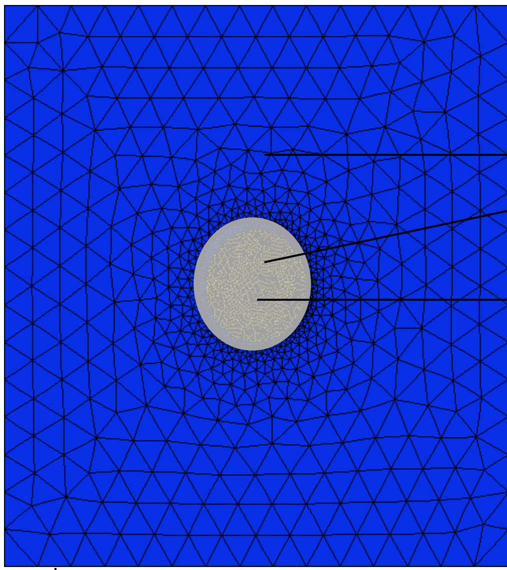
Armature Cross-Section



- Filaments are twisted and magnetically decoupled
  - Analysis can be done on 1 filament
  - Model implemented in COMSOL Multiphysics
  - Sc. Modeled using a power law
  - No current yet

Model description (1/2)

- Geometry and model description:



r filament radius

H-Formulation:
$$\begin{cases} \overrightarrow{rot}\vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t} \\ \vec{J} = \overrightarrow{rot}\vec{H} \end{cases}$$

Electrical behavior, power-law:

$$\vec{E} = \frac{E_c}{J_c} \cdot \left(\frac{|\vec{J}|}{J_c} \right)^{n-1} . \vec{J}$$

n n-value

J_c critical current @ E_c = 100 μV/m

Boundary conditions:

$$\vec{B}_{rot} = \begin{cases} kB_0 \sin(2\pi ft) \\ kB_0 \cos(2\pi ft) \end{cases}$$

$$\vec{B}_{alt} = \begin{cases} 0 \\ (1-k)B_0 \cos(2\pi ft + \varphi) \end{cases}$$

B_0 field amplitude

k = [0,1] → field configuration: *"ellipticity"*

f frequency

φ = [-π,π]

7 variables

Model description (2/2)

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- Loss [J/m/cycle]:

$$Q = \oint dt \int_S E_z J_z dS$$

Over 1 cycle

Filament cross-section (πr^2)

- Dimensional Analysis:

Variable	Usual unit	[kg]	[m]	[s]	[A]	
J_c	$A.m^{-2}$	0	-2	0	1	n1
μ_0	$H.m^{-1}$	1	1	-2	-2	n2
r	m	0	1	0	0	n3
E_c	$V.m^{-1}$	1	1	-3	-1	n4
Q (per cycle)	$J.m^{-1}$	1	1	-2	0	n5
B_0	T	1	0	-2	-1	n6
f	Hz	0	0	-1	0	n7
φ	rad	0	0	0	0	
k	adim	0	0	0	0	
n	adim	0	0	0	0	

$$\left(\frac{r^2 f J_c \mu_0}{E_c}\right)^{n_7}\left(\frac{B_0}{J_c \mu_0 r}\right)^{n_6}\left(\frac{Q}{J_c^{\,2} \mu_0 r^4}\right)^{n_5}$$

3 dimensionless numbers

f* , b* , q*

Remark: adding a current I_t would lead to an additional dimensionless number.

$$\left(\frac{I_t}{J_c r^2}\right)^{n_8}$$

$$[J_c]^{n_1}[\mu_0]^{n_2}[r]^{n_3}[E_c]^{n_4}[Q]^{n_5}[B_0]^{n_6}[f]^{n_7} = [0]$$

This number must be dimensionless.

Dimensional Analysis (2/2)

- Our parameters:

$$b^* = \frac{B_0}{B_p} \quad \text{with} \quad B_p = \frac{2J_c\mu_0 r}{\pi}$$

$$f^* = \frac{f}{f_c} \quad \text{with} \quad f_c = \frac{E_c}{r^2 J_c \mu_0}$$

and n, k, φ

5 variables instead of 7

Those parameters can be called invariants since the reduced losses q do not change if they do not change.

Reduced losses:

$$q^* = \frac{Q}{Q_c} \quad \text{with} \quad Q_c = \mu_0 J_c^{\ 2} r^4 = rB_p I_c \quad \text{with} \quad I_c = J_c \pi r^2$$

Alternating field losses – SL (1/5)

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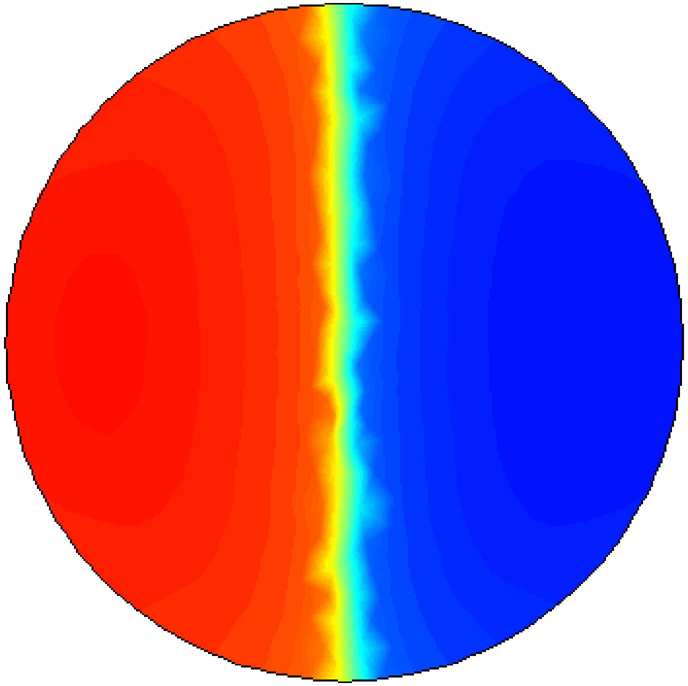
b*

f*

k=0

n



Alternating field losses – SL (1/5)

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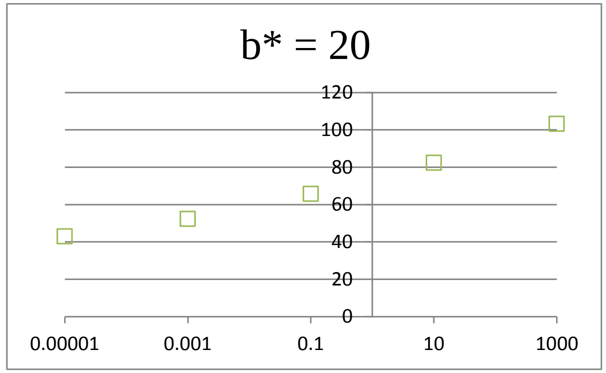
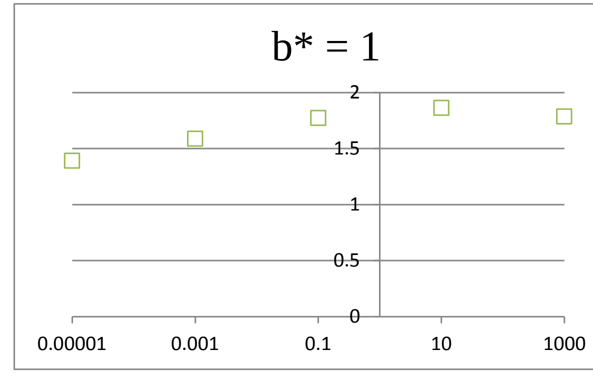
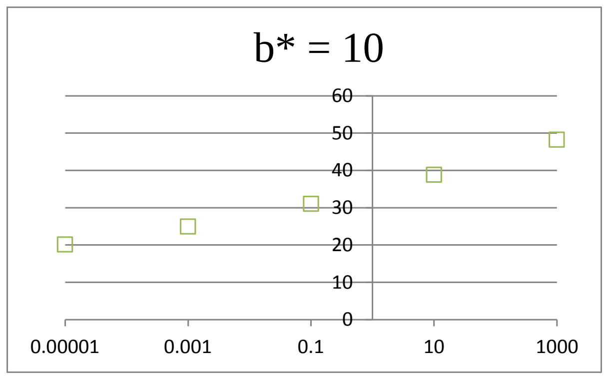
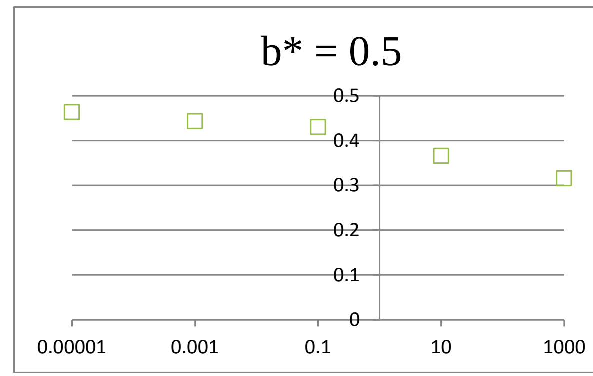
- n = 20

q* versus **f*** for various **b***

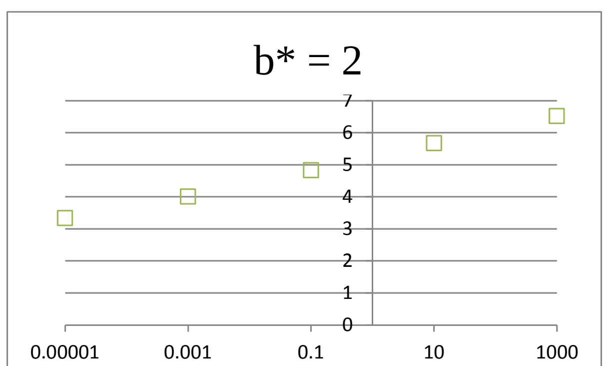


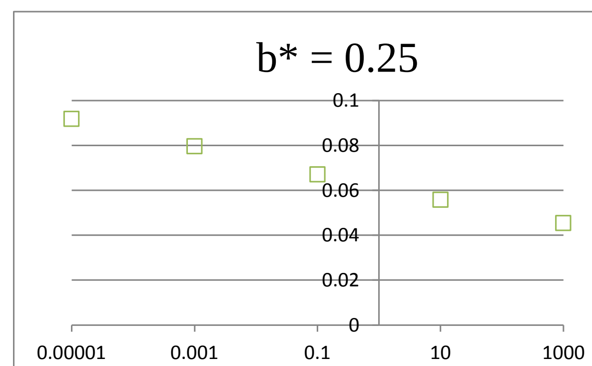






We need to reorder the charts to explain the thought process which led to a semi-analytical formula for AC losses under alternating field or rotating field





Alternating field losses – SL (2/5)

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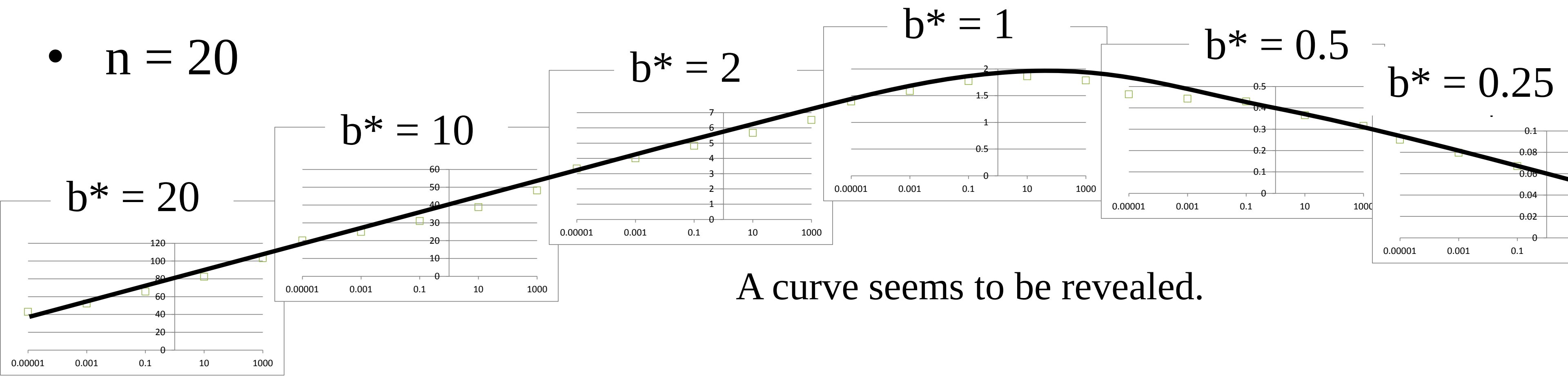
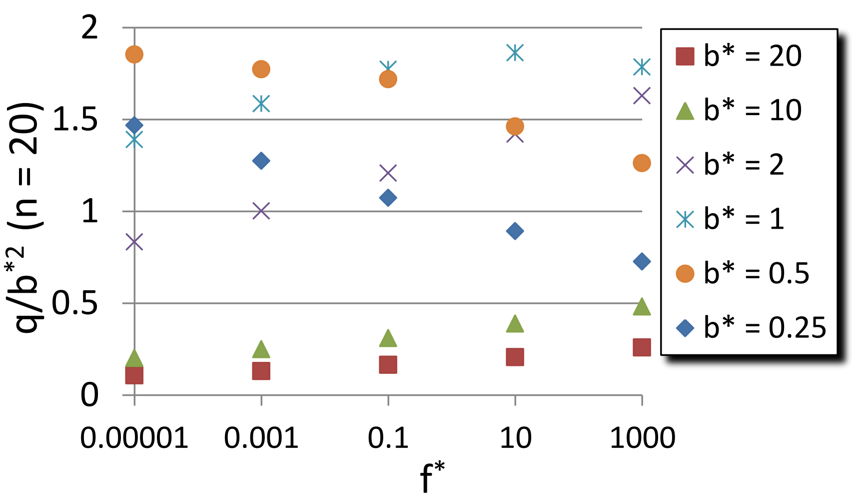
- n = 20



A curve seems to be revealed.

→We try to scale the ordinate to get reduced losses values in the same order of magnitude, to do so we divide q* by b^{*2}

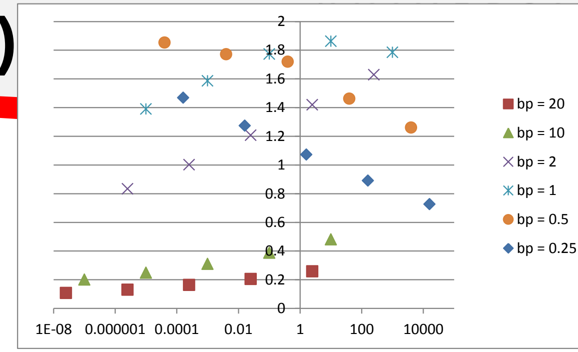


q*/b*² versus **f*** for multiple **b***
(b_p = b* former notation)

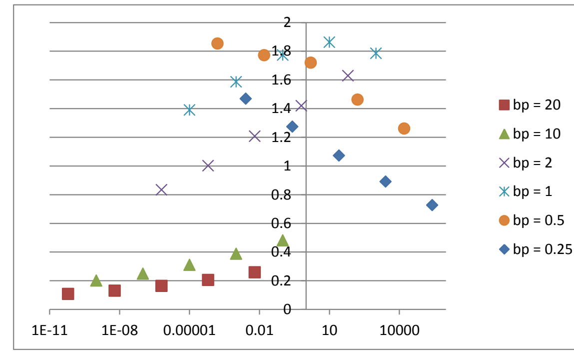
→ Then we try to adjust the abscissa values for each values of b* so as to create a curve with all the dots looking like the black curve above.

Alternating field losses (n=20) – SL (3/5)

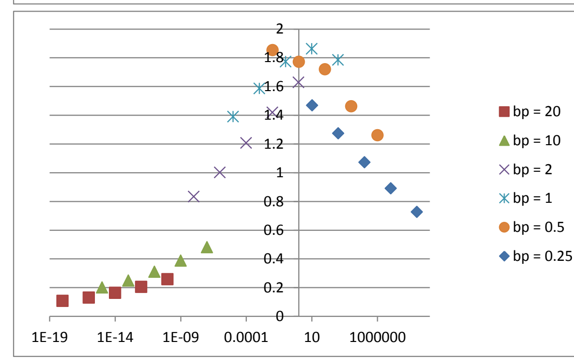
If we divide the abscissa by $b^{*2} \Rightarrow$



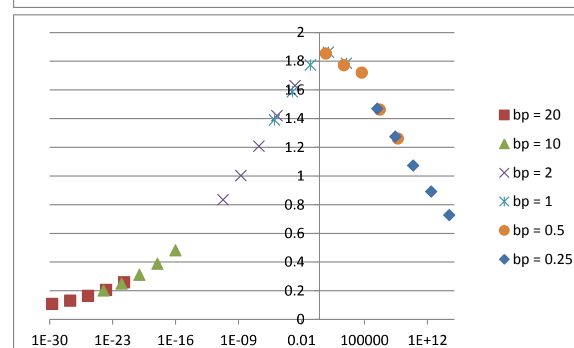
If we divide the abscissa by $b^{*4} \Rightarrow$



If we divide the abscissa by $b^{*10} \Rightarrow$

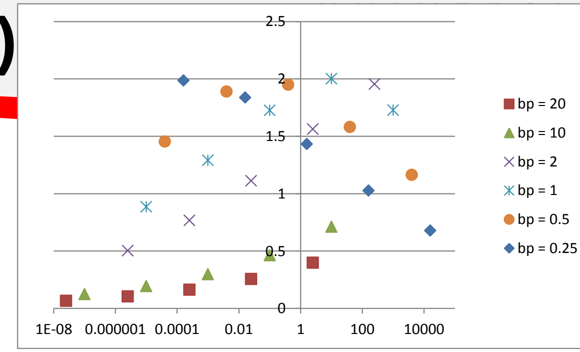


If we divide the abscissa by $b^{*19} \Rightarrow$

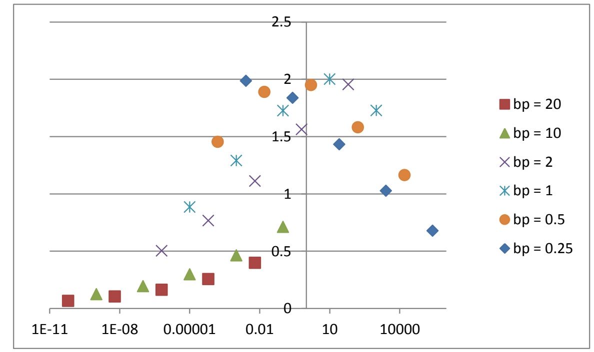


Alternating field losses (n=10) – SL (4/5)

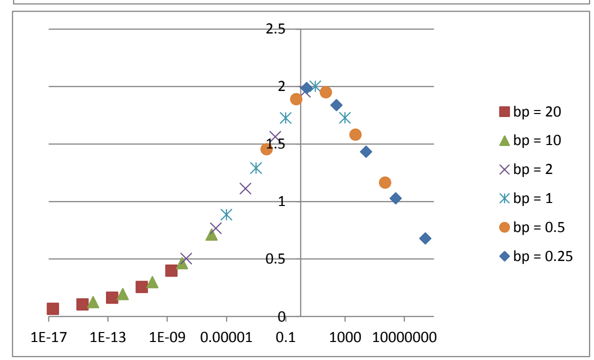
If we divide the abscissa by b^{*2} $\Rightarrow$



If we divide the abscissa by b^{*4} $\Rightarrow$

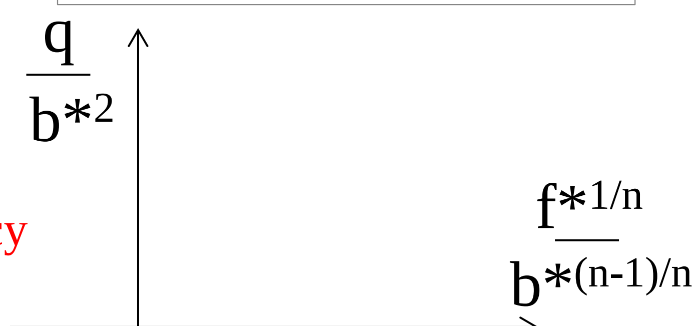


If we divide the abscissa by b^{*9} $\Rightarrow$



We did the same for n = 2, n= 3, n = 5, n = 40, n = 50

Dividing by b^{*n-1} aligns the data points. Instead of an abscissa f^*/b^{*n-1} we can use $(f^*/b^{*n-1})^{1/n}$ to match Critical State Model (i.e. no frequency dependency when $n \to \infty$).



Alternating field losses – Scaling Law (5/5)

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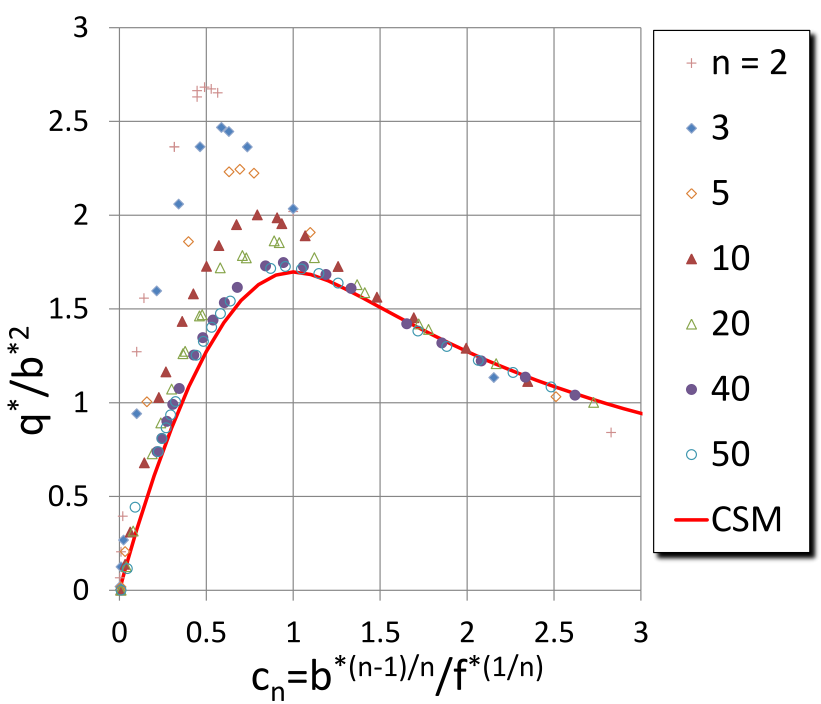
$$\frac{q^*}{b^{*2}} = SL(c_n)$$

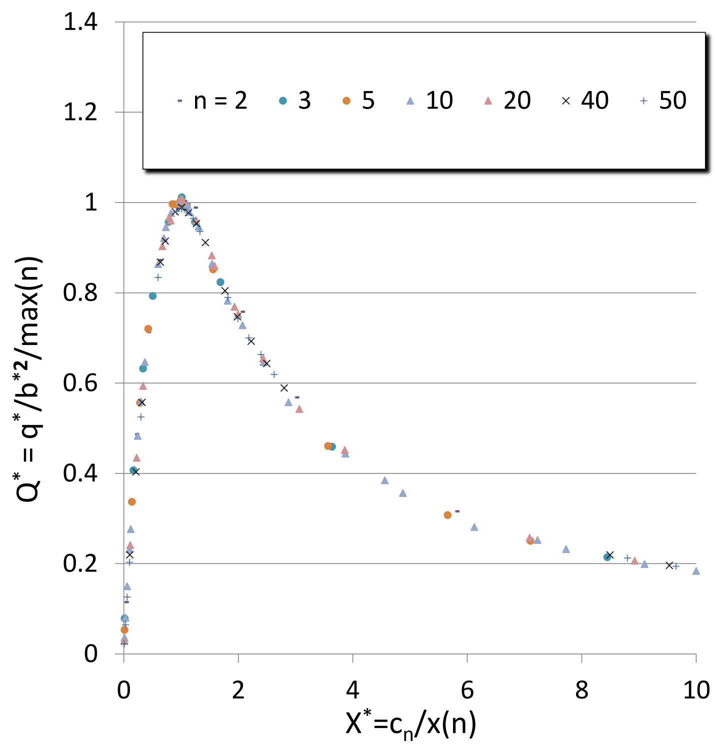
$$\text{with} \quad c_n = \left(\frac{b^{*n-1}}{f^*} \right)^{\frac{1}{n}}$$

- Data can be normalized with respect to a function of n

$$X^* = \frac{c_n}{x(n)} \quad \text{with} \quad x(n) = 1 - \frac{\alpha_1}{n^{\alpha_2}}$$

$$Q^* = \frac{q^*}{b^{*2}\, max(n)} \quad \text{with} \quad max(n) = \alpha_3 + \frac{\alpha_4}{n^{\alpha_5}}$$





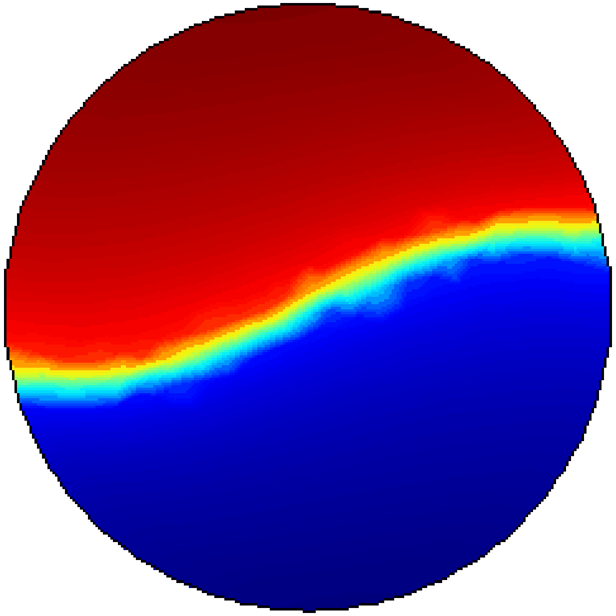
Losses in Rotating field Scaling Law

b*

f*

k=1

n



Losses in Rotating field Scaling Law

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Rotating field: k = 1

$$\vec{B}_{rot} = \begin{cases} kB_0 \sin(2\pi f t) \\ kB_0 \cos(2\pi f t) \end{cases}$$

$$\frac{q^*}{b_p^{\ 2}} = SL_{rot}(c_n)$$

Same method

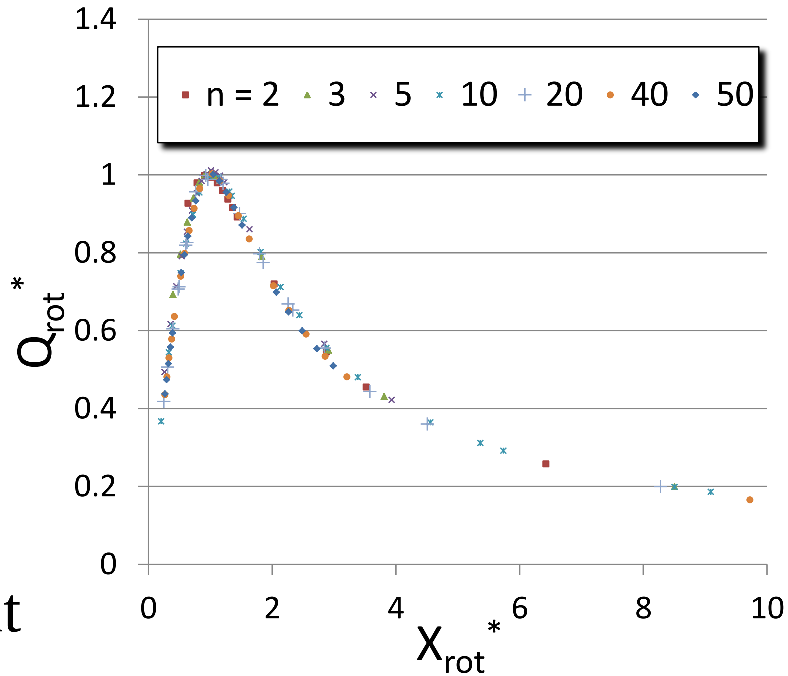
with $c_n = \left(\frac{b^{*\,n-1}}{f^*} \right)^{\frac{1}{n}}$

- Data can be normalized with respect to a function of n

$$X_{rot}^* = \frac{c_n}{x_{rot}(n)} \quad \text{with} \quad x_{rot}(n) = 1 - \frac{\alpha_1}{n^{\alpha_2}}$$

$$Q_{rot}^* = \frac{q^*}{b^{*2}\, max_{rot}(n)} \quad \text{with} \quad max_{rot}(n) = \alpha_3 + \frac{\alpha_4}{n^{\alpha_5}}$$

Coefficients α_1 α_2 α_3 α_4 α_5 are different



Elliptical field

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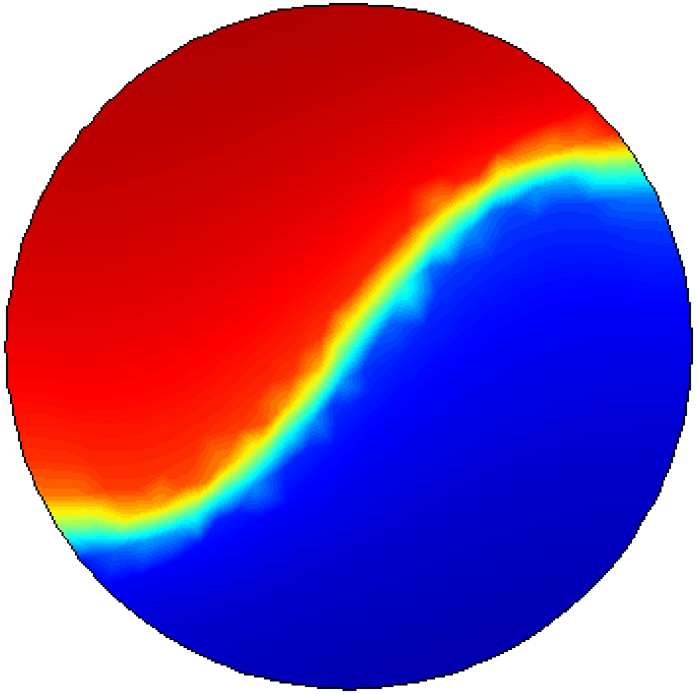
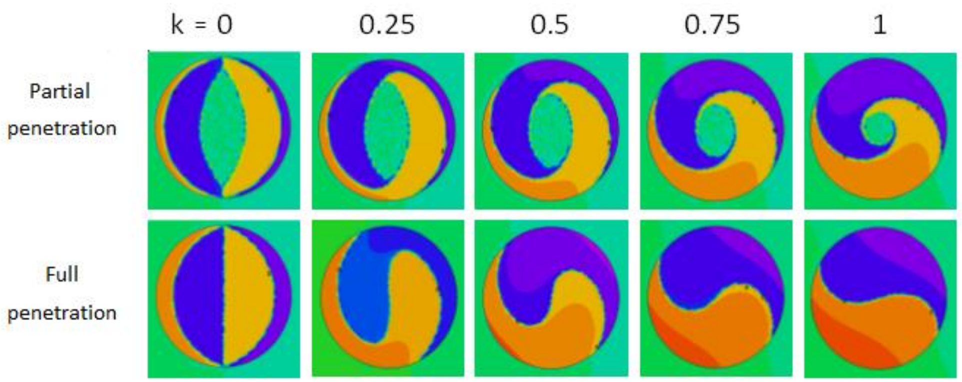
b*
f*
k
n



Elliptical field

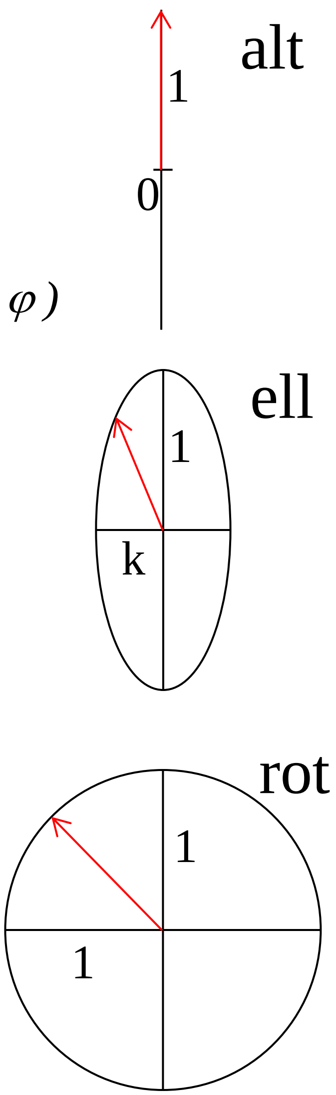
- k = 0.25, 0.5, 0.75



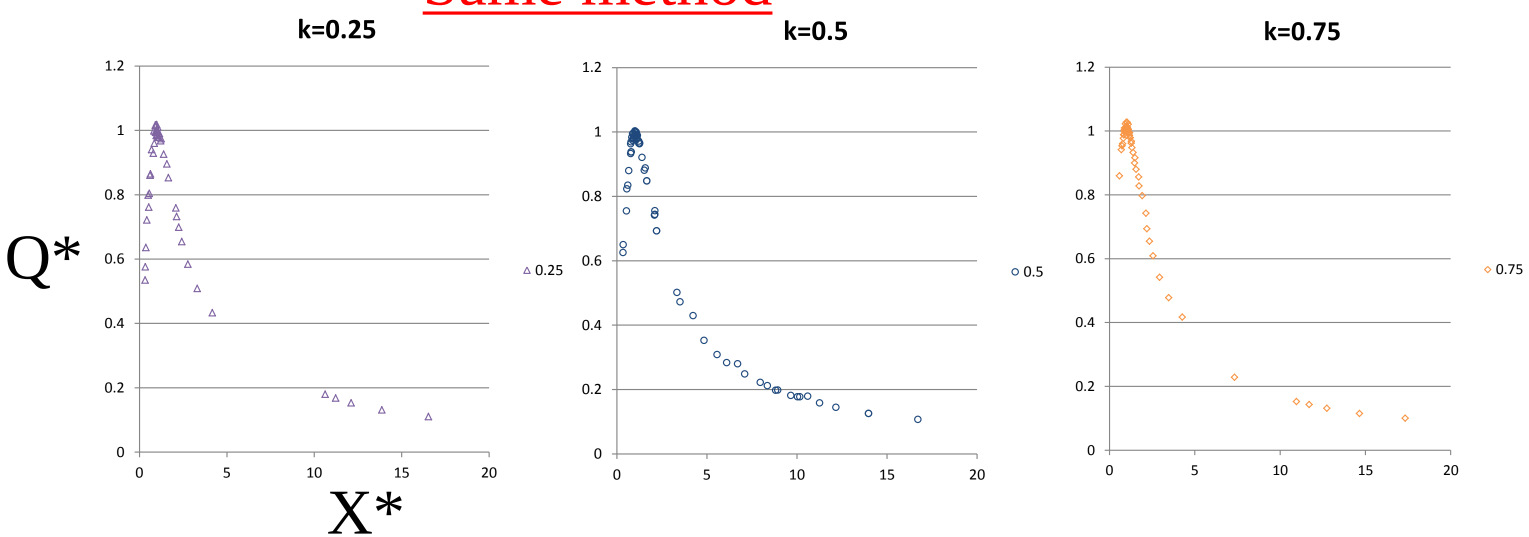
$$\vec{B}_{rot} = \begin{cases} kB_0 \sin(2\pi ft) \\ kB_0 \cos(2\pi ft) \end{cases}$$

$$\vec{B}_{alt} = \begin{cases} 0 \\ (1-k)B_0 \cos(2\pi ft + \varphi) \end{cases}$$

$$\vec{B}_{total} = \vec{B}_{rot} + \vec{B}_{alt}$$



Same method



Fit(b*,f*,n,k) or fit (c_n,n,k)

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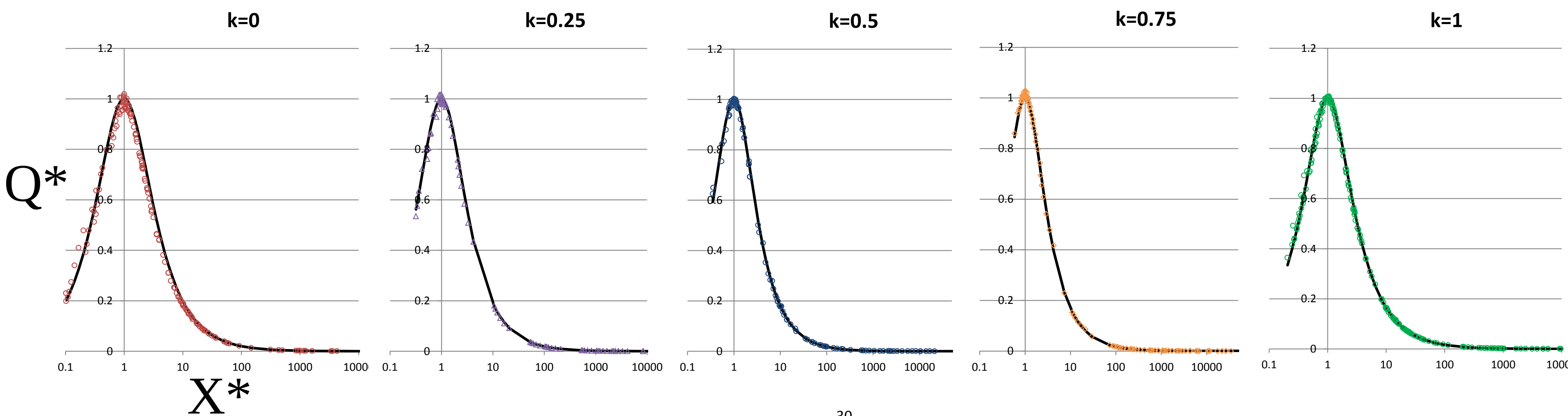
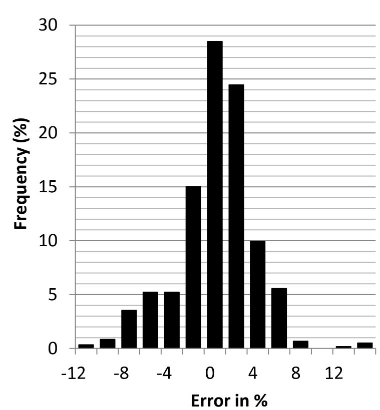
- Fit curves

General expression: $$Q^* = \frac{X^*}{\left[0.5\left(1 + X^{*\beta_{in}}\right)\right]^{\beta_{out}}}$$



Error on data used to get the fit (~600):



Elliptical field – with phase angle

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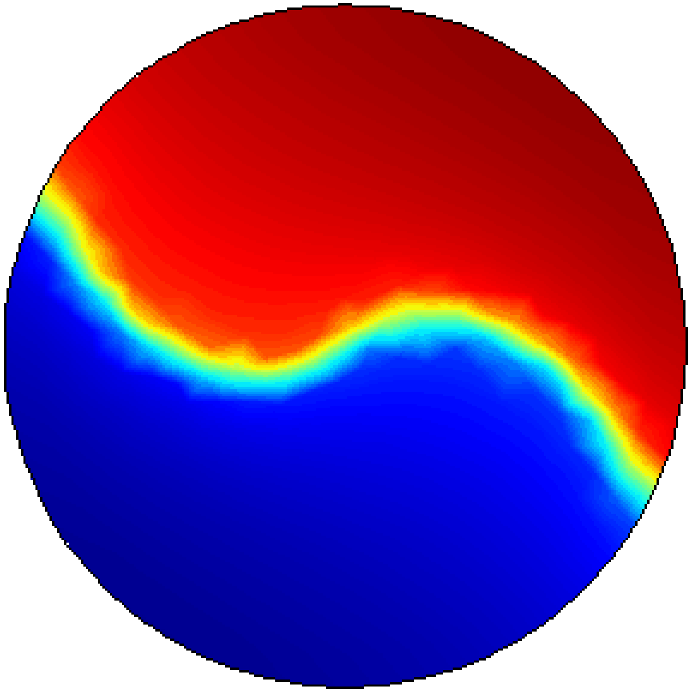
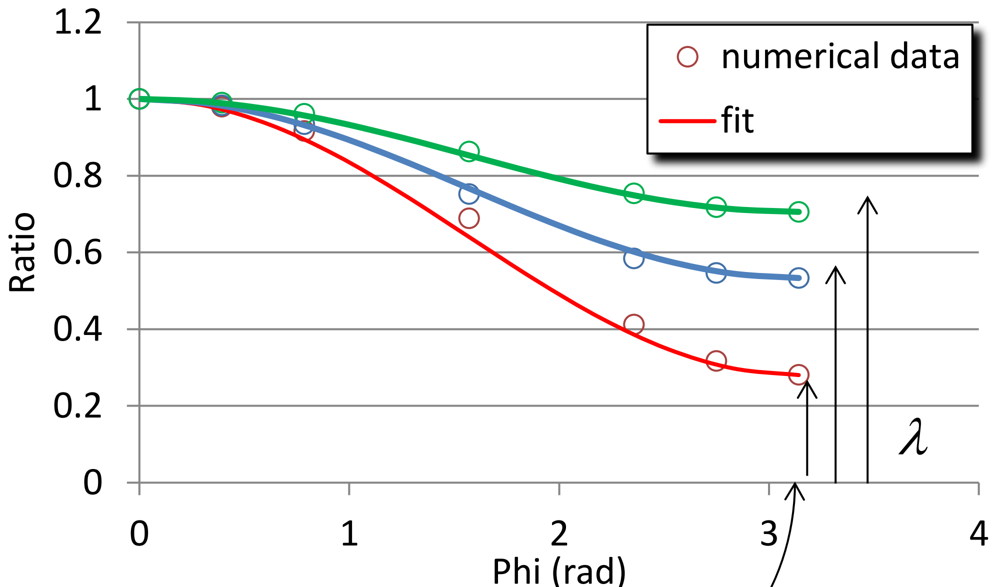
b*
f*
k
n
φ



Elliptical field – with phase angle

- Ratio between losses with and without phase angle can be approximate by:

$$\frac{Q^*(k,n,b^*,f^*,\varphi)}{Q^*(k,n,b^*,f^*,\varphi=0)} = \frac{(1-\lambda)}{2}(1+\cos\varphi)+\lambda$$



$$\lambda = \frac{Q^*(k,n,b^*,f^*,\varphi=\pi)}{Q^*(k,n,b^*,f^*,\varphi=0)}$$

$$Q^*(k,n,b^*,f^*,\varphi=\pi)$$

Elliptical field – with phase angle

- Transform☺ $Q^*(k,\varphi=\pi,B_0,\ldots)$ to $Q^*(k_1,\varphi=0,B_1,\ldots)$
- k_1 must be in [0,1] to use our previous fit.

TABLE I
LOSSES EQUIVALENCE BETWEEN PI- AND ZERO-LOAD ANGLE

k range	k_1	B_1
$0 < k < \frac{1}{3}$	$\frac{k}{1-2k}$	$(1-2k)B_0$
$\frac{1}{3} \le k < \frac{1}{2}$	$\frac{1-2k}{k}$	kB_0
$\frac{1}{2} \le k \le 1$	$\frac{2k-1}{k}$	kB_0

Based on geometrical considerations on the applied field.

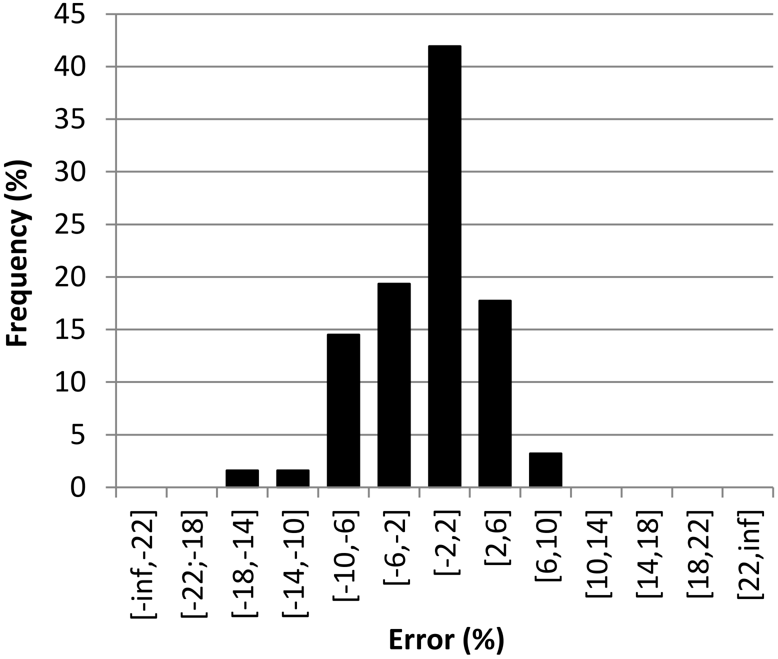
So $\lambda = \frac{W(k,\varphi=\pi,B_0)}{W(k,\varphi=0,B_0)} = \frac{W(k_1,\varphi=0,B_1)}{W(k,\varphi=0,B_0)}$ is known.

☺ Lorin C. , Masson P.J., Numerical Analysis of the Impact of Elliptical Fields on Magnetization Losses, IEEE TAS, 23, 3, 8201405 (2013)

Random test

- Results

60 simulations randomly taken in the whole domain



Conclusion

- Investigation of elliptical field losses in smooth power-law electrical behavior round-shaped filaments:
  - Reveal a fundamental variable:

$$c_n = \frac{b_0 \pi}{2\mu_0 j_c r}\left(\frac{2E_c}{b_0 f \pi^2 r}\right)^{\frac{1}{n}}$$

  - Semi-analytical formulae give *fast* and reliable magnetization losses values depending on:

r filament radius

n n-value

J_c critical current @ E_c = 100 μV/m

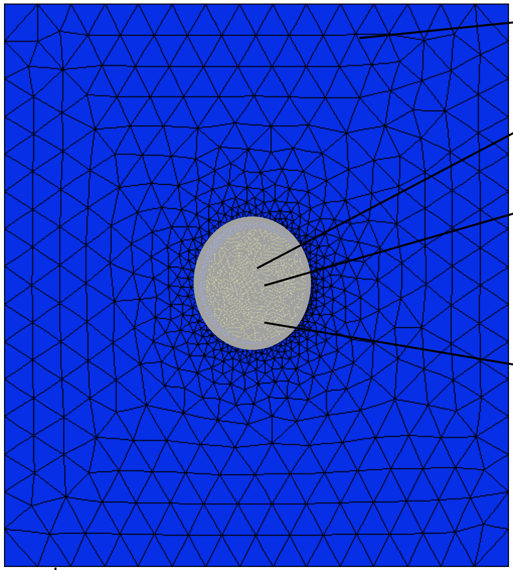
B_0 field amplitude

k = [0,1] → field configuration: *"ellipticity"*

f frequency

φ = [-π,π]

Next Steps (1/2)



H-Formulation: $\begin{cases} \overrightarrow{rot}\vec{E} = -\mu_0 \dfrac{\partial \vec{H}}{\partial t} \\ \vec{J} = \overrightarrow{rot}\vec{H} \end{cases}$

Electrical behavior, power-law:

$$\vec{E} = \frac{E_c}{J_c} \cdot \left(\frac{|\vec{J}|}{J_c} \right)^{n-1} . \vec{J}$$

where $J_c = 100 A/mm^2$, $E_c = 100 \mu V/m$, $n = 20$

Transport current constrain:

$\iint_A j dA = I(t)$ where A = filament area

Boundary conditions:

$$\frac{\vec{B}_{rot}}{B_p} = \begin{cases} k b_p \sin(\omega t) \\ k b_p \cos(\omega t) \end{cases}$$

$$\frac{\vec{B}_{alt}}{B_p} = \begin{cases} 0 \\ (1-k) b_p \cos(\omega t + \varphi) \end{cases}$$

$$\frac{I}{I_p} = i_p \cos(\omega t + \theta)$$

bp penetration field ratio

k = [0,1] → field configuration

φ = [-π,π] → rotor load angle

ip = [0,1] → penetration current ratio

θ = [0,2π] → current/field phase angle

Next Steps (2/2)

UNIVERSITY of HOUSTON

- Implement model in Amber HTS Machine Sizing Code
- Perform experimental validation
  - Tests will start in January 2014
    - AC field
    - Transport current
  - Upgrade to include elliptical field scheduled for 2015