Implementation of Advanced Numerical Methods for the Optimization of Accelerators

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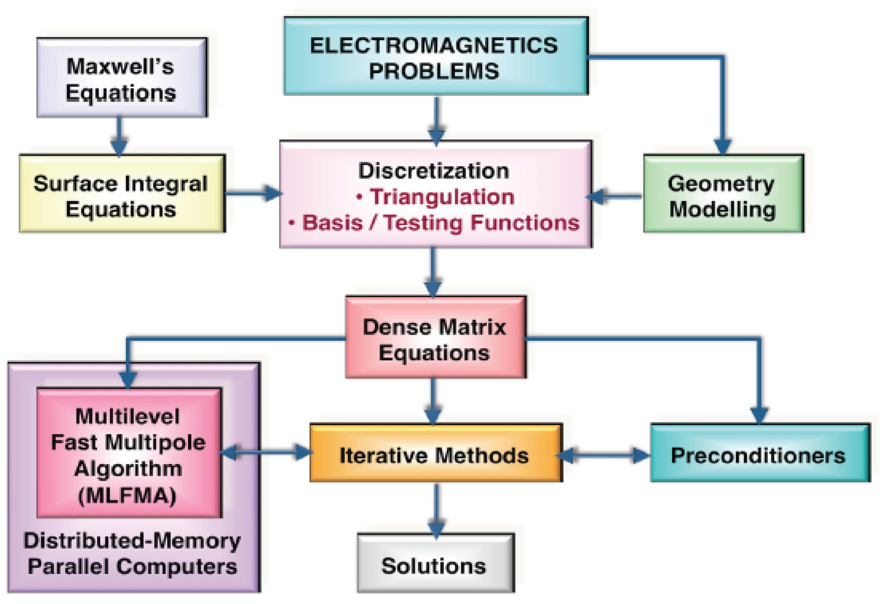
Abstract

For complex electromagnetic structures discretization of 3D space leads to dense matrix equations constituting millions of unknowns. These matrix equations are solved iteratively where the matrix vector multiplications (MVMs) result in the need for large memories and processor speeds that often exceed the performance of even the best computers. The Multilevel Fast Multipole Method (MLFMM) could be more cost efficient than the commonly used codes for particle accelerators as it might greatly speed up the computation and reduce the memory requirements by accelerating the matrix vector multiplication. The code will be used to calculate the fields for guiding and accelerating structures in existing and future accelerators, such as the LHC and CLIC, and also for the simulations of electromagnetic fields of beam diagnostic instruments, such as beam position monitors for the ELENA ring at CERN, and will be benchmarked against other, more conventional codes.

Introduction

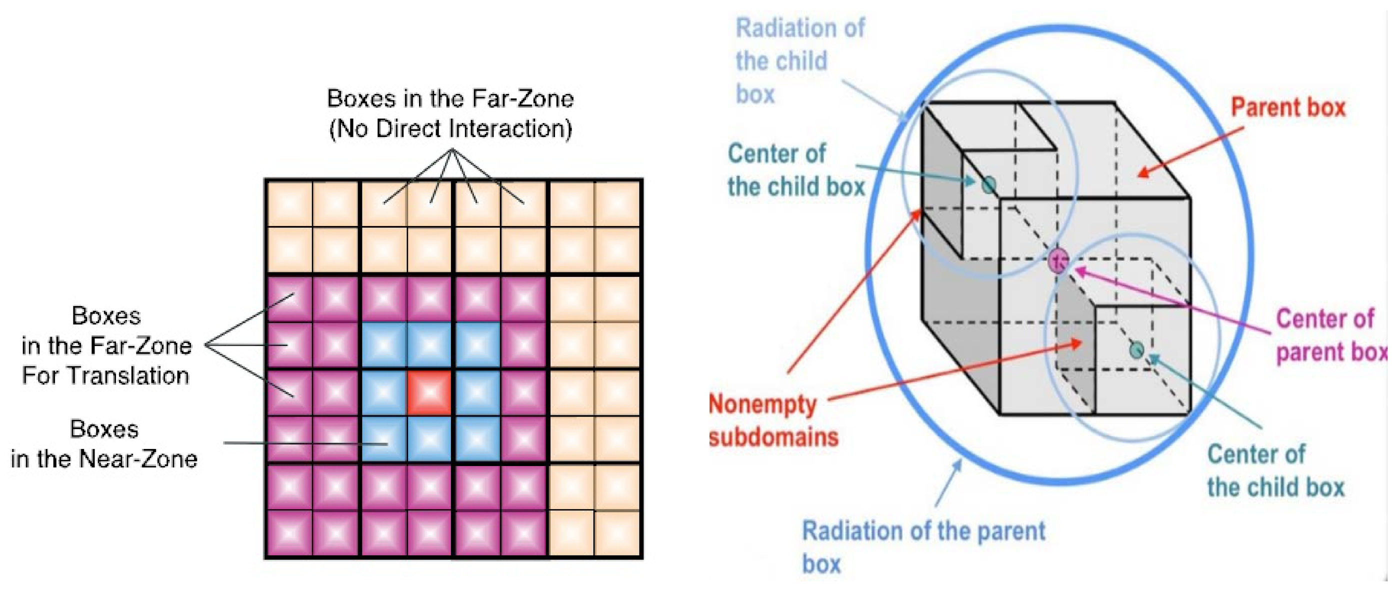
The Fast Multipole Method based on the single level grouping strategy was developed to speed up the calculations of long-ranged forces in the n-body problem. FMM provides an efficient way of performing the Matrix Vector Multiplication (MVMs) which could be evaluated in $O(N^{3/2})$ flops. The clustering idea of FMM can be extended and applied in a recursive manner, leading to MLFMA [1] which enables the solution of even large problems by reducing the complexity of MVMs to $O(N \log^2 N)$[2] or $O(N \log N)$[3].



A typical simulation environment based on MLFMA [2].

How the Algorithm Works?

- Based on a hierarchical decomposition of a cube named as the *oct-tree* grouping.
- Group center to center distances are fixed at each level.
- Only one set of translation operators need to be cached, dramatically reducing memory.



Boxes in the near and far zones for a given box (red) according to the one-box buffer scheme.[2]

Formulation of the Method

MLFMM calculates the interactions between the radiating (basis) and receiving (testing) elements in a group-by-group manner consisting of three stages:

Aggregation: Radiated fields of boxes are calculated from the lowest level of the tree structure to the highest level.

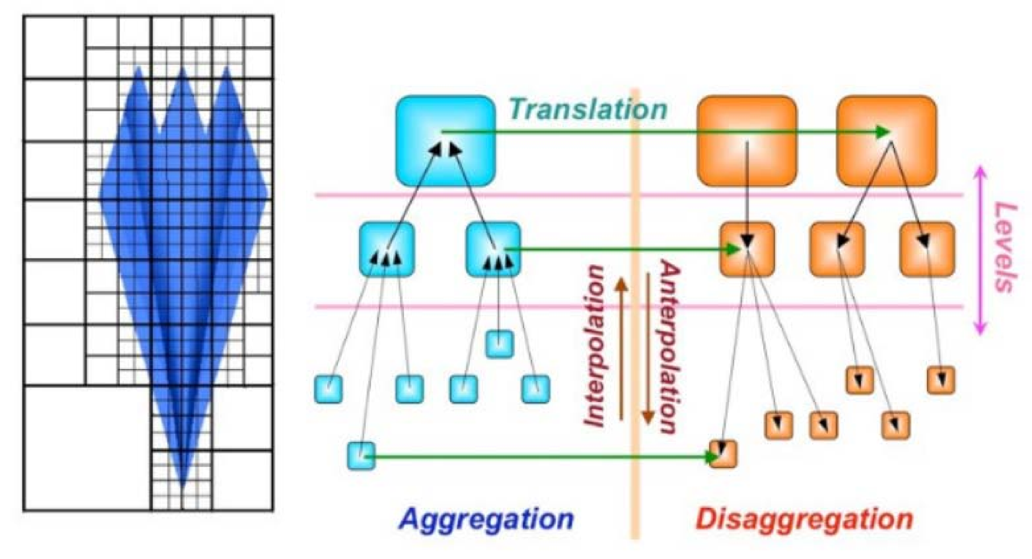
$$F_{G'}(\hat{k}) = \sum\nolimits_{n \in G'} F_{nG'}(\hat{k}) a_n$$

Translation: Radiated fields computed during the aggregation stage are translated into incoming fields.

$$F^T{}_G(\hat{k}) = \sum_{G' \notin N(G)} T_L(k, D) F_{G'}(\hat{k})$$

Disaggregation: Total incoming fields at the box centers are calculated from the top of the tree structure to the bottom.

$$\sum_{n=1}^{N} Z_{mn} a_n = \frac{1}{4\pi} \int d^2\hat{k} F^C_{mG}(\hat{k}).F^T_G(\hat{k})$$

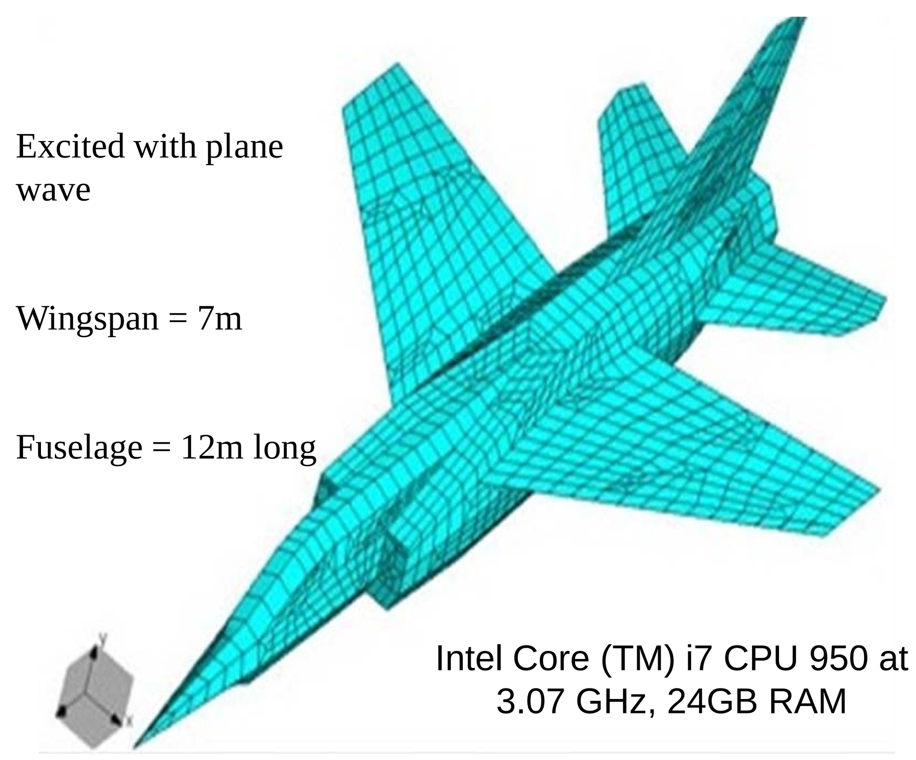


Recursive clustering of an arbitrary object and the construction of the multilevel tree structure [2]

A Case Study

The fighter airplane is excited with a plane wave, coming in from 30° under the horizon. The airplane is simulated at 3 GHz (120 λ long) and 4 GHz (160 λ long).

Higher order Method-of-Moments (MoM) formulation results in:
- 1.5 million RWG unknowns at 3GHz
- 3 million RWG unknowns at 4GHz



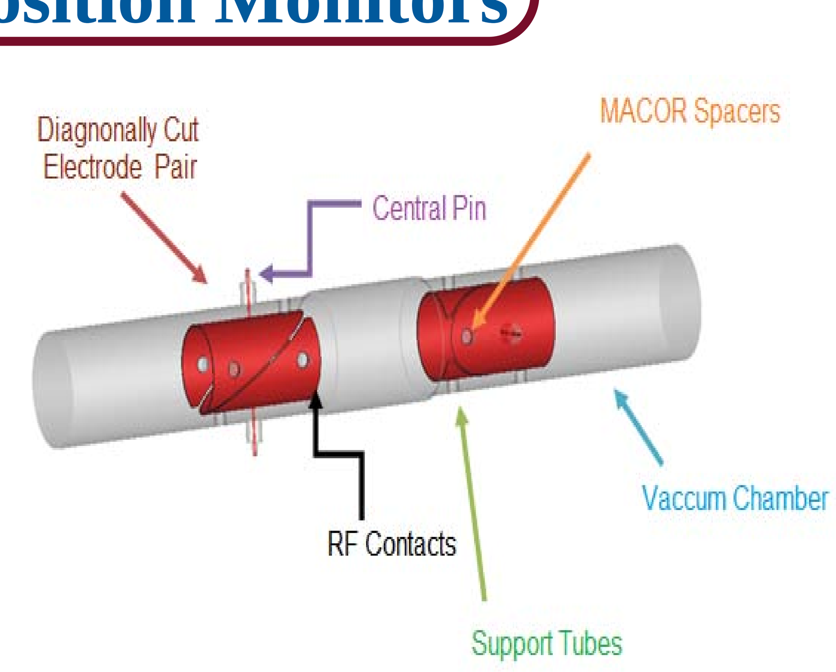
No. of Unknowns	Memory Allocation GB	No. of Iterations
153646	188.9	100
307170	754	102
MoM Formulation		

No. of Unknowns	Memory Allocation GB	No. of Iterations
153646	3.2	65
307170	7.2	83
MLFMM Formulation		

A Comparative Analysis: By applying the MLFMM, memory requirements are reduced to 3.2 GB and 7.2 GB [4]

Optimization of Beam Position Monitors

For beams traveling at velocities much smaller than the speed of light, the time to carry out simulations of electromagnetic fields increases with decreasing value of β. As the computed volume needs to be significantly extended, not only can it lead to an excessive number of mesh cells but also make the algorithm fail to find a stable step size required to perform the calculations. The implementation of faster algorithms for the optimization of BPMs could potentially help speed up such studies considerably.



ELENA pickup in CST Microwave Studio

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References

[1] J. Song, C.-C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propag., vol. 45, no. 10, pp. 1488–1493, Oct. 1997.*
[2] M. F. Gyure and M. A. Stalzer, "A prescription for the multilevel Helmholtz FMM," *IEEE Comput. Sci. Eng., vol. 5, no. 3, pp. 39–47, Jul.-Sep. 1998.*
[3] O.Ergul, "The Multilevel Fast Multipole Algorithm for Solving Large-Scale Computational Electromagnetics Problems"
[4] Multilevel Fast Multipole Algorithm: http://www.wipl-d.com/products/mlfmm.php

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