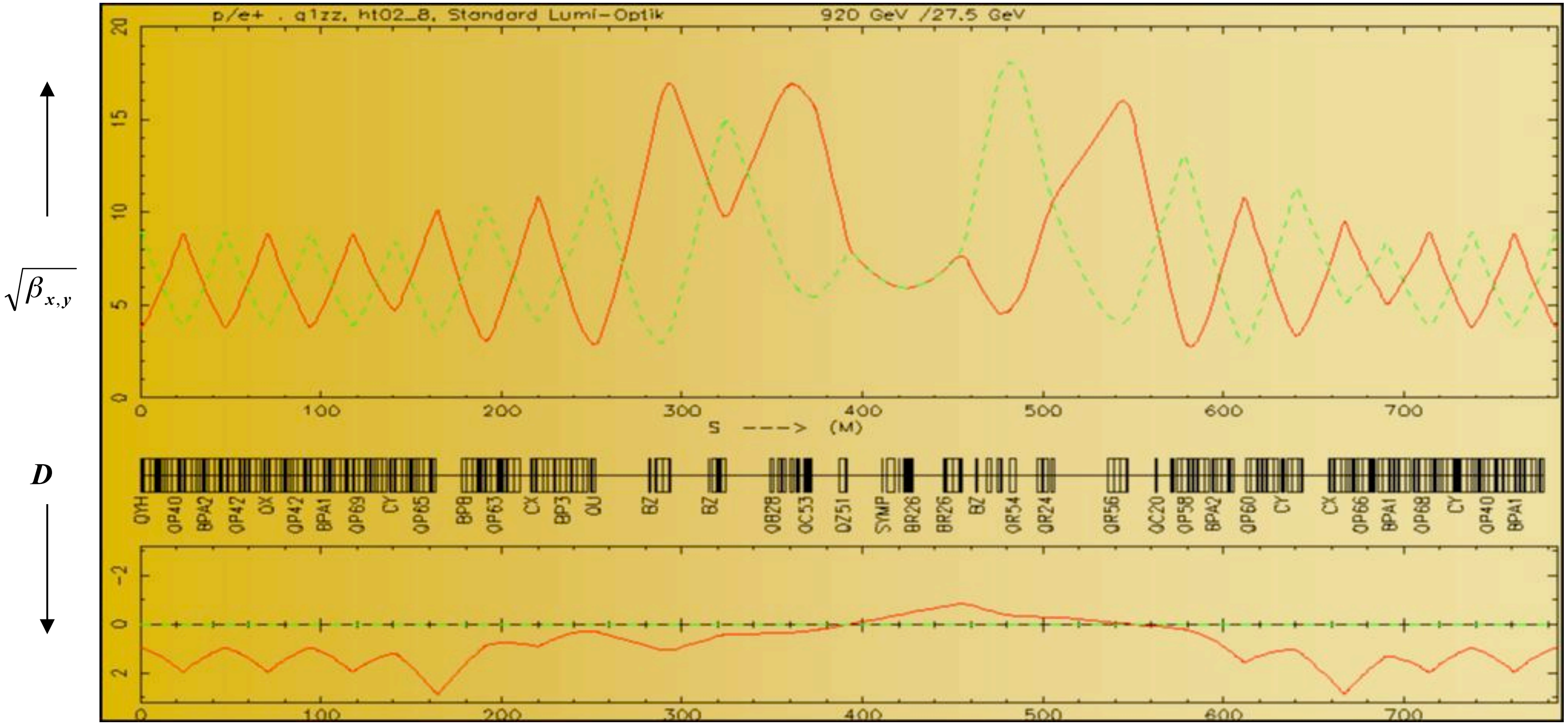
III Storage Rings

Lattice Design and Acceleration

III.) The „not so ideal“ World

Lattice Design in Particle Accelerators



1952: Courant, Livingston, Snyder:

Theory of strong focusing in particle beams

Recapitulation: ...the story with the matrices !!!

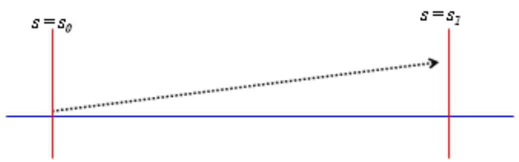
Equation of Motion:

$$x'' + K\,x = 0 \qquad K = 1/\rho^2 - k \;\;\text{... hor. plane:}$$

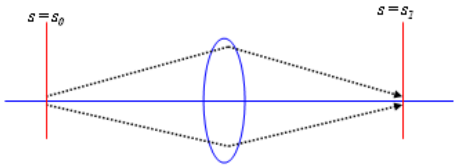
$$K = k \qquad \text{... vert. Plane:}$$

Solution of Trajectory Equations

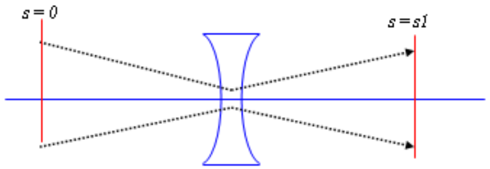
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$



$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}\,l) & \frac{1}{\sqrt{|K|}}\sin(\sqrt{|K|}\,l) \\ -\sqrt{|K|}\sin(\sqrt{|K|}\,l) & \cos(\sqrt{|K|}\,l) \end{pmatrix}$$



$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}\,l) & \frac{1}{\sqrt{|K|}}\sinh(\sqrt{|K|}\,l) \\ \sqrt{|K|}\sinh(\sqrt{|K|}\,l) & \cosh(\sqrt{|K|}\,l) \end{pmatrix}$$

$$M_{total} = M_{QF} * M_D * M_B * M_D * M_{QD} * M_D * \quad \dots$$

8.) Lattice Design: *„… how to build a storage ring“*

Geometry of the ring: $B * \rho = p / e$

p = momentum of the particle,
ρ = curvature radius

Bρ= beam rigidity

Circular Orbit: bending angle of one dipole

$$\alpha = \frac{ds}{\rho} \approx \frac{dl}{\rho} = \frac{Bdl}{B\rho}$$

The angle run out in one revolution must be 2π, so for a full circle

$$\alpha = \frac{\int Bdl}{B\,\rho} = 2\pi$$

$$\int Bdl = 2\pi \frac{p}{q}$$

… defines the integrated dipole field around the machine.

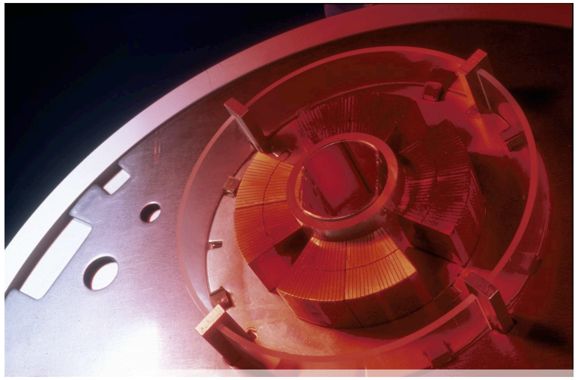
Example LHC:

7000 GeV Proton storage ring
dipole magnets N = 1232
l = 15 m
q = +1 e

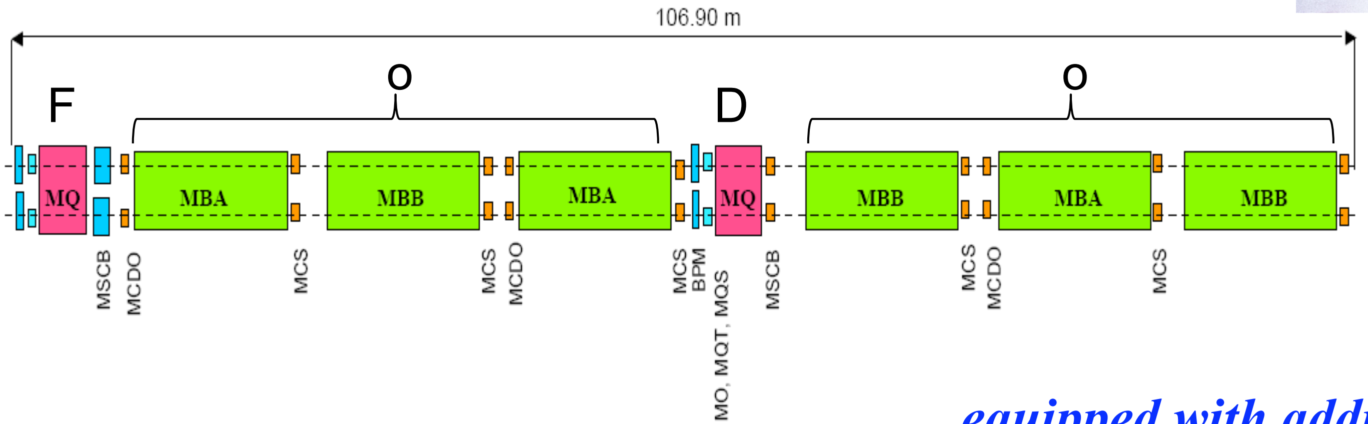
$$\int B\, dl \approx N\, l\, B = 2\pi\; p / e$$

$$B \approx \frac{2\pi \;\; 7000 \;\; 10^{9}\, eV}{1232 \;\; 15\, m \;\; 3\; 10^{8}\, \frac{m}{s} \;\; e} = 8.3\, Tesla$$

LHC: Lattice Design
the ARC 90° FoDo in both planes



MQ: main quadrupole



equipped with additional corrector coils

MB: main dipole
MQ: main quadrupole
MQT: Trim quadrupole
MQS: Skew trim quadrupole
MO: Lattice octupole (Landau damping)
MSCB: Skew sextupole
Orbit corrector dipoles
MCS: Spool piece sextupole
MCDO: Spool piece 8 / 10 pole
BPM: Beam position monitor + diagnostics

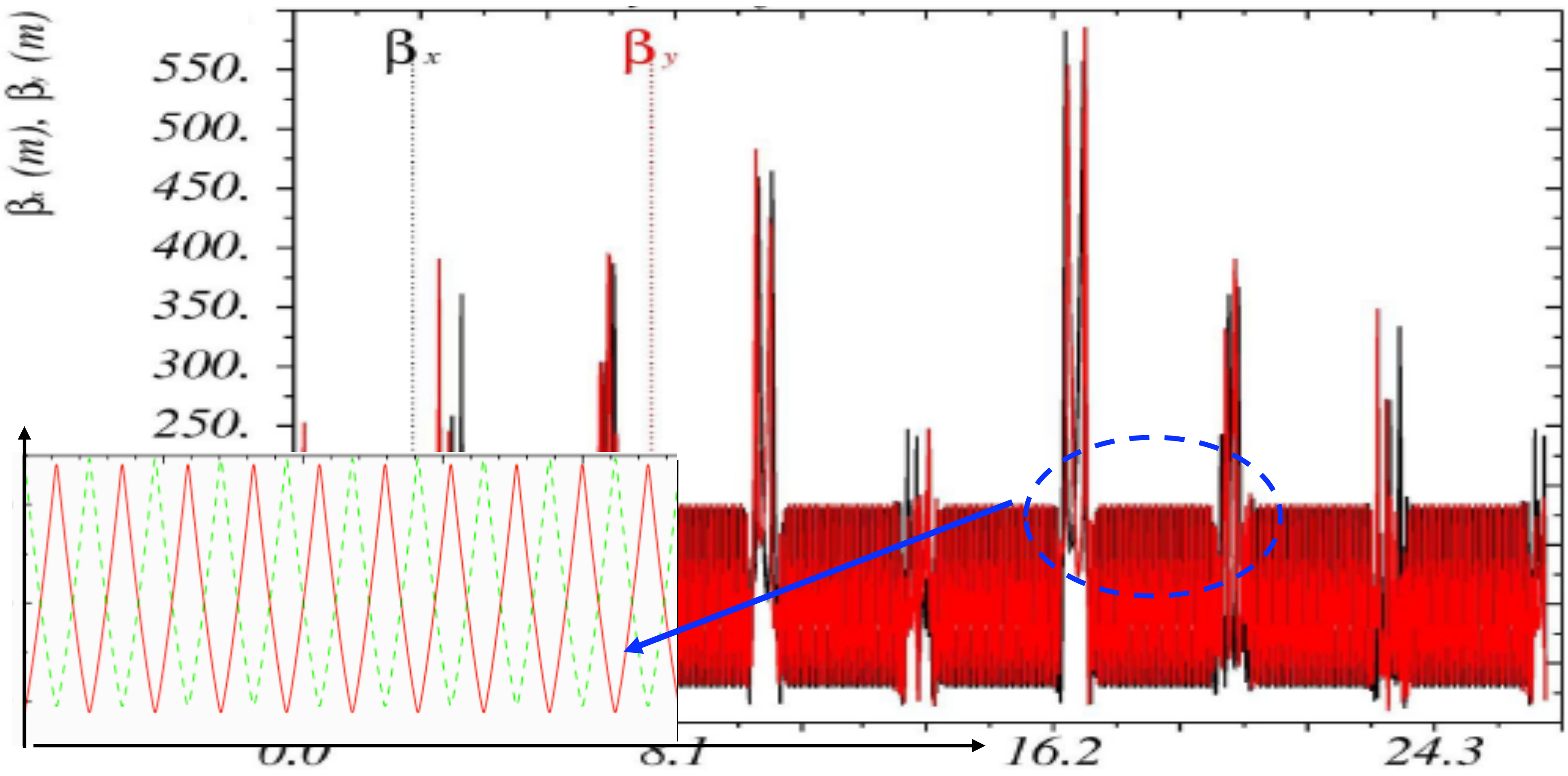
Magnets for the LHC, total budget, every magnet has a role in the optics design

Name	Quantity	Purpose
MB	1232	Main dipoles
MQ	400	Main lattice quadrupoles
MSCB	376	Combined chromaticity/ closed orbit correctors
MCS	2464	Dipole spool sextupole for persistent currents at injection
MCDO	1232	Dipole spool octupole/decapole for persistent currents
MO	336	Landau octupole for instability control
MQT	256	Trim quad for lattice correction
MCB	266	Orbit correction dipoles
MQM	100	Dispersion suppressor quadrupoles
MQY	20	Enlarged aperture quadrupoles

In total 6628 cold magnets ...

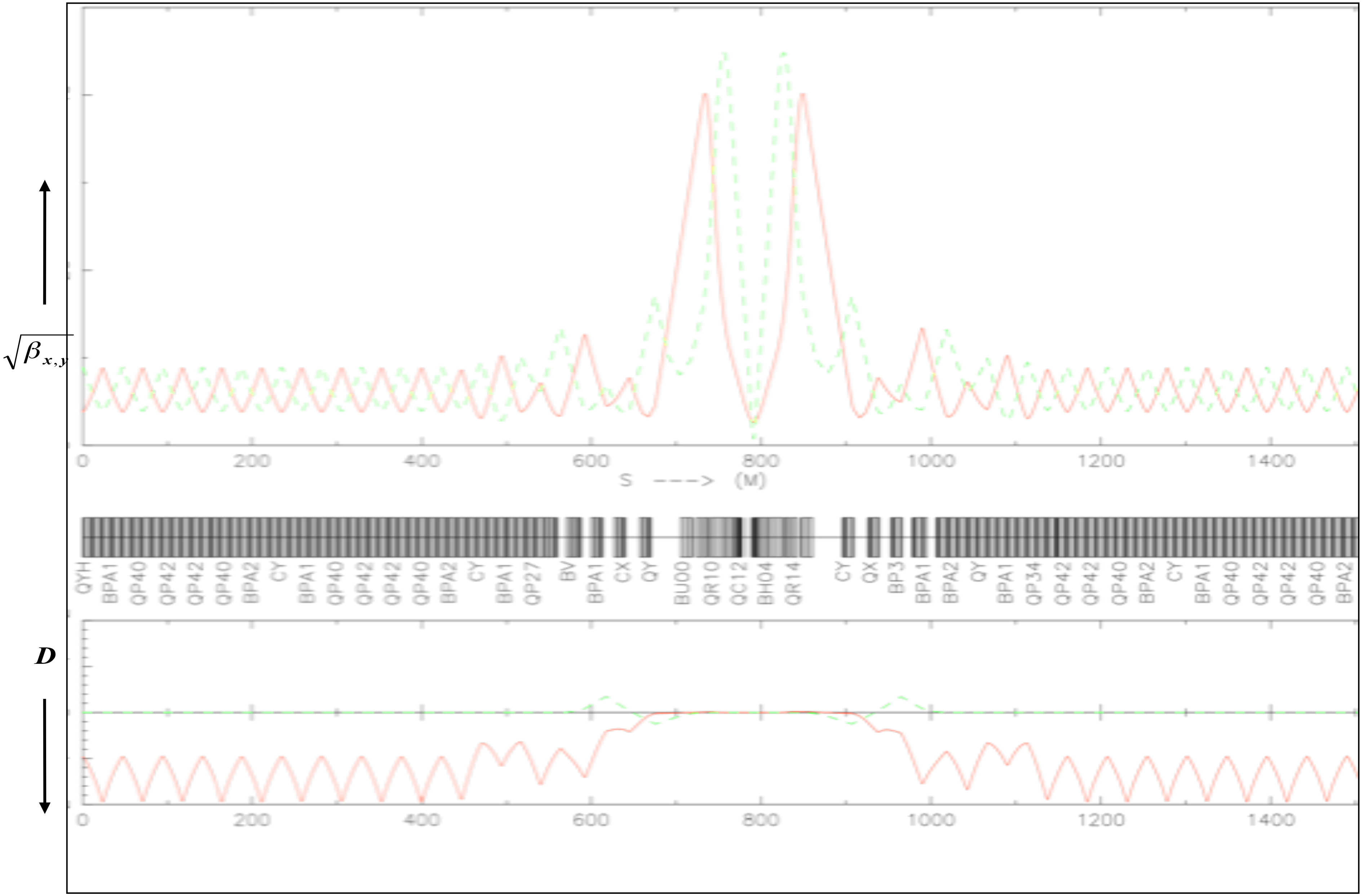
FoDo-Lattice

A magnet structure consisting of focusing and defocusing quadrupole lenses in alternating order with nothing in .
(Nothing = elements that can be neglected on first sight: drift, bending magnets, RF structures ... and especially experiments...)



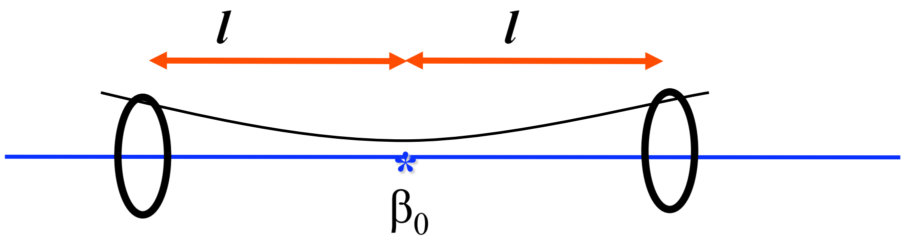
Starting point for the calculation: in the middle of a focusing quadrupole
Phase advance per cell μ = 45°,
→ calculate the twiss parameters for a periodic solution

9.) Insertions



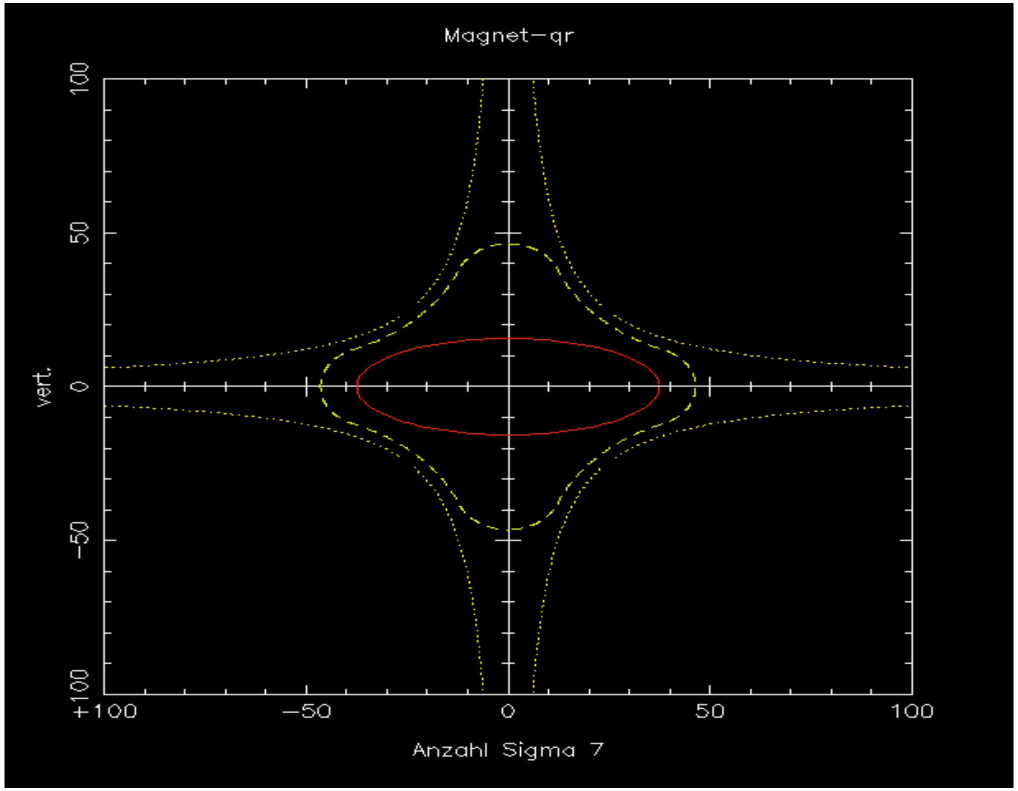
β-Function in a Drift:

$$\beta(\ell) = \beta_0 + \frac{\ell^2}{\beta_0}$$



At the end of a long symmetric drift space the beta function reaches its maximum value in the complete lattice.
-> here we get the largest beam dimension.

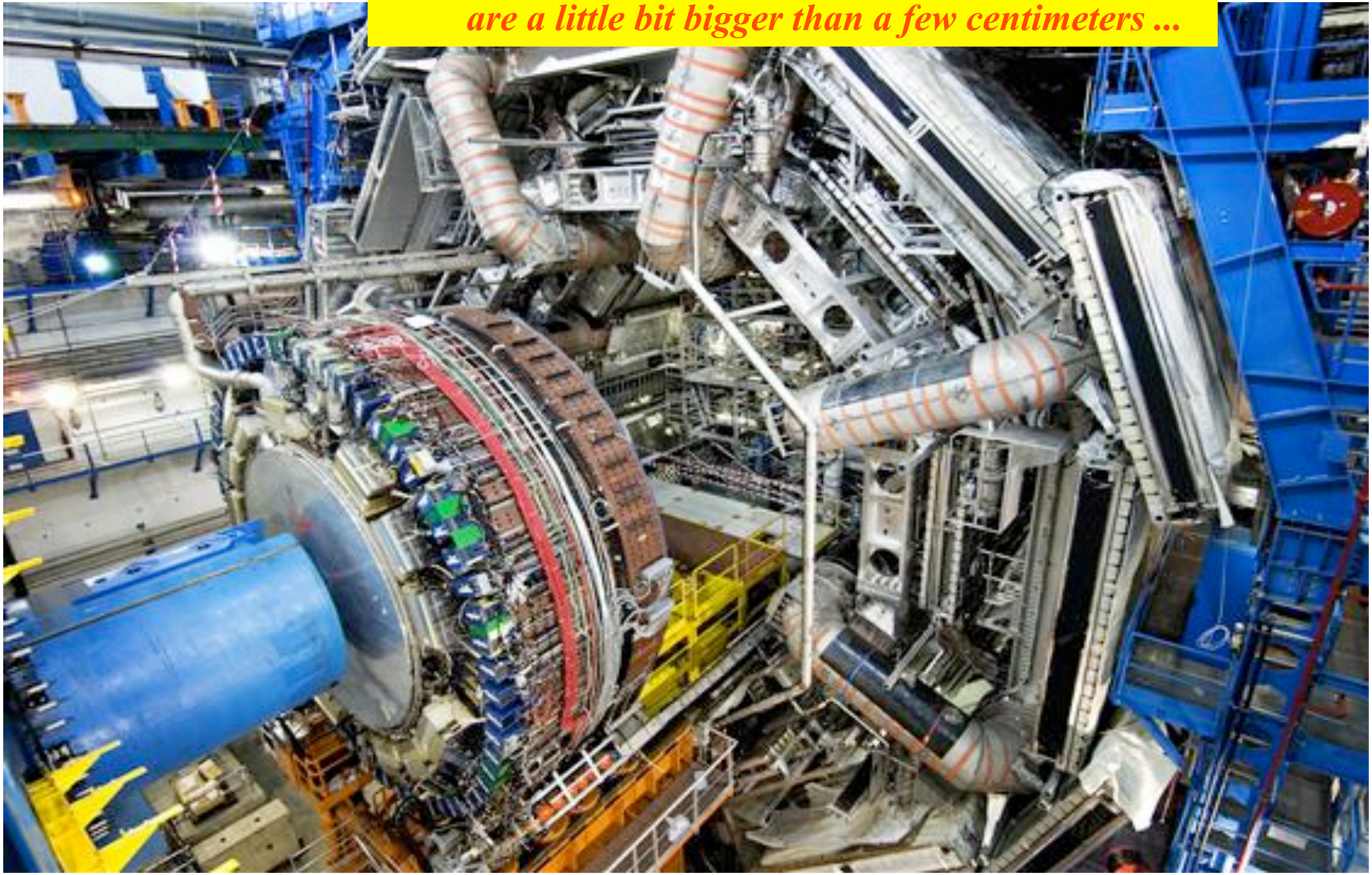
-> keep l as small as possible



7 sigma beam size inside a mini beta quadrupole

... clearly there is an

... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

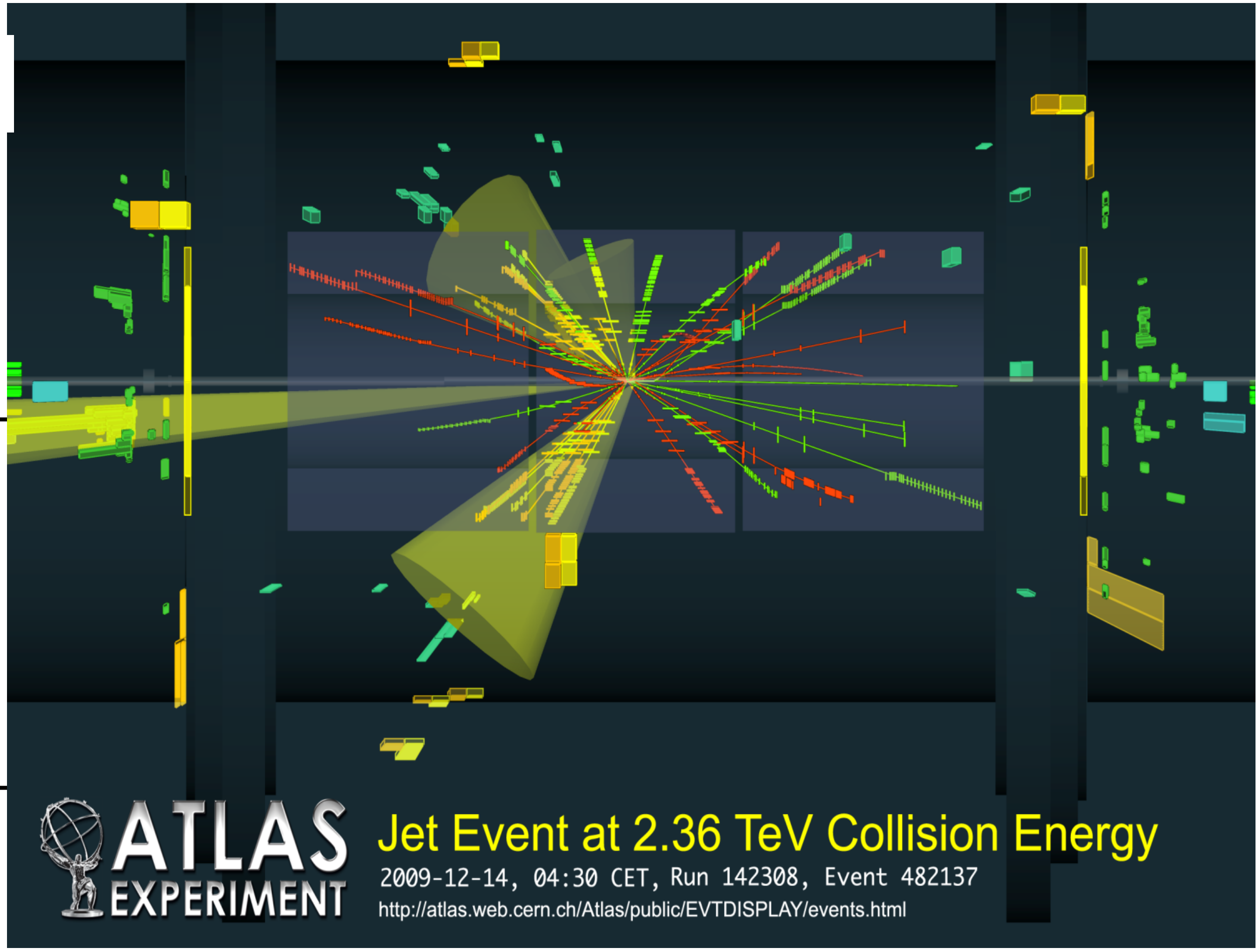


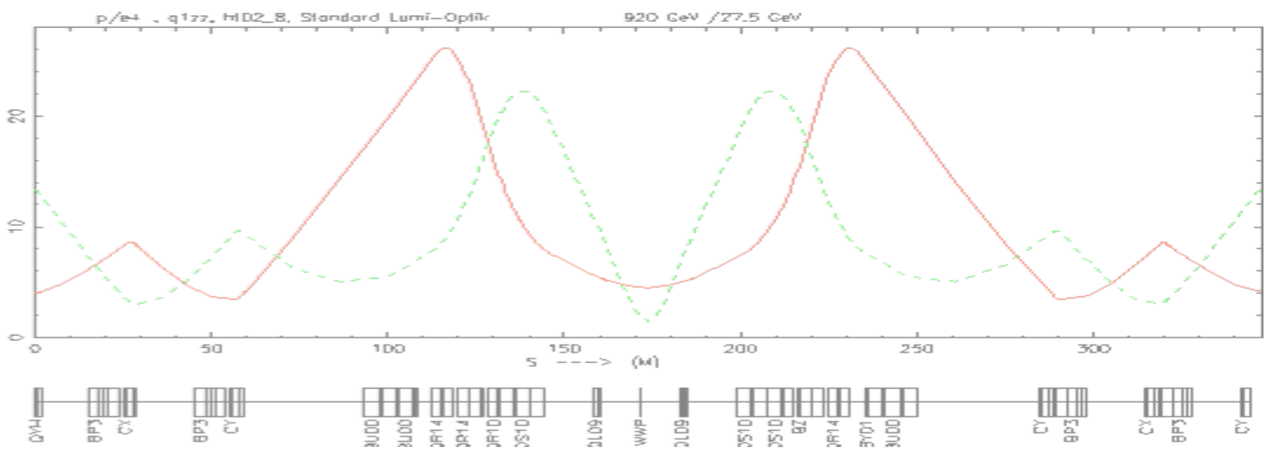
The Mini-β Insertion:

$$R = L * \Sigma_{react}$$

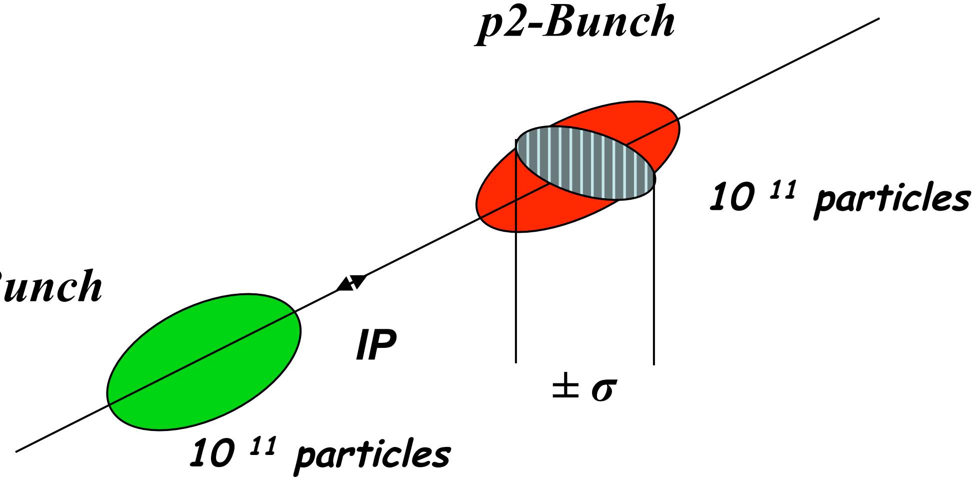
production rate of events
is determined by the
cross section Σ_{react}
and a parameter L that is given
by the design of the accelerator:
… the luminosity

$$L = \frac{1}{4\pi e^2 f_0 b} * \frac{I_1 * I_2}{\sigma^*_x * \sigma^*_y}$$





10.) Luminosity



Example: Luminosity run at LHC

$\beta_{x,y} = 0.55\ m$

$\varepsilon_{x,y} = 5*10^{-10}\ rad\ m$

$\sigma_{x,y} = 17\ \mu m$

$f_0 = 11.245\ kHz$

$n_b = 2808$

$I_p = 584\ mA$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$L = 1.0*10^{34}\ \frac{1}{cm^2 s}$$

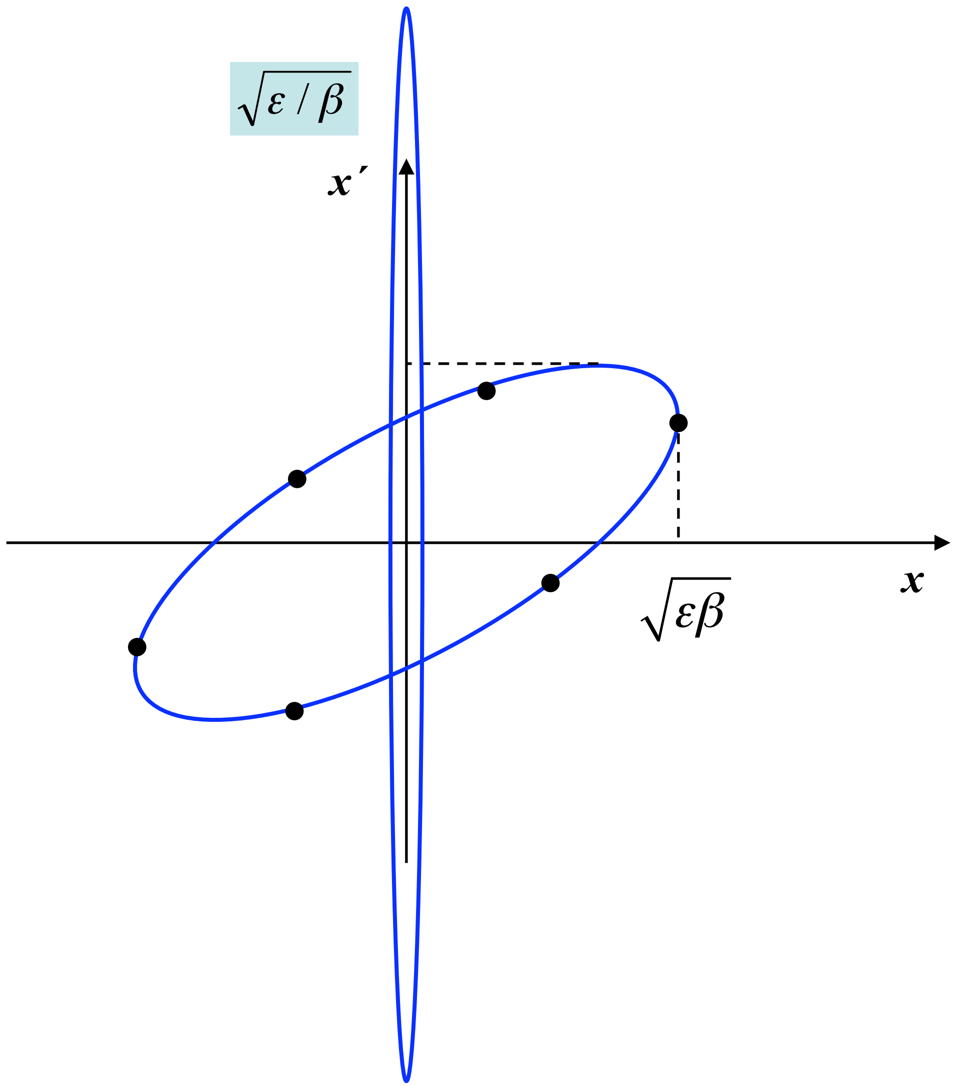
Mini-β Insertions: Betafunctions

A mini-β insertion is always a kind of special symmetric drift space.

→greetings from Liouville

the smaller the beam size

the larger the bam divergence



Mini-β Insertions: some guide lines

** calculate the periodic solution in the arc*

** introduce the drift space needed for the insertion device (detector ...)*

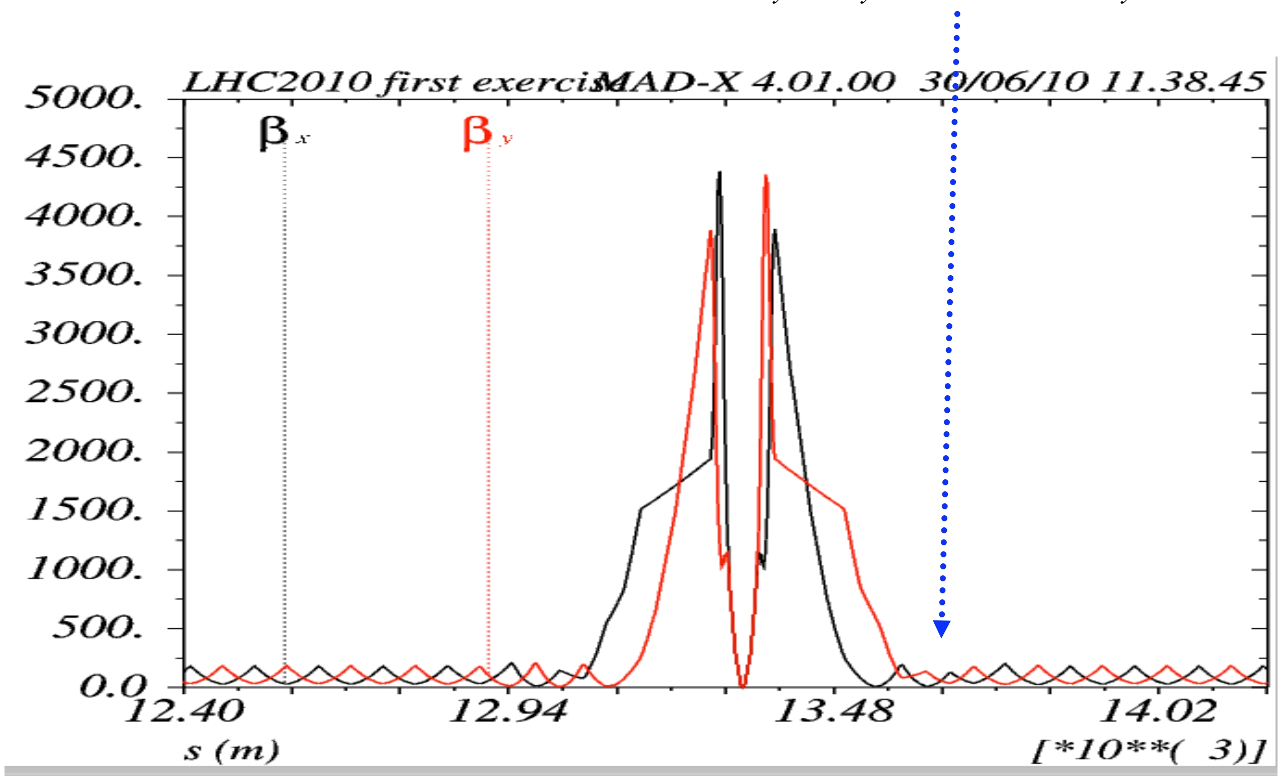
** put a quadrupole doublet (triplet ?) as close as possible*

** introduce additional quadrupole lenses to match the beam parameters to the values at the beginning of the arc structure*

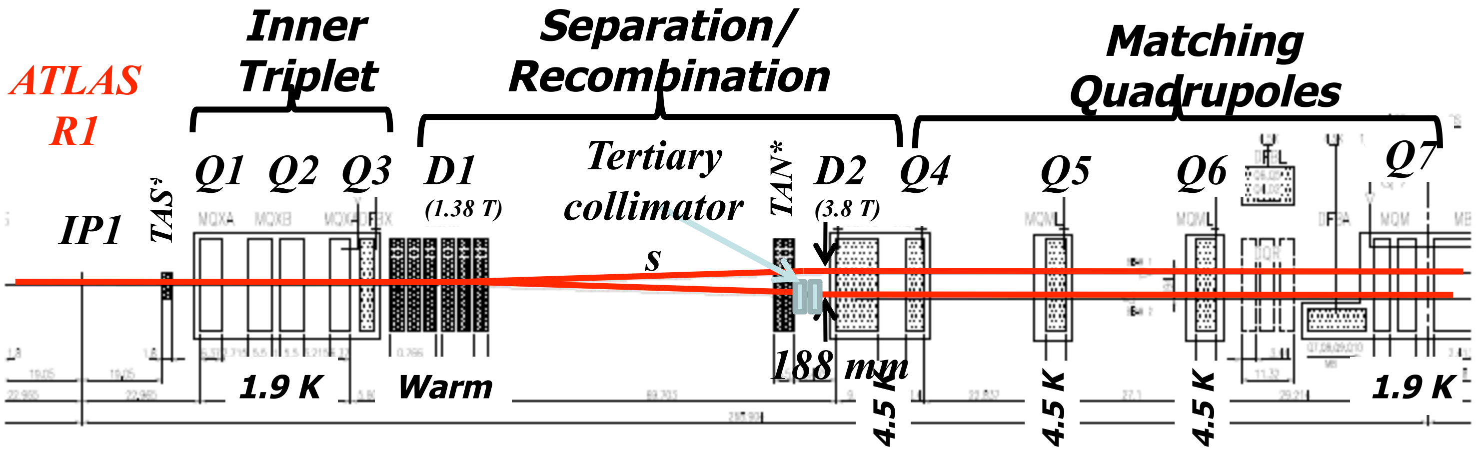
parameters to be optimised & matched to the periodic solution:

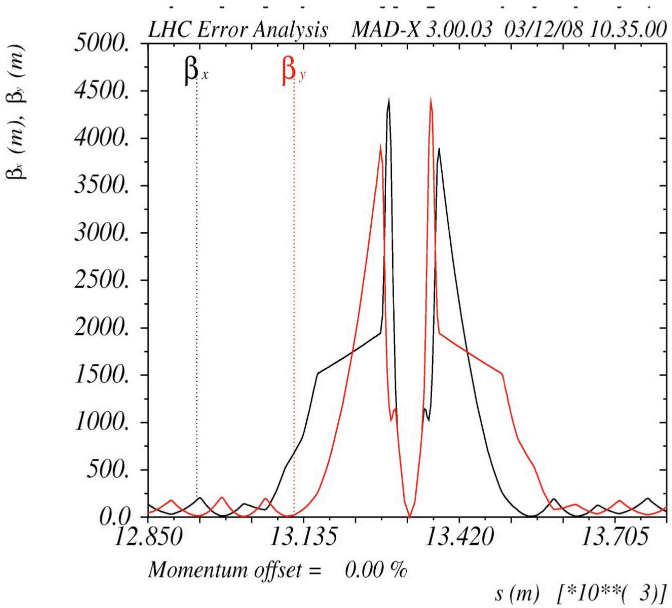
$$\alpha_x,\ \beta_x \qquad D_x,\ D_x'$$
$$\alpha_y,\ \beta_y \qquad Q_x,\ Q_y$$

8 individually powered quad magnets are needed to match the insertion
(... at least)

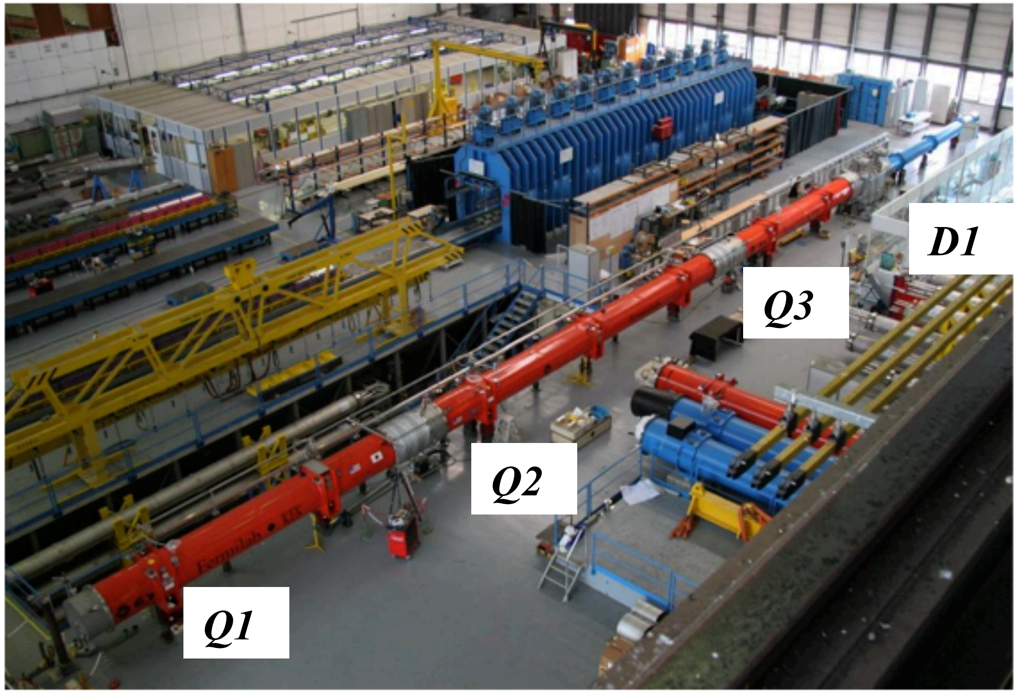


The LHC Insertions





mini β optics



Acceleration: Energy Gain

... we have to start again from the basics

Lorentz force $\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$ — *in long. direction the B-field creates no force* ($\boldsymbol{v \parallel B}$)

$$\vec{F} = \frac{d\vec{p}}{dt} = e\vec{E}$$

acc. force is given by the electr. Field

In relativistic dynamics, energy and momentum satisfy the relation:

$$E^2 = E_0^2 + p^2c^2 \qquad (E = E_0 + W)$$

Hence:

$$dE = \int F ds = v dp$$

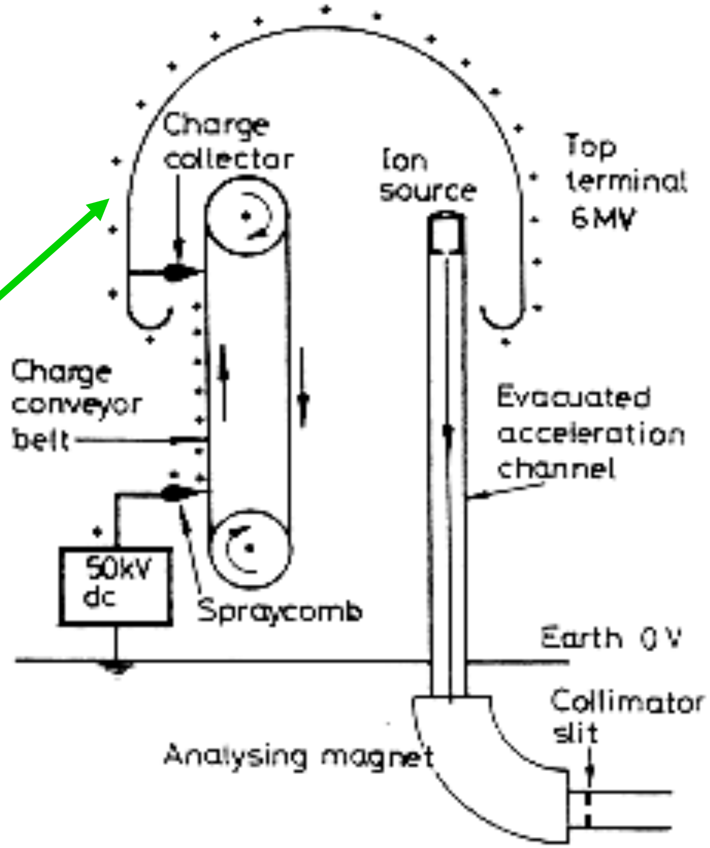
and the kinetic energy gained from the field along the z path is:

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

11.) Electrostatic Machines

(Tandem –) van de Graaff Accelerator

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the
structure

$$dW = dE = eE_z ds \quad \Rightarrow \quad W = e\int E_z ds = eV$$

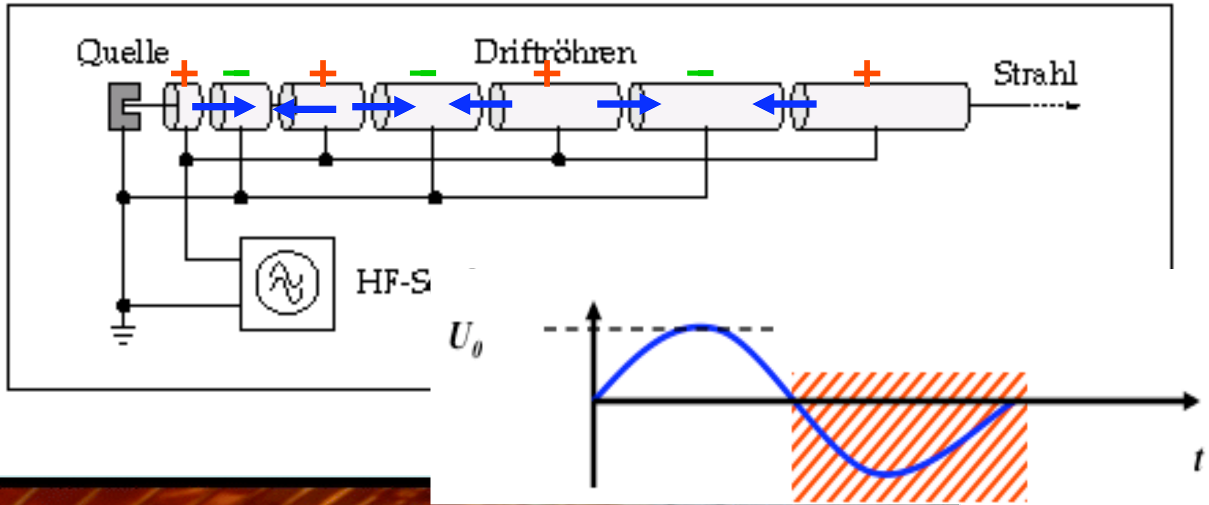
nota bene: all particles are “synchron” with the acceleration potential

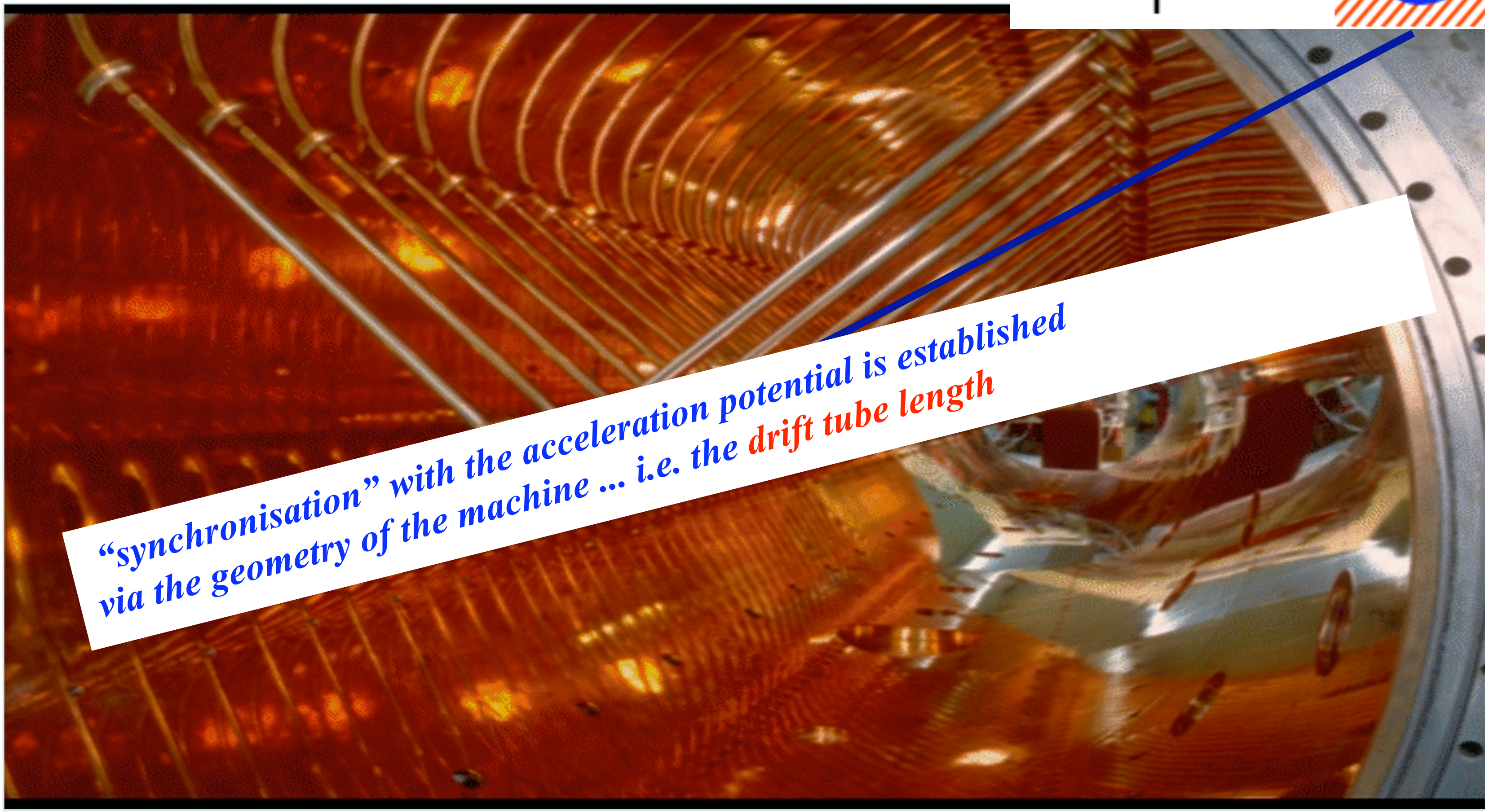
Electro Static Accelerator: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

12.) Linear Accelerator 1928, Wideroe

Energy Gain per „Gap“:

$$W = q\,U_0 \sin \omega_{RF} t$$





drift tube structure at a proton linac (GSI Unilac)

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\gamma * m} * B_z$$

1.) if $v << c \Rightarrow \gamma \cong 1$

2.) γ increases with the energy

$\Rightarrow$ no exact synchronism

Cyclotron SPIRAL at GANIL

"synchronisation" with the acceleration potential is established via the spiraling orbit length

Syn… …on

B = constant

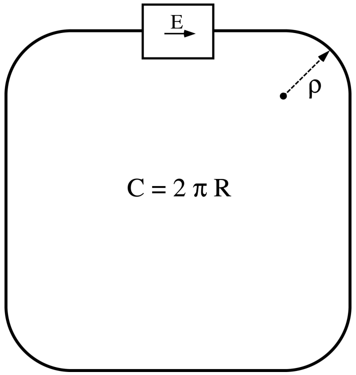
$\gamma\, \omega_{RF}$ = constant

ω_{RF} decreases with time

$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) * m_0} * B$$

keep the synchronisation condition by varying the rf frequency

The Synchrotron (Mac Millan, Veksler, 1945)



The synchrotron: Ring Accelerator of const. R where the increase in momentum (i.e. B-field) is automatically synchronised with the correct synchronous phase of the particle in the rf cavities

"synchronisation" as basic principle of the particle dynamics

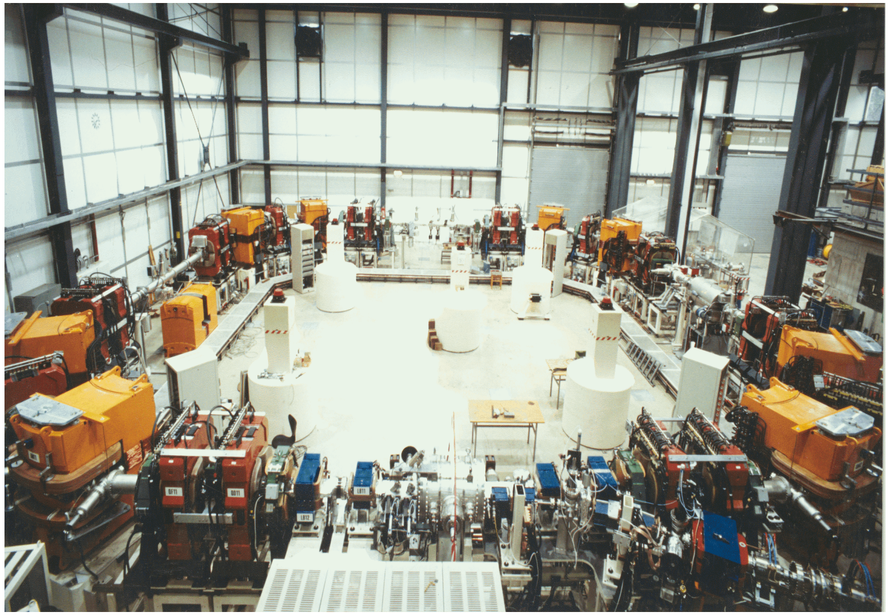
$e\hat{V}$ ... → ***...nergy gain per turn***

$\Phi = \Phi_s = cte$ → ***Synchronous particle***

$\omega_{RF} = h\omega_r$ → ***RF synchronism***

$\rho = cte \quad R = cte$ → ***Constant orbit***

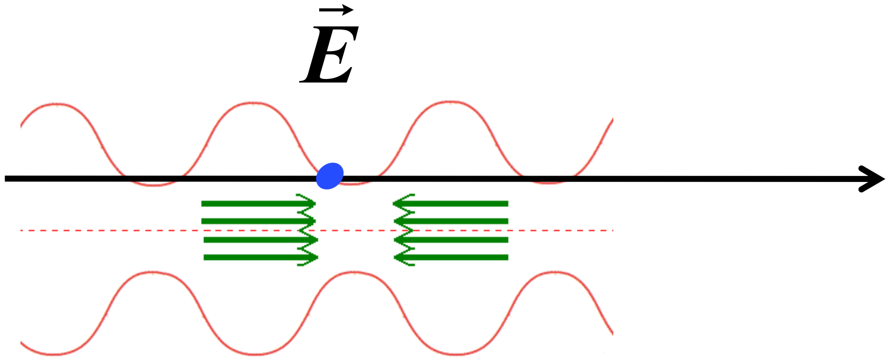
$B\rho = P/e \Rightarrow B$ → ***Variable magnetic field***



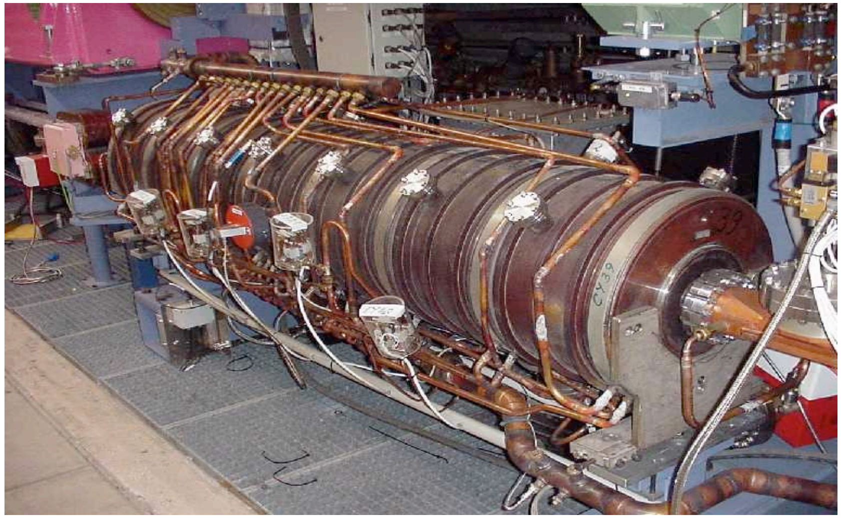
13.) The Acceleration

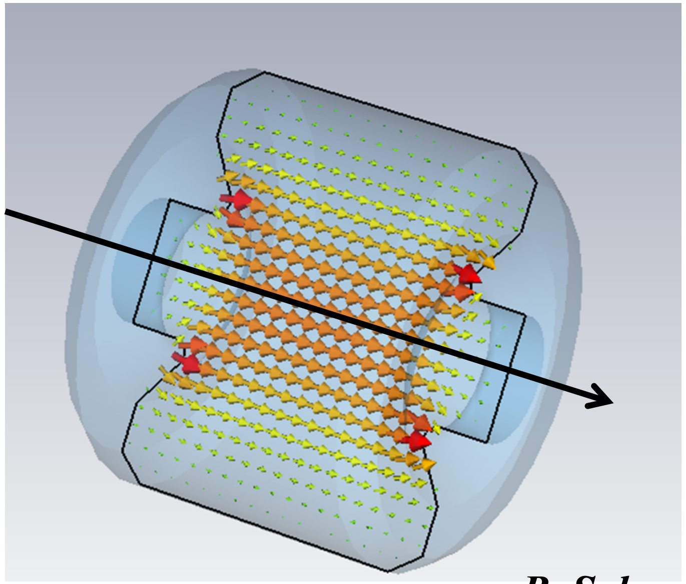
Where is the acceleration?

Install an RF accelerating structure in the ring and adjust the phase (the timing) between particle and RF-Voltage in the right way: "Synchronisation"



500 MHz cavities in an electron storage ring





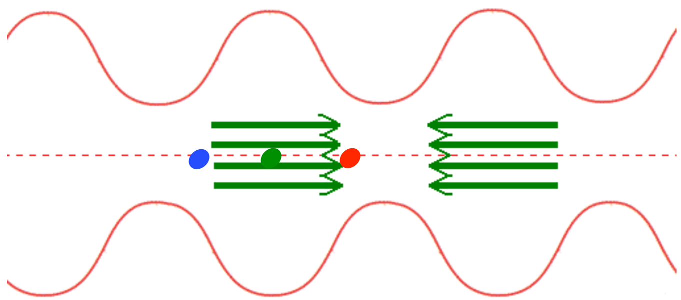
B. Salvant
N. Biancacci

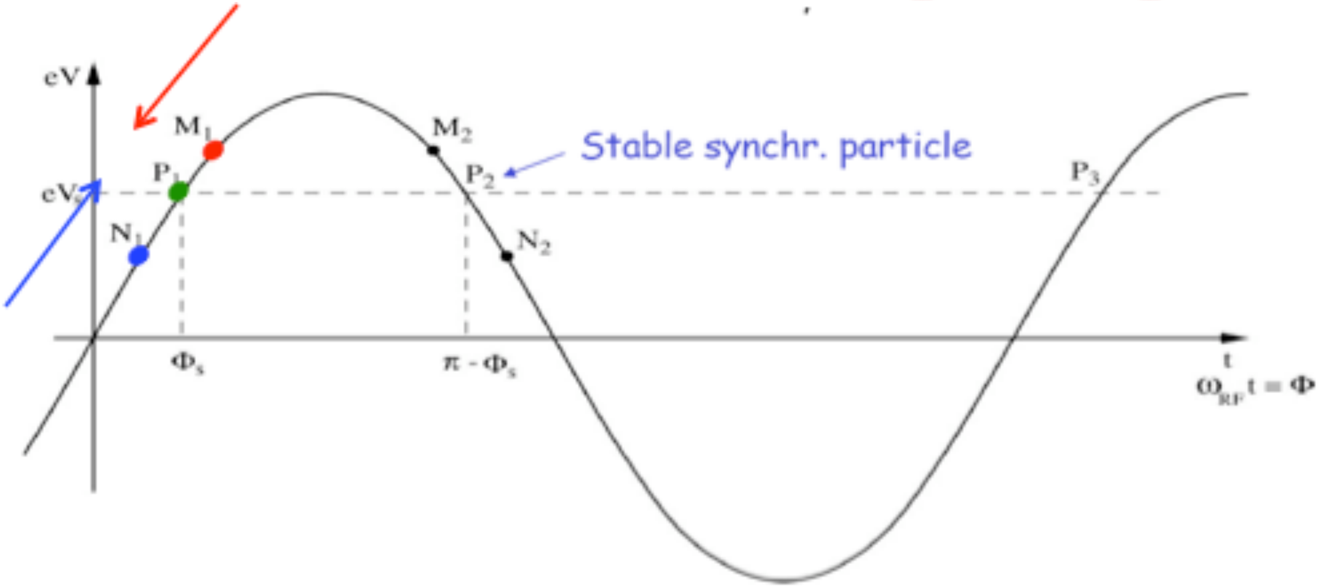
14.) The Acceleration for Δp/p≠0
"Phase Focusing" below transition

ideal particle •

particle with Δp/p > 0 • *faster*

particle with Δp/p < 0 • *slower*



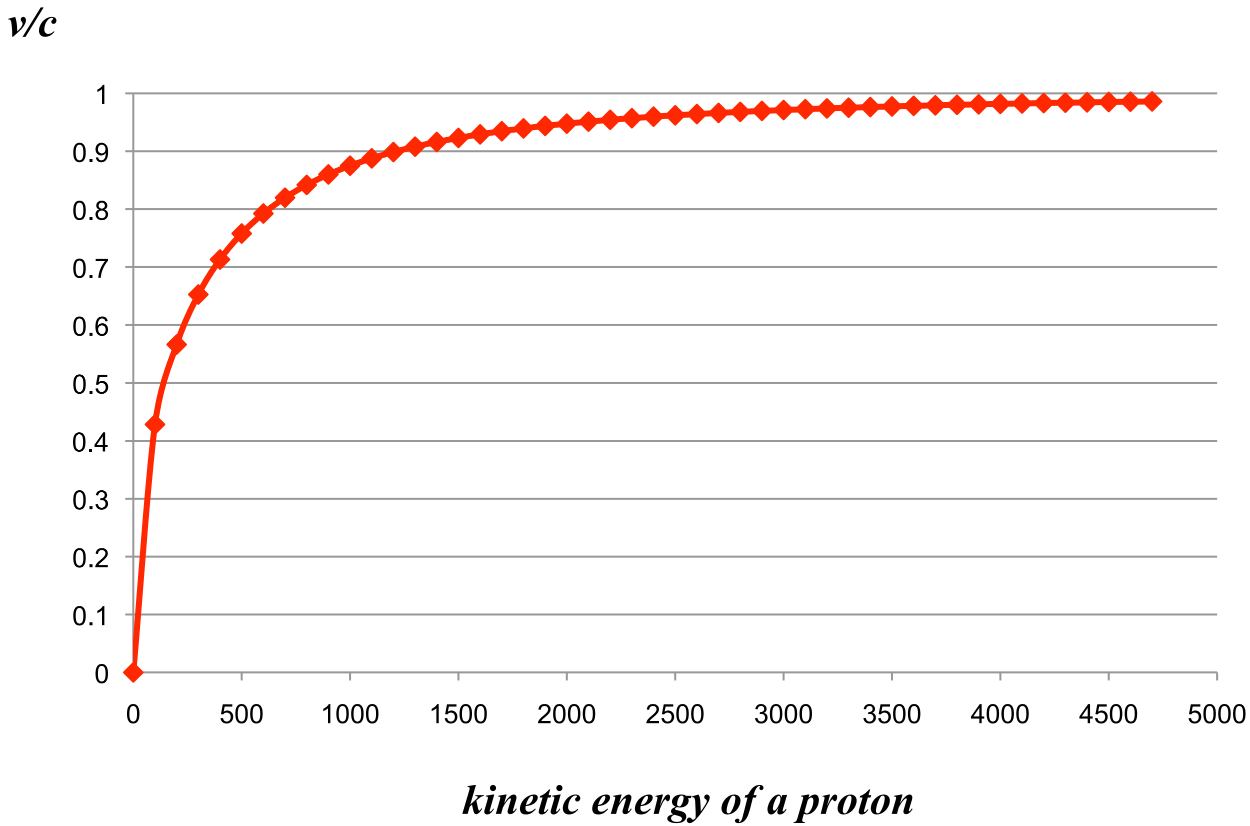


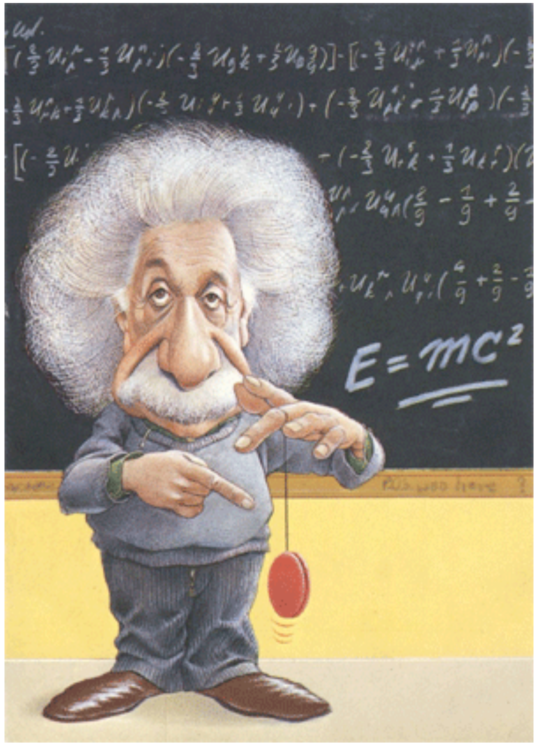
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

.... but heavier !

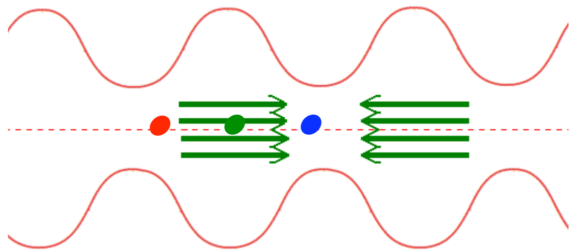
15.) The Acceleration for $\Delta p/p \neq 0$

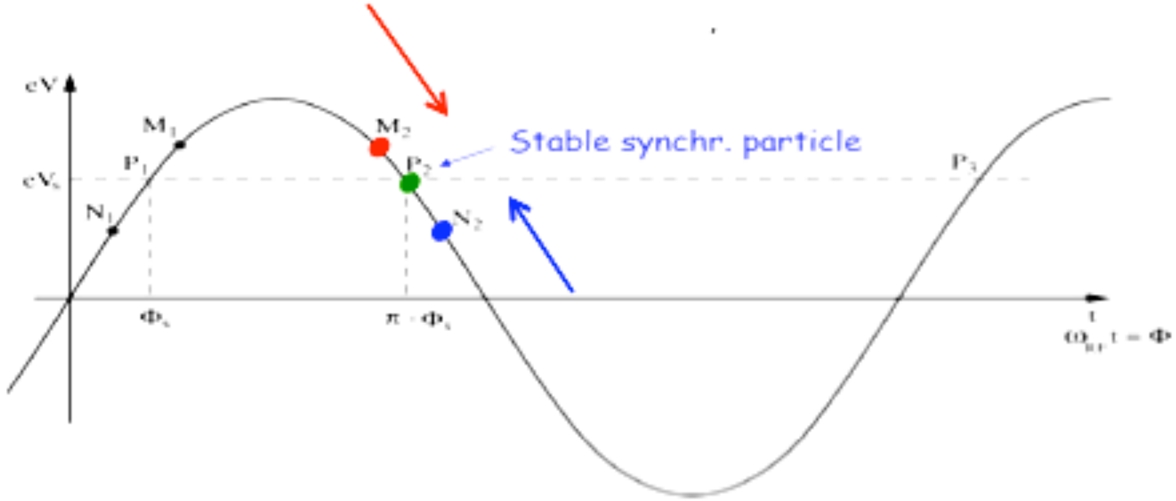
"Phase Focusing" above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *heavier*

particle with $\Delta p/p < 0$ ● *lighter*



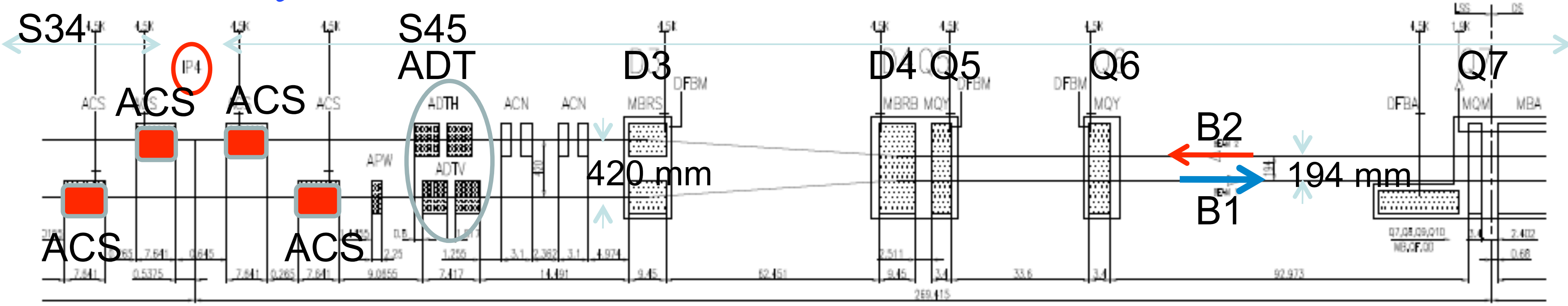


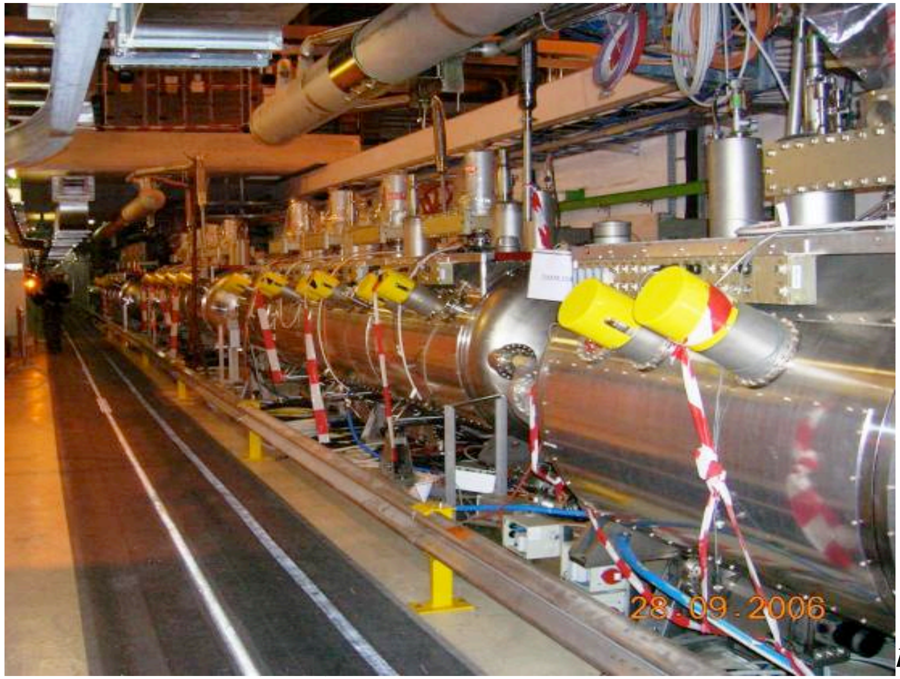
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4



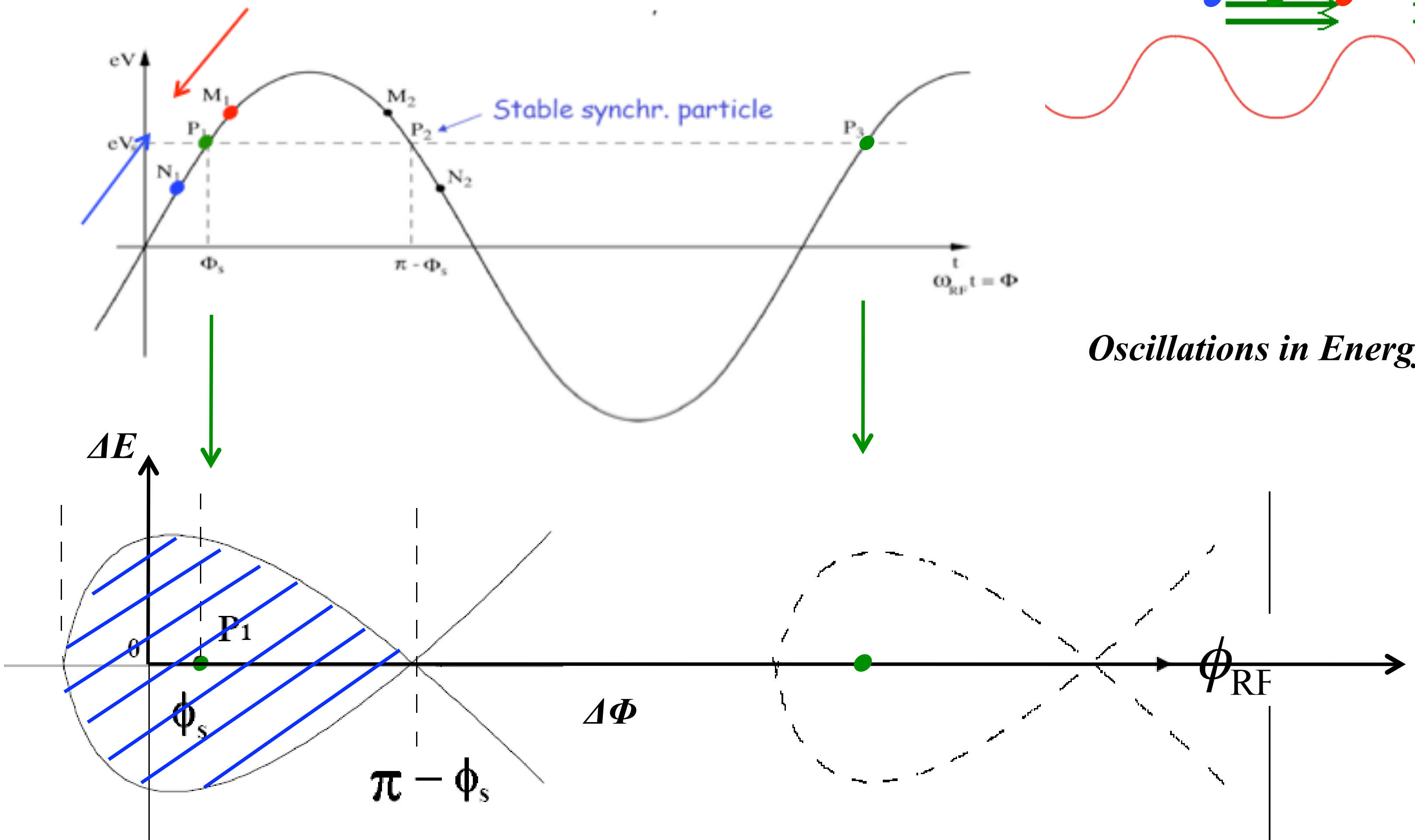


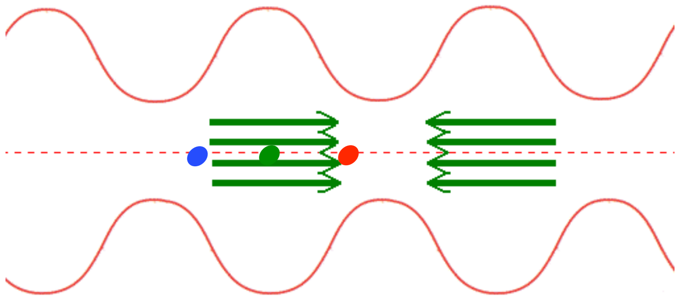
m

Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	10^{-3}	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

RF Buckets & long. dynamics in phase space



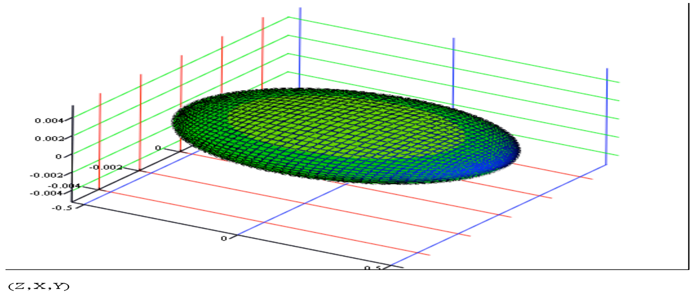


Oscillations in Energy and Phase

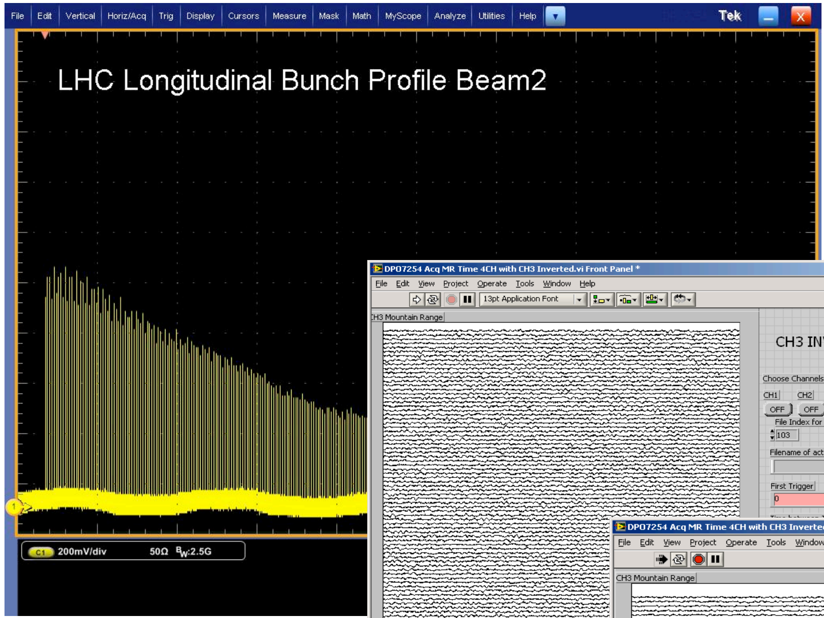
35640 Possible Bunch Positions ("buckets")

2808 Bunches

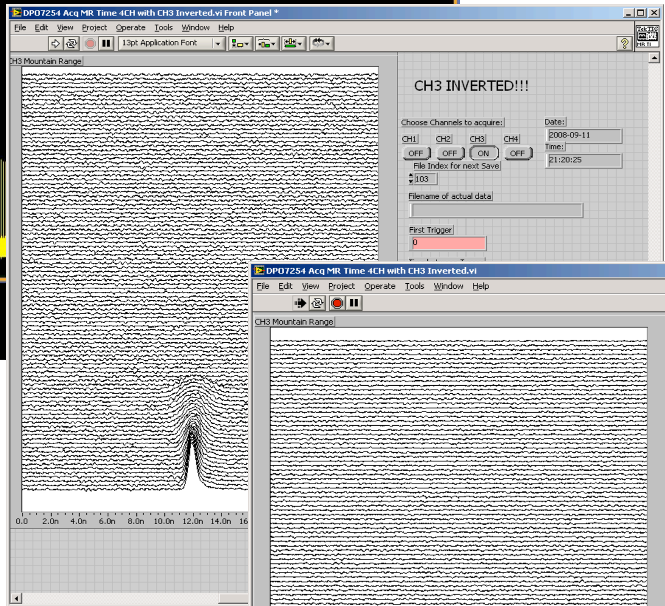
LHC Commissioning: RF



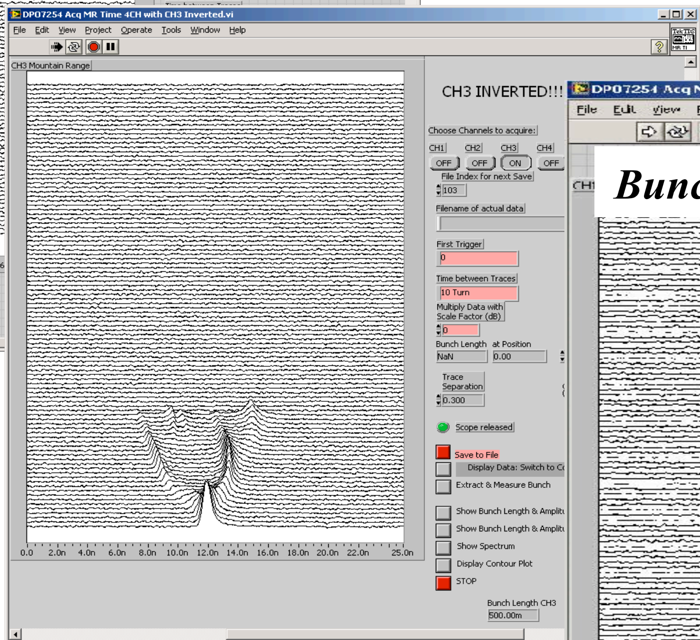
a proton bunch: focused longitudinal by the RF field



RF off



RF on,
phase optimisation



RF on, phase adjusted,
beam captured