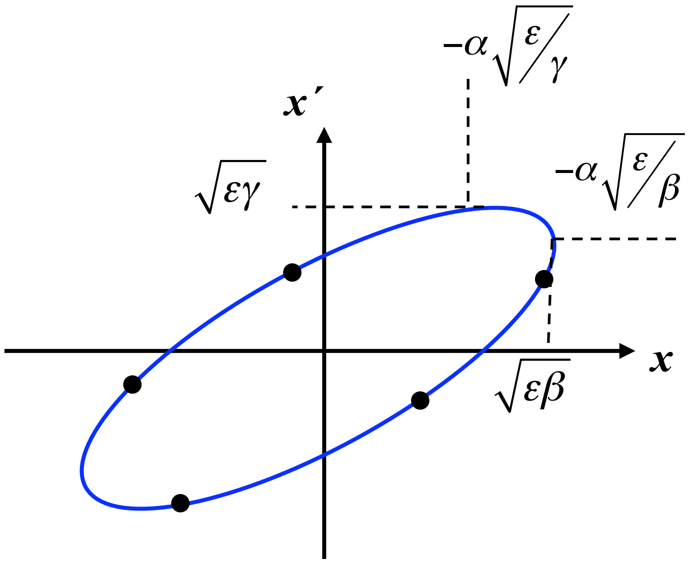
IV.) Are there Any Problems ???

sure there are

Liouville during Acceleration

$$\varepsilon = \gamma(s)\,x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)\,x'^2(s)$$



***Beam Emittance** corresponds to the area covered in the x, x´ Phase Space Ellipse*

***Liouville:** Area in phase space is constant.*

But so sorry ... $\varepsilon \neq$ const !

Classical Mechanics:

***phase space** = diagram of the two canonical variables*

position** & **momentum

x $\qquad p_x$

$$p_j = \frac{\partial L}{\partial \dot{q}_j} \quad ; \quad L = T - V = kin.\,Energy - pot.\,Energy$$

According to Hamiltonian mechanics:
phase space diagram relates the variables q and p

$$q = position = x$$
$$p = momentum = \gamma m v = m c \gamma \beta_x$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ; \quad \beta_x = \frac{\dot{x}}{c}$$

Liouvilles Theorem: $\int p\, dq = const$

for convenience (i.e. because we are lazy bones) we use in accelerator theory:

$$x' = \frac{dx}{ds} = \frac{dx}{dt}\frac{dt}{ds} = \frac{\beta_x}{\beta} \qquad \text{where } \beta_x = v_x / c$$

$$\int p\, dq = mc \int \gamma \beta_x dx$$

$$\int p\, dq = mc\gamma\beta \underbrace{\int x' dx}_{\varepsilon}$$

$$\Rightarrow \quad \varepsilon = \int x' dx \propto \frac{1}{\beta\gamma}$$

the beam emittance shrinks during acceleration $\varepsilon \sim 1/\gamma$

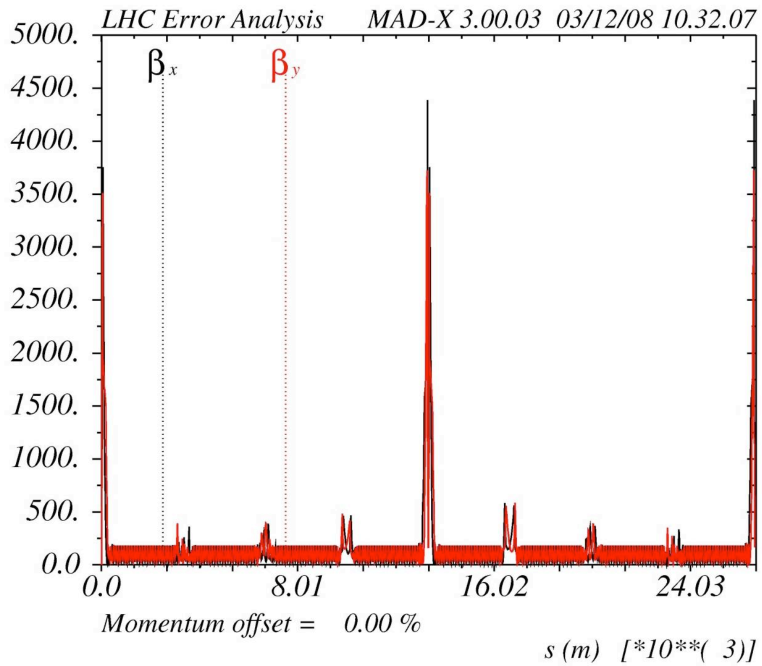
Nota bene:

1.) *A proton machine … or an electron linac … needs the highest aperture at injection energy !!!*
as soon as we start to accelerate the beam size shrinks as $\gamma^{-1/2}$ in both planes.

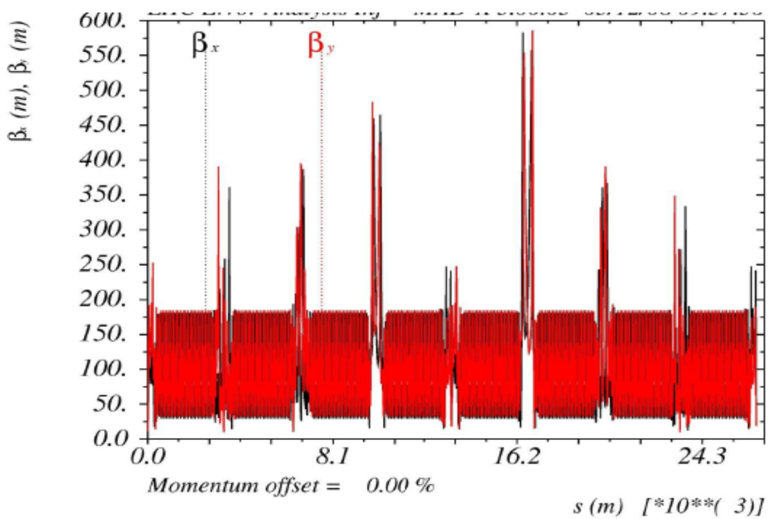
$$\sigma = \sqrt{\varepsilon\beta}$$

2.) *At lowest energy the machine will have the major aperture problems,*
→ here we have to minimise $\hat{\beta}$

3.) *we need different beam optics adopted to the energy:*
A Mini Beta concept will only be adequate at flat top.



LHC mini beta optics at 7000 GeV

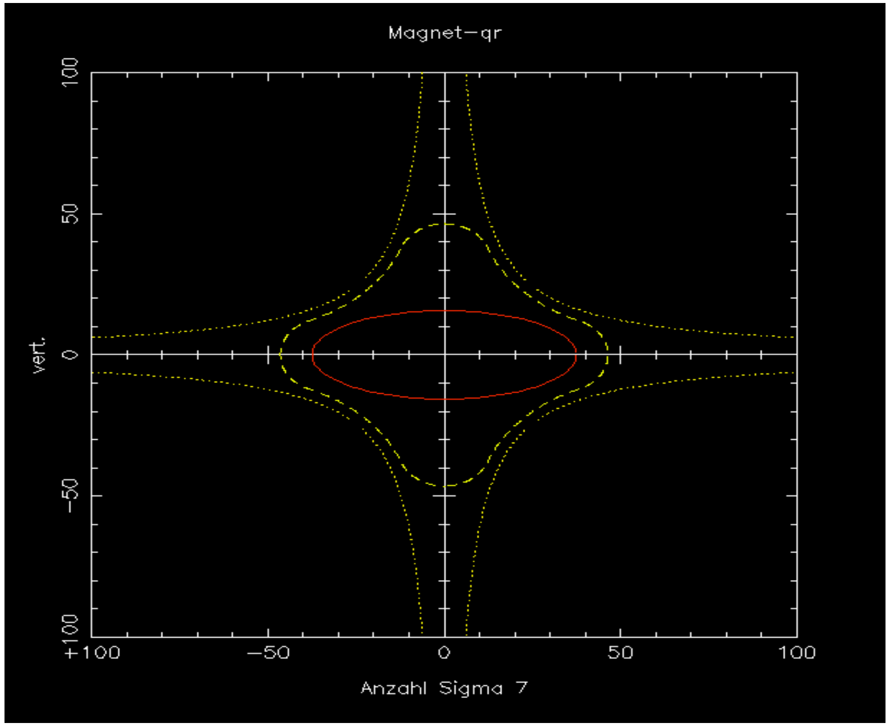


LHC injection optics at 450 GeV

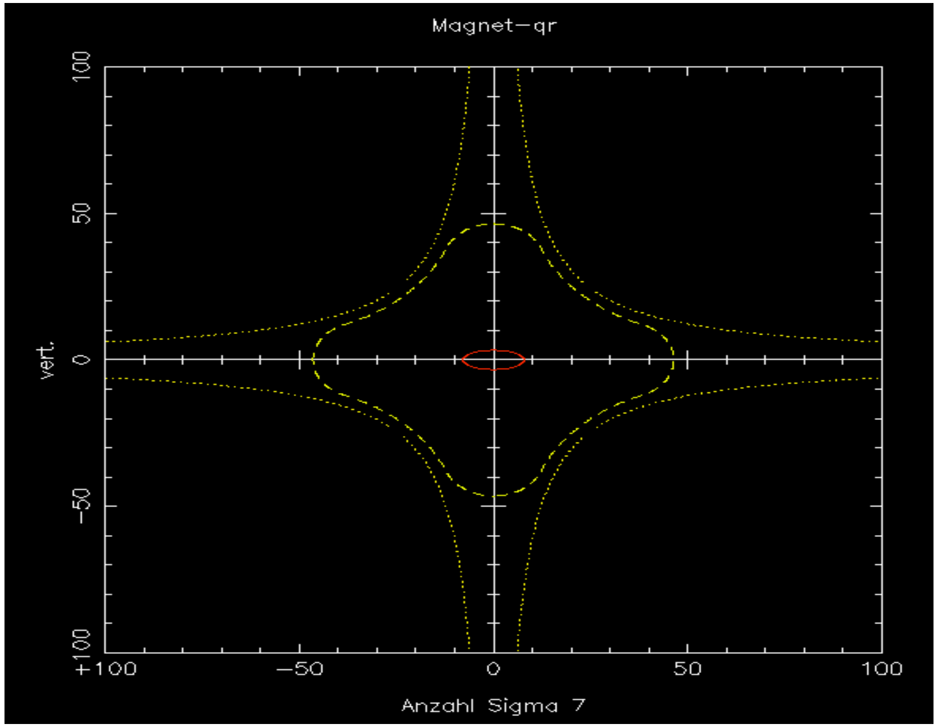
Example: HERA proton ring

injection energy: 40 GeV $\gamma = 43$
flat top energy: 920 GeV $\gamma = 980$

emittance $\varepsilon\ (40GeV) = 1.2 * 10^{-7}$
$\varepsilon\ (920GeV) = 5.1 * 10^{-9}$



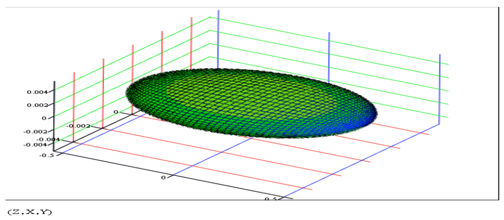
7 σ beam envelope at E = 40 GeV



… and at E = 920 GeV

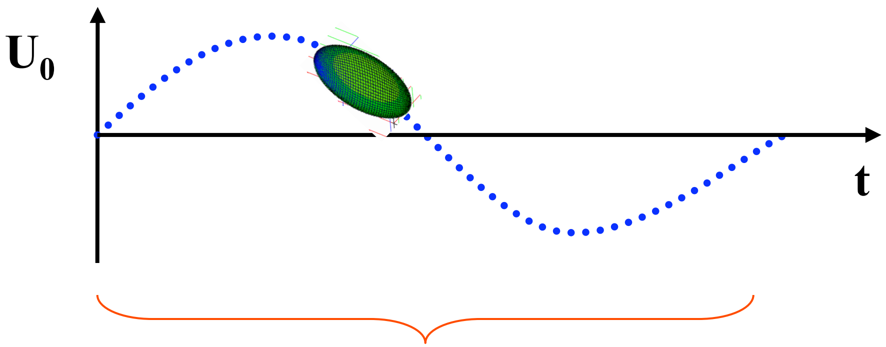
RF Acceleration-Problem:
panta rhei !!!

(Heraklit: 540-480 v. Chr.)



Bunch length of Electrons ≈ 1cm

just a stupid (and nearly wrong) example)



$\lambda = 75\ cm$

$$\nu = 400\,MHz$$
$$c = \lambda\,\nu$$
$$\lambda = 75\ cm$$

$$\sin(90^o) = 1$$
$$\sin(84^o) = 0.994$$

$$\frac{\Delta U}{U} = 6.0\ \ 10^{-3}$$

typical momentum spread of an electron bunch:

$$\frac{\Delta p}{p} \approx 1.0\ \ 10^{-3}$$

Dispersive and Chromatic Effects: $\Delta p/p \neq 0$

Are there any Problems ???

Sure there are !!!

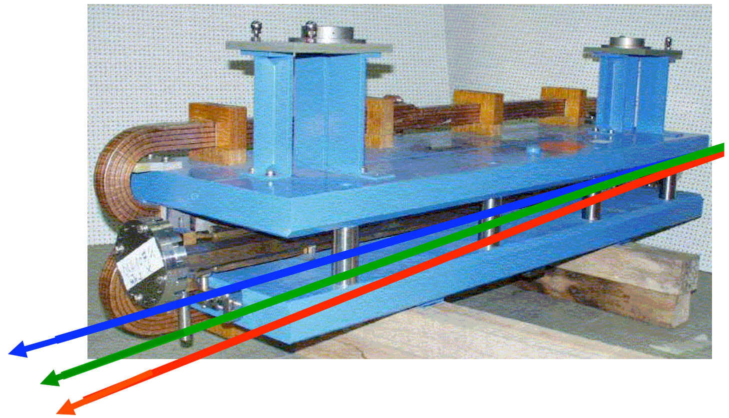
font colors due to pedagogical reasons

17.) Dispersion and Chromaticity:
Magnet Errors for Δp/p ≠ 0

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

dipole magnet

$$\alpha = \frac{\int B\,dl}{p/e}$$



$$x_D(s) = D(s)\,\frac{\Delta p}{p}$$

focusing lens

$$k = \frac{g}{p/e}$$

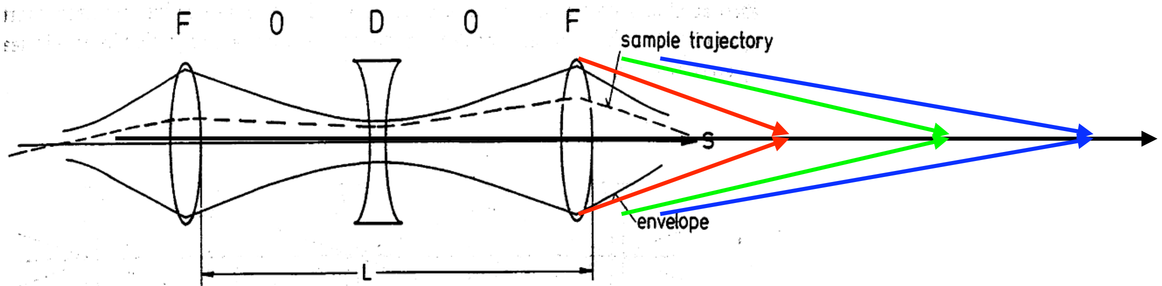


Figure 29: FODO cell

particle having ...

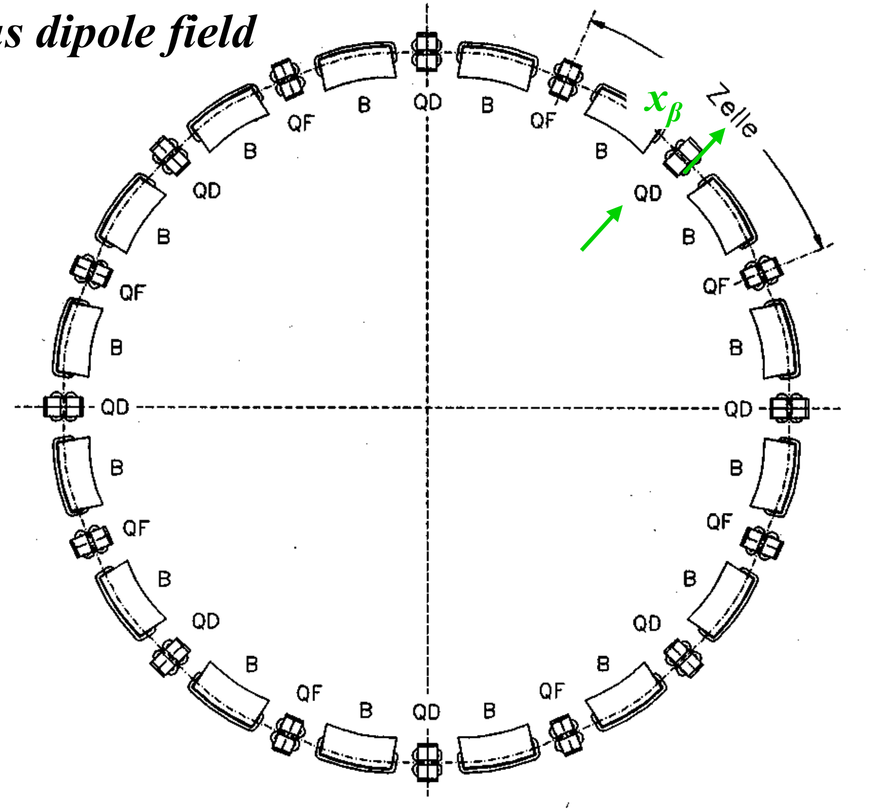
to high energy

to low energy

ideal energy

Dispersion

Example: homogeneous dipole field



it for $\Delta p/p > 0$

$: D(s)\cdot\frac{\Delta p}{p}$

Matrix formalism:

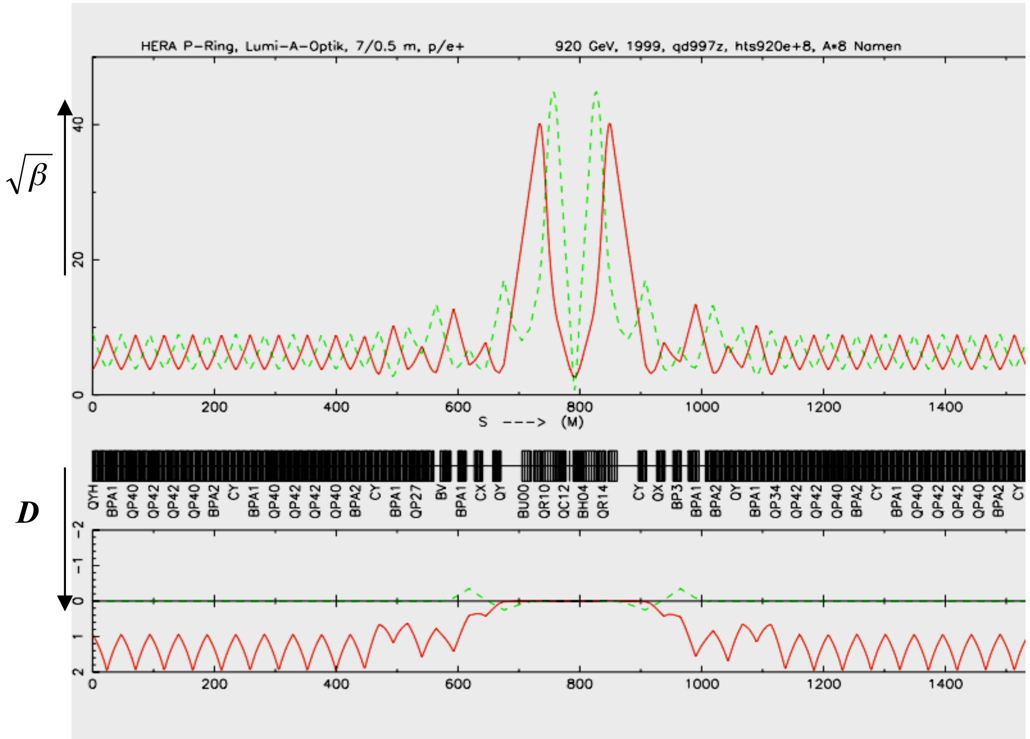
$$x(s) = x_\beta(s) + D(s)\cdot\frac{\Delta p}{p}$$

$$x(s) = C(s)\cdot x_0 + S(s)\cdot x_0' + D(s)\cdot\frac{\Delta p}{p}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_s = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix}_0 + \frac{\Delta p}{p}\begin{pmatrix} D \\ D' \end{pmatrix}_0$$

or expressed as 3x3 matrix

$$\begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_s = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ \Delta p / p \end{pmatrix}_0$$



Example

$$x_\beta = 1 \ldots 2\ mm$$
$$D(s) \approx 1 \ldots 2\ m$$
$$\Delta p / p \approx 1 \cdot 10^{-3}$$

Amplitude of Orbit oscillation
contribution due to Dispersion ≈ beam size
→ Dispersion must vanish at the collision point

!

Calculate D, D´: ... takes a couple of sunny Sunday evenings !

26.) Chromaticity:
A Quadrupole Error for $\Delta p/p \neq 0$

Influence of external fields on the beam: *prop. to magn. field & prop. zu 1/p*

focusing lens

$$k = \frac{g}{p/e}$$

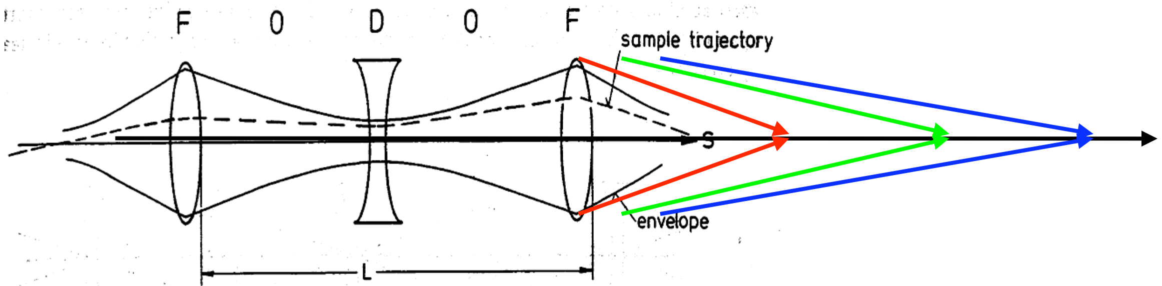


Figure 29: FODO cell

particle having ...
to high energy
to low energy
ideal energy

... which acts like a quadrupole error in the machine
and leads to a tune spread:

$$\Delta Q = -\frac{1}{4\pi}\frac{\Delta p}{p_0} k_0 \beta(s) ds$$

definition of chromaticity:

$$\Delta Q = Q' * \frac{\Delta p}{p}$$

… what is wrong about Chromaticity:

Problem: chromaticity is generated by the lattice itself !!

Q' is a number indicating the size of the tune spot in the working diagram,
Q' is always created if the beam is focussed
→ *it is determined by the focusing strength k of all quadrupoles*

$$Q' = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$$

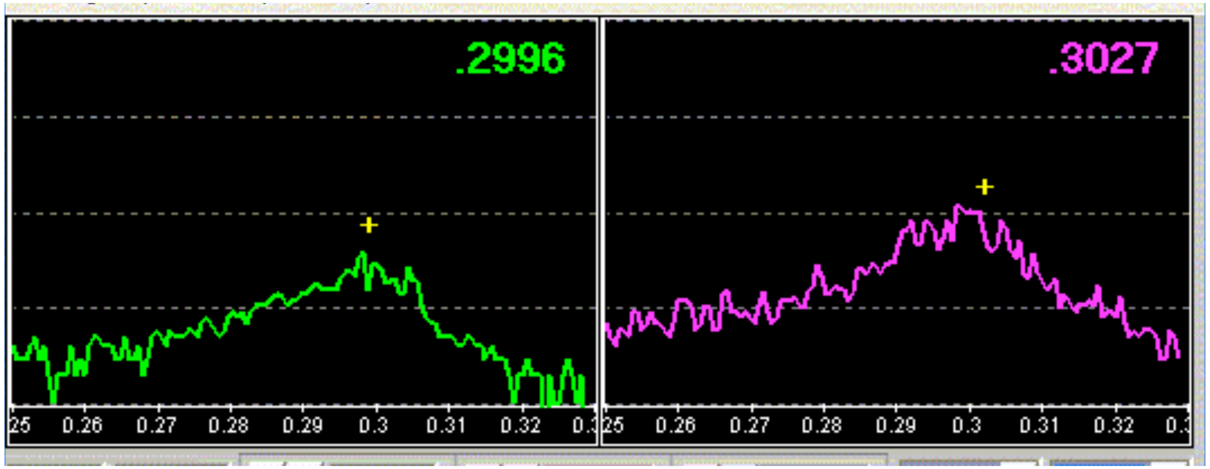
k = quadrupole strength
β = betafunction indicates the beam size … and even more the sensitivity of the beam to external fields

Example: LHC

$Q' = 250$
$\Delta p/p = \pm 0.2 \cdot 10^{-3}$
$\Delta Q = 0.256 \ldots 0.36$

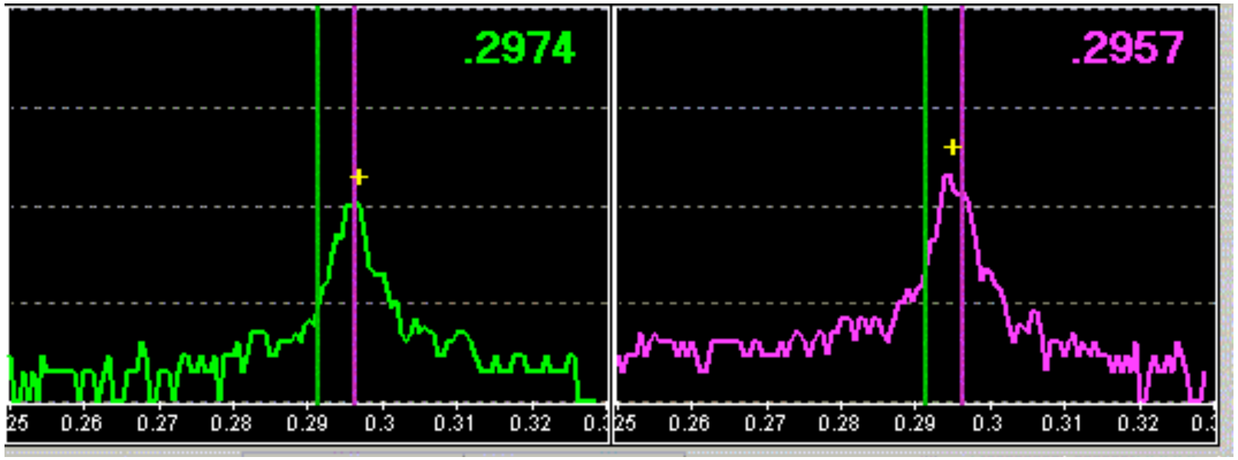
→ *Some particles get very close to resonances and are lost*

in other words: the tune is not a point
it is a pancake



Tune signal for a nearly uncompensated cromaticity (Q' ≈ 20)

Ideal situation: cromaticity well corrected, (Q' ≈ 1)

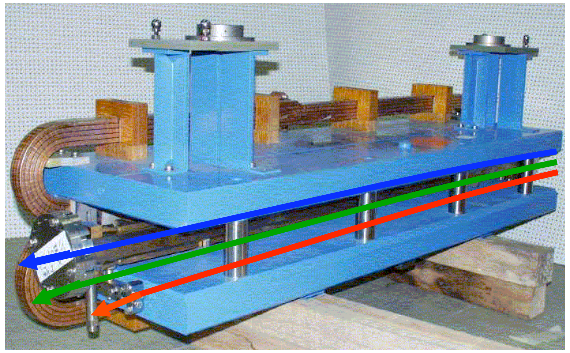


Correction of Q':

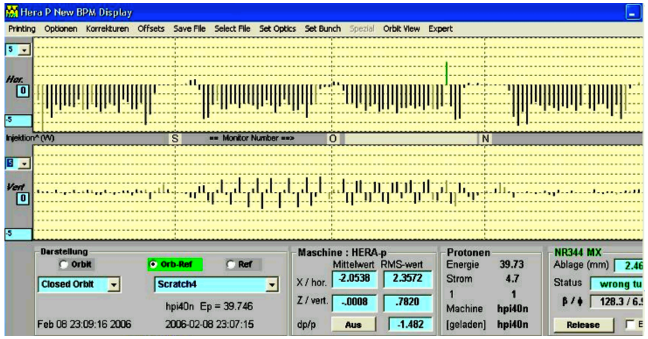
Need: additional quadrupole strength for each momentum deviation Δp/p

1.) sort the particles acording to their momentum

$$x_D(s) = D(s)\frac{\Delta p}{p}$$



… using the dispersion function



2.) apply a magnetic field that rises quadratically with x (sextupole field)

$$B_x = \tilde{g}xz$$

$$B_z = \frac{1}{2}\tilde{g}(x^2 - z^2)$$

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x} = \tilde{g}x$$

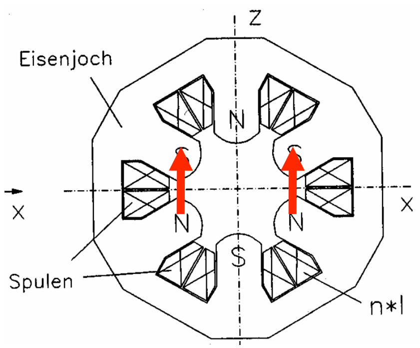
linear rising „gradient":

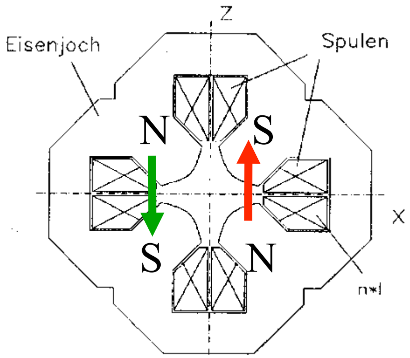
Correction of Q':

k_1 normalised quadrupole strength

k_2 normalised sextupole strength

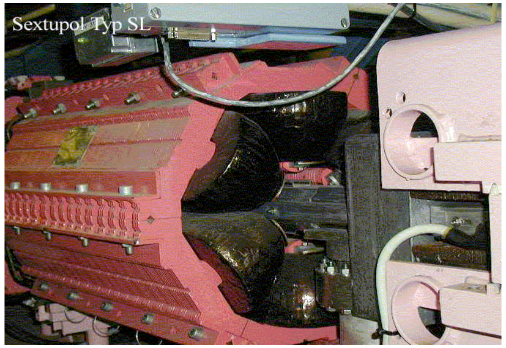
Sextupole Magnets:





$$k_1(sext) = \frac{\tilde{g}\, x}{p/e} = k_2 * x$$

$$k_1(sext) = k_2 * D * \frac{\Delta p}{p}$$



corrected chromaticity

considering a single cell:

$$Q'_{cell\_x} = -\frac{1}{4\pi}\left\{k_{qf}\hat{\beta}_x l_{qf} - k_{qd}\breve{\beta}_x l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

$$Q'_{cell\_y} = -\frac{1}{4\pi}\left\{-k_{qf}\breve{\beta}_y l_{qf} + k_{qd}\hat{\beta}_y l_{qd}\right\} + \frac{1}{4\pi}\sum_{F\,sext} k_2^F l_{sext} D_x^F \beta_x^F - \frac{1}{4\pi}\sum_{D\,sext} k_2^D l_{sext} D_x^D \beta_x^D$$

Some Golden Rules to Avoid Trouble

I.) Golden Rule number one: do not focus the beam !

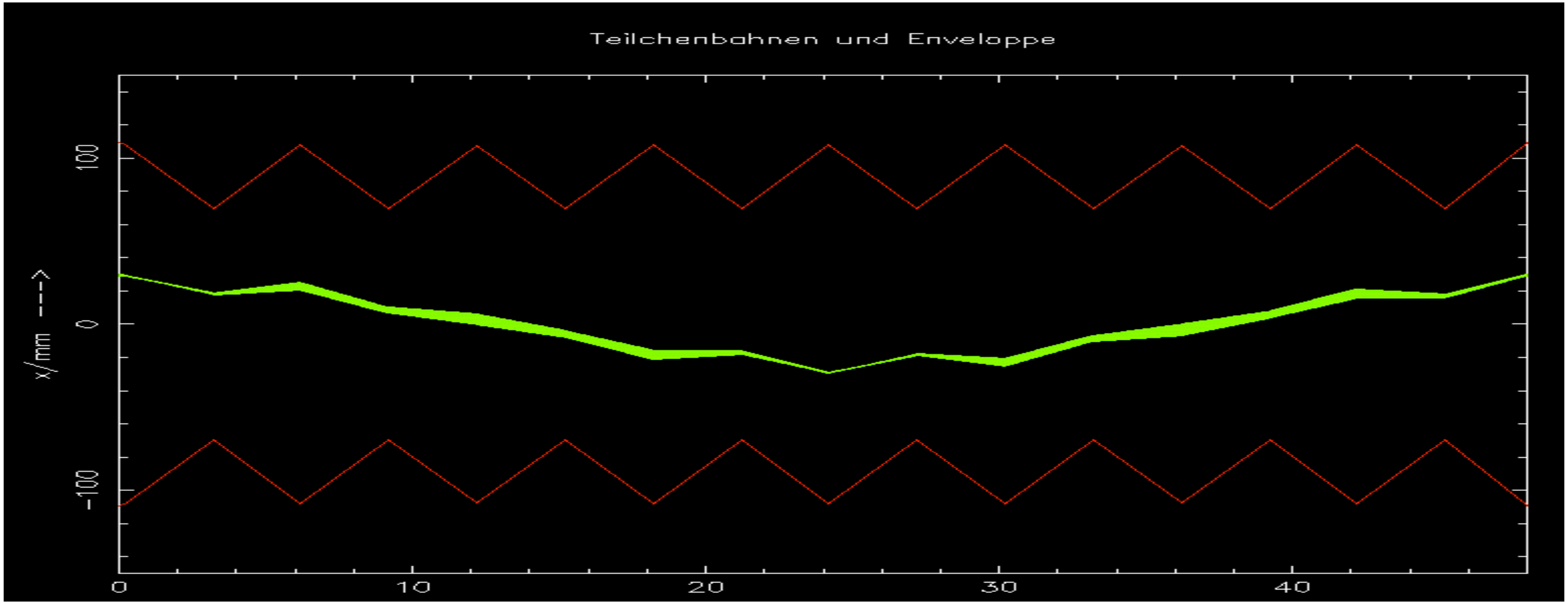
Problem: Resonances

$$x_{co}(s) = \frac{\sqrt{\beta(s)} * \int \frac{1}{\rho_{s1}} \sqrt{\beta_{s1}} * \cos(\psi_{s1} - \psi_s - \pi Q)\, ds}{2 \sin \pi Q}$$

Assume: Tune = integer $\quad Q = 1 \quad \rightarrow \quad 0$

Qualitatively spoken:

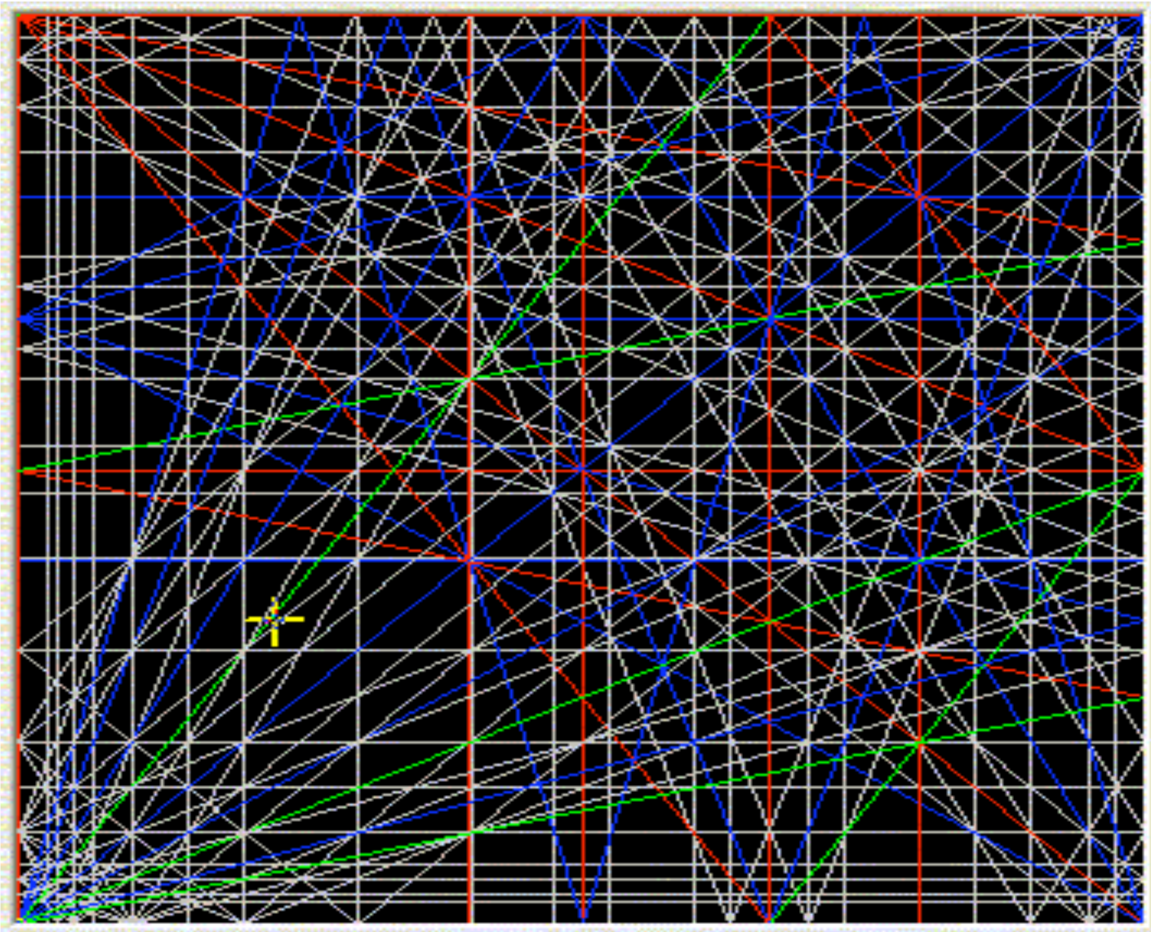
Integer tunes lead to a resonant increase of the closed orbit amplitude in presence of the smallest dipole field error.



Tune and Resonances

$$m*Q_x+n*Q_y+l*Q_s = integer$$

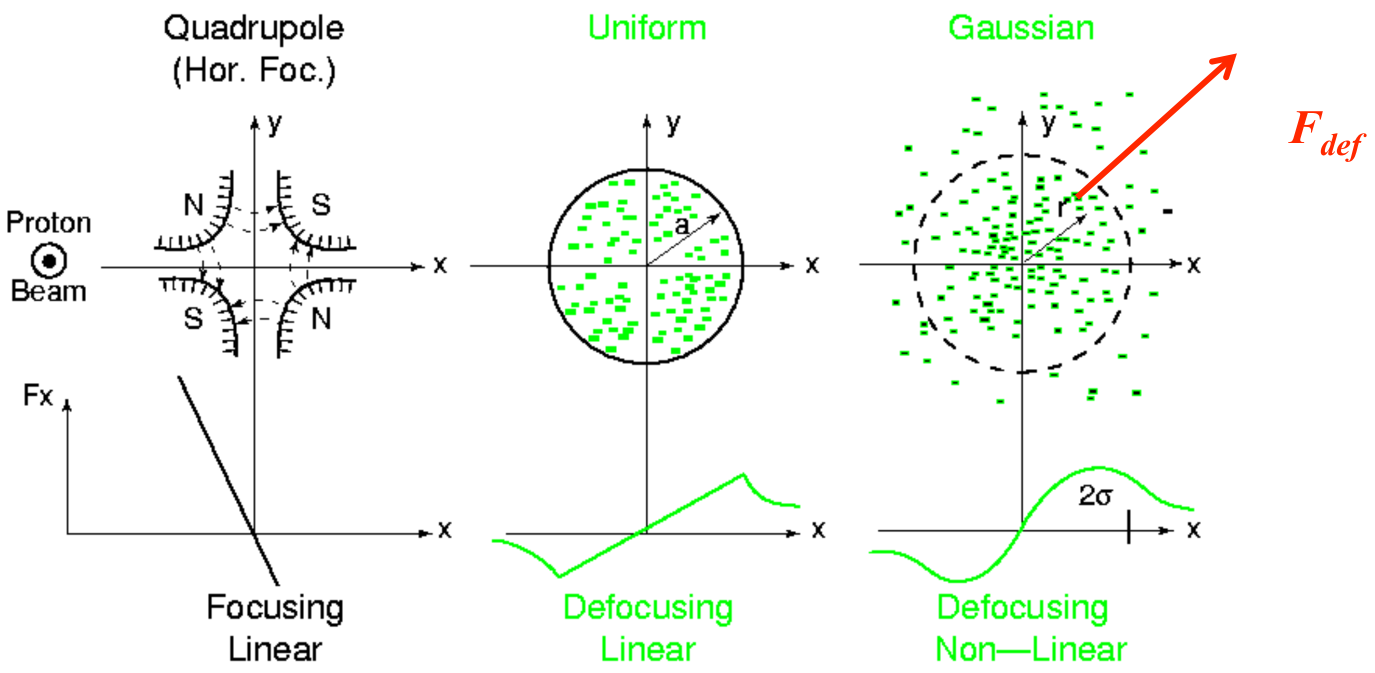
Tune diagram up to 3rd order



… and up to 7th order

Homework for the operateurs: find a nice place for the tune where against all probability the beam will survive

II.) Golden Rule number two: Never accelerate charged particles !



Transport line with quadrupoles

$$x'' + K(s)x = 0$$

Transport line with quadrupoles and space charge

$$x'' + (K(s) + K_{SC}(s))x = 0$$

$$x'' + \left(K(s) - \frac{2r_0 I}{e a^2 \beta^3 \gamma^3 c} \right) x = 0$$

K_{SC}

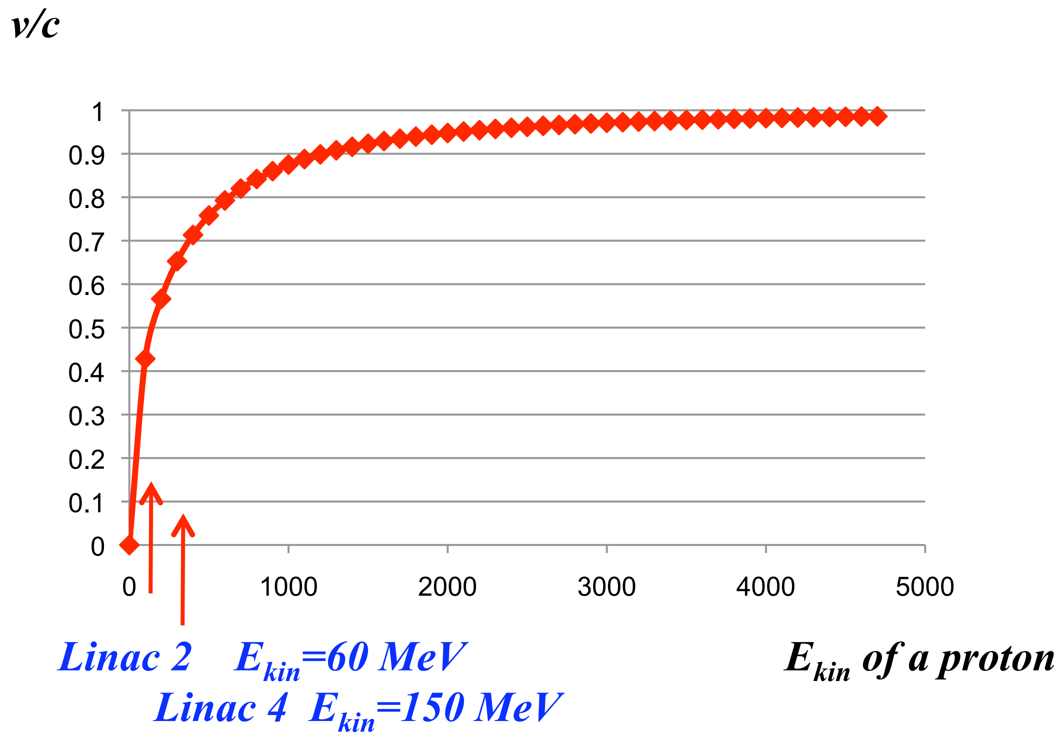
Golden Rule number two: ***Never accelerate charged particles !***

Tune Shift due to Space Charge Effect

Problem at low energies

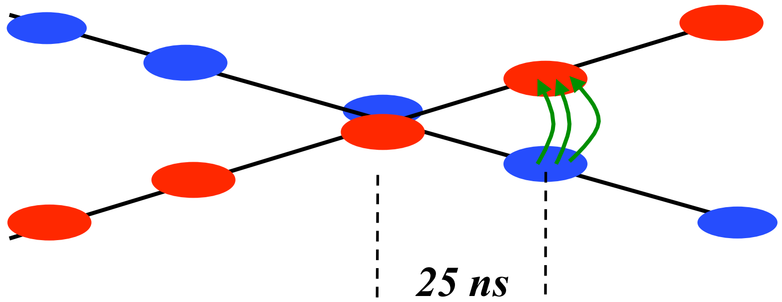
$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi \varepsilon_{x,y} \beta \gamma^2}$$

... at low speed the particles repel each other

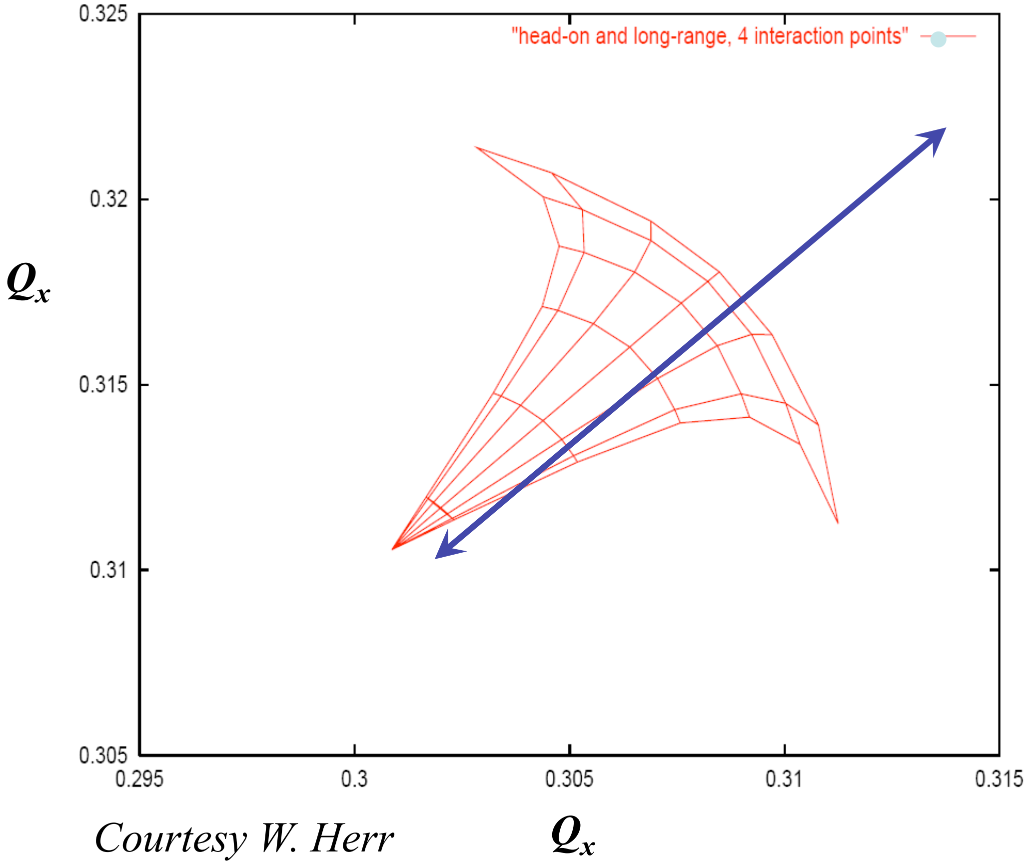


III.) Golden Rule number three:

Never Collide the Beams !



the colliding bunches influence each other
→ change the focusing properties of the ring !!

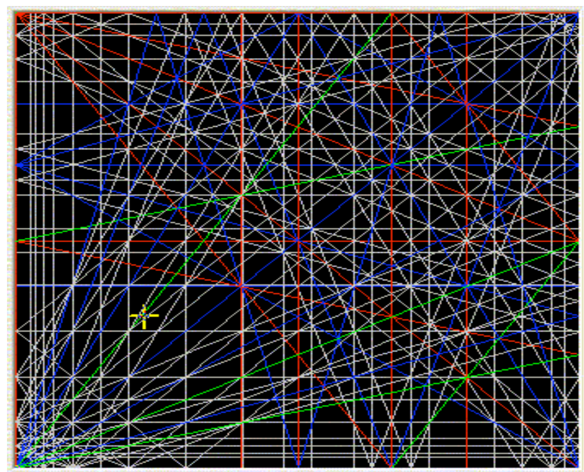


Courtesy W. Herr

most simple case:
linear beam beam tune shift

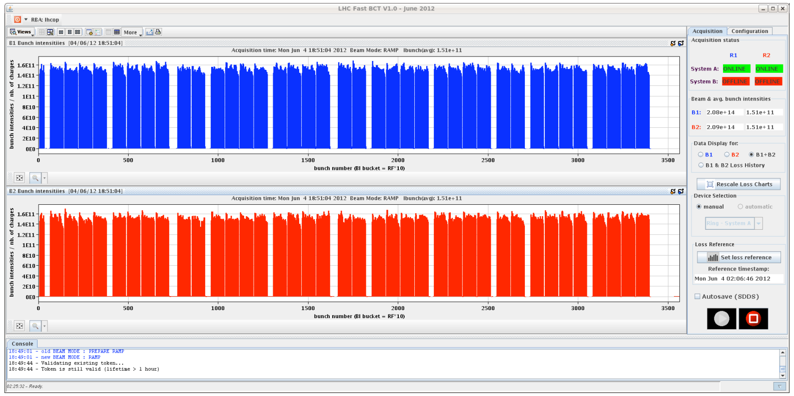
$$\Delta Q_x = \frac{\beta_x^* * r_p * N_p}{2\pi \gamma_p (\sigma_x + \sigma_y) * \sigma_x}$$

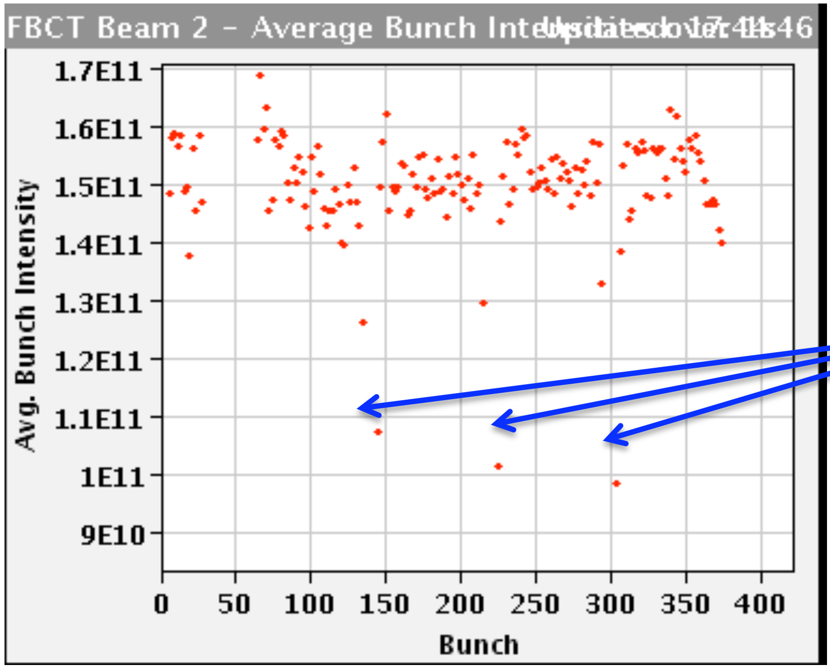
and again the resonances !!!



LHC logbook: Sat 9-June “Late-Shift”

18:18h injection for physics
clean injection !

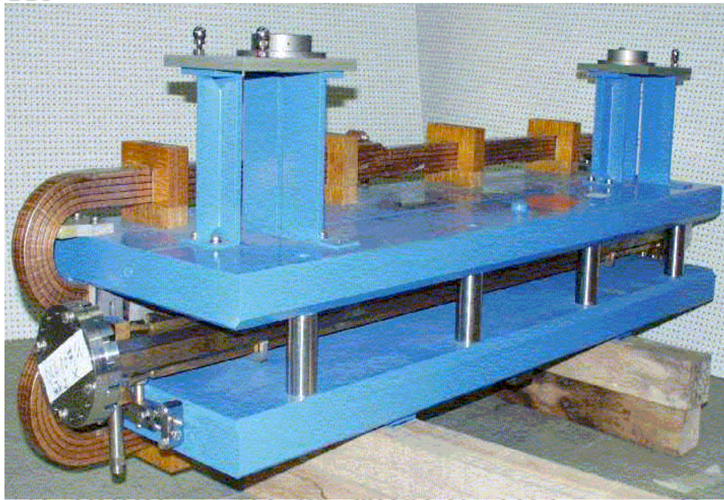




but particle losses when beams are brought into collision

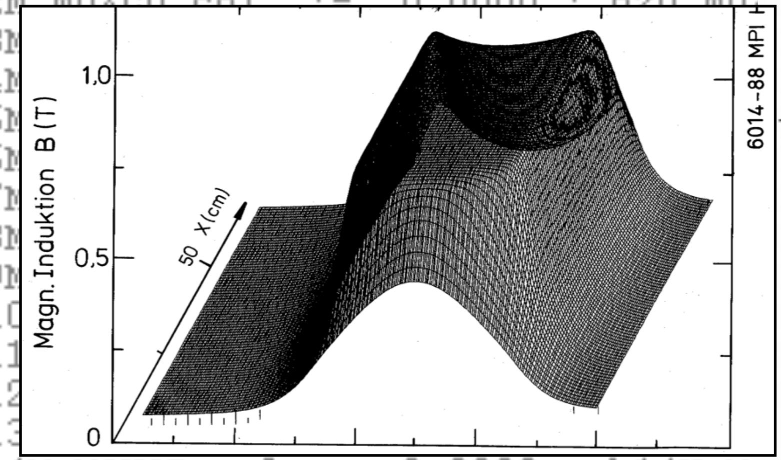
IV.) Golden Rule Number 4: Never use Magnets

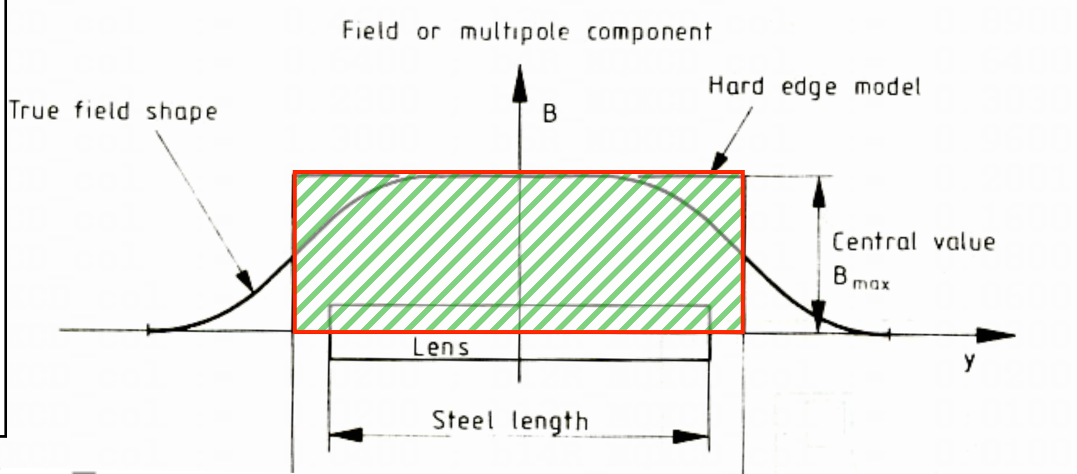
$$B_y + iB_x = B_{ref} * \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$



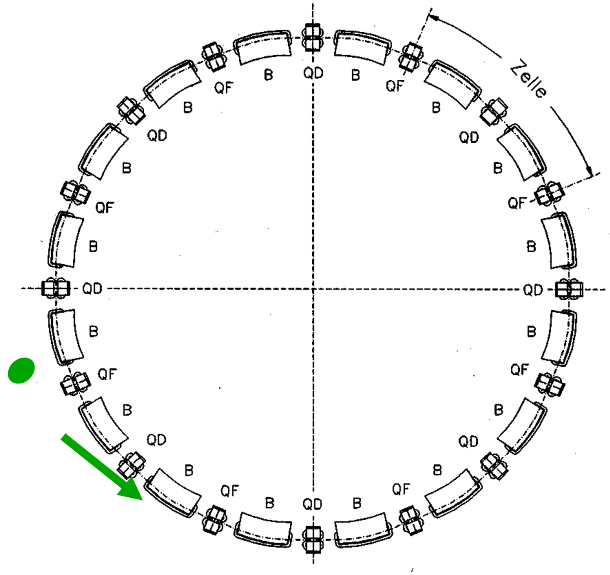
"effective magnetic length"

$$B * l_{eff} = \int_0^{l_{mag}} B ds$$





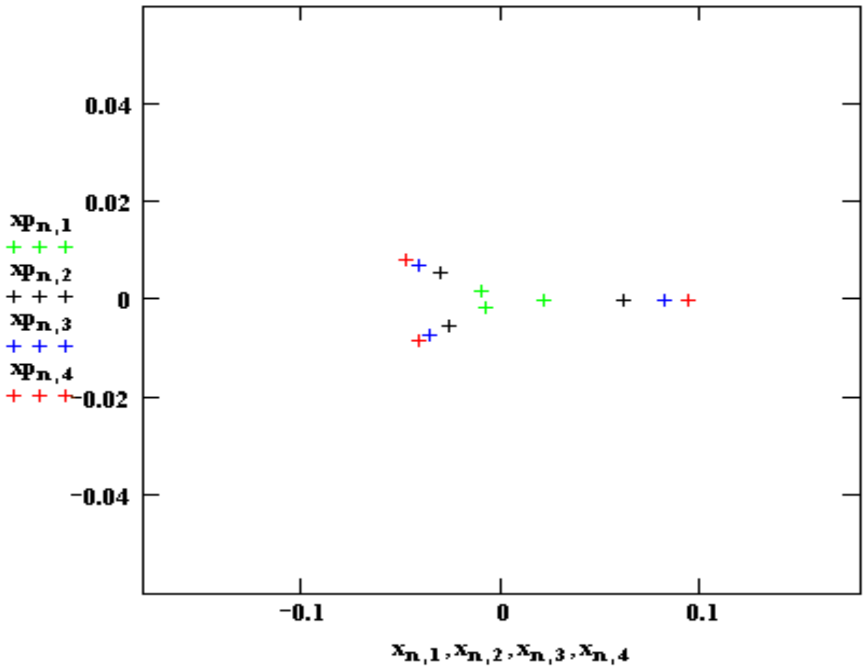
Clearly there is another problem ...
... if it were easy everybody could do it



Again: the phase space ellipse

for each turn write down – at a given position „s" in the ring – the single partilce amplitude x and the angle x´... and plot it.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

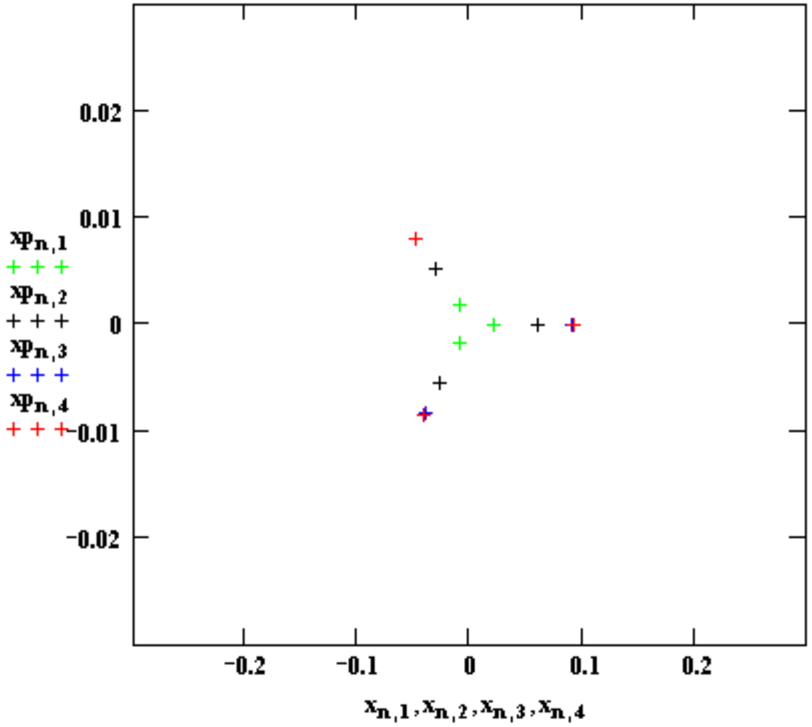


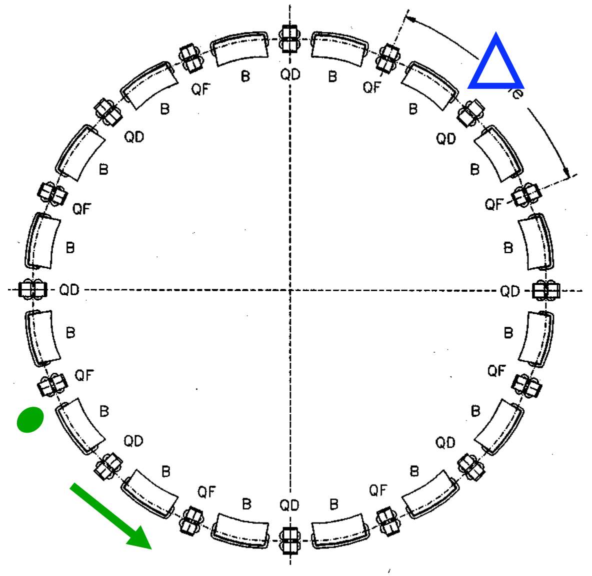
A beam of 4 particles

– each having a slightly different emittance:

Installation of a weak (!!!) sextupole magnet

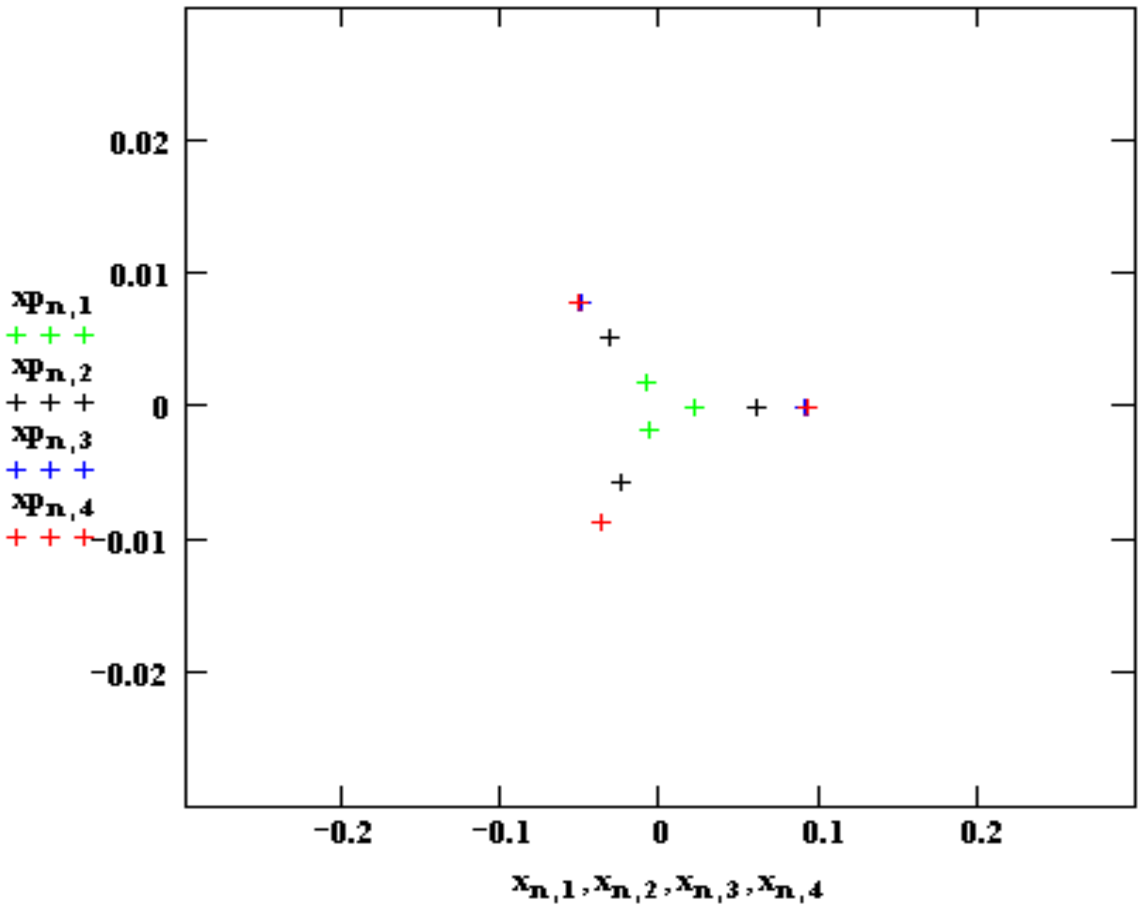
The good news: sextupole fields in accelerators cannot be treated analytically anymore.
→ no equatiuons; instead: Computer simulation
„ particle tracking "

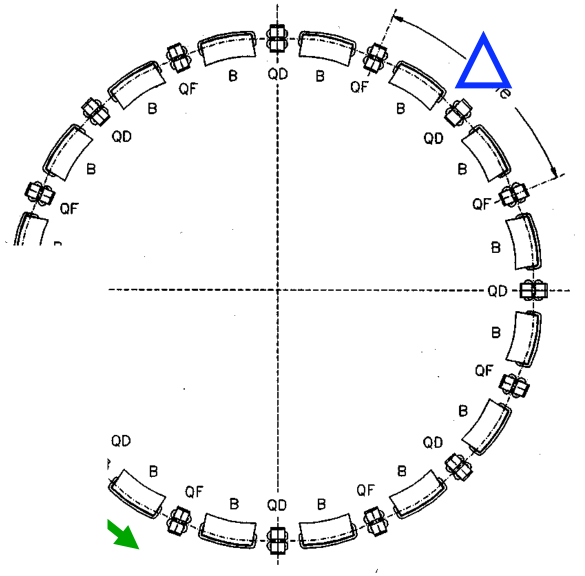




Effect of a strong (!!!) Sextupole …

→ *Catastrophy !*

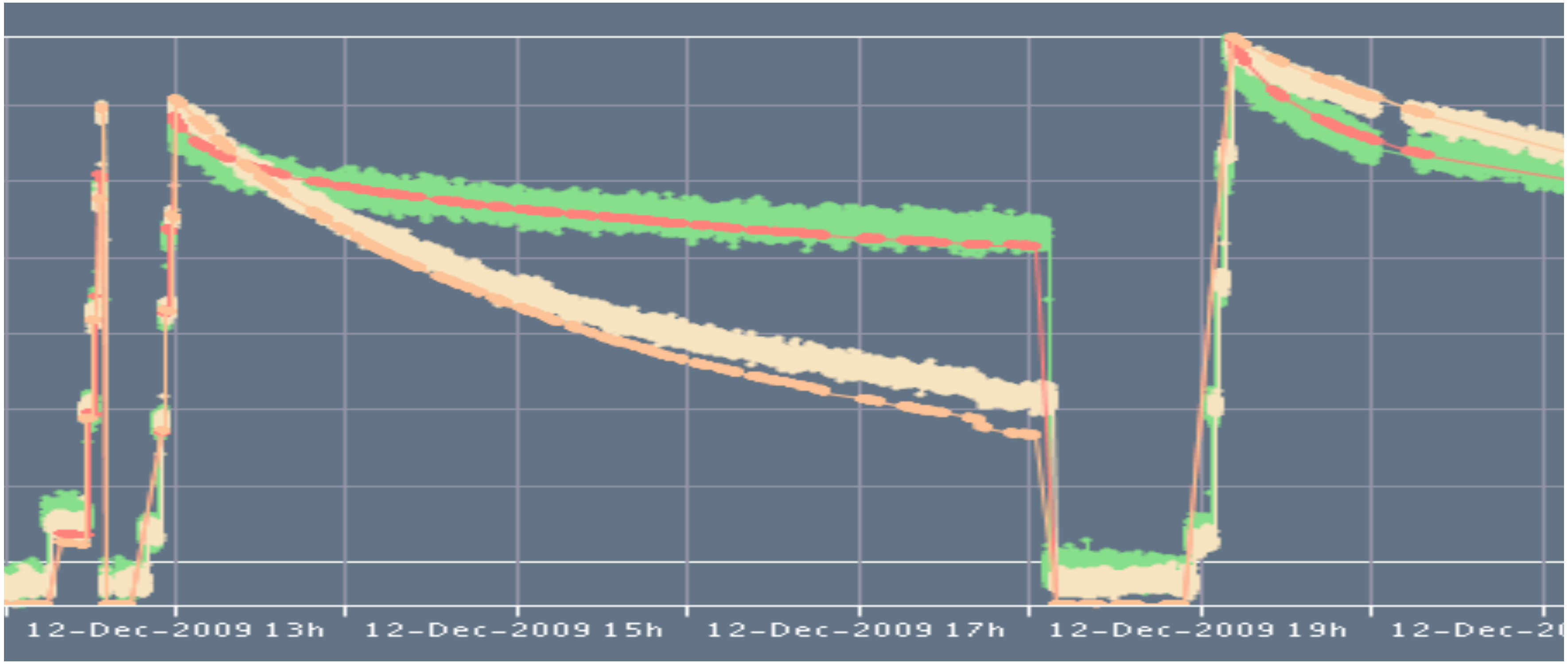




„dynamic aperture“

Golden Rule XXL: COURAGE

and with a lot of effort from Bachelor / Master / Diploma / PhD and Summer-Students the machine is running !!!



thank'x for your help and have a lot of fun

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