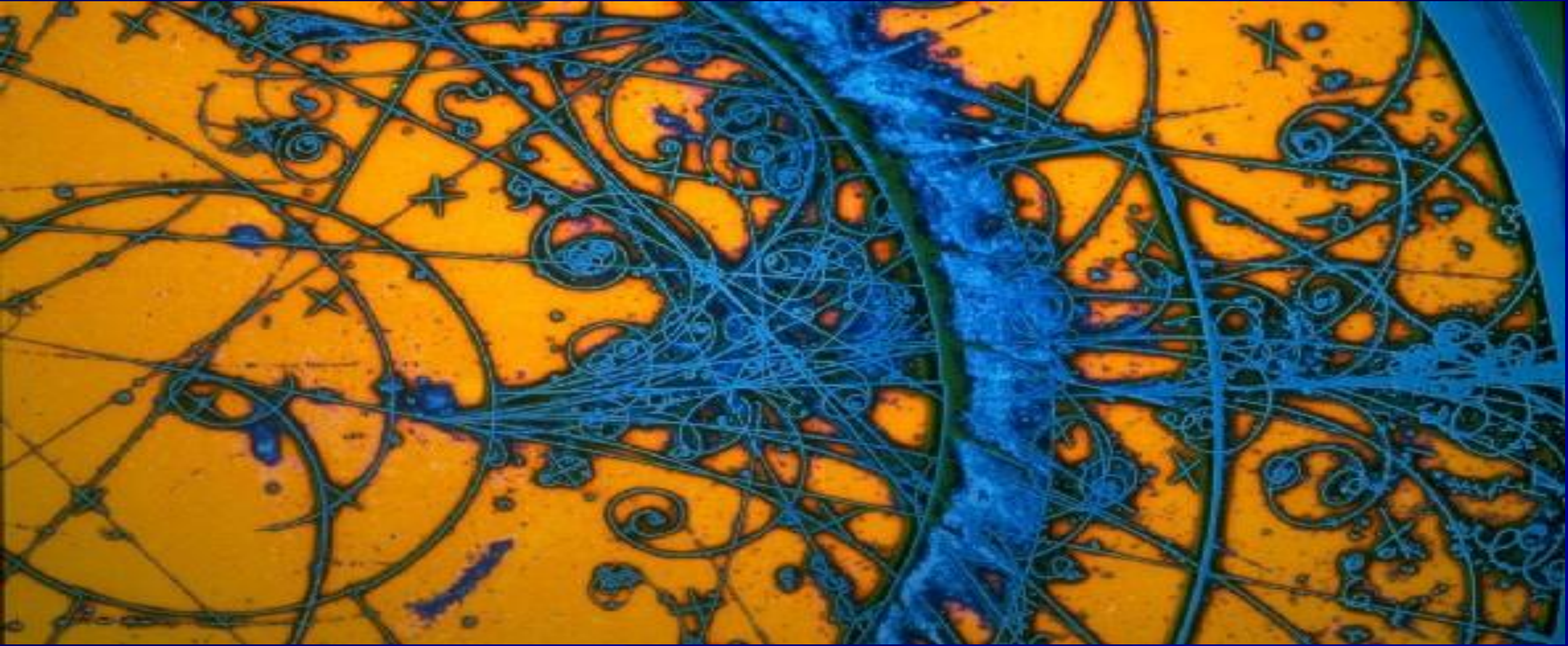


Detectors for Particle Physics

Interaction with Matter

D. Bortoletto
Purdue University

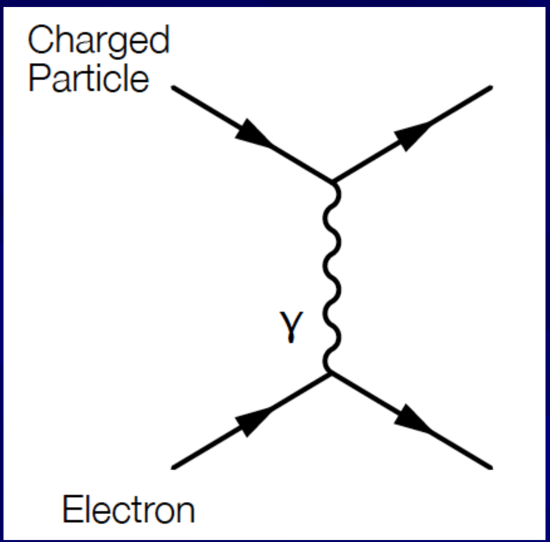
Detecting particles

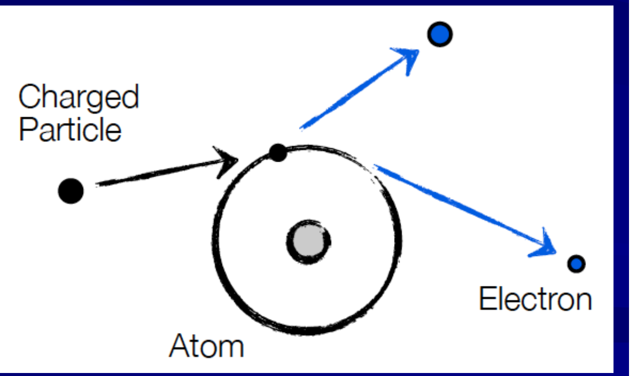
- Every effect of particles or radiation can be used as a working principle for a particle detector.

Claus Grupen

Example of particle interactions

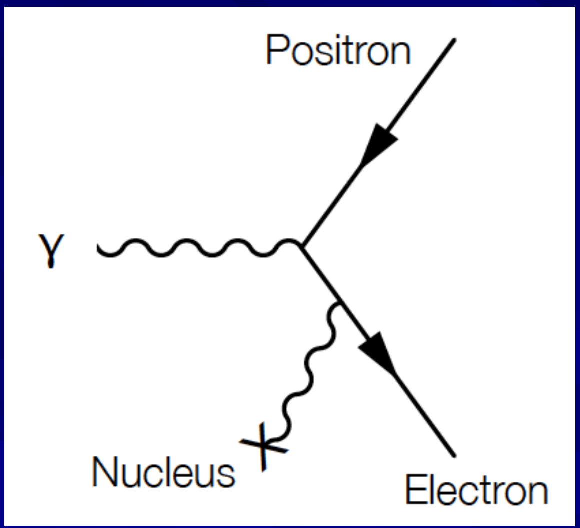
- Ionization

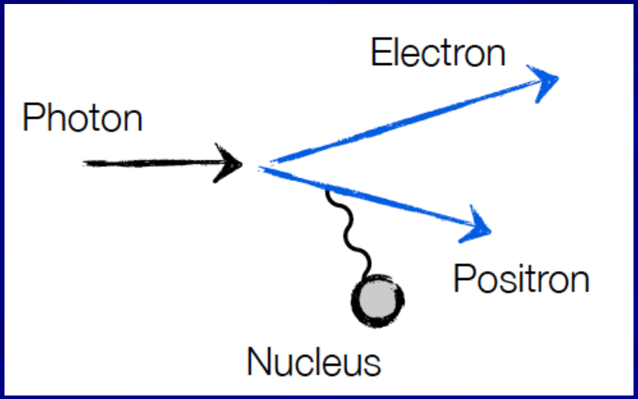




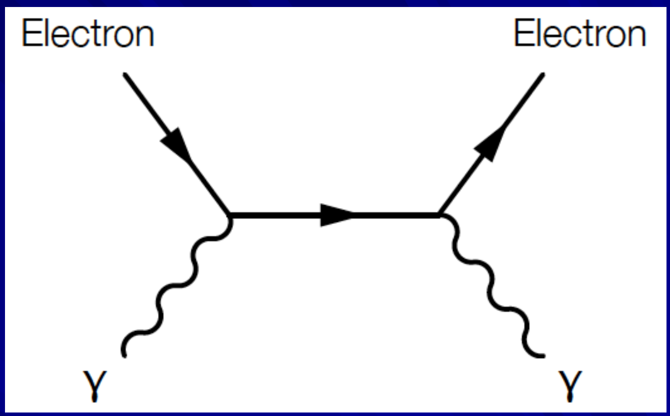
Delta-electrons

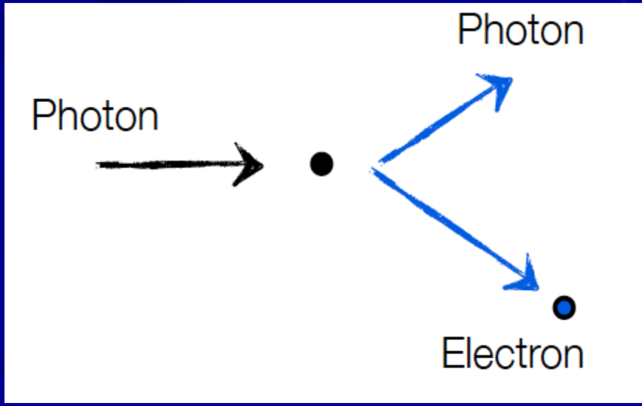
- Pair production



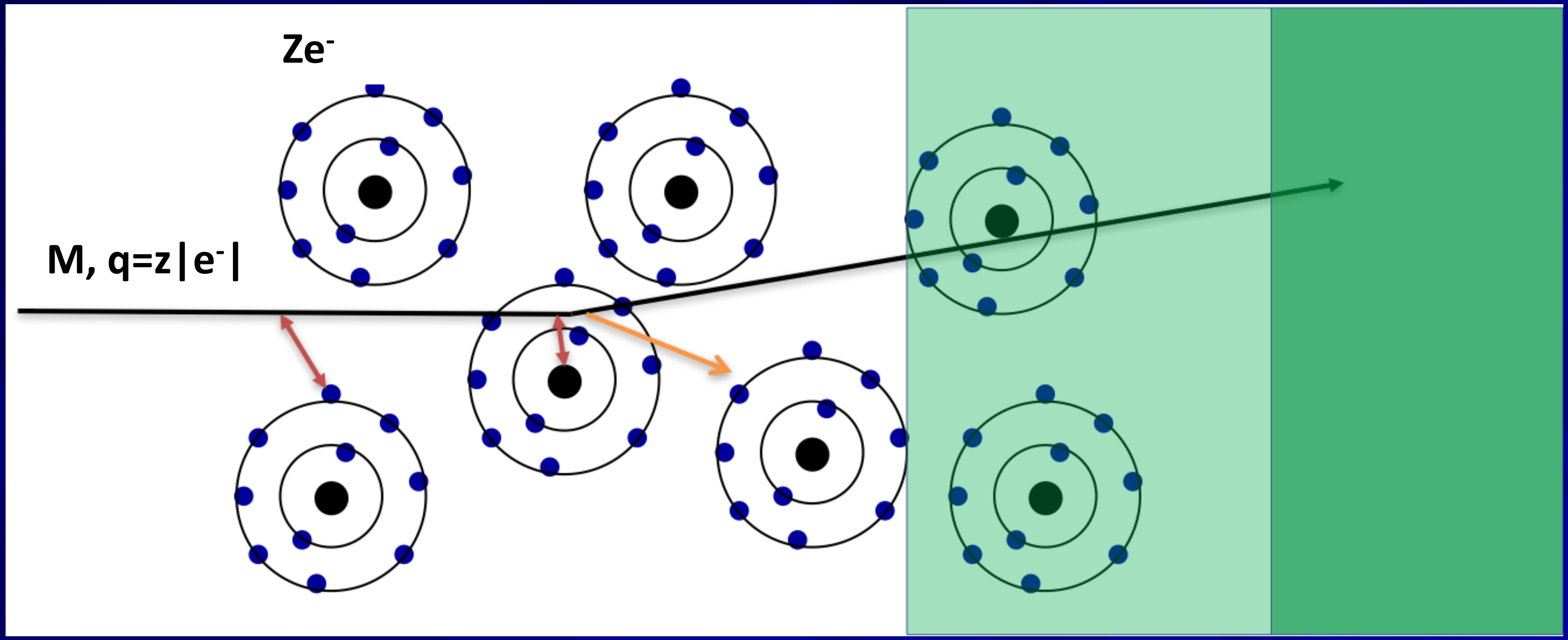


- Compton scattering





EM interaction of particles with matter



Interaction with the atomic electrons. Incoming particles lose energy and the atoms are excited or ionized.

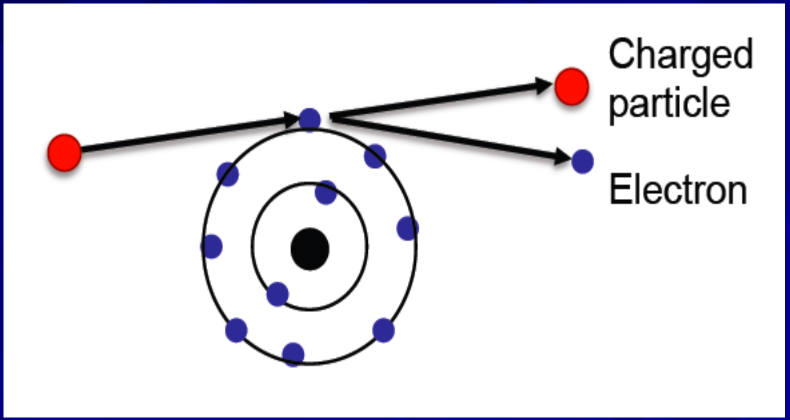
Interaction with the atomic nucleus. Particles are deflected and a Bremsstrahlung photon can be emitted.

If the particle's velocity is > the velocity of light in the medium ➔ Cherenkov Radiation.

When a particle crosses the boundary between two media, there is a probability ≈1% to produce an X ray photon➔Transition radiation.

Energy Loss by Ionization

- Assume: $Mc^2 \gg m_e c^2$ (calculation for electrons and muons are more complex)
- Interaction is dominated by elastic collisions with electrons
  - The trajectory of the charged particle is unchanged after scattering
- Energy is transferred to the δ-electrons



Energy loss (- sign)

Bethe-Bloch Formula

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Classical derivation in backup slides agrees with QM within a factor of 2

$\propto 1/\beta^2 \cdot \ln(\text{const} \cdot \beta^2\gamma^2)$

Energy loss by ionization

- The Bethe-Bloch equation for energy loss

Valid for heavy charged particles ($m_{incident} >> m_e$), e.g. proton, k, π, μ

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2}{I^2} W_{max} \right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z} \right]$$

$2\pi N_a r_e^2 m_e c^2$ = 0.1535 MeV cm^2/g

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

Fundamental constants
r_e = classical radius of electron
m_e = mass of electron
N_a = Avogadro's number
c = speed of light

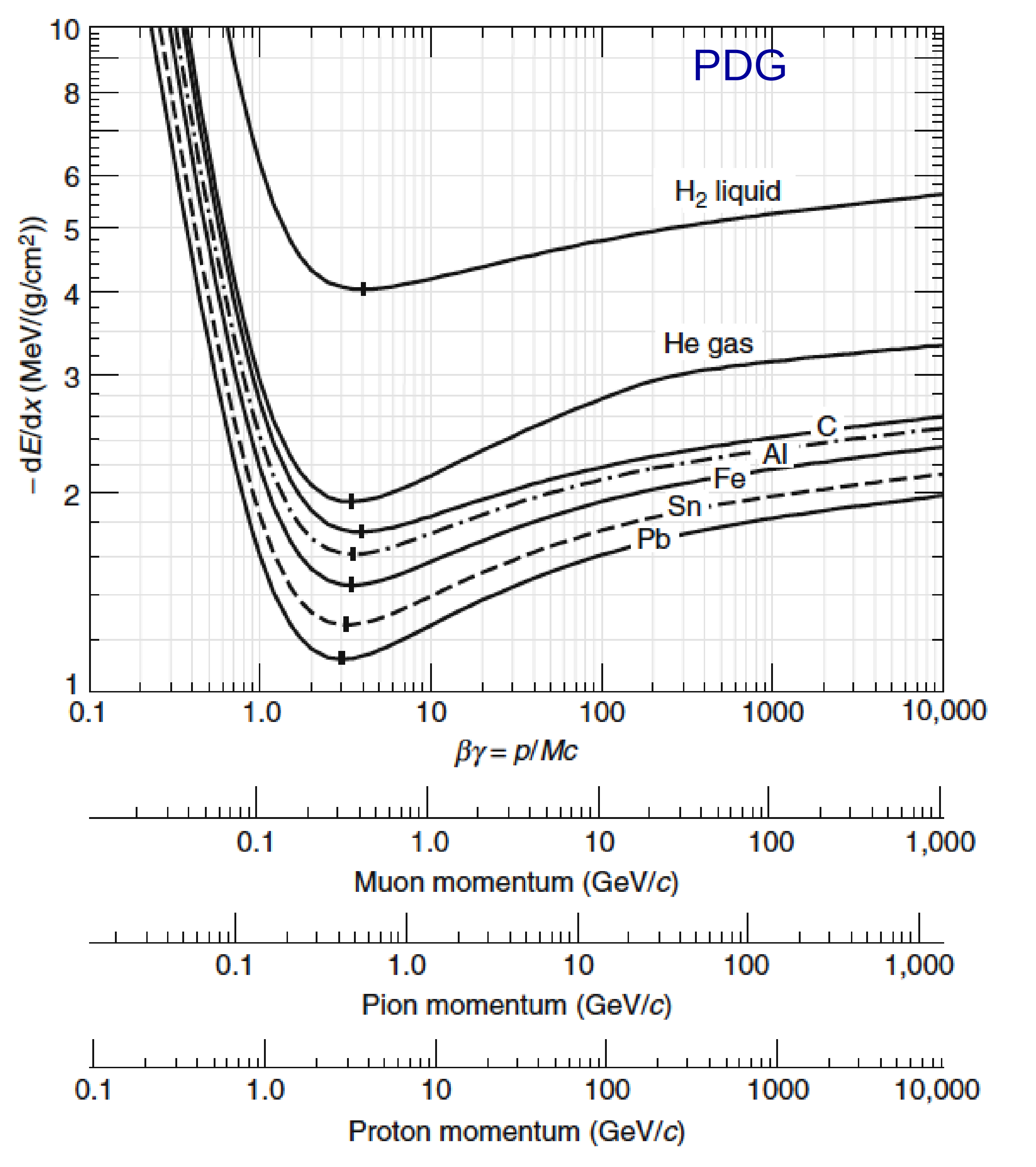
$$r_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2}$$

Absorber medium
I = mean ionization potential
Z = atomic number of absorber
A = atomic weight of absorber
ρ = density of absorber
δ = density correction
C = shell correction

Incident particle
z = charge of incident particle
β = v/c of incident particle
γ = $(1-\beta^2)^{-1/2}$
W_{max} = max. energy transfer in one collision

The Bethe-Bloch Formula

- Common features:
  - fast growth, as **$1/\beta^2$**, at low energy
  - wide minimum in the range $3 \leq \beta\gamma \leq 4$,
  - slow increase at high **βγ**.
- A particle with d**E/**dx near the minimum is a minimum-ionizing particle or mip.
- The mip's ionization losses for all materials except hydrogen are in the range 1-2 MeV/(g/cm^2)
  - increasing from large to low Z of the absorber.

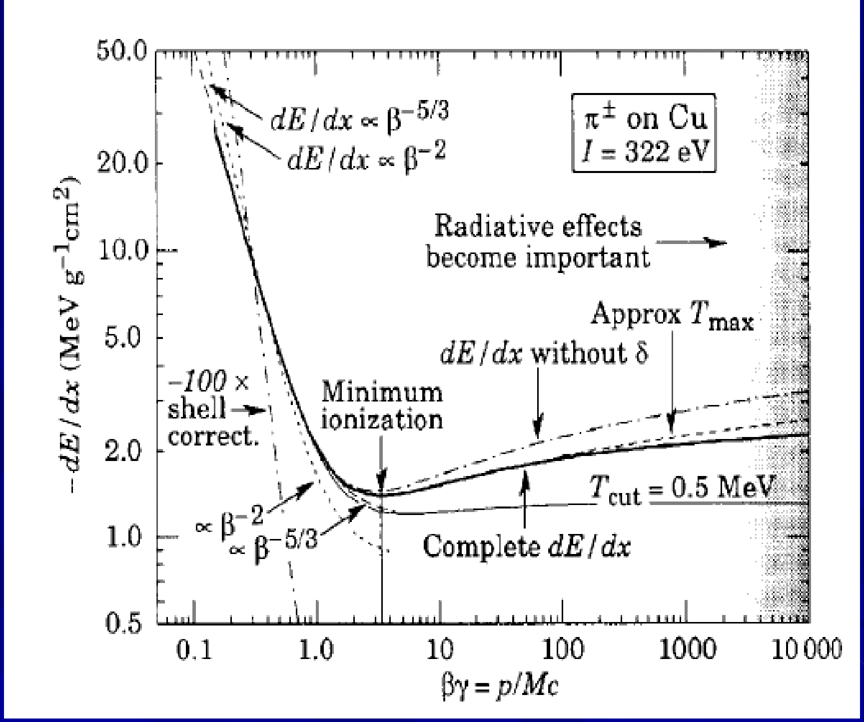


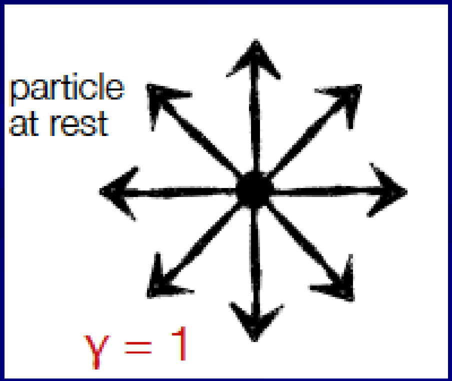
Understanding Bethe-Bloch

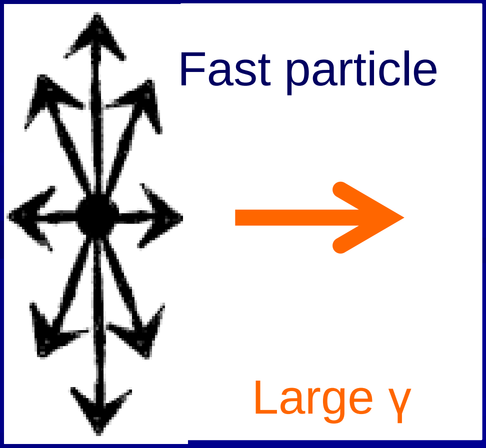
- dE/dx falls like $1/\beta^2$ [exact dependence $\beta^{-5/3}$]
  - Classical physics: slower particles “feel“ the electric force from the atomic electron more

$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dt}{dx} dx = \int F_\perp \frac{dx}{v}$$

- Relativistic rise as ®©>4
  - Transversal electric field increases due to Lorentz boost



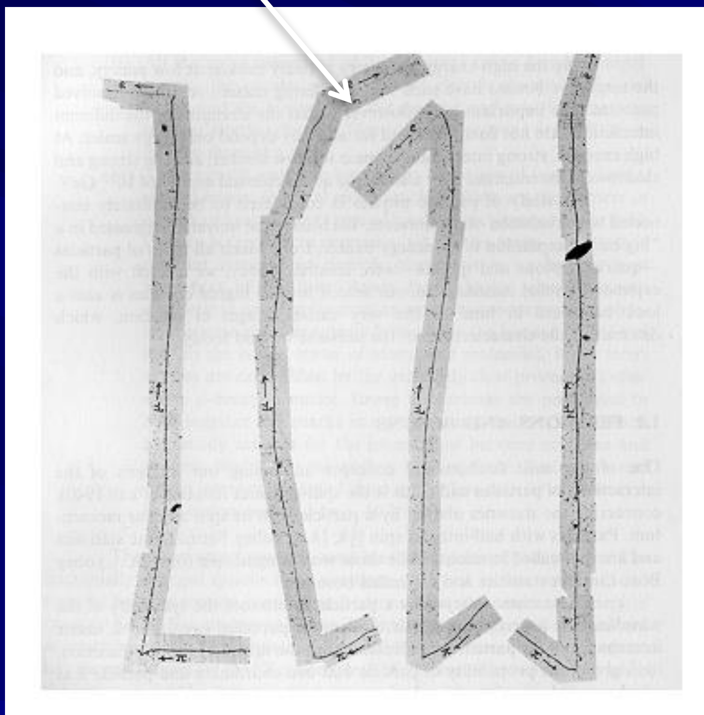




- Shell corrections
  - if particle v ≈ orbital velocity of electrons, i.e. **β**c ~ v_e. Assumption that electron is at rest breaks down ➔capture process is possible .
- Density effects due to medium polarization (shielding) increases at high ©

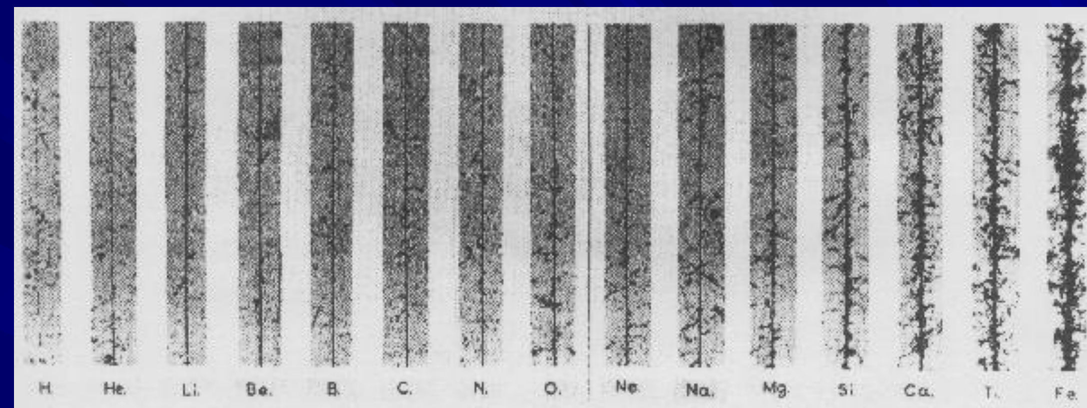
Understanding Bethe-Bloch

Small energy loss
→ Fast Particle



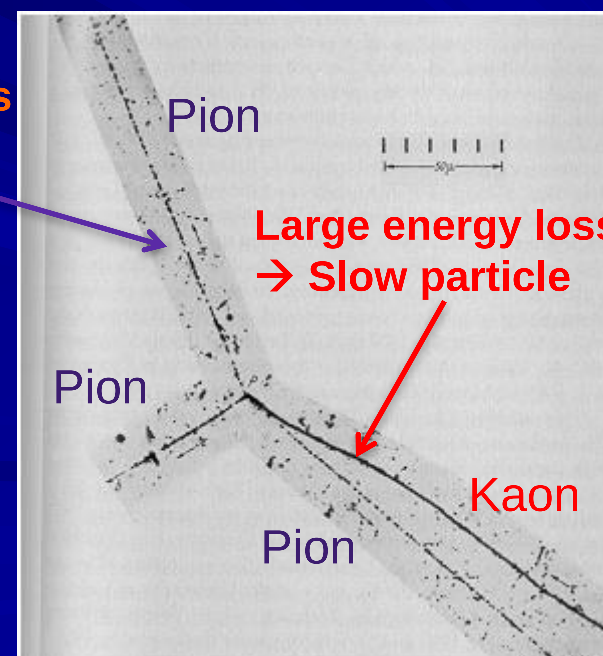
Discovery of muon and pion

Cosmic rays: $dE/dx \approx z^2$



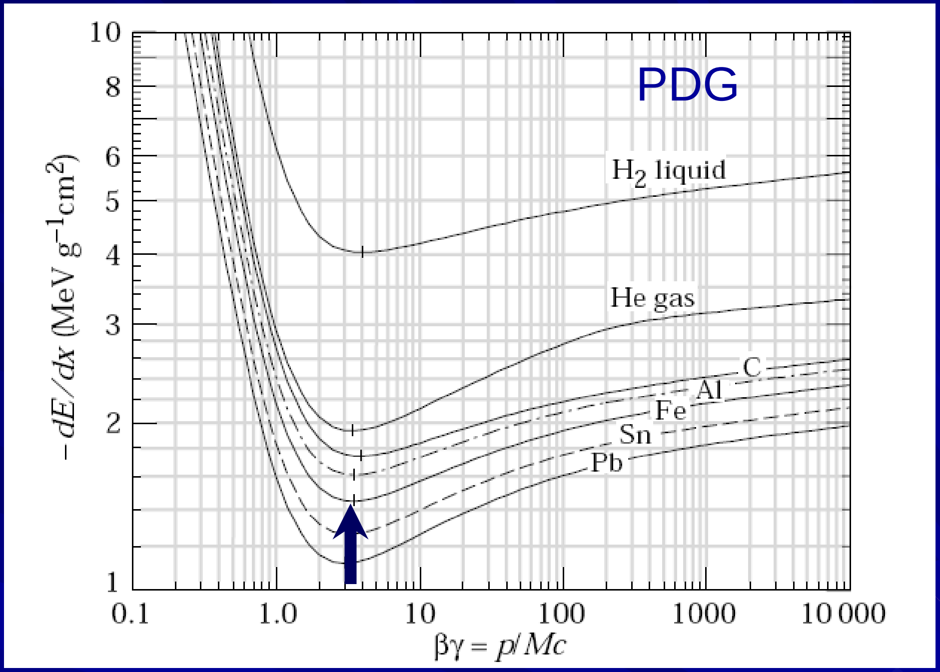
Small energy loss
→ Fast particle

Large energy loss
→ Slow particle



Bethe-Bloch: Order of magnitude

- For Z ≈ 0.5 A
  - 1/ρ dE/dx ≈ 1.4 MeV cm ^{2}/g for ®©≈ 3
- Can a 1 GeV muon traverse 1 m of iron ?
  - Iron: Thickness = 100 cm; 〉 = 7.87 g/cm ^{3}
  - dE ≈ 1.4 MeV cm ^{2}/g × 100 cm × 7.87g/cm ^{3}= 1102 MeV
- dE/dx must be taken in consideration when you are designing an experiment

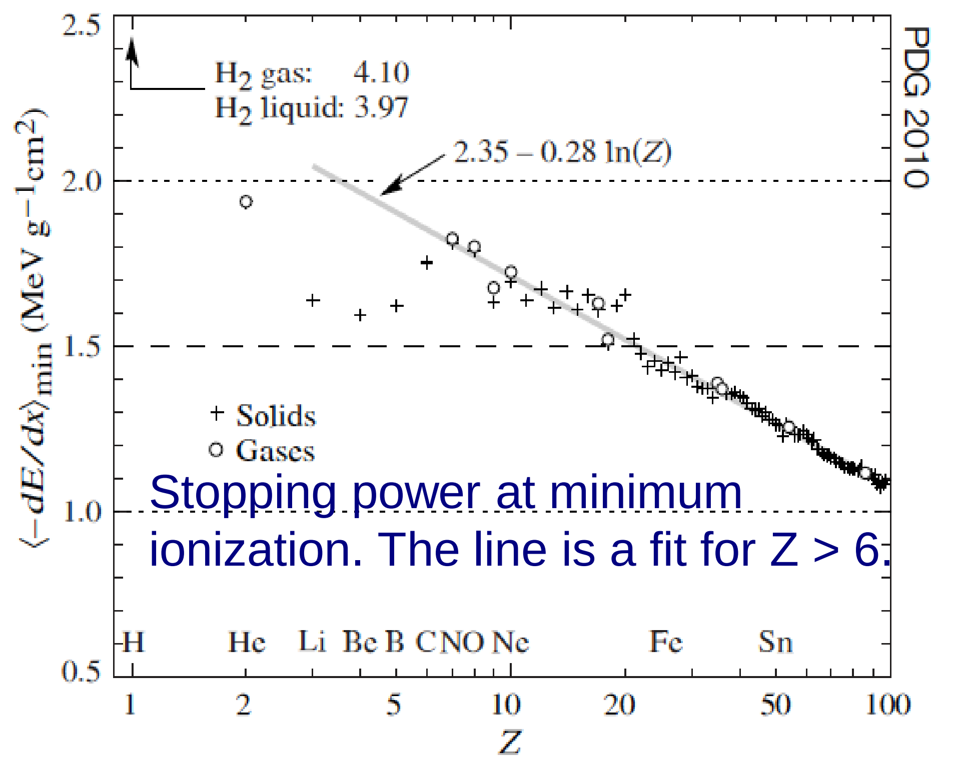


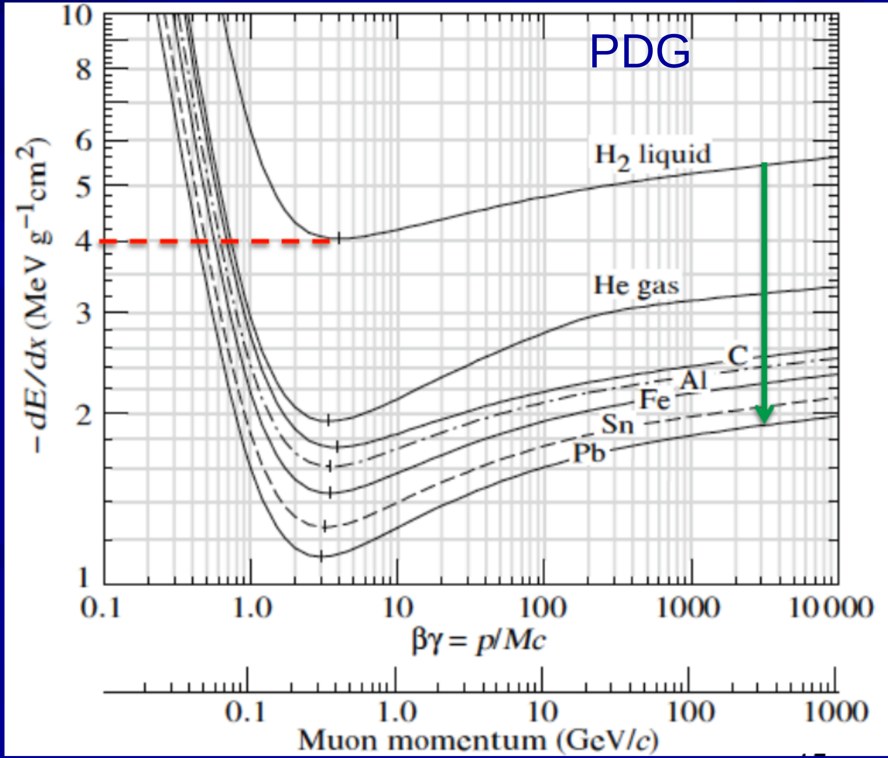
This number must be multiplied with ρ [g/cm ^{3}] of the Material → dE/dx [MeV/cm]

Bethe-Bloch dependence on Z/A

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2}{I^2} W_{max}\right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z} \right]$$

- Minimum ionization ≈ 1 - 2 MeV/g cm^{-2}. For H_2: 4 MeV/g cm^{-2}
- Linear decrease as a function of Z of the absorber



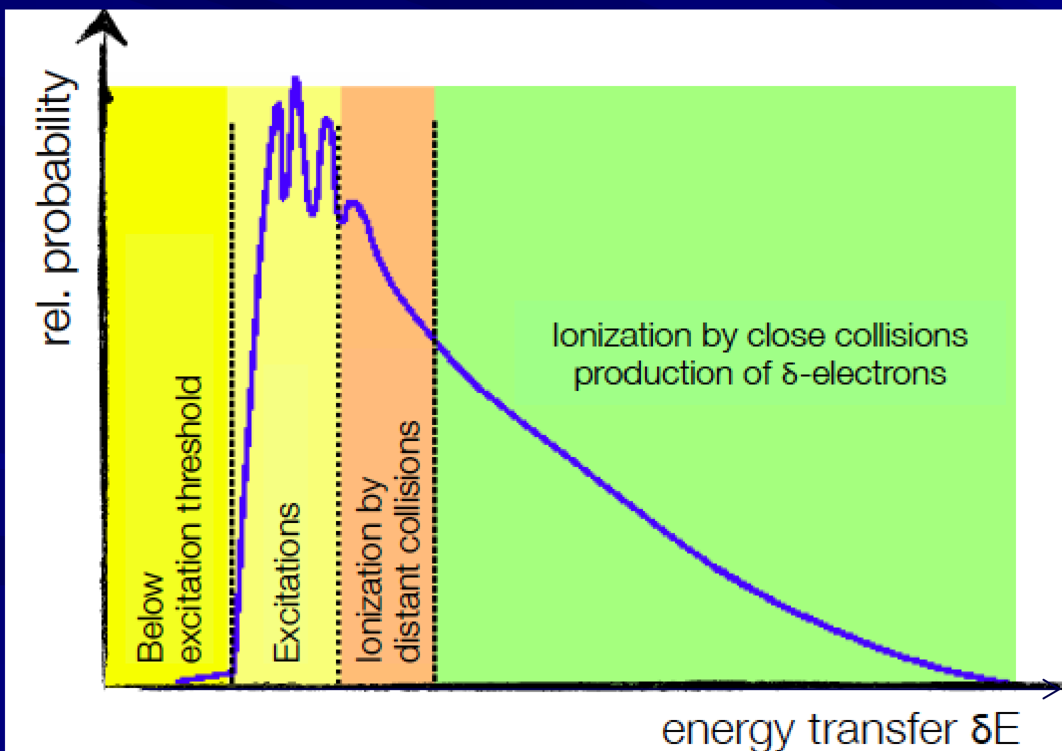


dE/dx Fluctuations

- The statistical nature of the ionizing process results in a large fluctuations of the energy loss (Δ) in absorber which are thin compared with the particle range.

$$\Delta E = \sum_{n=1}^{N} \delta E_n$$

N= number of collisions
δE=energy loss in a single collision



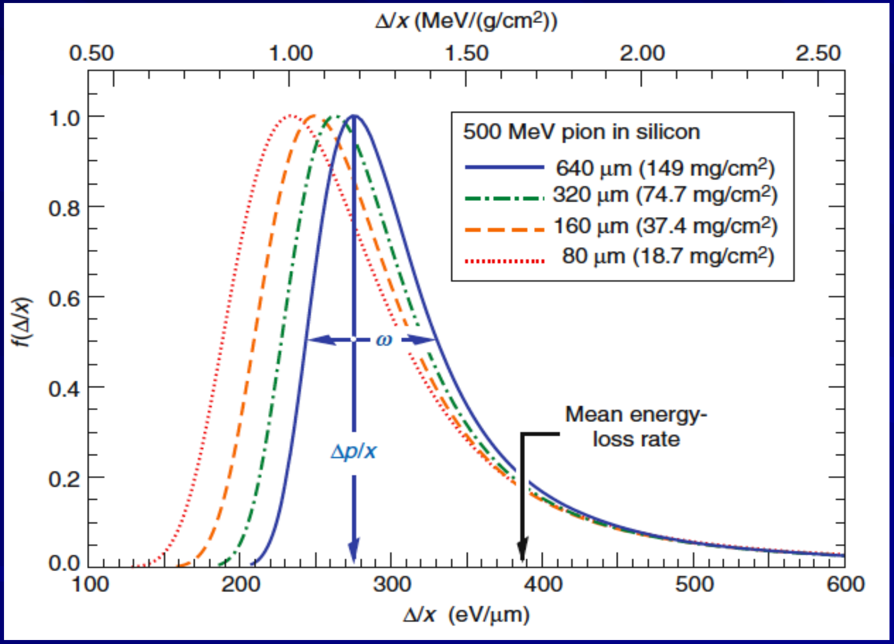
- Ionization loss is distributed statistically
- Small probability to have very high energy delta-rays

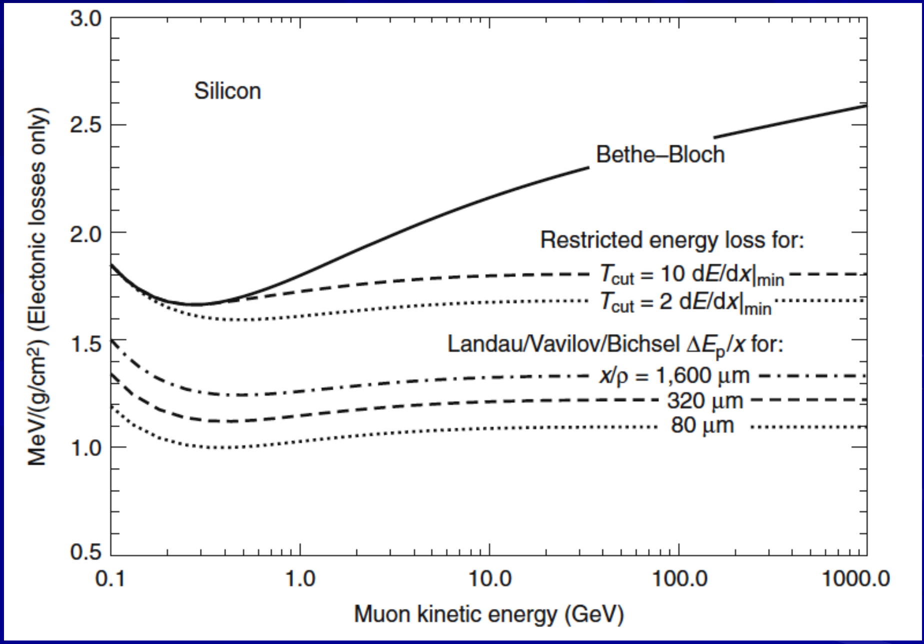
Landau Distribution

- For thin (but not too thin) absorbers the Landau distribution offers a good approximation (standard Gaussian + tail due to high energy delta-rays)

Landau distribution

Normalized energy loss probability





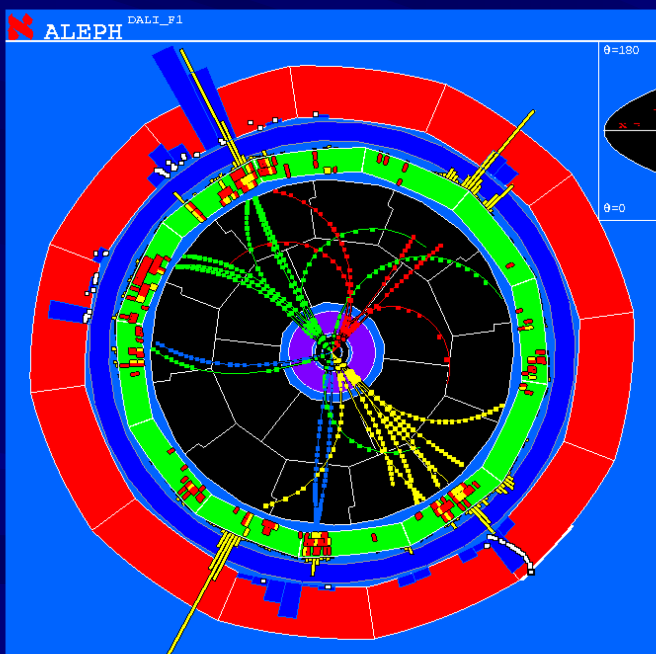
Approximation:

$$f(\Delta/x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Delta/x - a(\Delta/x)_{\mathrm{mip}}}{\xi}\right) + e^{-\left(\frac{\Delta/x - a(\Delta/x)_{\mathrm{mip}}}{\xi}\right)}\right]$$

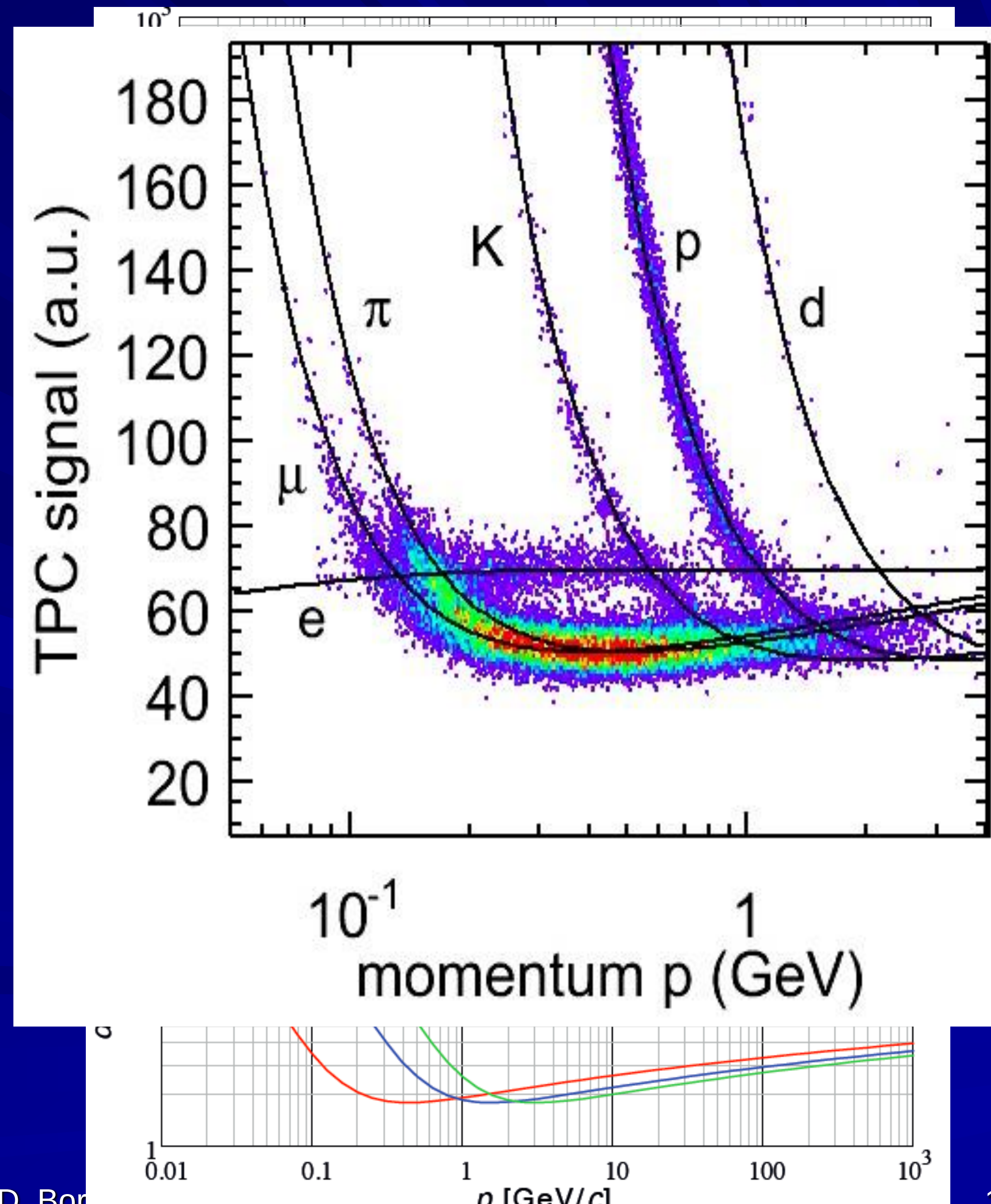
ξ: material constant

dE/dx and particle ID

- Energy loss is a function of momentum P=Mcβγ and it is independent of M.
- By measuring P and the energy loss independently → Particle ID in certain momentum regions

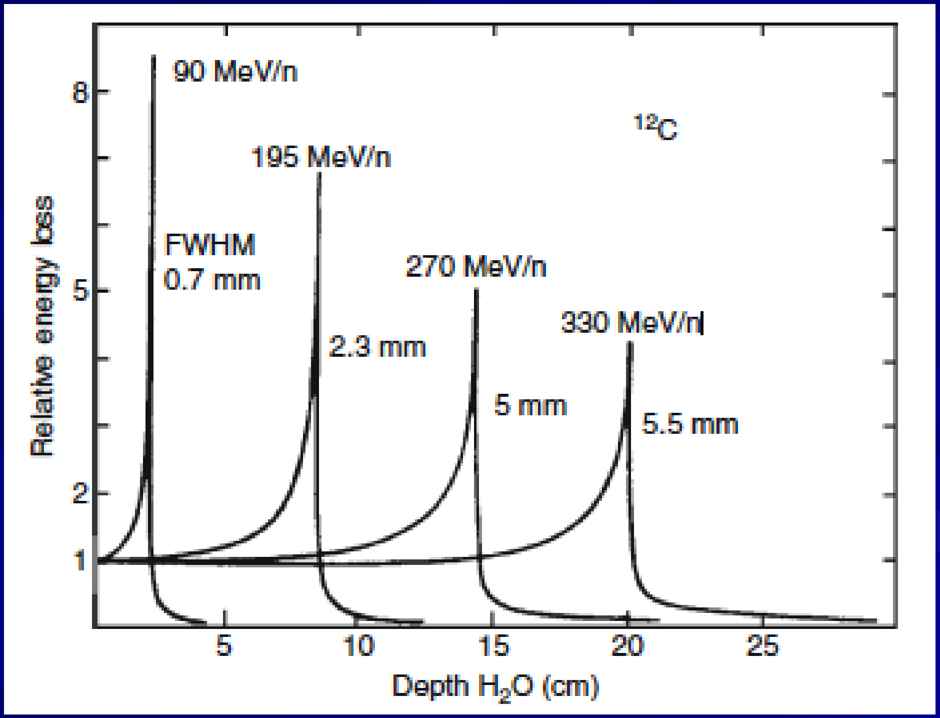


$$P_T\left[GeV/c\right]=0.3B[T]r[m]$$



Energy loss at small momenta

- If the energy of the particle falls below $\beta\gamma=3$ the energy loss rises as $1/\beta^2$ ➔Particles deposit most of their energy at the end of their track➔ Bragg peak



Great important for radiation therapy

Range of particles in matter

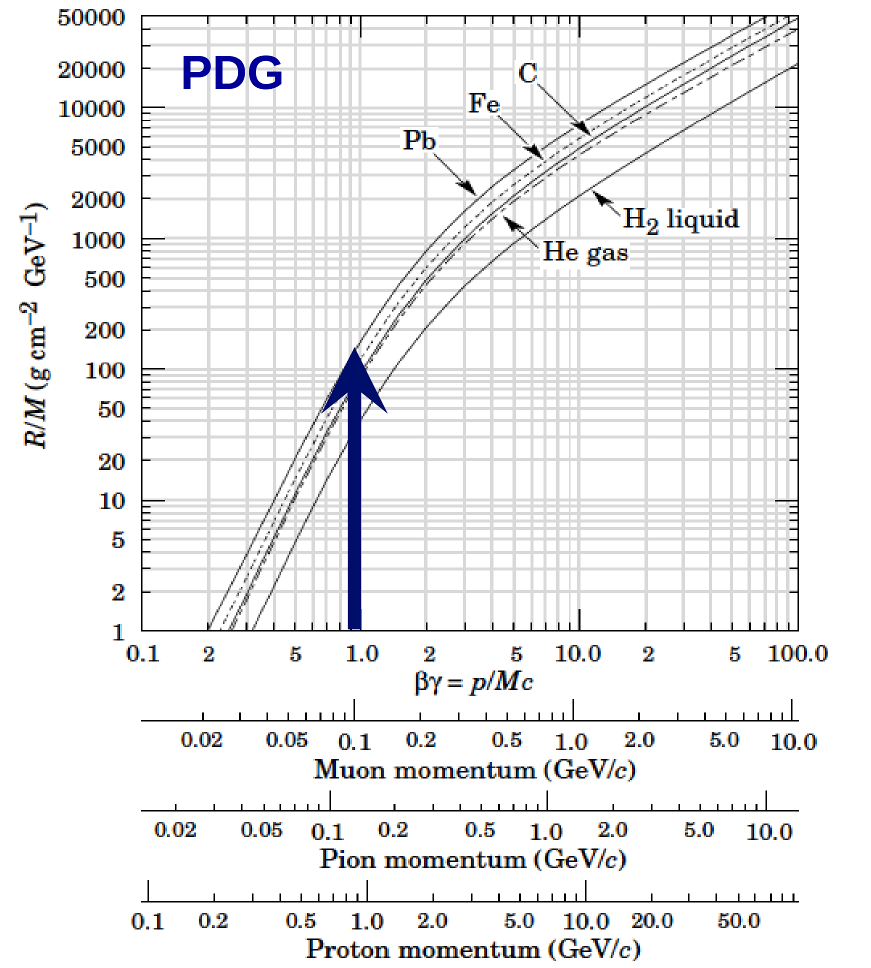
- Particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f\left(\beta_0\gamma_0\right)$$

$$\frac{\rho R(\beta_0\gamma_0)}{Mc^2} = \frac{1}{Z_1^2} \frac{A}{Z} f\left(\beta_0\gamma_0\right)$$

- R/M is ≈ independent of the material
- R is a useful concept only for low-energy hadrons (R <λ_I =the nuclear interaction length)



1GeV p in Pb ρ(Pb)= 11.34 g/cm^3

R/M(Pb)=200 g cm^{-2} GeV^{-1}

R=200/11.34/1cm≈ 20 cm

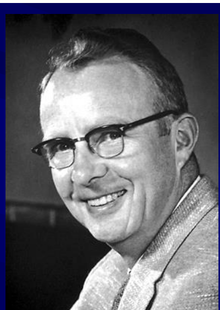
Search for Hidden Chambers in the Pyramids

The structure of the Second Pyramid of Giza is determined by cosmic-ray absorption.

Luis W. Alvarez, Jared A. Anderson, F. El Bedwei, James Burkhard, Ahmed Fakhry, Adib Girgis, Amr Goneid, Fikhry Hassan, Dennis Iverson, Gerald Lynch, Zenab Miligy, Ali Hilmy Moussa, Mohammed-Sharkawi, Lauren Yazolino

Fig. 2 (bottom right). Cross sections of (a) the Great Pyramid of Cheops and (b) the Pyramid of Chephren, showing the known chambers: (A) Smooth limestone cap, (B) the Belzoni Chamber, (C) Belzoni's entrance, (D) Howard-Vyse's entrance, (E) descending passageway, (F) ascending passageway, (G) underground chamber, (H) Grand Gallery, (I) King's Chamber, (J) Queen's Chamber, (K) center line of the pyramid.

6 FEBRUARY 1970



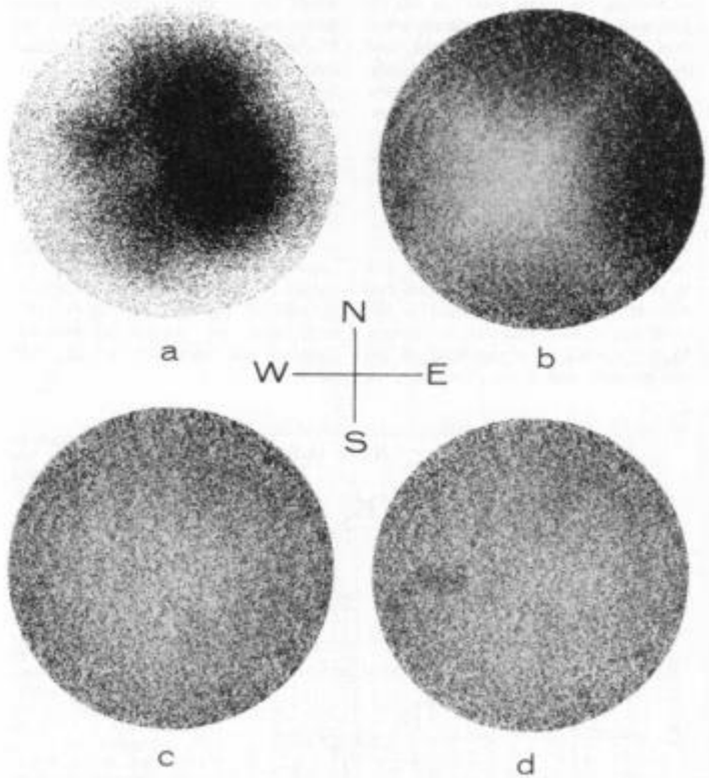
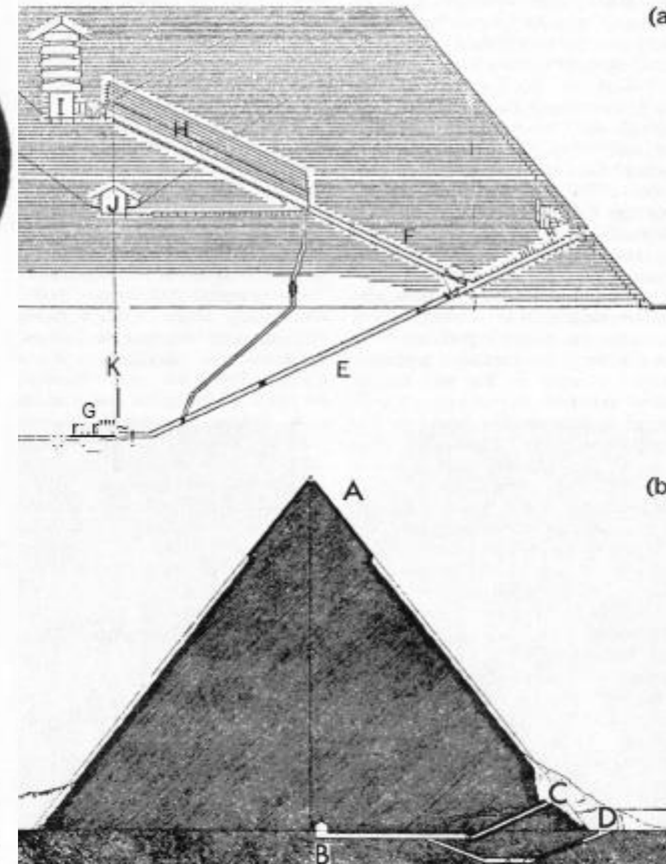


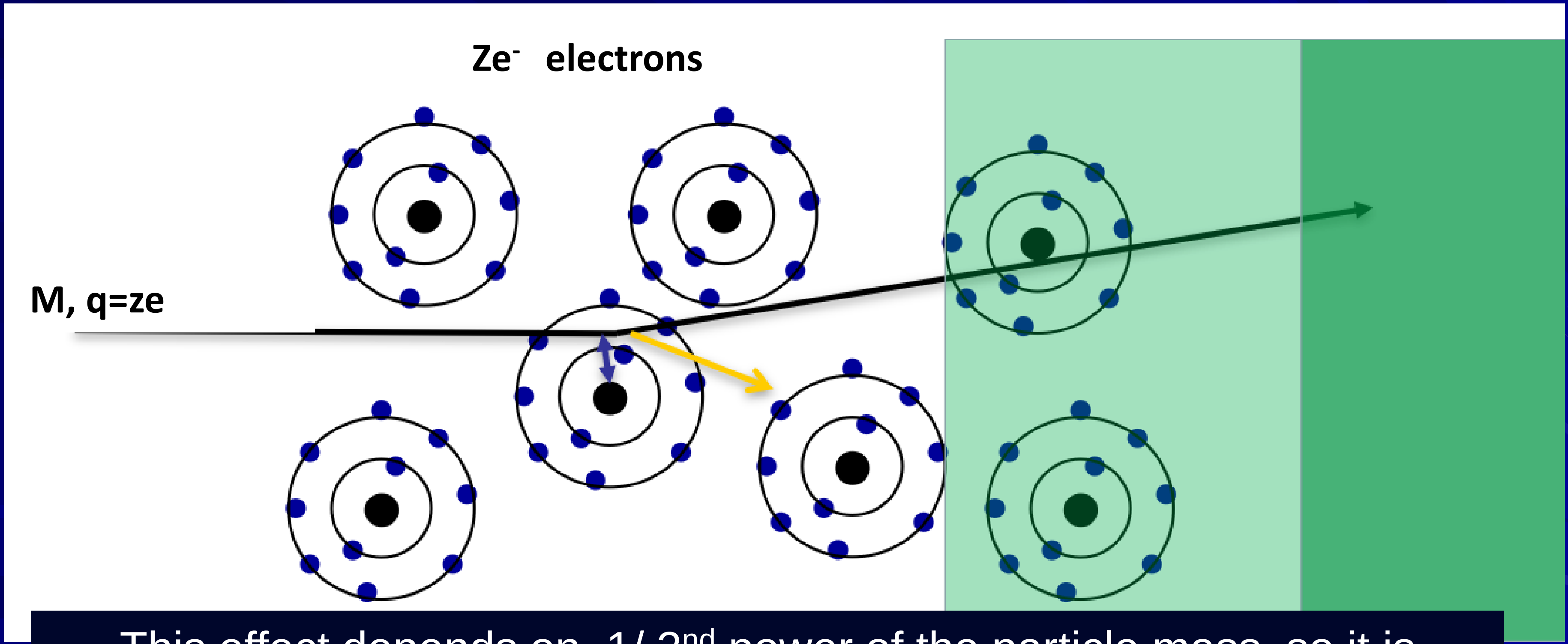
Fig. 13. Scatter plots showing the three stages in the combined analytic and visual analysis of the data and a plot with a simulated chamber. (a) Simulated "x-ray photograph" of uncorrected data. (b) Data corrected for the geometrical acceptance of the apparatus. (c) Data corrected for pyramid structure as well as geometrical acceptance. (d) Same as (c) but with simulated chamber, as in Fig. 12.



- Luis Alvarez used the attenuation of muons to look for chambers in the Second Giza Pyramid → Muon Tomography
- He proved that there are no chambers present.

Bremsstrahlung

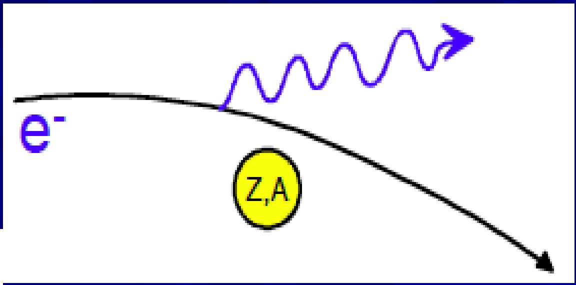
A charged particle of mass M and charge q=ze is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and therefore it can radiate a photon → Bremsstrahlung.



This effect depends on 1/ 2nd power of the particle mass, so it is relevant for electrons and very high energy muons

Energy loss for electrons and muons

- Bremsstrahlung, photon emission by an electron accelerated in Coulomb field of nucleus, is the dominant process for E_e > 10-30 MeV



$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

  - energy loss proportional to $1/m^2$
  - Important mainly for electrons and h.e. muons

$$-\left\langle \frac{dE}{dx} \right\rangle_{brem} \mu \ \frac{E}{m^2}$$

- For electrons

$$\frac{dE}{dx} = 4a N_A \frac{Z^2}{A} r_e^2 E \ln \frac{183}{Z^{1/3}}$$

$$\text{If} \quad X_0 \gg \frac{A}{4a N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

X_0 = radiation length in [g/cm^2]

$$\frac{dE}{dx} = \frac{E}{X_0}$$

$$E = E_0 e^{-x/X_0}$$

After passing a layer of material of thickness $\mathbf{X_0}$ the electron has 1/e of its initial energy.

Bremsstrahlung critical energy

- Critical energy

$$\frac{dE}{dx}(E_c)\bigg|_{brems} = \frac{dE}{dx}(E_c)\bigg|_{ion}$$

For solid and liquids

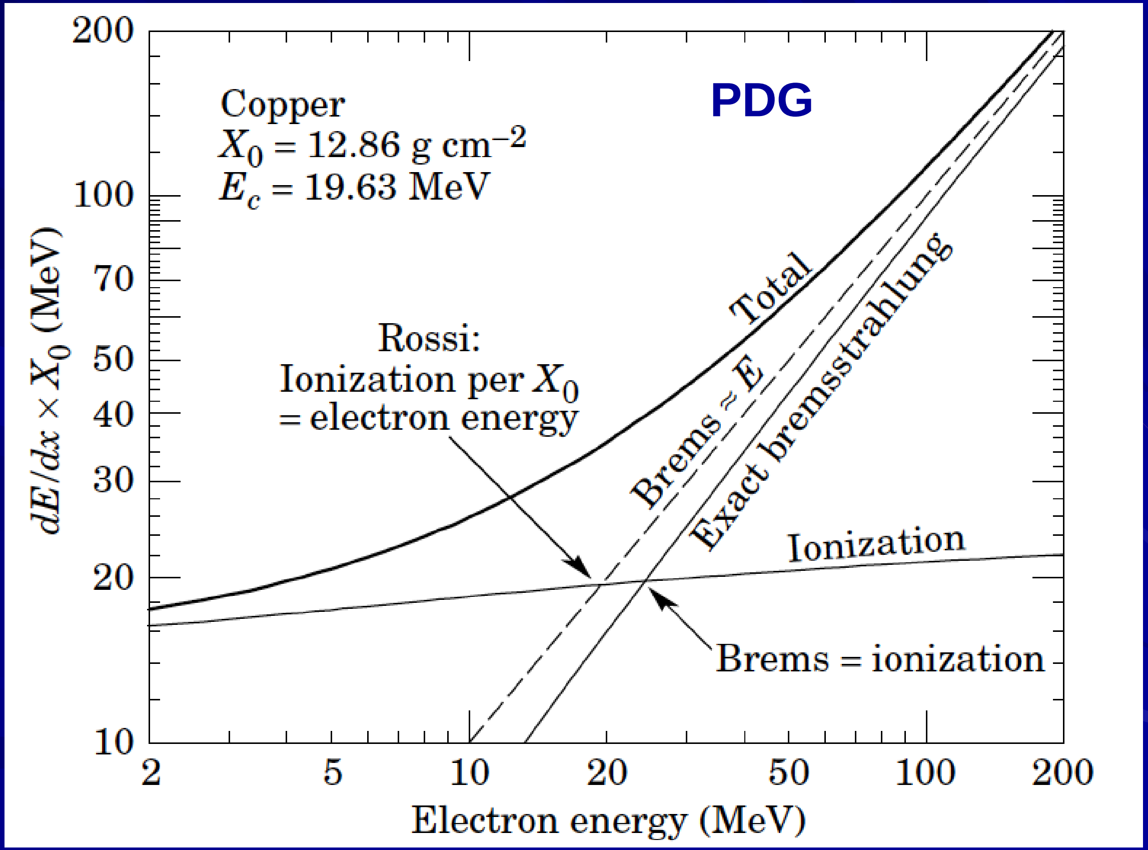
$$E_c = \frac{610\ \mathrm{MeV}}{Z+1.24}$$

For gasses

$$E_c = \frac{710\ \mathrm{MeV}}{Z+0.92}$$

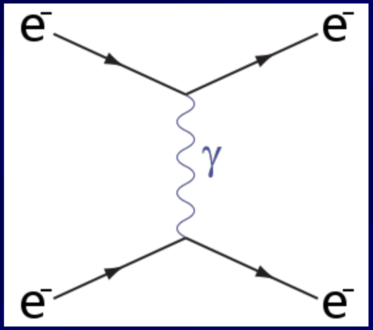
Example Copper:
$E_c \approx 610/30$ MeV ≈ 20 MeV

$$\left(\frac{dE}{dx}\right)_{\mathrm{Tot}} = \left(\frac{dE}{dx}\right)_{\mathrm{Ion}} + \left(\frac{dE}{dx}\right)_{\mathrm{Brems}}$$

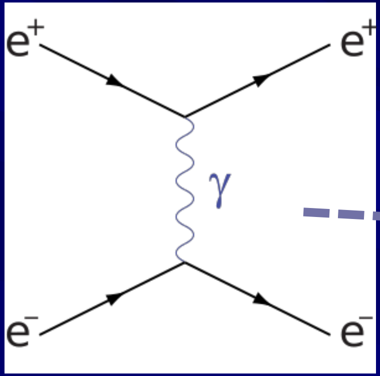


Electron energy loss

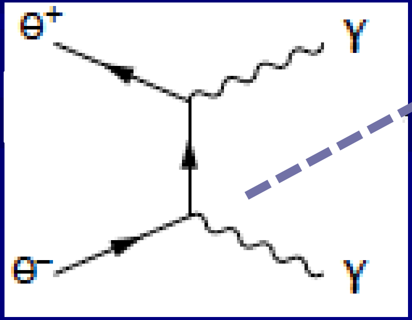
- Møller scattering

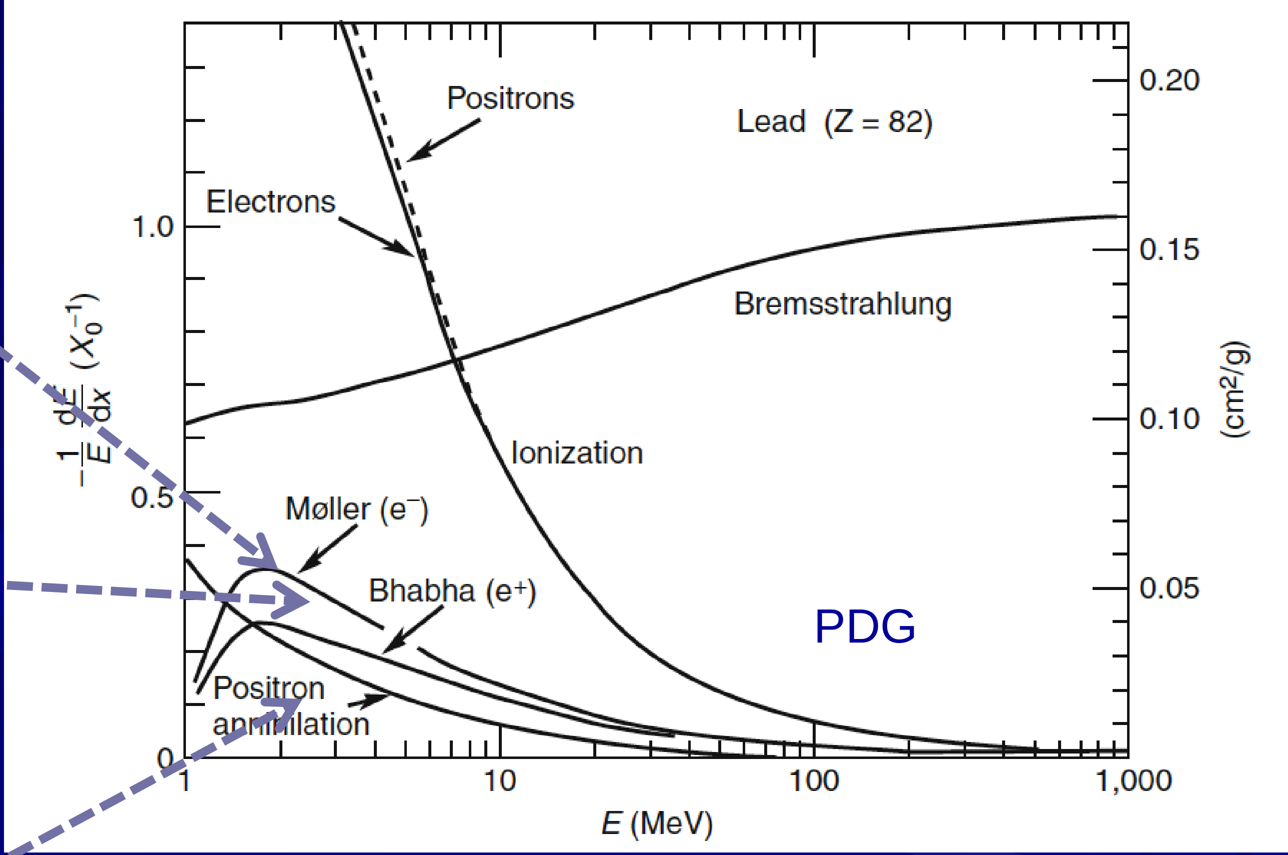


- Bhabha scattering



- Positron annihilation

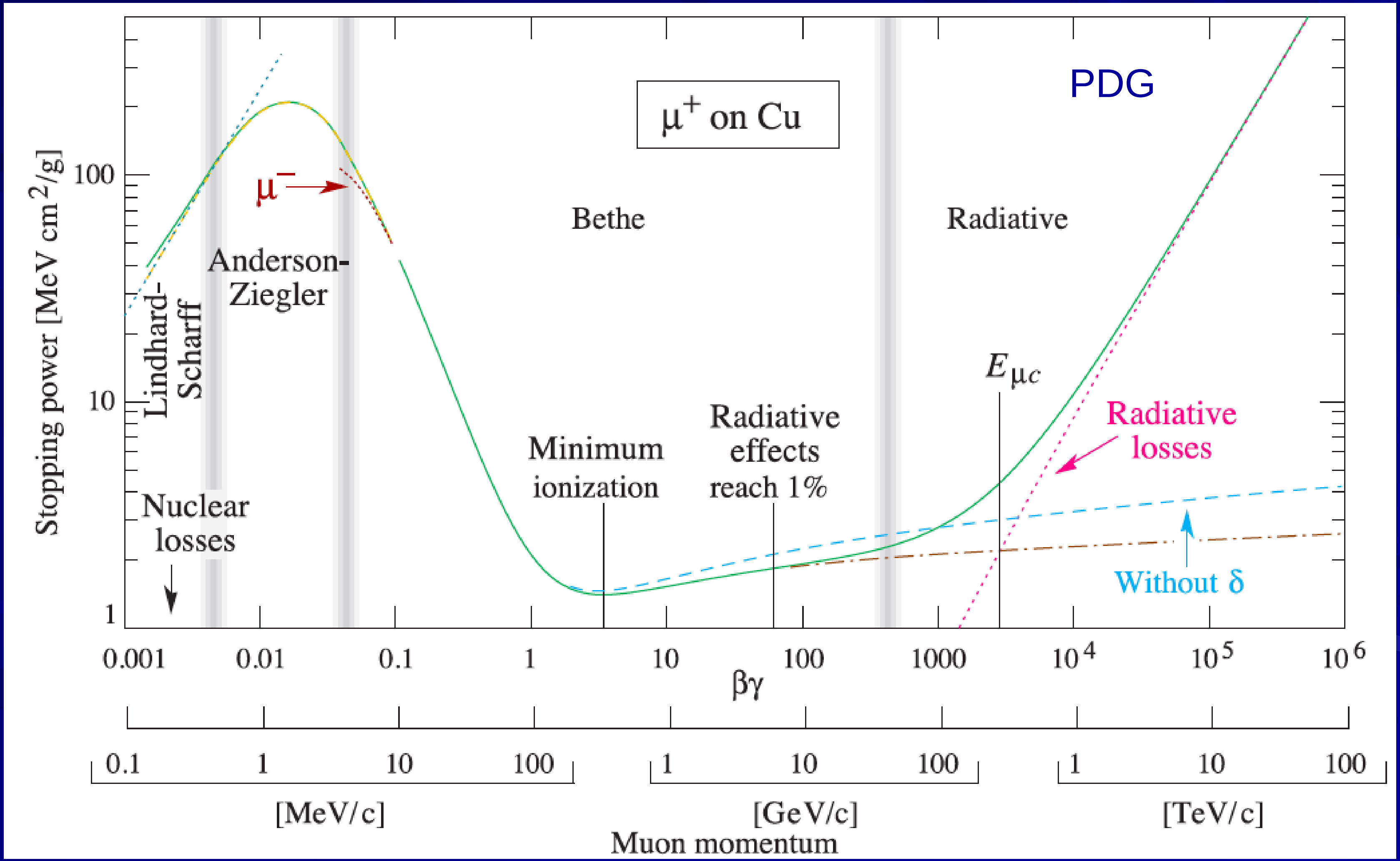




- Fractional energy loss per radiation length in lead as a function of the electron or positron energy

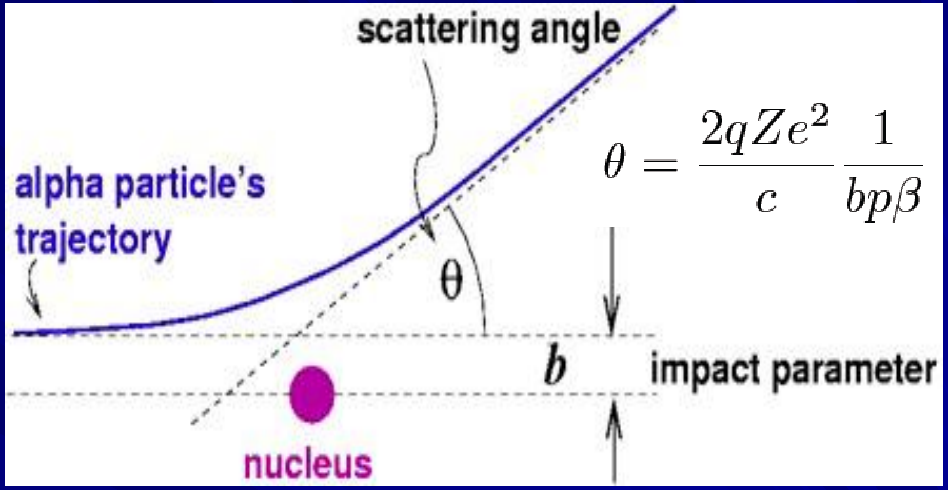
Energy loss summary

For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.



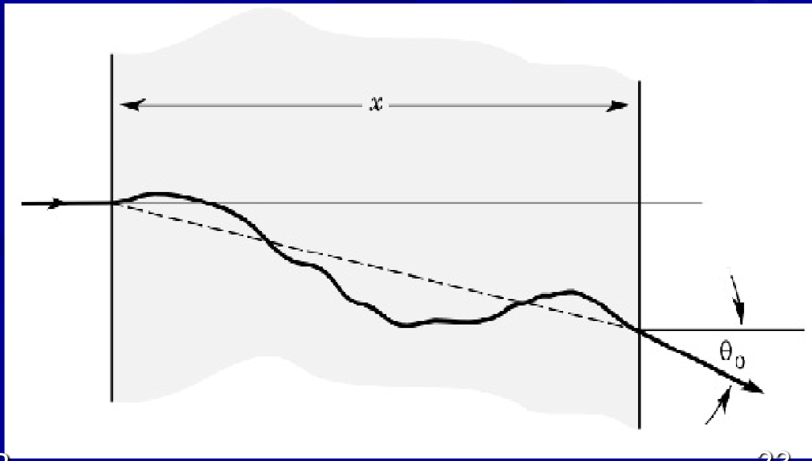
Multiple scattering

- A particle passing through material undergoes multiple small-angle scattering due to large-impact-parameter interactions with nuclei
  - The scattering angle as a function of thickness is



$$\theta_{\rm rms}^{\rm proj} = \sqrt{\langle \theta^2 \rangle} = \frac{13.6\,\mathrm{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}}\left[1 + 0.038 \ln(x/X_0)\right]$$

- Where:
  - p (in MeV/c) is the momentum,
  - βc the velocity,
  - z the charge of the scattered particle
  - x/X_0 is the thickness of the medium in units of radiation length (X_0).



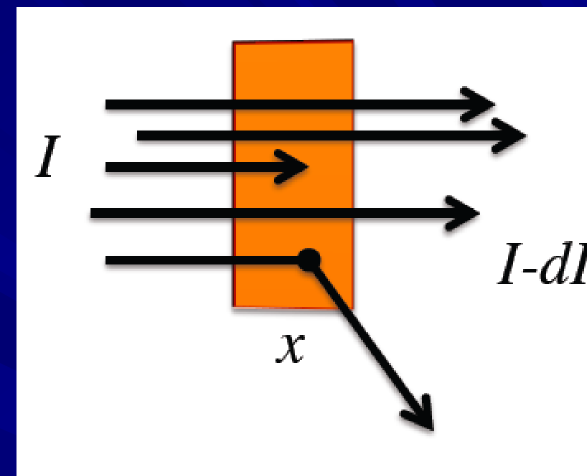
Interaction of photons with matter

- A photon can disappear or its energy can change dramatically at every interaction

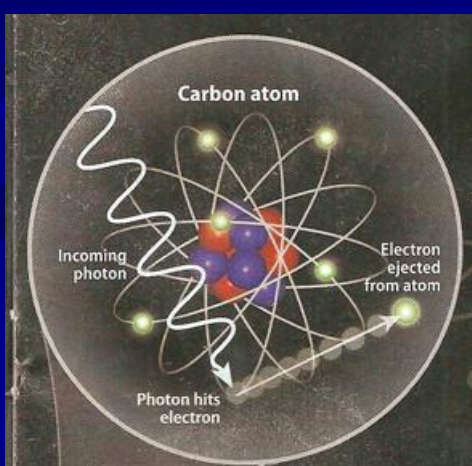
$$I(x) = I_0 e^{-\mu x} \qquad \mu = \frac{N_A}{A}\sum_{i=1}^{3}\sigma_i$$

$$\lambda = \frac{1}{\mu}$$

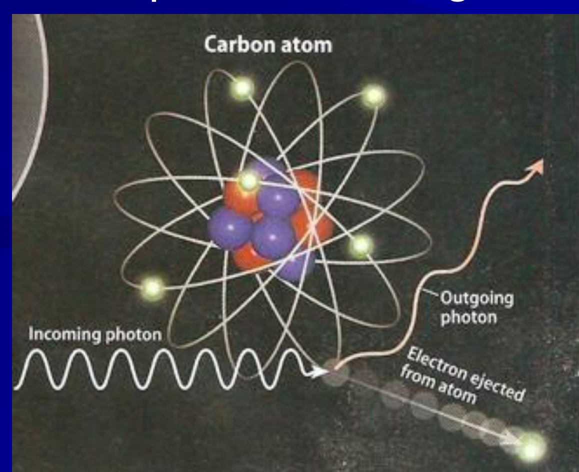
μ=total attenuation coefficient
σ_i=cross section for each process



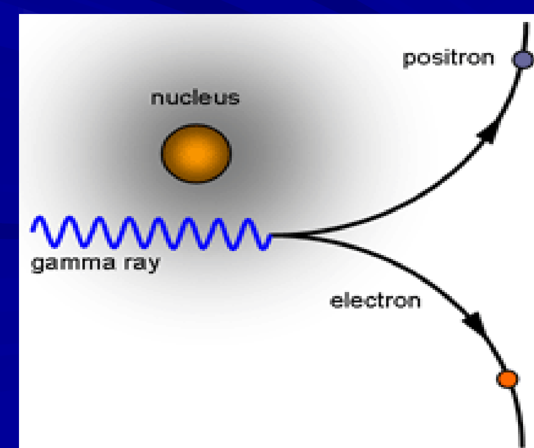
Photoelectric Effect



Compton Scattering



Pair production



Photoelectric effect

- Absorption of a photon by an electron bound to the atom and transfer of the photon energy to this electron.
  - From energy conservation:

    $E_e=E_{©}-E_N=h\{ -I_b$

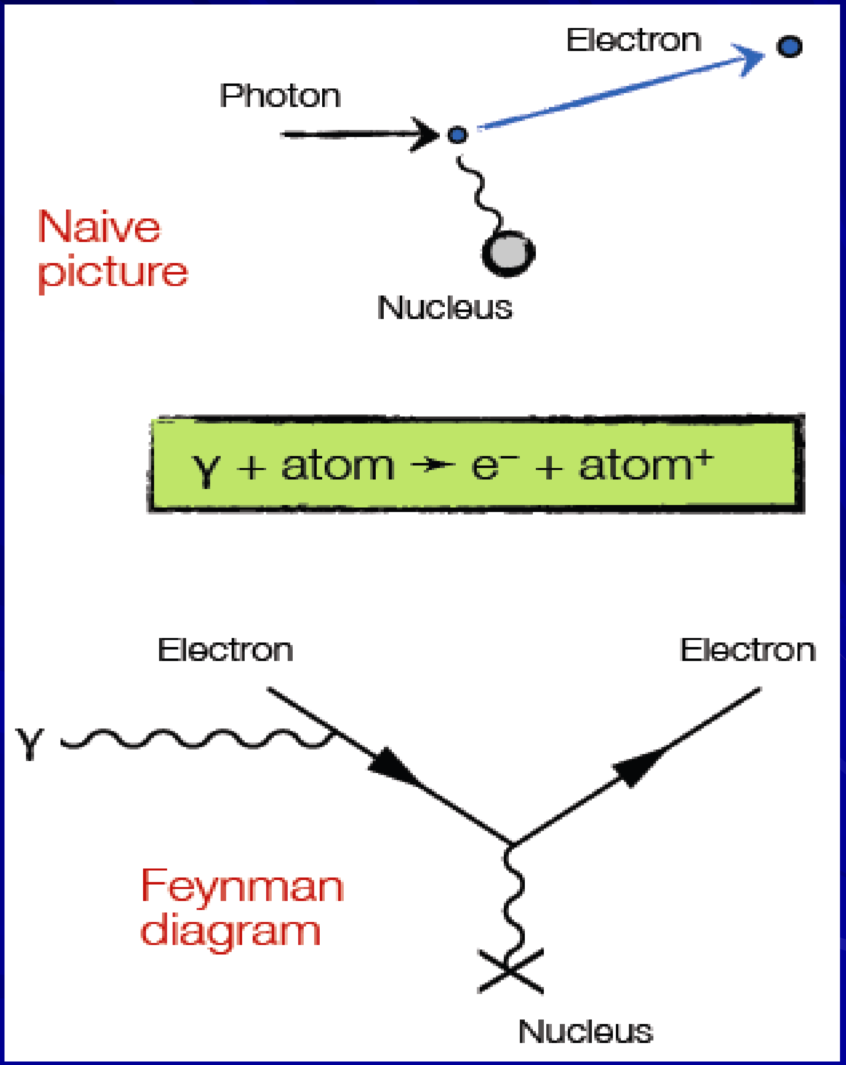
    Where I_b=Nucleus binding energy
  - E depends strongly on Z
- Effect is large for K-shell electrons or when $E_\gamma \approx$ K-shell energy
- E_γ dependence for $\mathbf{I_0} < \mathbf{E_\gamma} < m_e c^2$

$$s_{ph} = apa_B Z^5 \left(I_0 / E_g\right)^{7/2}$$

- E dependence for $\mathbf{E_\gamma} > m_e c^2$

$$s_{ph} = 2pr_e^2 a^4 Z^5 (mc)^2 / E_g$$

I_0=13.6 eV and a_B=0.5 3A



$\boldsymbol{\sigma}_{ph}$(Fe) = 29 barn

$\boldsymbol{\sigma}_{ph}$(Pb)= 5000 barn

Compton scattering

- Best known electromagnetic process (Klein–Nishina formula)
  - for $\mathbf{E_\lambda << m_e c^2}$

$$\sigma_c \propto \sigma_{Th}(1-\varepsilon)$$

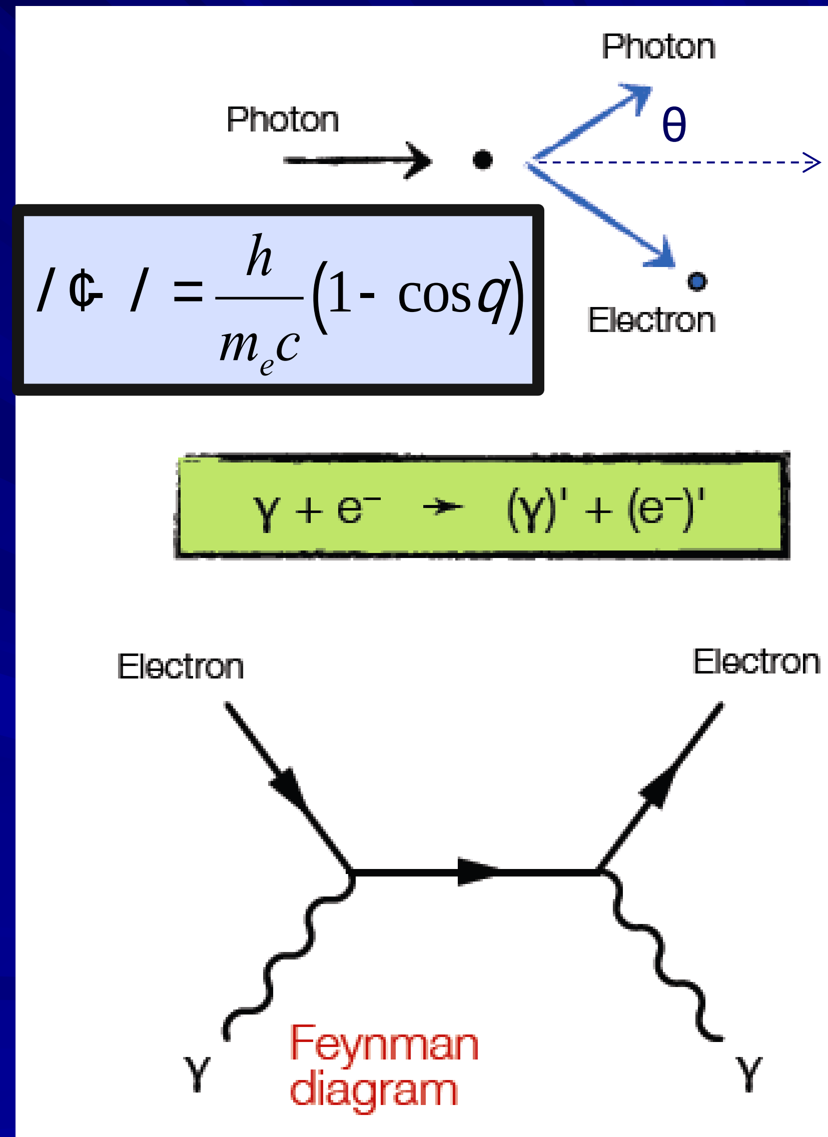
  - for $\mathbf{E_\lambda >> m_e c^2}$

$$\sigma_c \propto \frac{\ln \varepsilon}{\varepsilon} Z$$

where

$$\sigma_{Th} = \frac{8\pi}{3} r_e^2 = 0.66\ barn$$

$$\varepsilon \propto \frac{E_\lambda}{m_e c^2}$$



Compton scattering

- From E and p conservation get the energy of the scattered photon

$$E'_\gamma = \frac{E_\gamma}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{E_\gamma}{m_e c^2}$$

- Kinetic energy of the outgoing electron:

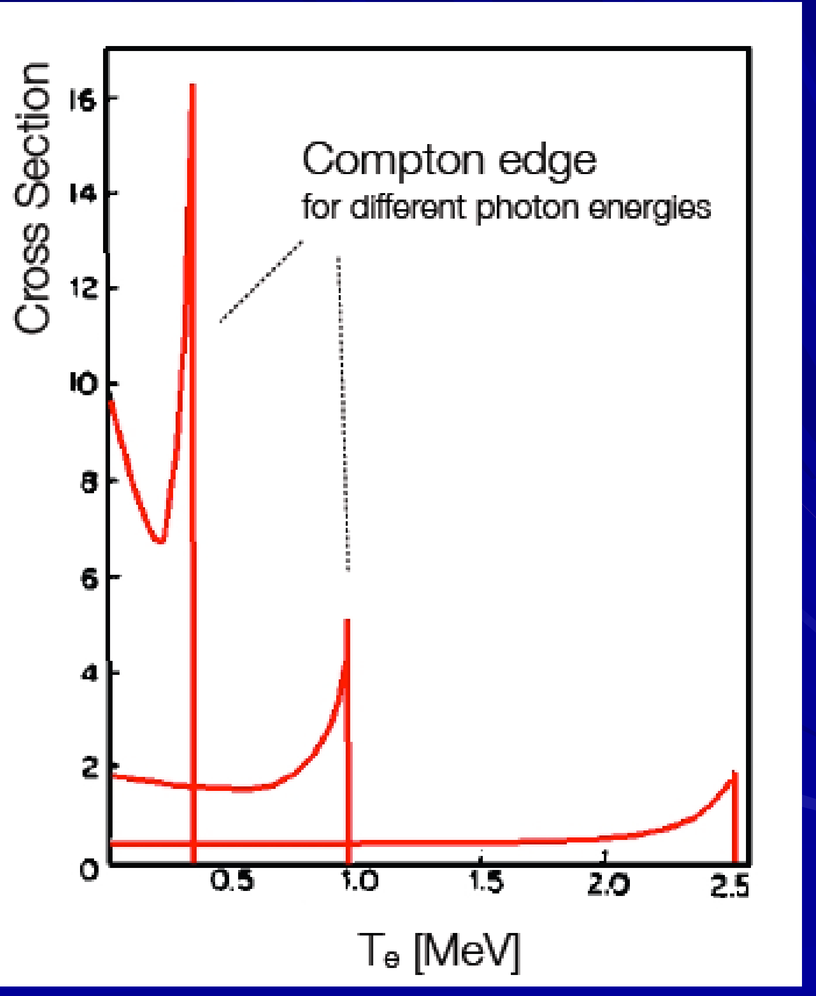
$$T_e = E_\gamma - E'_\gamma = E_\gamma \frac{\varepsilon(1-\cos\theta)}{1+2\varepsilon}$$

- The max. electron recoil is for θ=π

$$T_{\max} = E_\gamma \frac{2\varepsilon}{1+2\varepsilon}$$

$$\Delta E = E_\gamma - T_{\max} = E_\gamma \frac{1}{1+2\varepsilon}$$

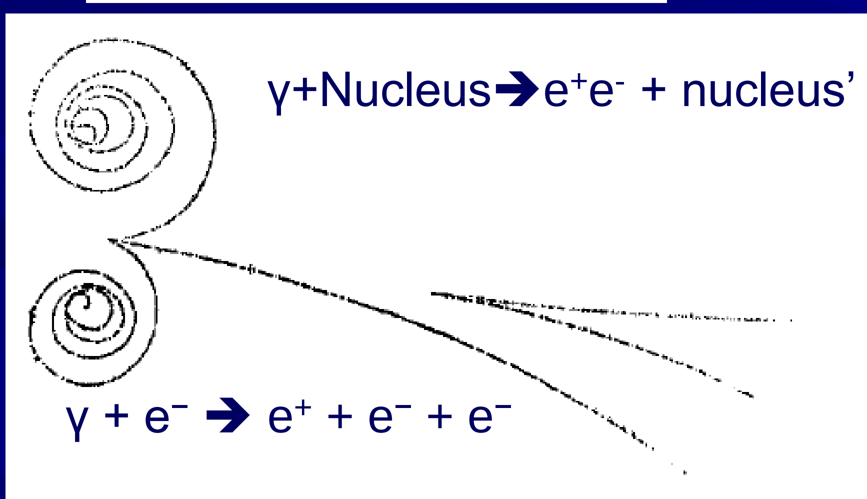
- Transfer of complete γ-energy via Compton scattering not possible

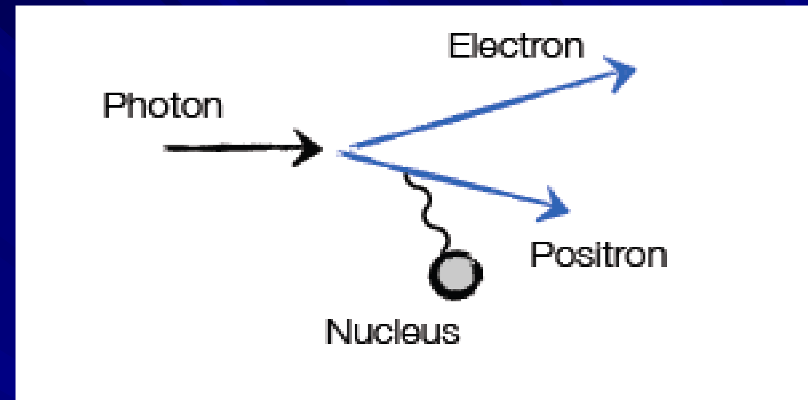


Pair production

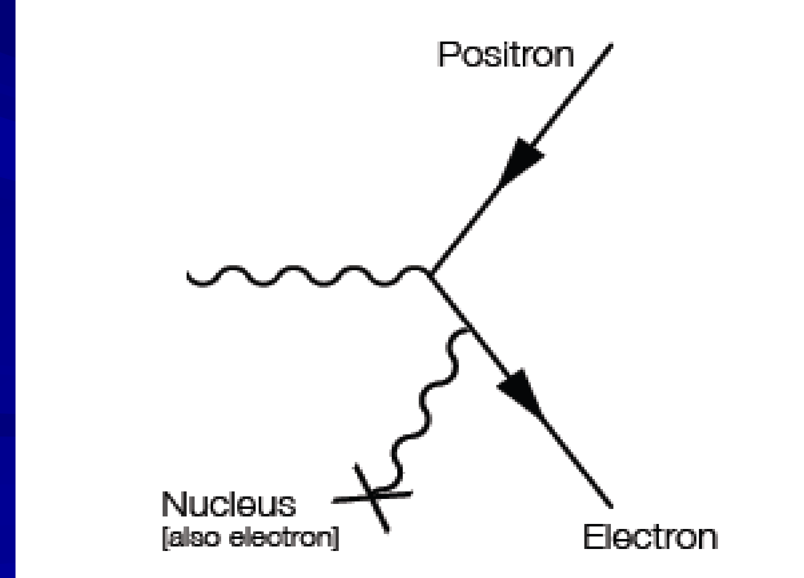
- At E>100 MeV, electrons lose their energy almost exclusively by bremsstrahlung while the main interaction process for photons is electron–positron pair production.
- Minimum energy required for this process 2 m_e + Energy transferred to the nucleus

$$E_\gamma \geq 2m_ec^2 + \frac{2m_ec^2}{M_{Nuleus}}$$





γ + atom → $e^+ + e^-$



Pair production

- If $E_\lambda \gg m_e c^2$

$$\sigma_{pair} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) [\mathrm{cm}^2/\mathrm{atom}]$$

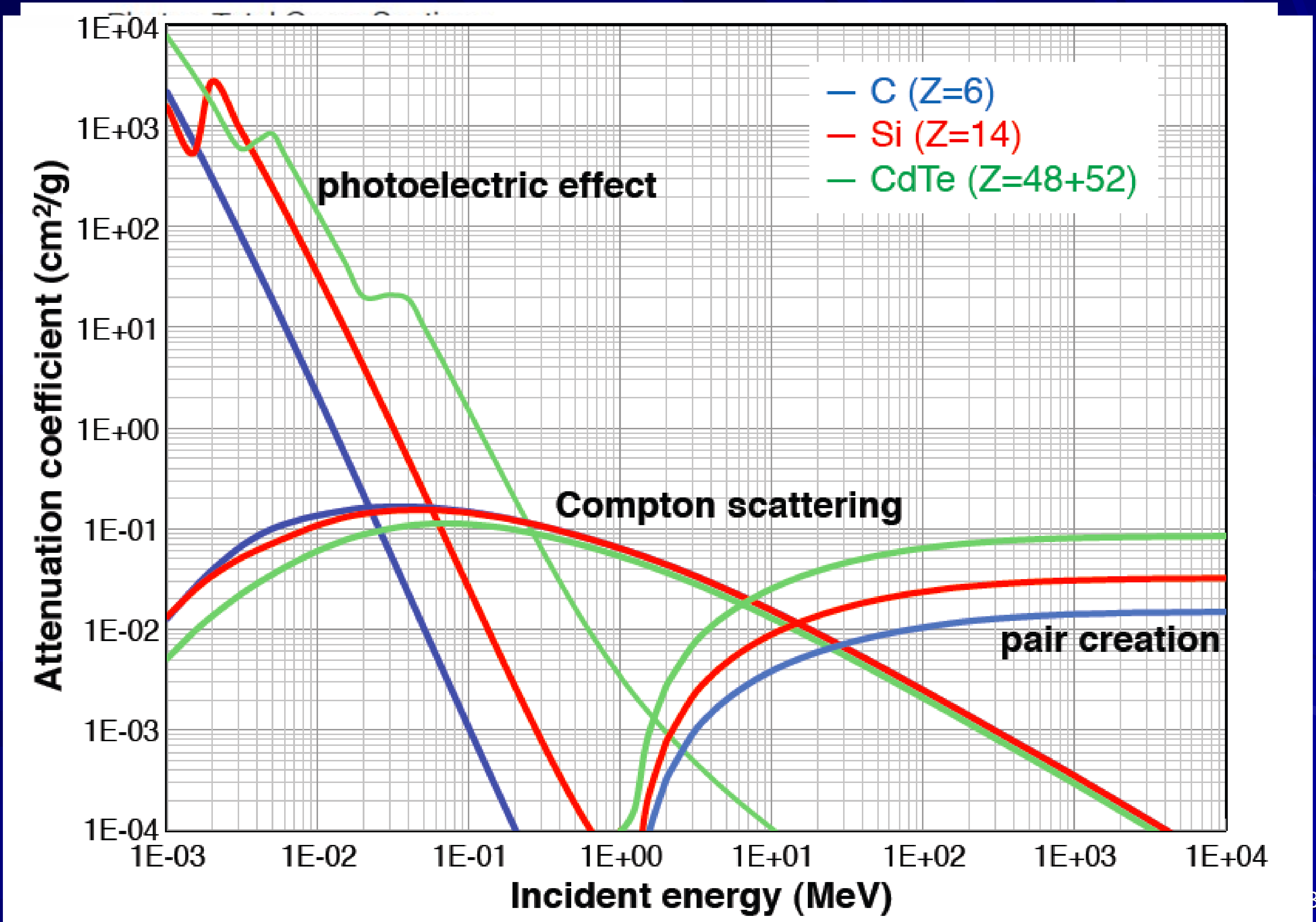
- Using as for Bremsstrahlung the radiation length

$$X_0 = \frac{A}{4\rho N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

$$\sigma_{pair} = \frac{7}{9} \frac{N_A}{A} \frac{1}{X_0}$$

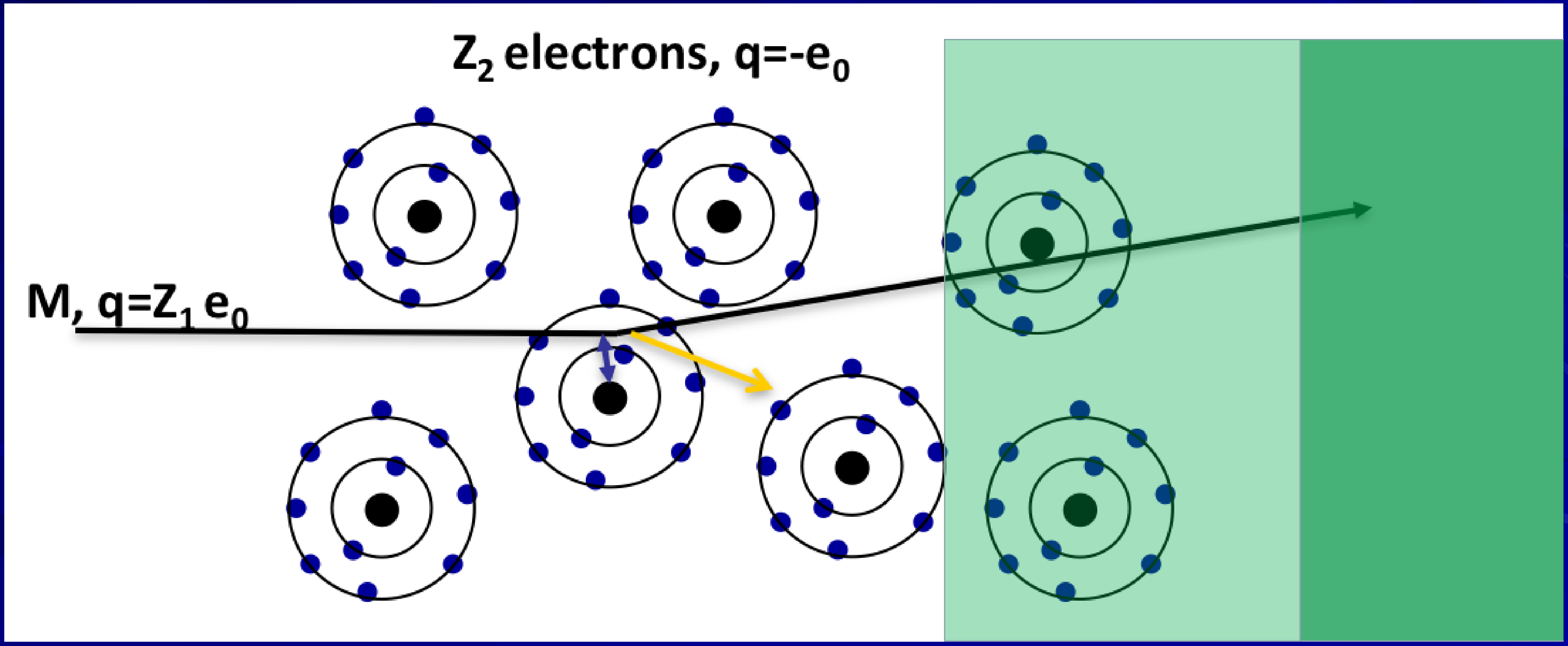
	ρ [g/cm³]	X_0 [cm]
H_2 [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Luft	$1.2 \cdot 10^{-3}$	$30 \cdot 10^{3}$

Interaction of photons with matter



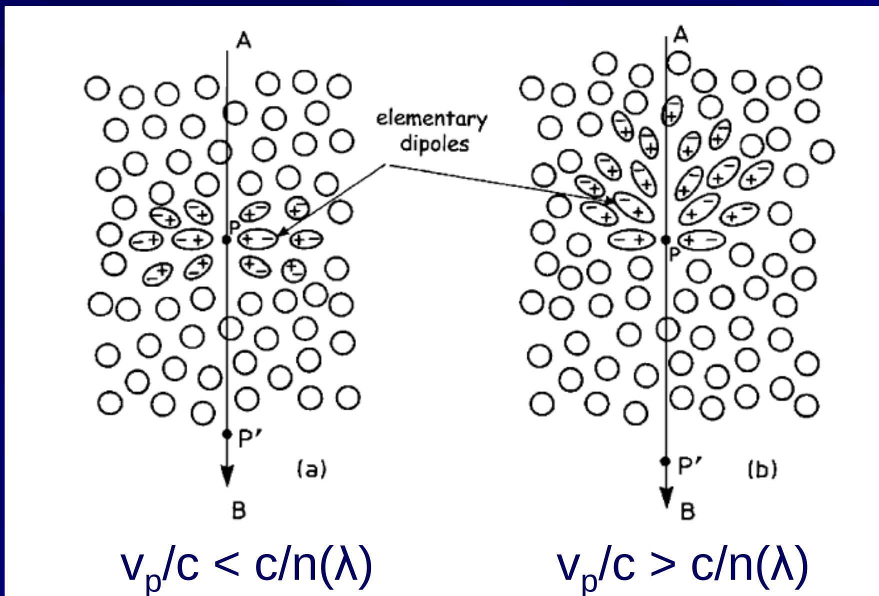
Energy loss by photon emission

- Emission of Cherenkov light
- Emission of transition radiation



Cherenkov photon emission

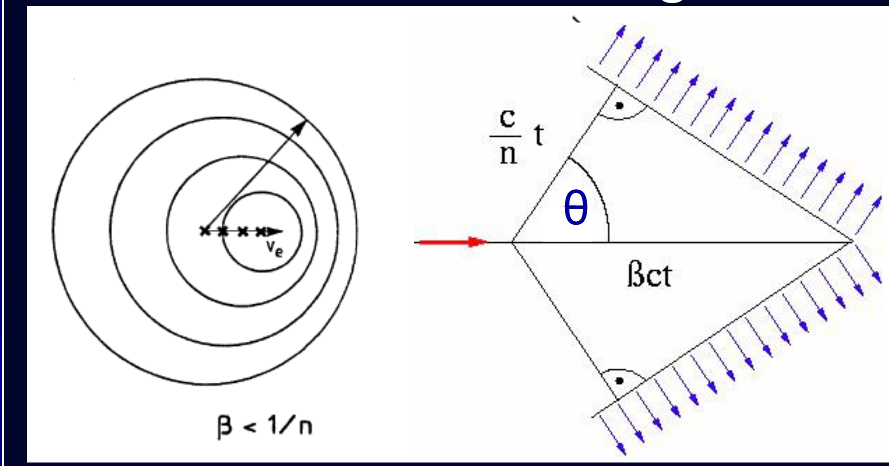
- If the velocity of a particle is such that **$\beta = v_p/c > c/n(\lambda)$** where $n(\lambda)$ is the index of refraction of the material, a pulse of light is emitted around the particle direction with an opening angle (**θ_c**)



Symmetric dipoles

coherent wavefront

Cherenkov angle

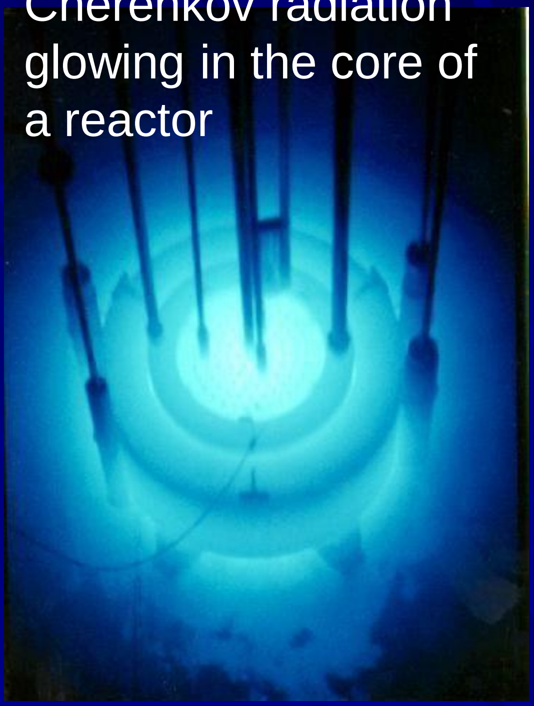


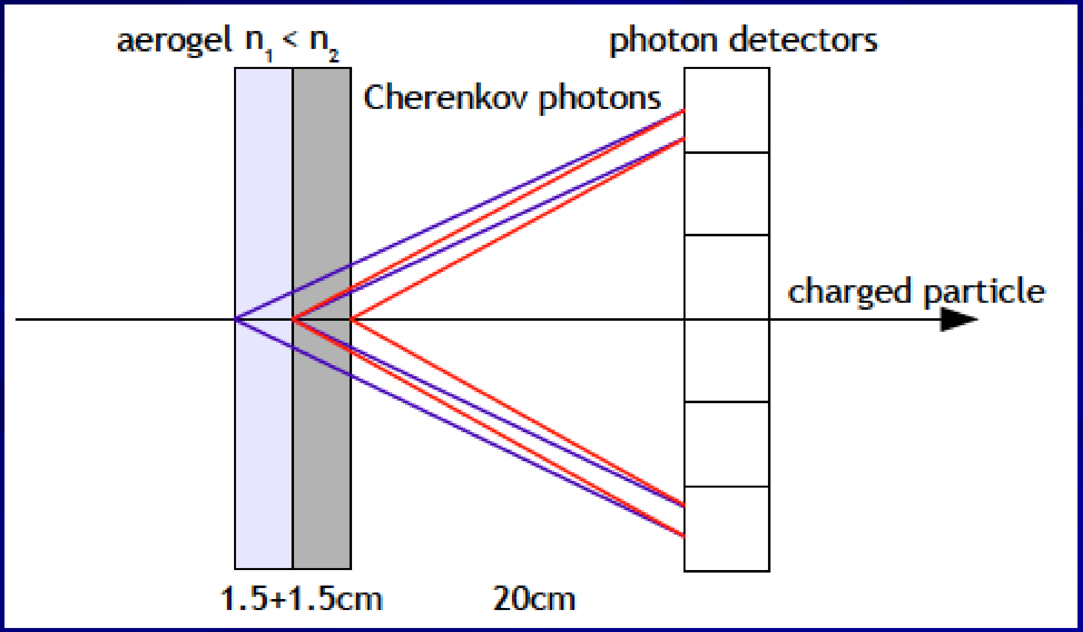
$$\cos q_c = \frac{1}{n(/)b}$$

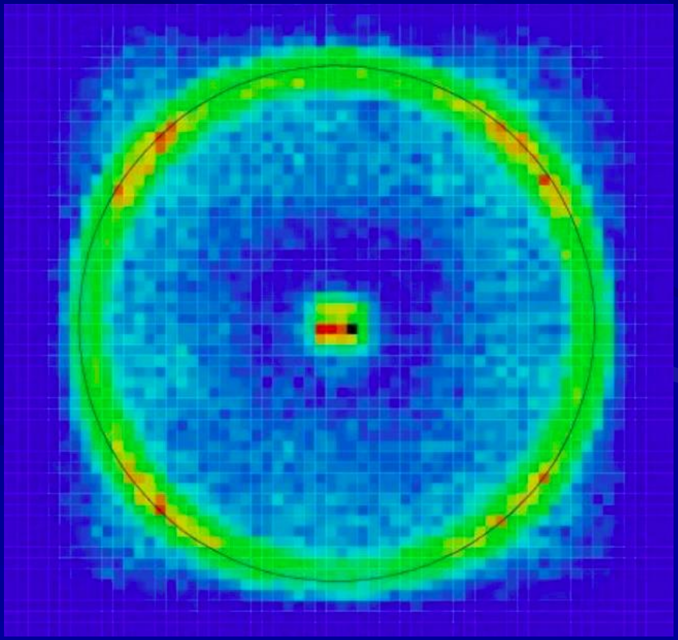
- The threshold velocity is $\beta_c = 1/n$
- At velocity below β_c no light is emitted

Cherenkov photon emission

- Cherenkov emission is a weak effect and causes no significant energy loss (<1%)
- It takes place only if the track L of the particle in the radiating medium is longer than the wavelength λ of the radiated photons.
- Typically O(1-2 keV / cm) or O(100-200) visible photons / cm

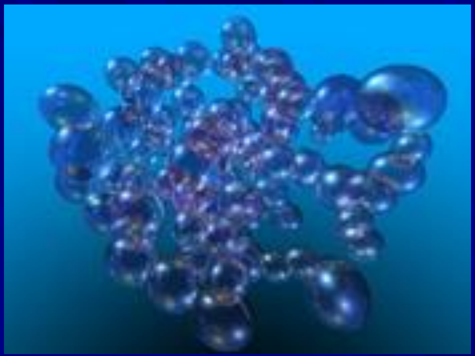






Cherenkov radiators

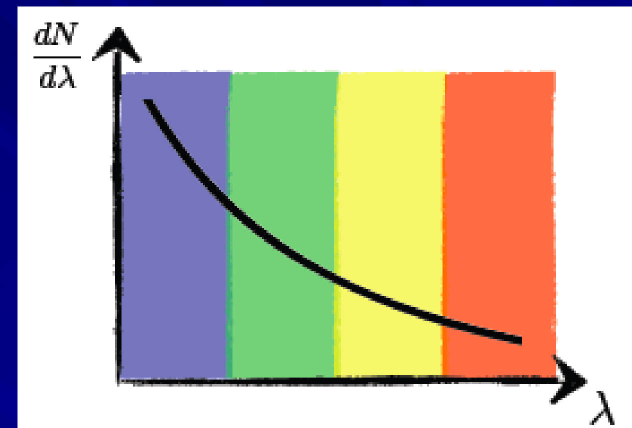
Material	n-1	β_c	θ_c	photons/cm
solid natrium	3.22	0.24	76.3	462
Lead sulfite	2.91	0.26	75.2	457
Diamond	1.42	0.41	65.6	406
Zinc sulfite	1.37	0.42	65	402
silver chloride	1.07	0.48	61.1	376
Flint glass	0.92	0.52	58.6	357
Lead crystal	0.67	0.6	53.2	314
Plexiglass	0.48	0.66	47.5	261
Water	0.33	0.75	41.2	213
Aerogel	0.075	0.93	21.5	66
Pentan	1.70E-03	0.9983	6.7	7
Air	2.90E-03	0.9997	1.38	0.3
He	3.30E-05	0.999971	0.46	0.03



Silica Aerogel

Cherenkov photon emission

- The number of Cherenkov photons produced by unit path length by a charged particle of charge **z is**



$$\frac{d^2N}{d\lambda\, dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_c$$

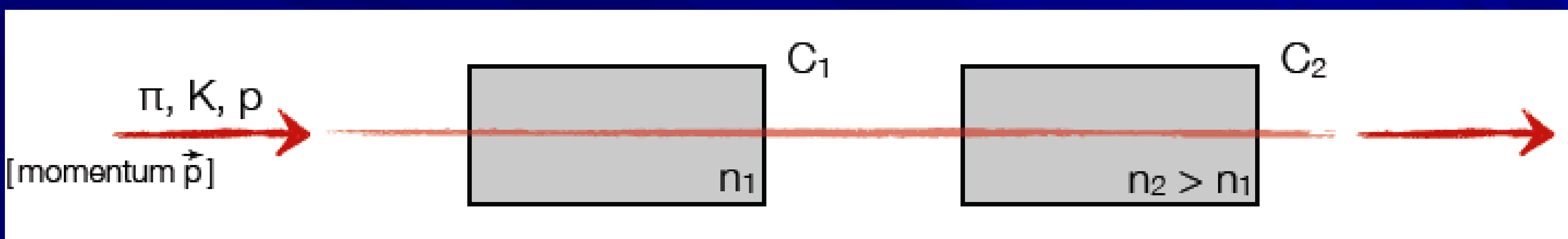
- Note the wavelength dependence ~ $1/\lambda^2$
- The index of refraction n is a function of photon energy E=hν, as is the sensitivity of the transducer used to detect the light.
- Therefore to get the number of photon we must integrate over the sensitivity range:

$$\frac{d^2N}{dx} = \int_{350nm}^{550nm} d\lambda \frac{dN}{d\lambda\, dx} = 475 z^2 \sin\theta_c \quad \text{photons/cm}$$

Threshold Cherenkov Counter Combination

- Combination of several threshold Cherenkov counters
- Separate different particles by choosing radiator such that

n_2: ®$_k$ and ®$_□$ >1/n_2 and ®$_p$<1/n_2
n_1: ®$_\pi$ >1/n_1 and ®$_p$, ®$_k$ and <1/n_1

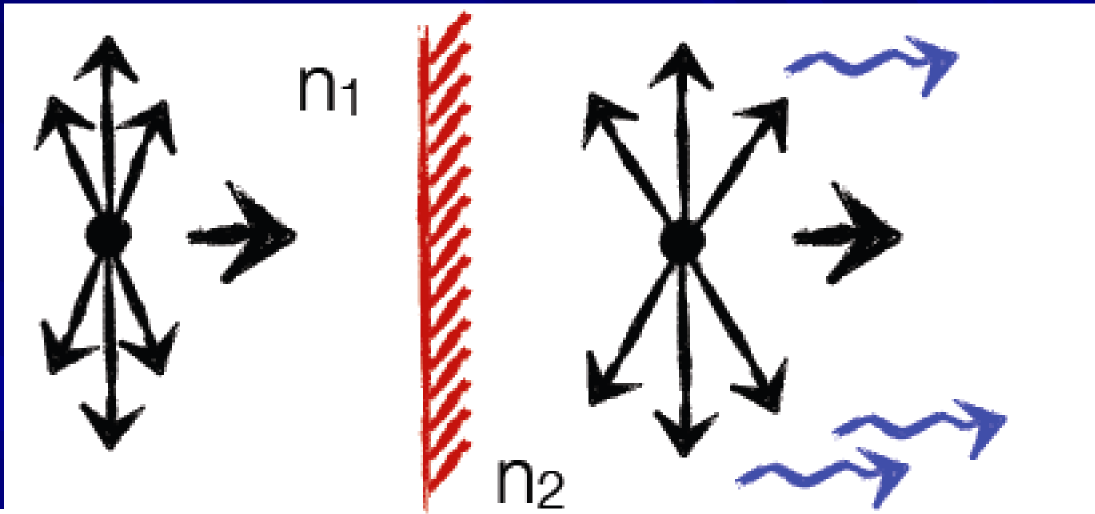


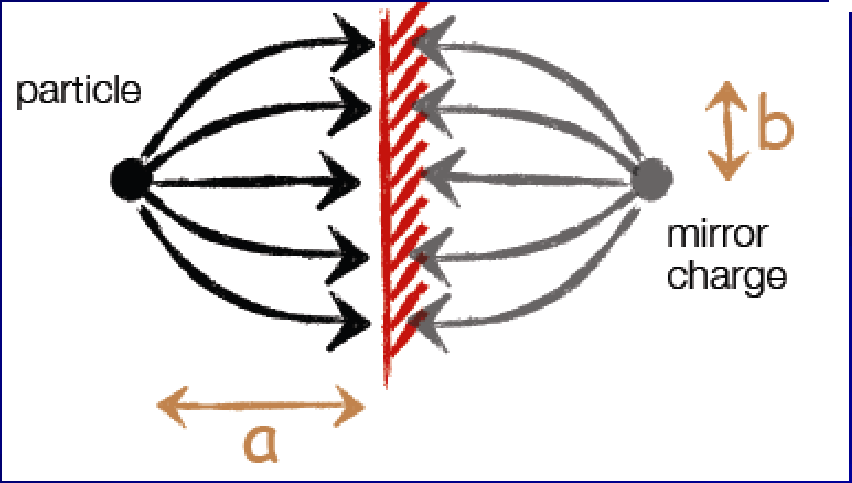
- Light in C1 and C2 identifies a pion
- Light in C2 and not C1 identifies a Kaon
- Light in neither C1 and C2 identifies a proton
- K-p-π separation up to 100 GeV

Transition radiation

- Transition radiation occurs if a relativist particle (large **γ**) passes the boundary between two media with different refraction indices (n1≠n2) [predicted by Ginzburg and Frank 1946; experimental confirmation 70ies]

- Effect can be explained by re-arrangement of electric field
- A charged particle approaching a boundary creates a dipole with its mirror charge

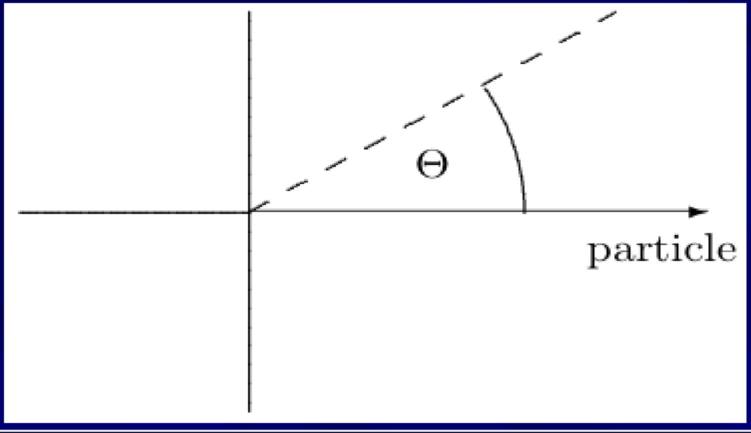




The time-dependent dipole field causes the emission of electromagnetic radiation

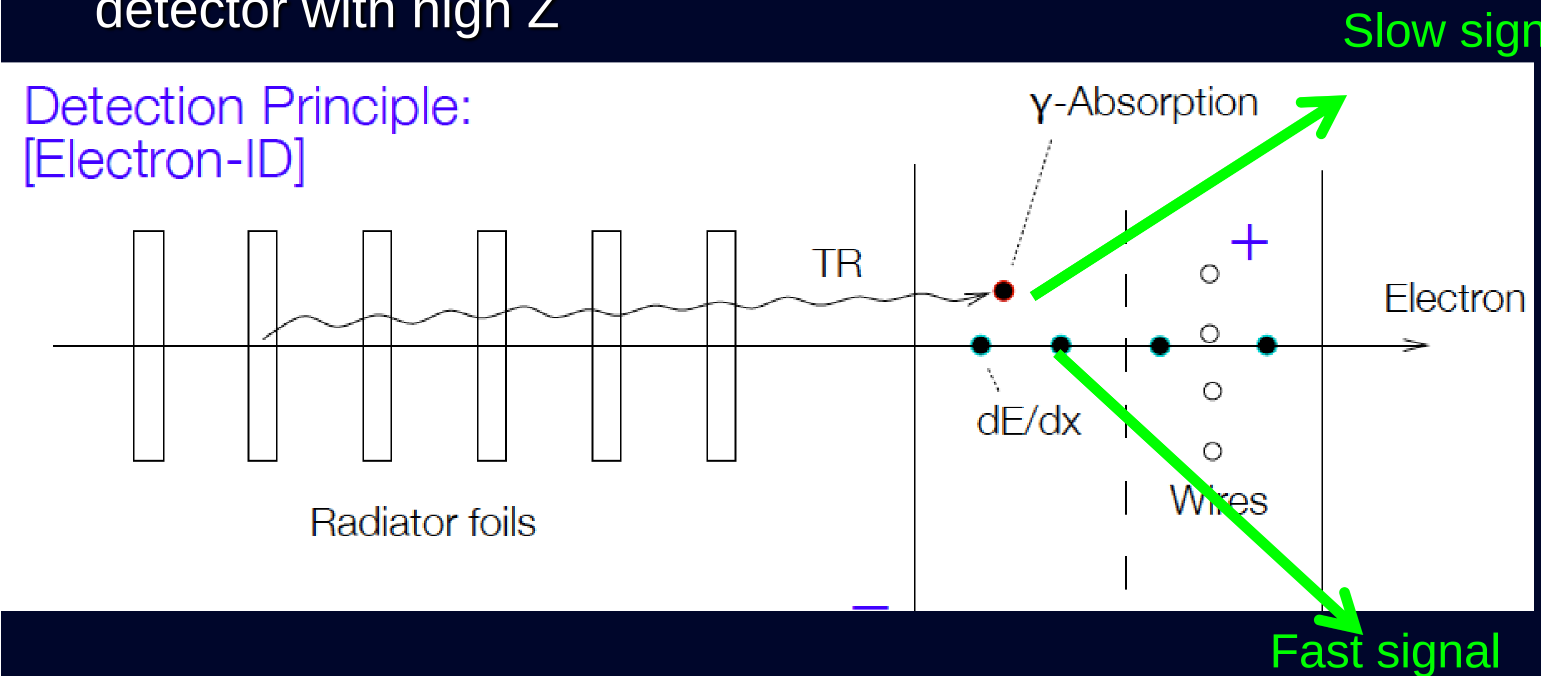
$$S = \frac{1}{3}\alpha z^2 \gamma \hbar\omega_P \quad \left(\hbar\omega_P \approx 28.8\sqrt{\frac{Z\rho}{A}}\,eV\right)$$

Transition Radiation



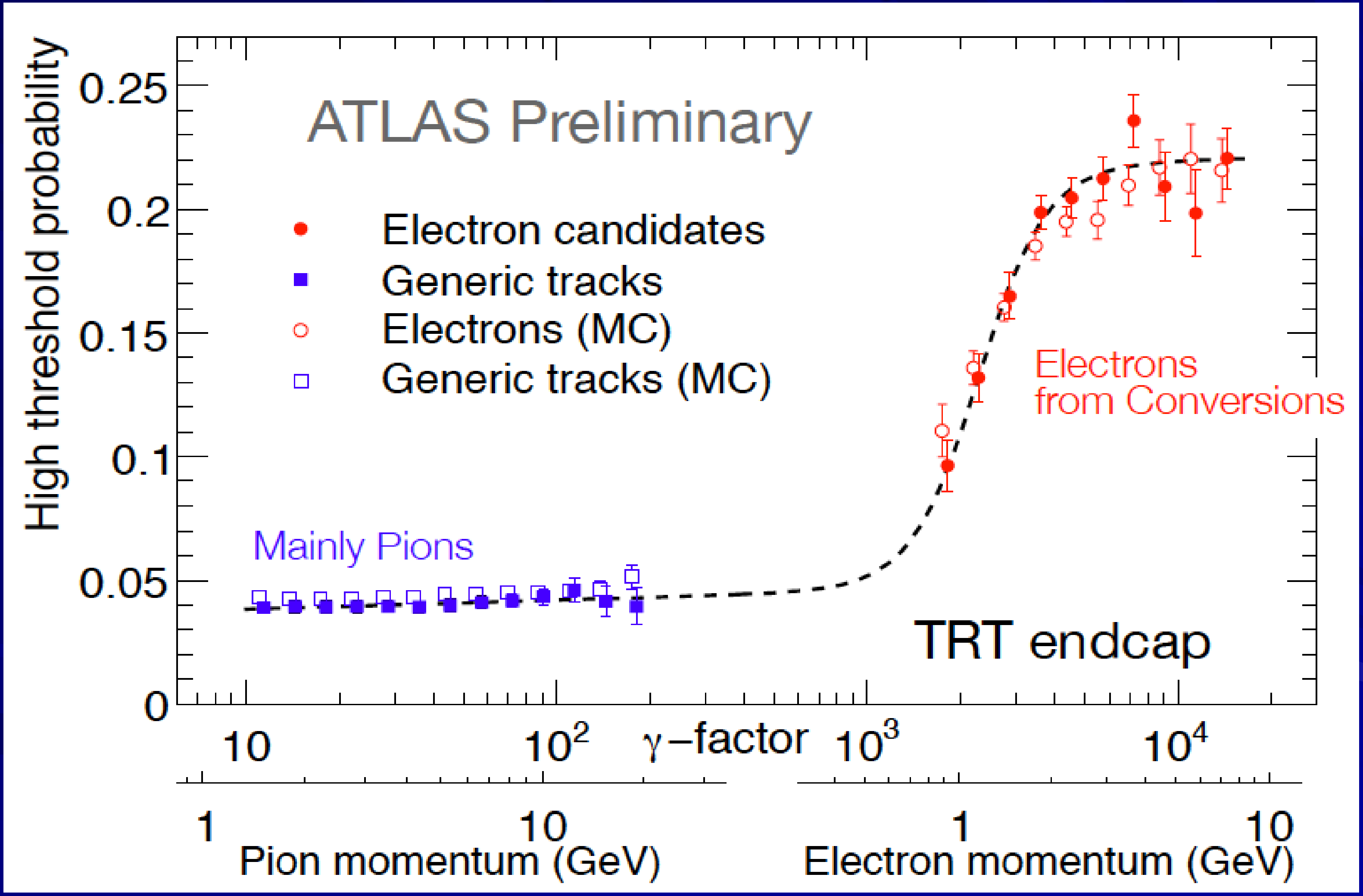
- Typical emission angle: θ=1/©
- Energy of radiated photons: ~ ©
- Number of radiated photons: αz^2
- Effective threshold: γ > 1000

- Use stacked assemblies of low Z material with many transitions and a detector with high Z



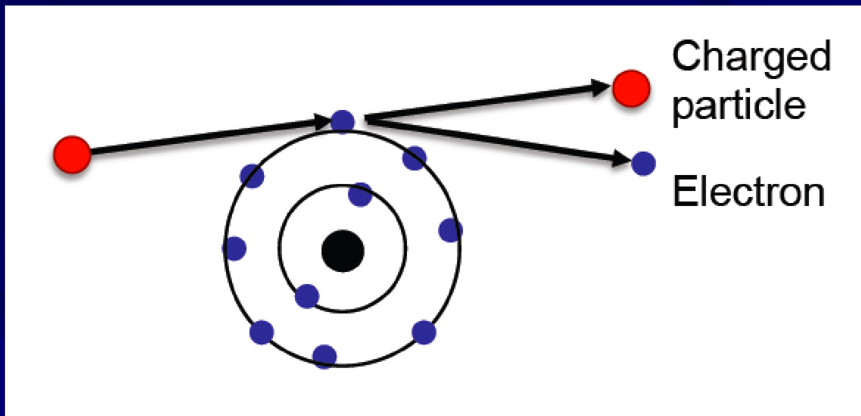
Note: Only X-ray (E>20keV) photons can traverse the many radiators without being absorbed

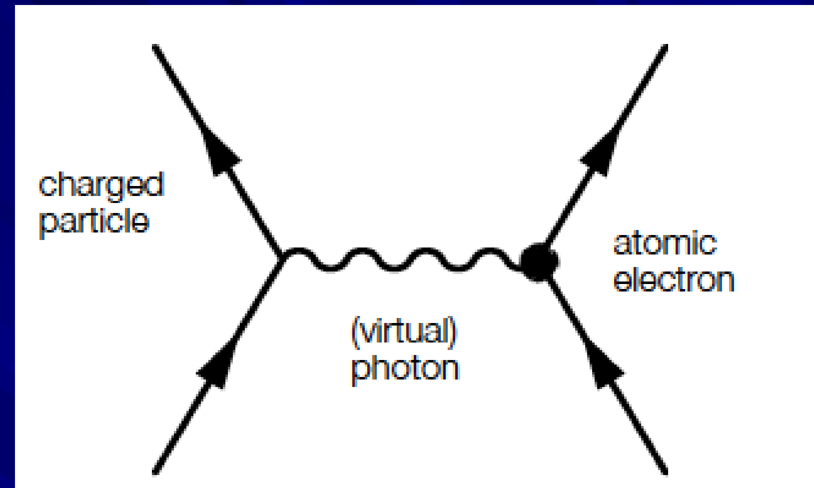
Transition radiation detector (ATLAS)



- BACKUP information

Energy loss by ionization





- First calculate for $Mc^2 \gg m_e c^2$:
- Energy loss for heavy charged particle [dE/dx for electrons more complex]
- The trajectory of the charged particle is unchanged after scattering

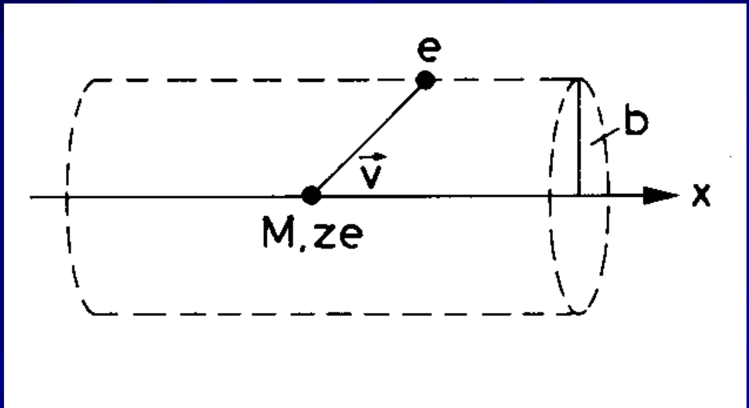
$$\frac{dE}{dx} \mu \frac{Z^2}{b^2} \ln\left(a b^2 g^2\right)$$

a= material dependent

Bohr's Classical Derivation

1913

- Particle with charge Ze and velocity v moves through a medium with electron density n.
- Electrons considered free and initially at rest
- The momentum transferred to the electron is:



$$\Delta p_{\perp} = \int F_{\perp} dt = \int F_{\perp} \frac{dt}{dx} dx = \int F_{\perp} \frac{dx}{v}$$

$\Delta p_{\parallel}$: averages to zero because of symmetry

$$\text{Gauss' Law:} \quad \int E_{\perp} (2\pi b) dx = 4\pi (ze)$$

$$\int E_{\perp} dx = \frac{4ze}{b}$$

$$F_{\perp} = eE_{\perp}$$

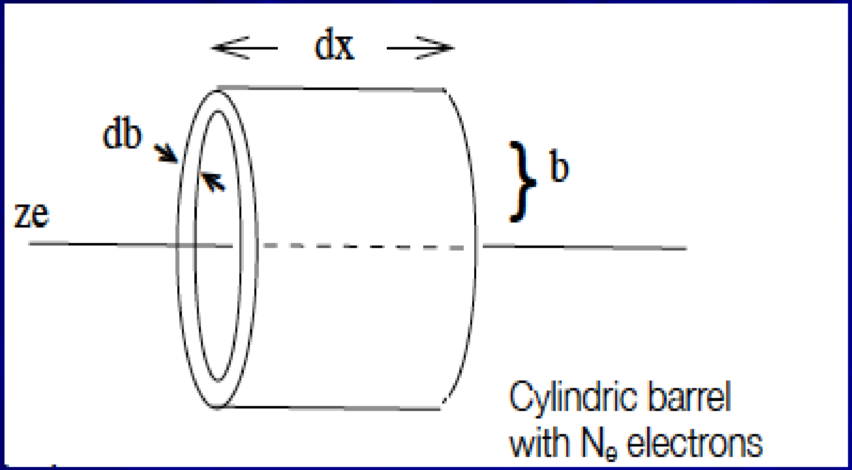
$$\Delta p_{\perp} = e \int E_{\perp} \frac{dx}{v}$$

$$\Delta p_{\perp} = \frac{2ze^2}{bv}$$

Bohr's Classical Derivation

- Energy transfer to a single electron with an impact parameter b

$$\Delta E(b) = \frac{\Delta p^2}{2m_e} \qquad \Delta p_\perp = \frac{2ze^2}{bv}$$



- Consider Cylindric barrel: N_e=**n**(2**π**b)·db dx
- Energy loss per path length dx for distance between b and b+db in medium with electron density n:

Energy loss

$$-dE(b) = \frac{\Delta p^2}{2m_e} 2\pi n b\,db\,dx = \frac{\left(2ze^2\right)^2}{2m_e (bv)^2} 2\pi n b\,db\,dx = \frac{4\pi n z^2 e^4}{m_e v^2}\frac{db}{b}dx$$

- Diverges for b→0. Integrate in [b_{min}, b_{max}]

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e v^2}\int_{b_{min}}^{b_{max}}\frac{db}{b} = \frac{4\pi n z^2 e^4}{m_e v^2}\ln\frac{b_{max}}{b_{min}}$$

Bohr’s Classical Derivation

- Determination of relevant range [b_{min}, b_{max}]:
- [Arguments: $b_{min} > \boldsymbol{\lambda}_e$, i.e. de Broglie wavelength; $b_{max} < \infty$ due to screening ...]

$$b_{\min} = \lambda_e = \frac{h}{p} = \frac{2\pi\hbar}{\gamma m_e v}$$

$$b_{\min} = \frac{\gamma v}{\langle v_e \rangle} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$-\frac{dE}{dx} = \frac{4\pi n z^2 e^4}{m_e c^2 \beta^2} n \ln \frac{m_e c^2 \beta^2 \gamma}{2\pi\hbar \langle v_e \rangle}$$

Deviates by factor 2 from QM derivation

Electron density n=**NA·ρ·Z/A**
Effective Ionization potential I=h <**v**$_e$>

Bohr Calculation of dE/dx

- Stopping power

$$-\frac{dE}{dx} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln\frac{b_{max}}{b_{min}}$$

- Determination of the relevant range [b_{min}, b_{max}]:
  - b_{min} : Maximum kinetic energy transferred Bohr formula

$$W_{max} = \frac{1}{2}\gamma^2 m_e (2v)^2 = 2 m_e c^2 \beta^2 \gamma^2 \qquad b_{min} = \frac{ze^2}{\gamma m_e v^2}$$

  - b_{max} :particle moves faster than e in the atomic orbit. Electrons are bound to atoms with average orbital frequency <v_e>. Interaction time has to be ≤ <$1/v_e$>

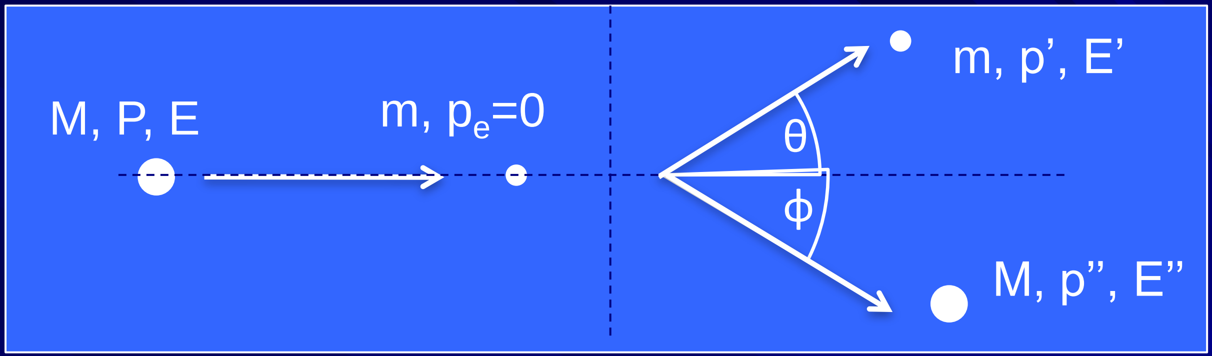
$$b_{max} = \frac{\gamma v}{\langle v_e \rangle}$$

or distance at which the kinetic energy transferred is minimum W_{min}= I (mean ionization potential)

- We can integrate in this interval an derive the classical Bohr formula

$$-\frac{dE}{dx} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln\frac{\gamma^2 m v^3}{ze^2 \langle v_e \rangle} = \frac{4\pi N_e z^2 r_e^2 m_e c^2}{\beta^2} \ln\left(\frac{2 m_e c \beta^2 \gamma^2}{I}\right)$$

Relativistic Kinematic



Energy conservation: $\sqrt{p^2c^2+M^2c^4}+mc^2=\sqrt{p''^2c^2+M^2c^4}+\sqrt{p'^2c^2+m^2c^4}$

momentum conservation: $p=p'\cos\theta+p''\cos\phi$

$$0=p'\sin\theta+p''\sin\phi$$

$$p''^2=p'^2+p^2-2pp'\cos\theta$$

- Using energy and momentum conservation we can find the kinetic energy

$$\varepsilon'=\sqrt{p'^2c^2+m^2c^4}-mc^2=\frac{2mc^2p^2c^2\cos^2\theta}{mc^2+\sqrt{p^2c^2+M^2c^4}-p^2c^2\cos^2\theta}$$

- The maximum energy transfer is

$$\varepsilon'_{max}=\frac{2mp^2}{m^2+M^2+2mE/c^2}$$

Cherenkov Radiation – Momentum Dependence

- Cherenkov angle θ and number of photons N grows with β
- Asymptotic value for β=1: $\cos\theta_{max} = 1/n$; $N_\infty = x \cdot 370 / \text{cm} \, (1-1/n^2)$