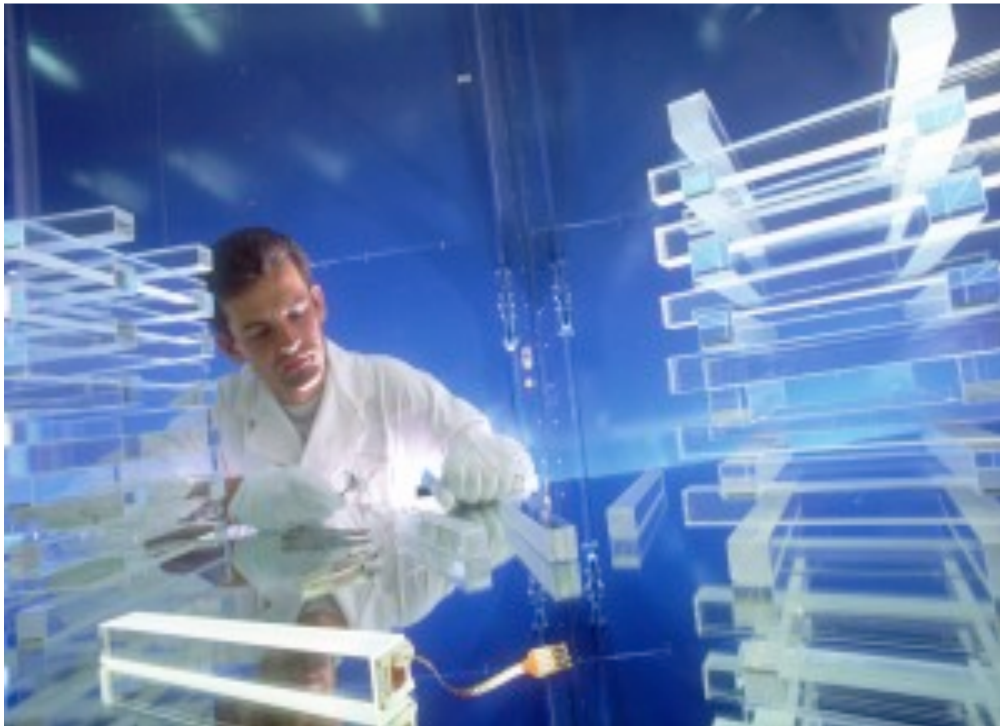
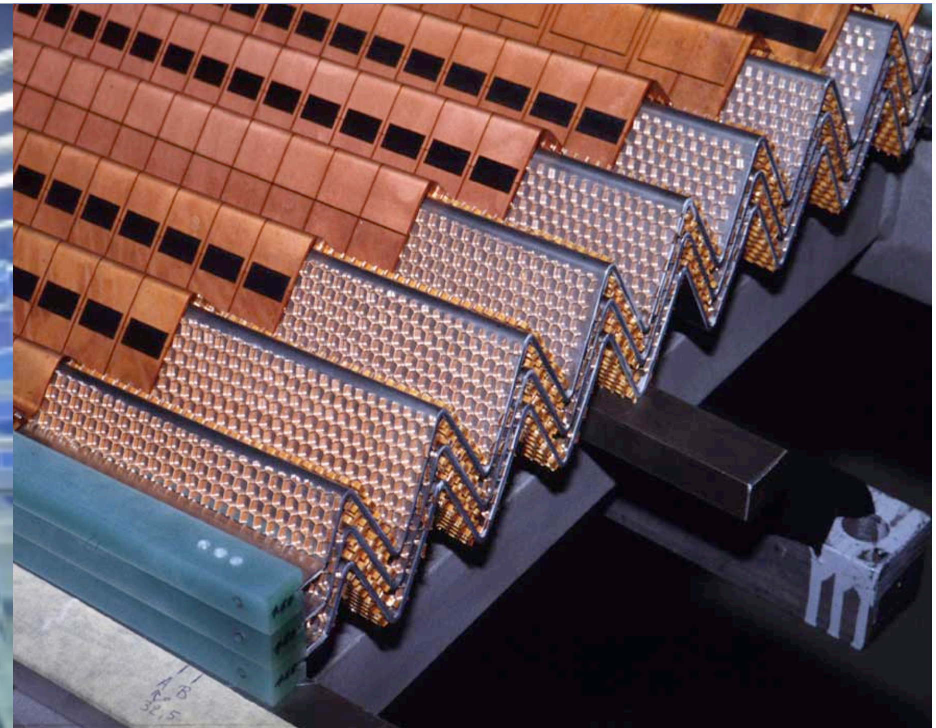



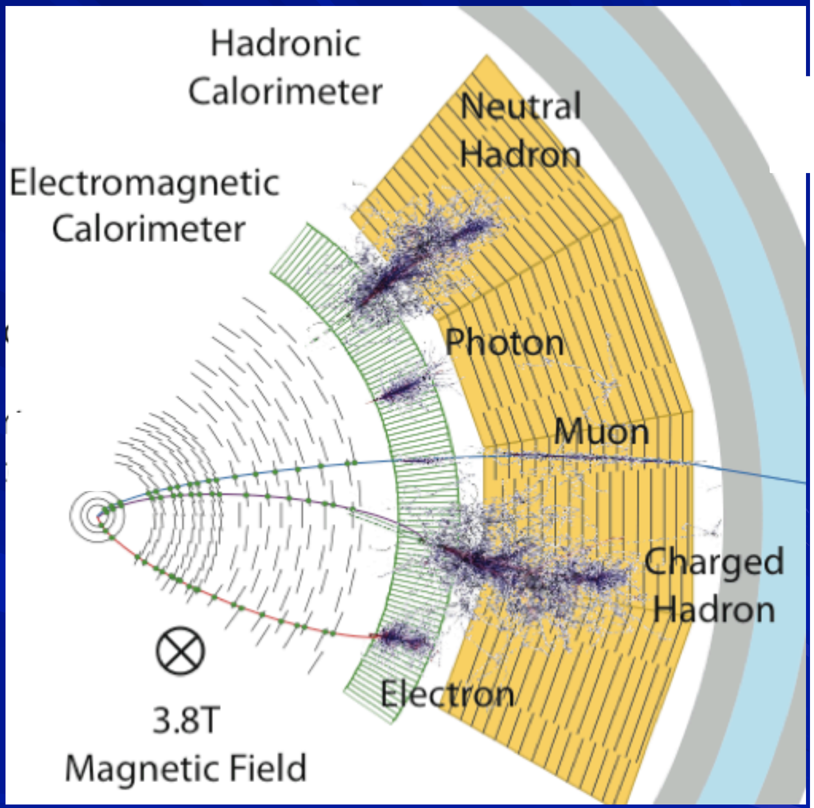
Detectors for Particle Physics

Calorimetry

D. Bortoletto
Purdue University

What is a calorimeter ?

- In nuclear and particle physics calorimetry refers to the detection of particles through total absorption in a block of matter
  - The measurement process is destructive for almost all particle
  - The exception are muons (and neutrinos) ➔ identify muons easily since they penetrate a substantial amount of matter
- In the absorption, almost all particle's energy is eventually converted to heat ➔ calorimeter
- Calorimeters are essential to measure neutral particles



Electromagnetic shower

- Dominant processes at high energies (E > few MeV) :

- Photons: Pair production

$$\sigma_{pair} \approx \frac{7}{9}\left(4\alpha r_e^2 Z^2 \ln\frac{183}{Z^{1/3}}\right) = \frac{7}{9}\frac{A}{N_A X_0}$$

$$I(x) = I_0 e^{-\mu x} \qquad \mu = \frac{7}{9}\frac{\rho}{X_0}$$

μ= attenuation coefficient
X_0 = radiation length in [cm] or [g/cm^2]

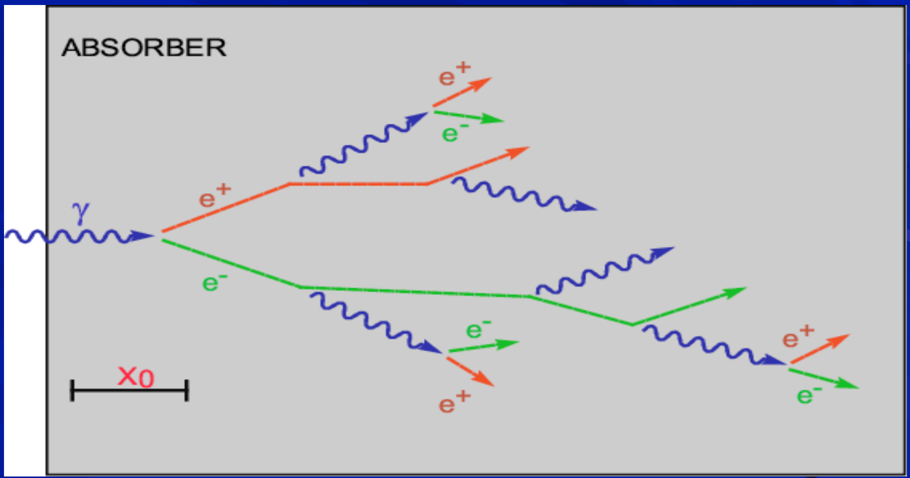
$$X_0 = \frac{A}{4\pi N_A Z^2 r_e^2 \ln\frac{183}{Z^{1/3}}}$$

- Electrons: Bremsstrahlung

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln\frac{183}{Z^{1/3}} = \frac{E}{X_0}$$

$$E = E_0 e^{-x/X_0}$$

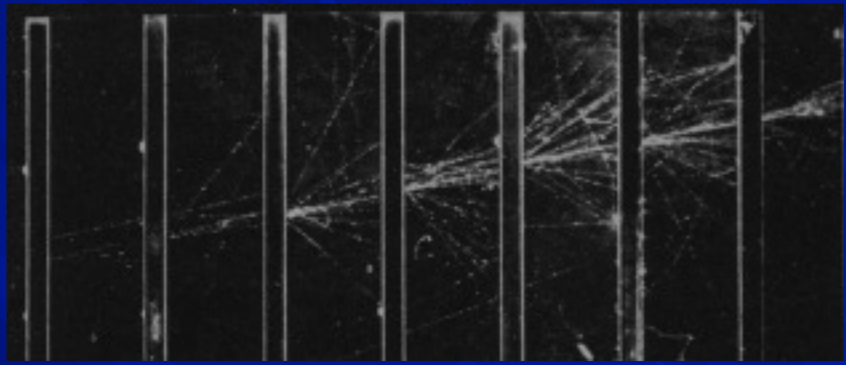
After traversing $x=X_0$ the electron has only 1/e=37% of its initial energy

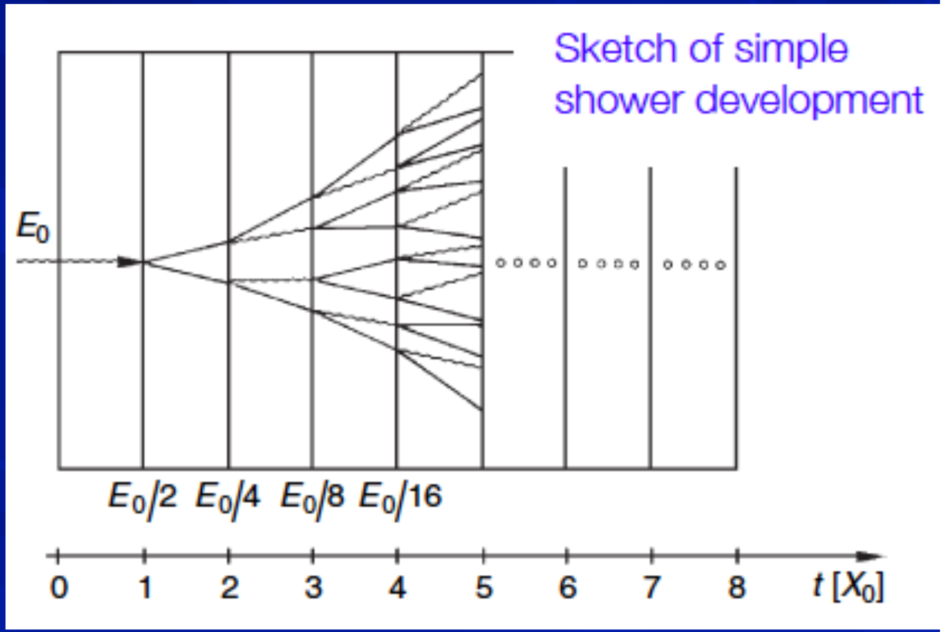


Analytic shower Model

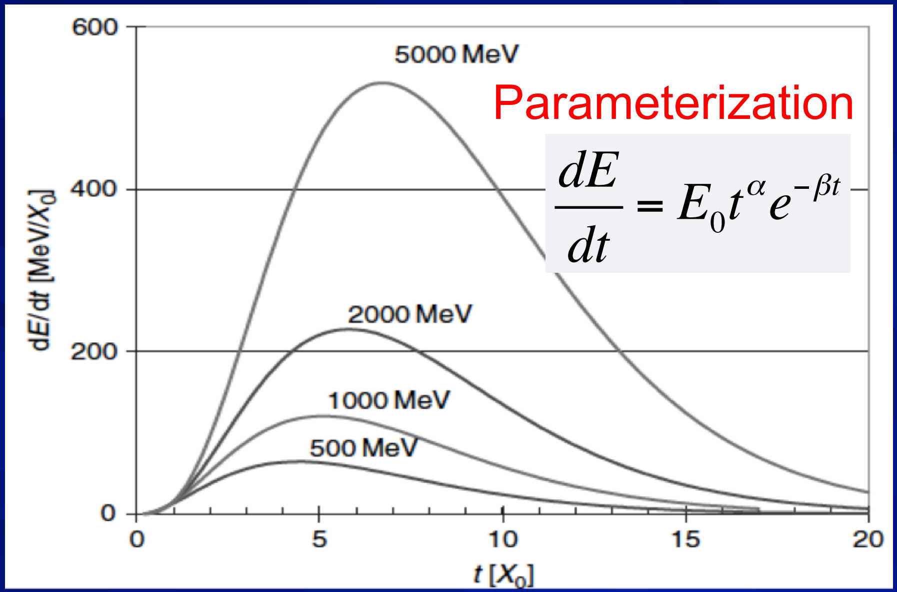
- Simplified model [Heitler]: shower development governed by X_0
  - e^- loses [1 - 1/e] = 63% of energy in 1 X_0 (Brems.)
  - the mean free path of a γ is 9/7 X_0 (pair prod.)
- Assume:
  - $E > E_c$: no energy loss by ionization/excitation
- Simple shower model:
  - 2t particles after t [X_0]
  - each with energy E/2t
  - Stops if $E < \mathbf{E_c}$
  - Number of particles $N = E/E_c$
  - Maximum at

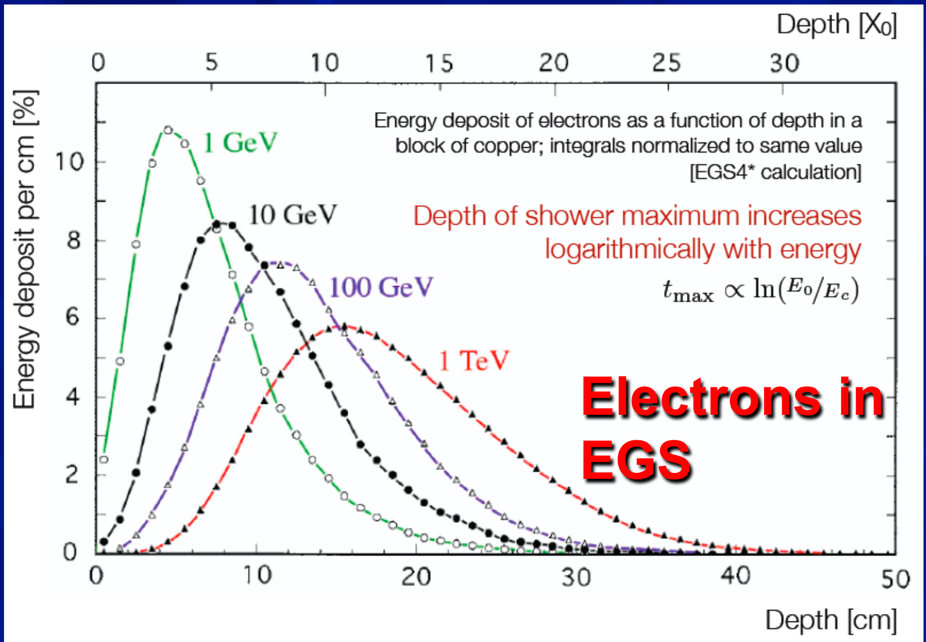
$$t_{\max} \propto \ln\left(\frac{E}{E_c}\right)$$

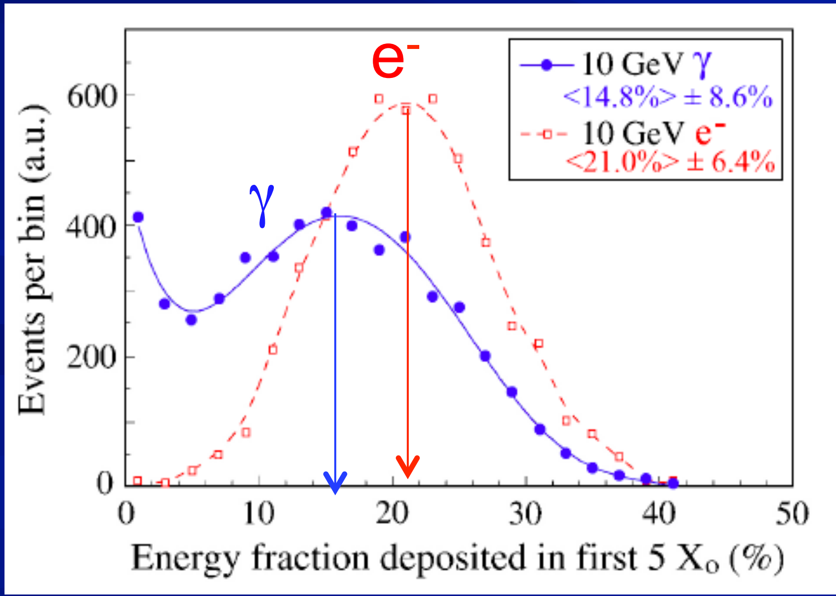




Longitudinal shower distribution







- Differences between electrons and photons generated showers
- Some photons penetrating (almost) the entire slab without interacting (peak at 0)

$$t_{max} = \ln\left(\frac{E_0}{E_c}\right) + C_{e\gamma}$$

$C_{e\gamma}$=-0.5 for photons
$C_{e\gamma}$=-1 for electrons

Longitudinal containment

- Longitudinal shower distribution increases only logarithmically with the primary energy of the incident particle, i.e. calorimeters can be compact
- L(95%) = t_{max} + 0.08 Z + 9.6 [X_0]

$$\text{Number of particle in shower} = N_{\max} = 2^{t_{\max}} = \frac{E_0}{E_c}$$

$$\text{Location of shower max} = t_{\max} \approx \ln\left(\frac{E_0}{E_c}\right)$$

$$\text{Longitudinal shower distribution} = L \approx \ln\left(\frac{E_0}{E_c}\right)$$

Transverse shower distribution

Example:

$$E_C \approx 10\,MeV \qquad E_0 = 1\,GeV \quad \Rightarrow t_{\max} = \ln 100 \approx 4.6 \qquad N_{\max} = 100$$

$$E_0 = 100\,GeV \Rightarrow t_{\max} = \ln 10{,}000 \approx 9.2 \quad N_{\max} = 10{,}000$$

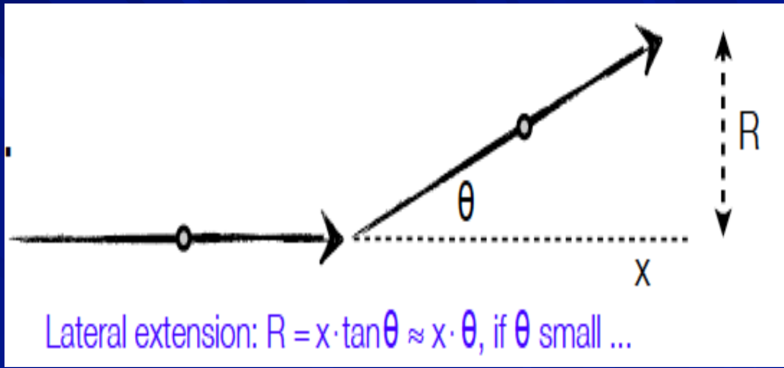
	Scint.	LAr	Fe	Pb	W
X_0(cm)	34	14	1.76	0.56	0.35

A 100 GeV electron is contained in 16 cm Fe or 5 cm Pb

Lateral development of EM shower

- Opening angle:
  - bremsstrahlung and pair production

$$\langle \theta^2 \rangle \approx \left(\frac{m_e c^2}{E_e}\right)^2 = \frac{1}{\gamma^2}$$



  - multiple coulomb scattering [**Molière** theory]

$$\langle \theta \rangle = \frac{E_s}{E_e}\sqrt{\frac{x}{X_0}} \quad \text{where} \quad E_s = \sqrt{\frac{4\pi}{\alpha}}\left(m_e c^2\right) = 21.2 MeV$$

- Main contribution from low energy electrons as <**θ**> ~ $1/E_e$, i.e. for electrons with $E < E_c$

- **Molière** Radius

$$R_M = \frac{E_s}{E_c} X_0 \approx \frac{21.2 MeV}{E_c} X_0$$

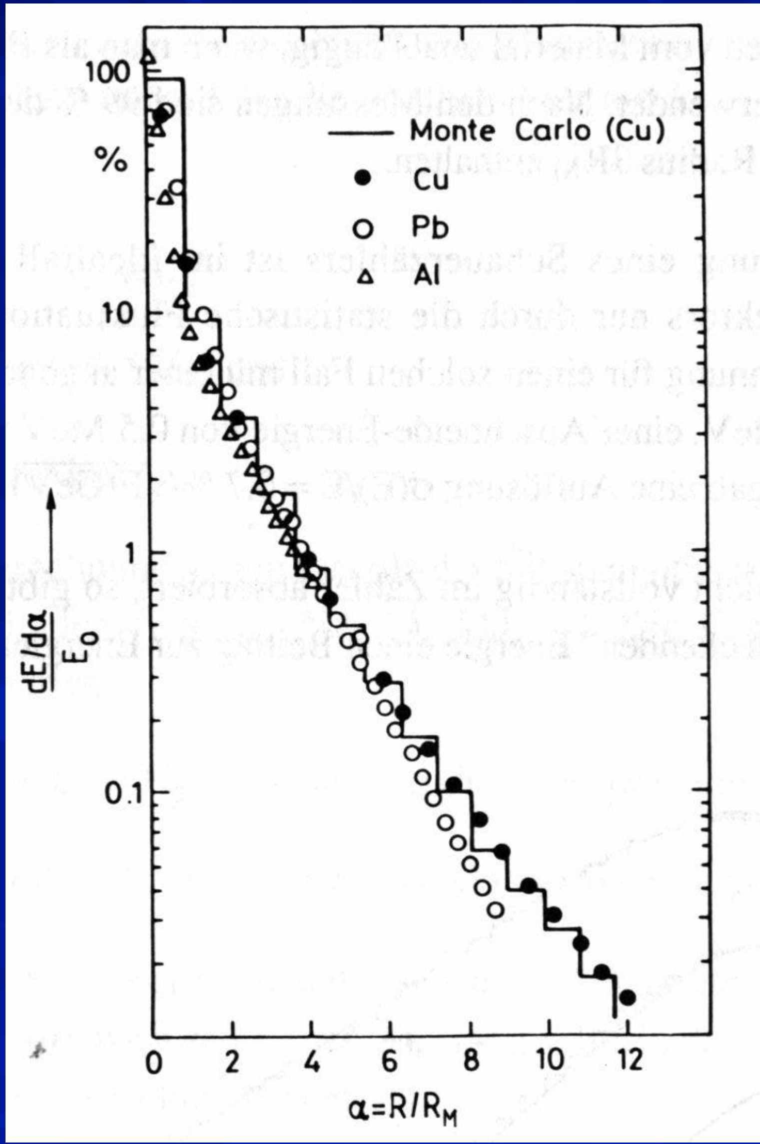
- Assuming the approximate range of electrons to be X_0 yields <**θ**>≈ 21.2 MeV/E_e → lateral extension: R =<**θ**>X_0

Lateral development of EM shower

- Inner part is due to Coulomb's scattering of electron and positron
- Outer part is due to low energy photons produces in Compton's scattering, photo-electric effect etc.
  - Predominant part after shower max especially in high Z absorbers

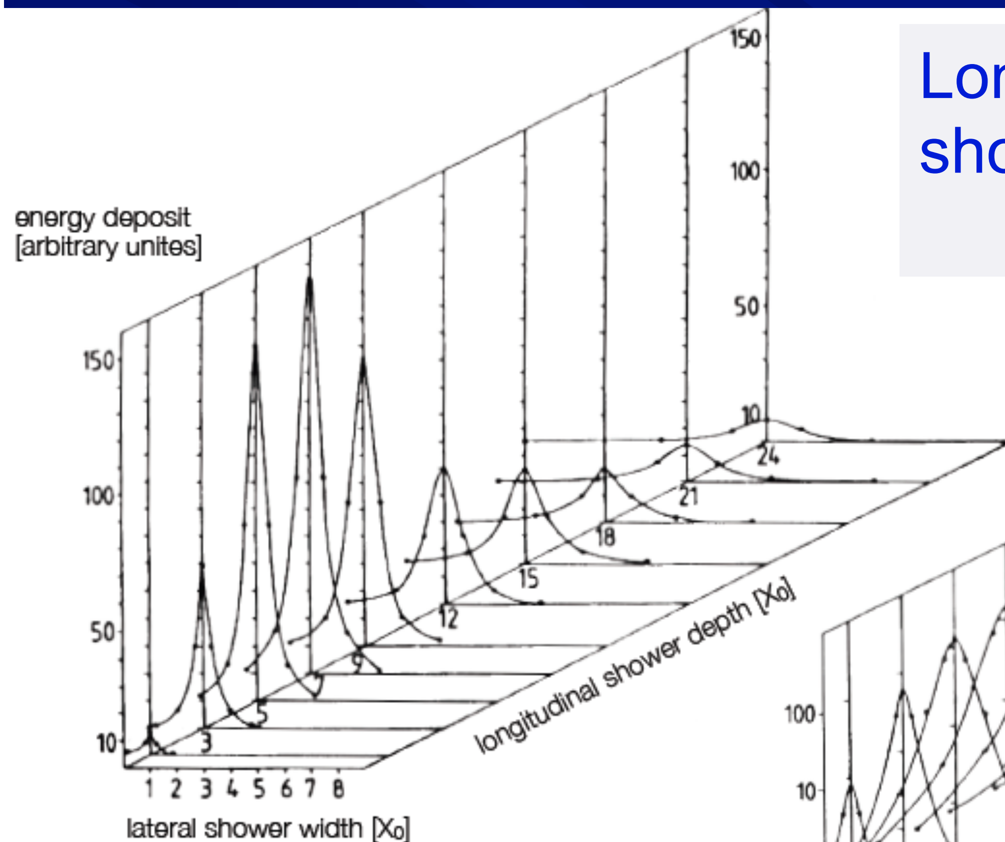
$$\frac{dE}{dr} = \alpha e^{-r/RM} + \beta e^{-r/\lambda_{\min}}$$

- The shower gets wider at larger depth
- An infinite cylinder of radius 1 R_M contains 90% of the shower

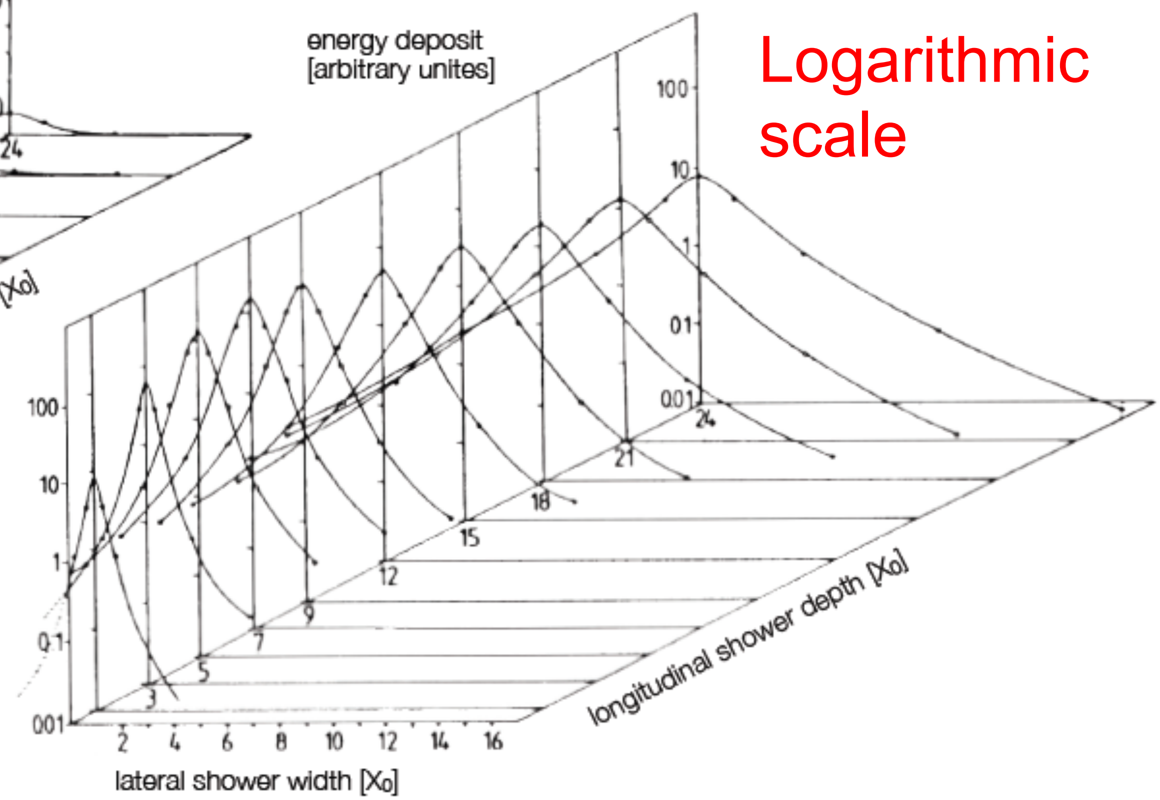


3D EM Shower development

Longitudinal and transfer EM shower profile of 6 GeV e^- in Lead



Linear scale



Logarithmic scale

Energy Measurement

- How we determine the energy of a particle from the shower?
  - Detector response ➔ Linearity
    - The average calorimeter signal vs. the energy of the particle
    - Homogenous and sampling calorimeters
    - Compensation (for hadronic showers)
  - Detector resolution ➔ Fluctuations
    - Event to event variations of the signal
    - What limits the accuracy at different energies?

“response = average

EM calorimeter are linear
Hadronic are not

Sources of Non Linearity

- Instrumental effects
  - Saturation of gas detectors, scintillators, photo-detectors, Electronics
- Response varies with something that varies with energy
- Examples:
  - Deposited energy “counts” differently, depending on depth
    - And depth increases with energy
- Leakage (increases with energy)

Signal linearity for electromagnetic showers

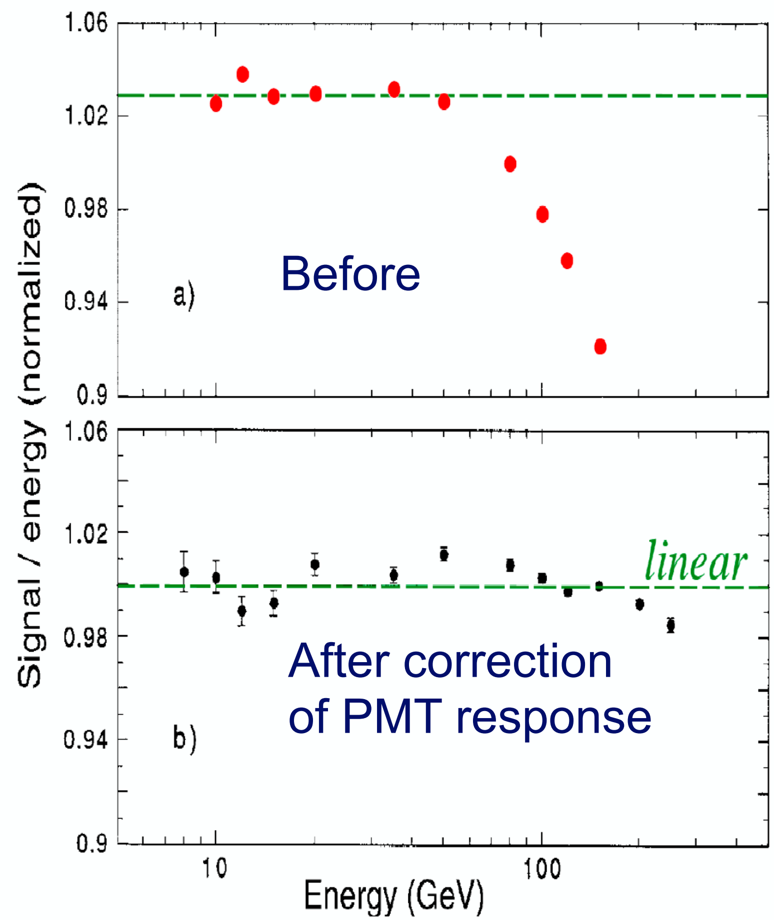


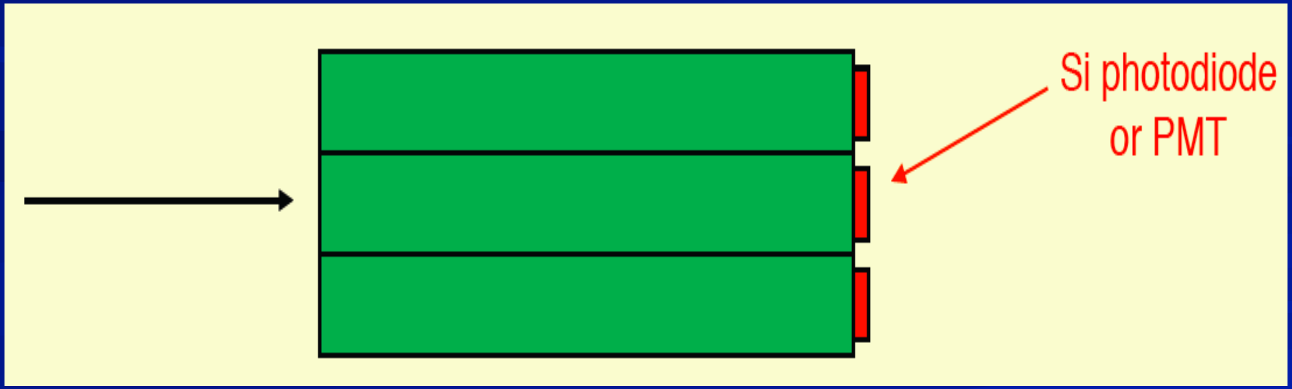
FIG. 3.1. The em calorimeter response as a function of energy, measured with the QFCAL calorimeter, before (*a*) and after (*b*) precautions were taken against PMT saturation effects. Data from [Akc 97].

EM Calorimeter configurations

- Total absorption
  - Electrons and photons stop in calorimeter
  - Scintillation proportional to energy of electron
  - Usually non-organic scintillator (BGO, $PbWO_{4,...}$) or liquid Xe
  - Advantage: Excellent energy resolution
    - see all charged particles in the shower (but for shower leakage) ➔best statistical precision
    - Uniform response ➔good linearity
  - Disadvantages:
    - cost and limited segmentation

If W is the mean energy required to produce a signal (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal)

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}}$$



- Examples:
  - B factories: small photon energies
  - CMS ECAL which was optimized for H➔γγ

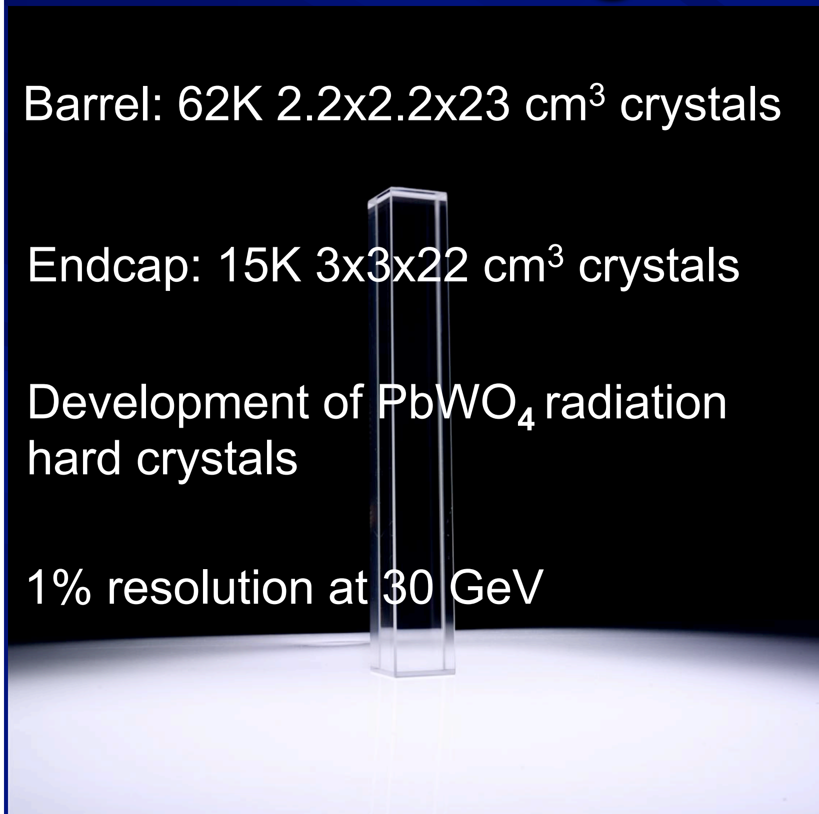
Homogenous calorimeters

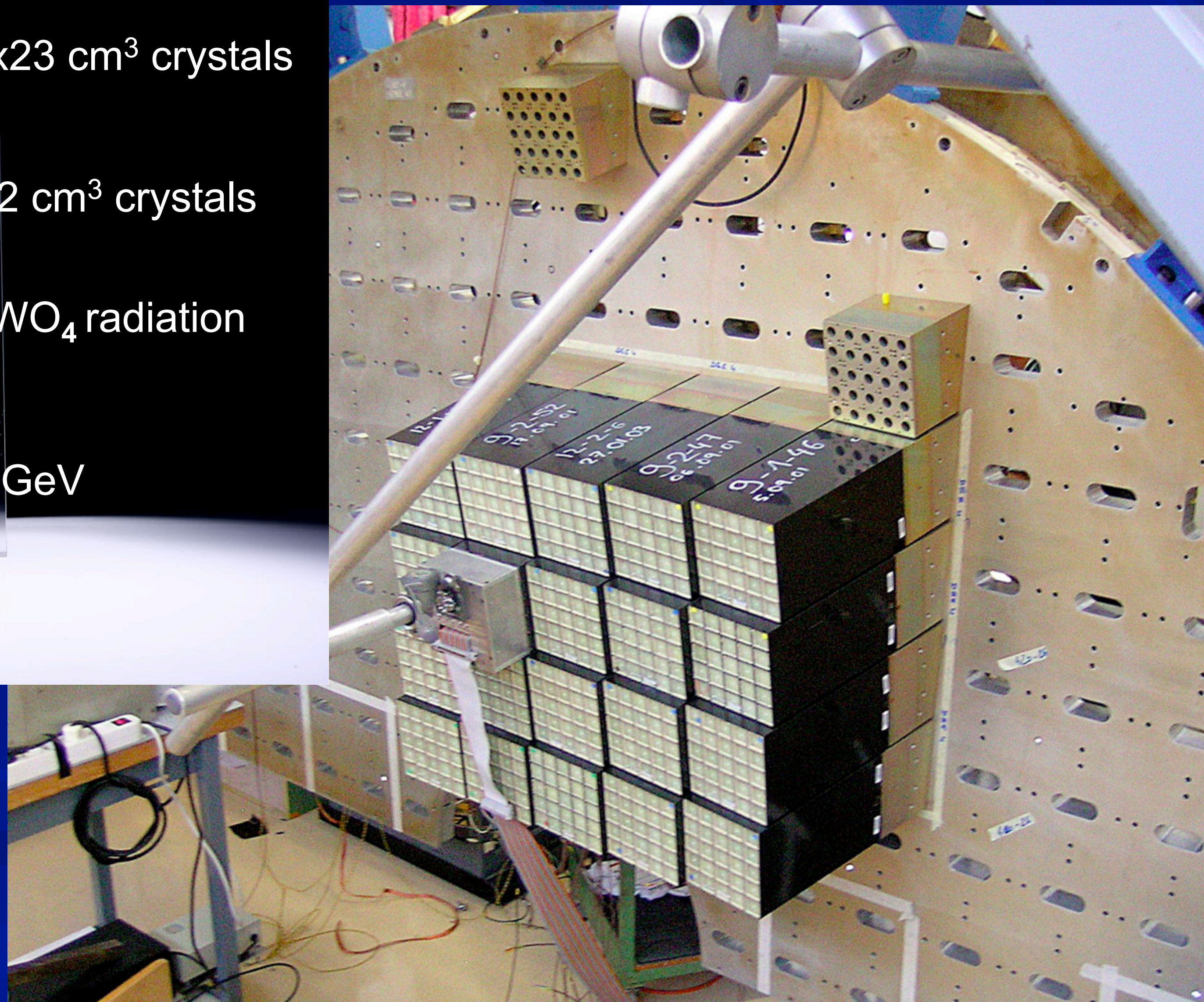
Barrel: 62K 2.2x2.2x23 cm^3 crystals

Endcap: 15K 3x3x22 cm^3 crystals

Development of $PbWO_4$ radiation hard crystals

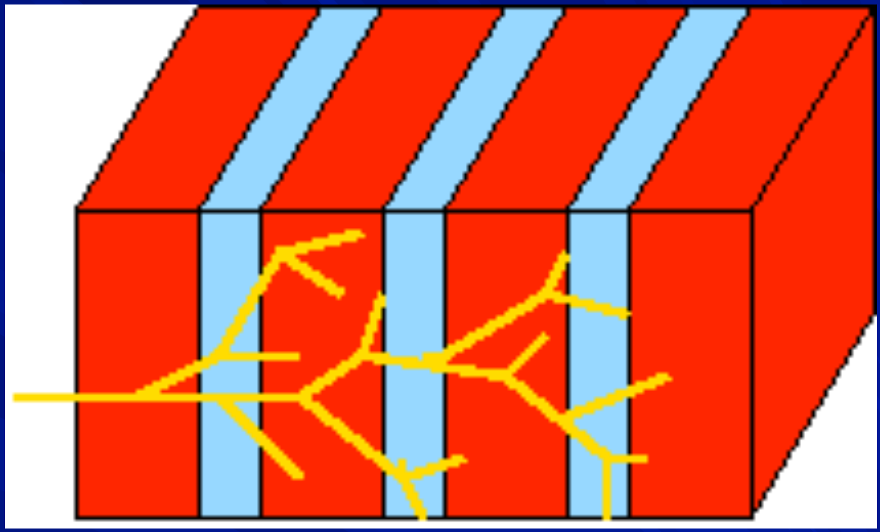
1% resolution at 30 GeV





EM Calorimeter configurations

- Sampling Calorimeter
  - One material to induce showering (high Z)
  - Another to detect particles (typically by counting number of charged tracks)
  - Many layers sandwiched together
  - Resolution $\propto E^{-1/2}$
  - Advantages: Can segment in depth; can have better spatial segmentation
  - Disadvantages:
    - Only part of shower seen, less precise
- Examples
  - ATLAS ECAL
  - Most HCALs

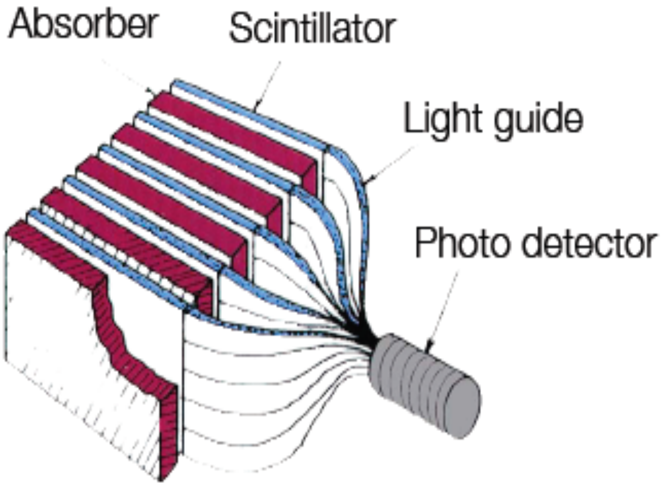


- Sampling fraction

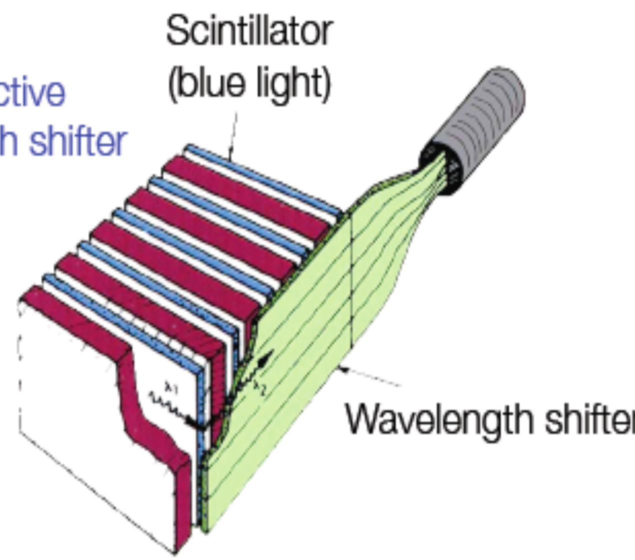
$$f_{sampling} = \frac{E_{visible}}{E_{deposited}}$$

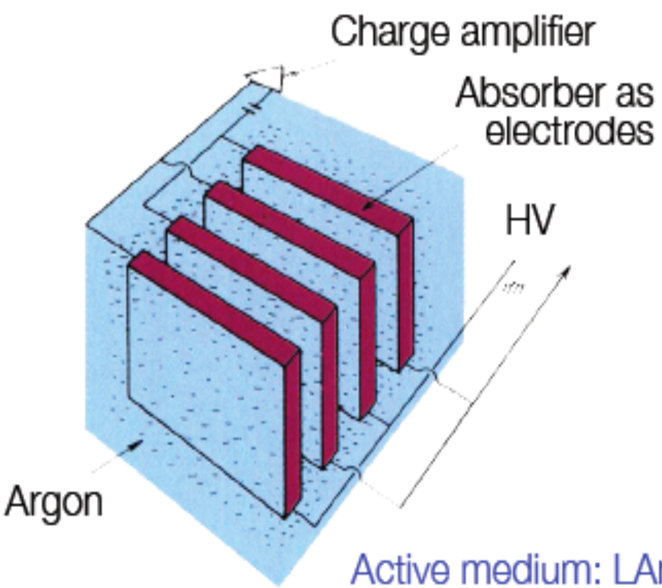
Possible setups

Scintillators as active layer;
signal readout via photo multipliers



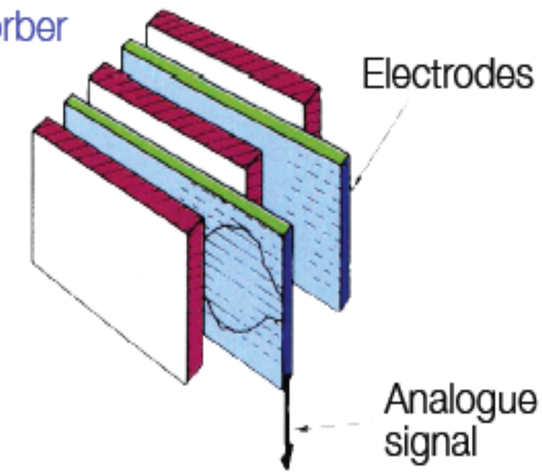
Scintillators as active layer; wave length shifter to convert light





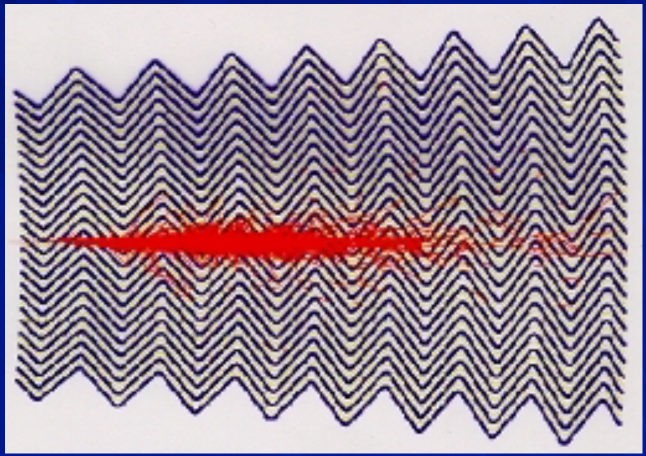
Active medium: LAr; absorber embedded in liquid serve as electrods

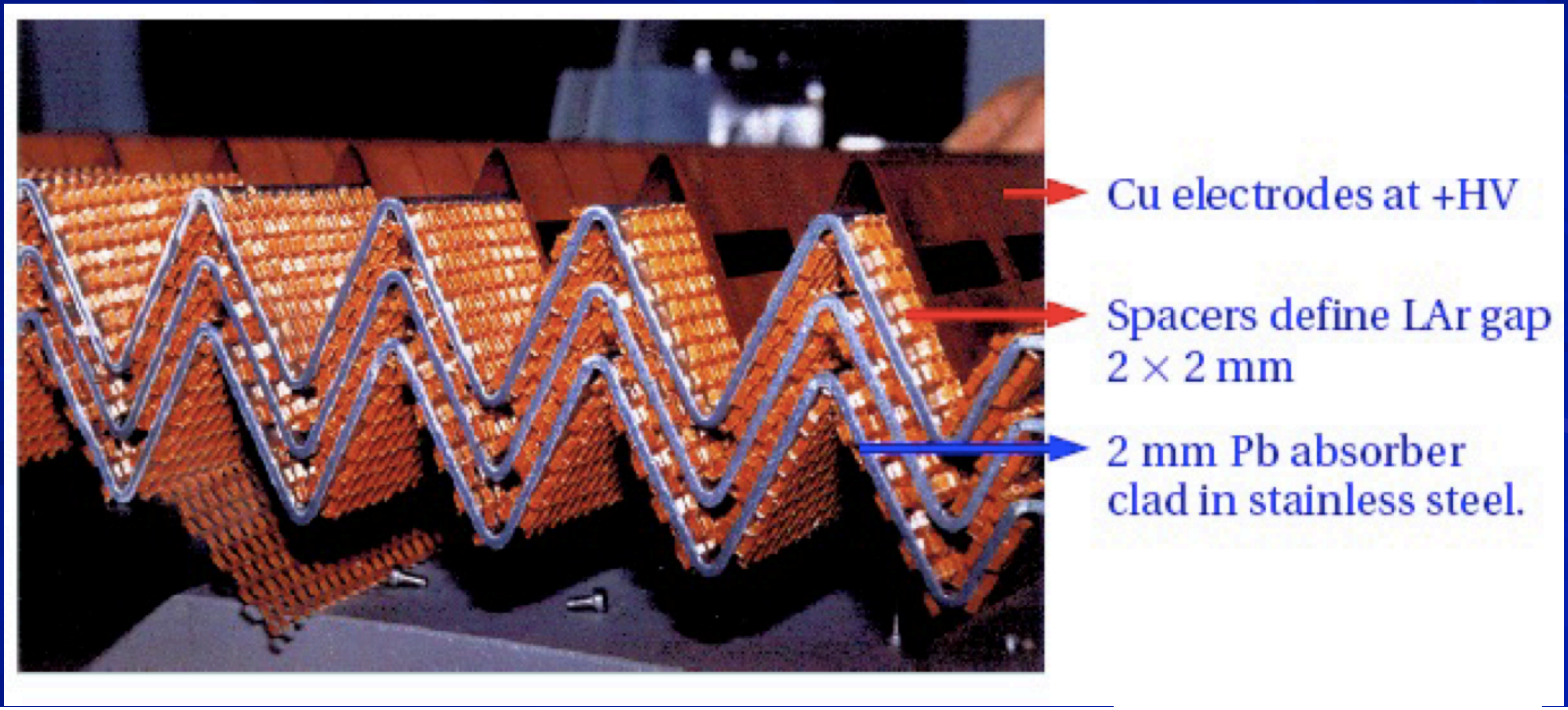
Ionization chambers between absorber plates



ATLAS Lar ECAL

- Accordion Design
  - Lead plates to initial showering
  - Ionization occurs liquid argon: drifts to sensors (electrodes on Cu/kapton sheets)
  - Fine segmentation transversely; 3 depths
  - Resolution: ~10%$E^{-1/2}$





Energy resolution

- Ideally, if all shower particles counted: $E \propto N \qquad \sigma_E \approx \sqrt{N} \approx \sqrt{E}$
- In practice

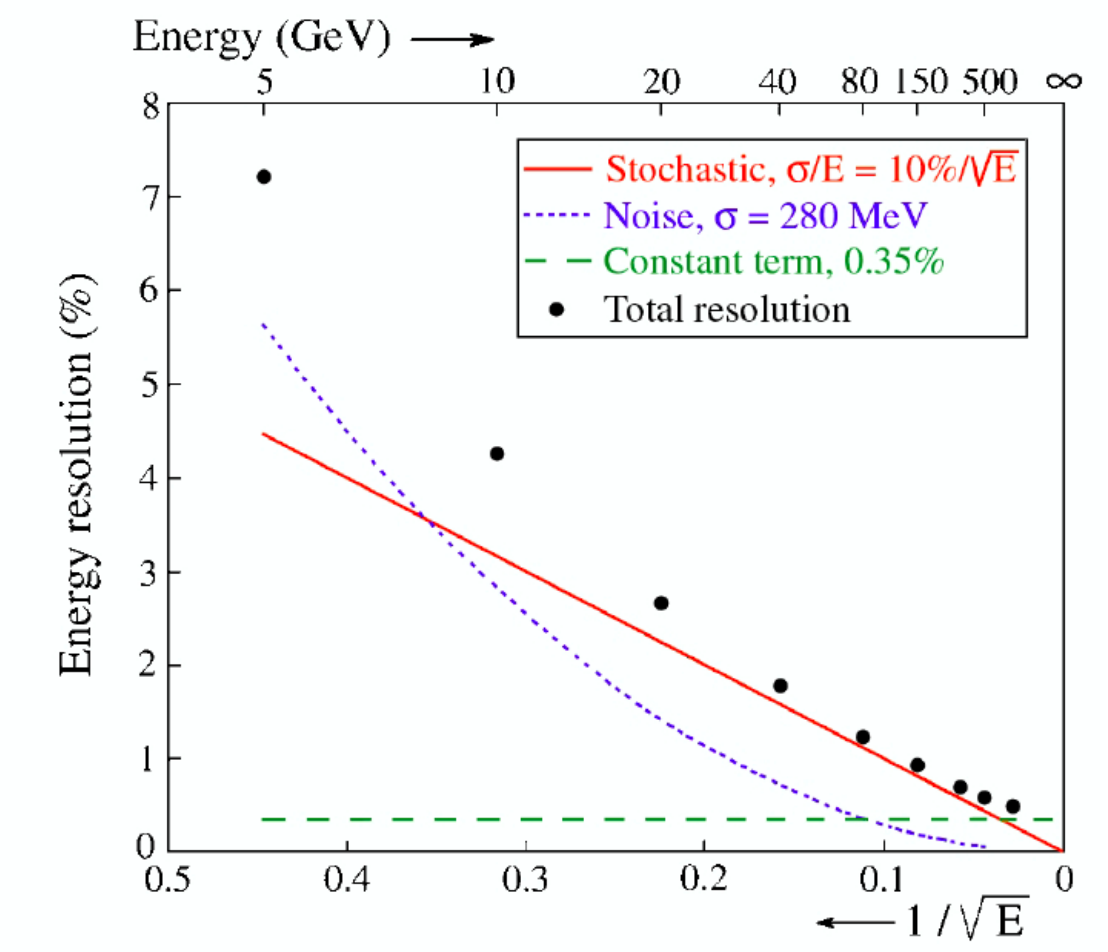
$$\sigma_E = a\sqrt{E} \oplus bE \oplus c \qquad \frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

- a: stochastic term
  - intrinsic statistical shower fluctuations
  - sampling fluctuations
  - signal quantum fluctuations (e.g. photo-electron statistics)
- b: constant term
  - inhomogeneities (hardware or calibration)
  - imperfections in calorimeter construction (dimensional variations, etc.)
  - non-linearity of readout electronics
  - fluctuations in longitudinal energy containment (leakage can also be ~ E-1/4)
  - fluctuations in energy lost in dead material before or within the calorimeter
- c: noise term
  - readout electronic noise
  - Radio-activity, pile-up fluctuations

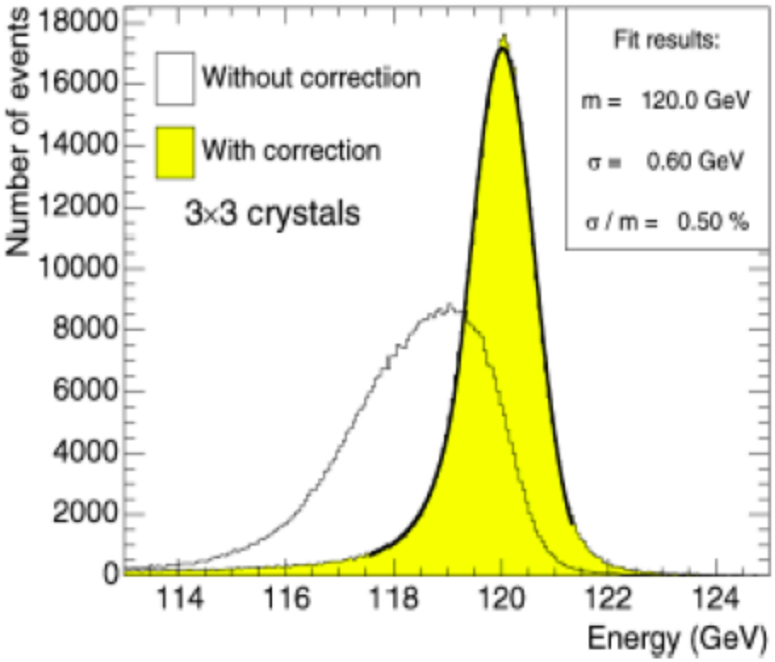
Effects on energy resolution

- Different effects have different energy dependence
  - Sampling fluctuations $\sigma/E \sim E^{-1/2}$
  - shower leakage $\sigma/E \sim E^{-1/4}$
  - electronic noise $\sigma/E \sim E^{-1}$
  - structural non-uniformities: σ/E = constant
- $\sigma^2_{tot} = \sigma^2_1 + \sigma^2_2 + \sigma^2_3 + \sigma^2_4 + \ldots$

ATLAS EM calorimeter



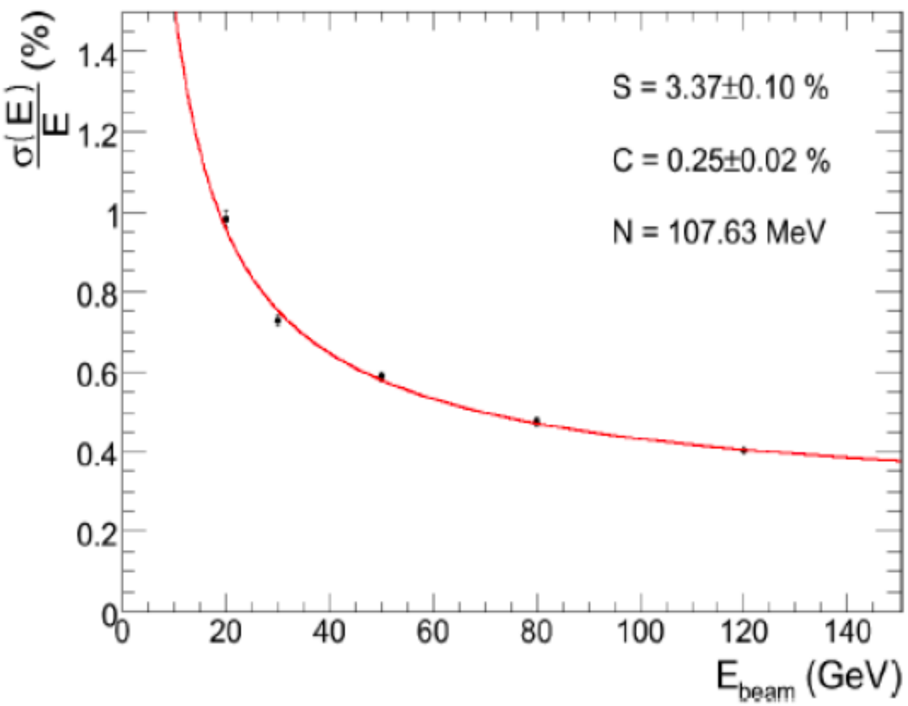
CMS ECAL resolution



Correction for radial loss

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{3.37\%}{\sqrt{E}}\right)^2 + \left(\frac{0.107}{E}\right)^2 + (0.25\%)^2$$

stoch. noise const.



The sampling term is 3 times smaller than ATLAS;
other terms are similar

Homogeneous vs Sampling

E in GeV

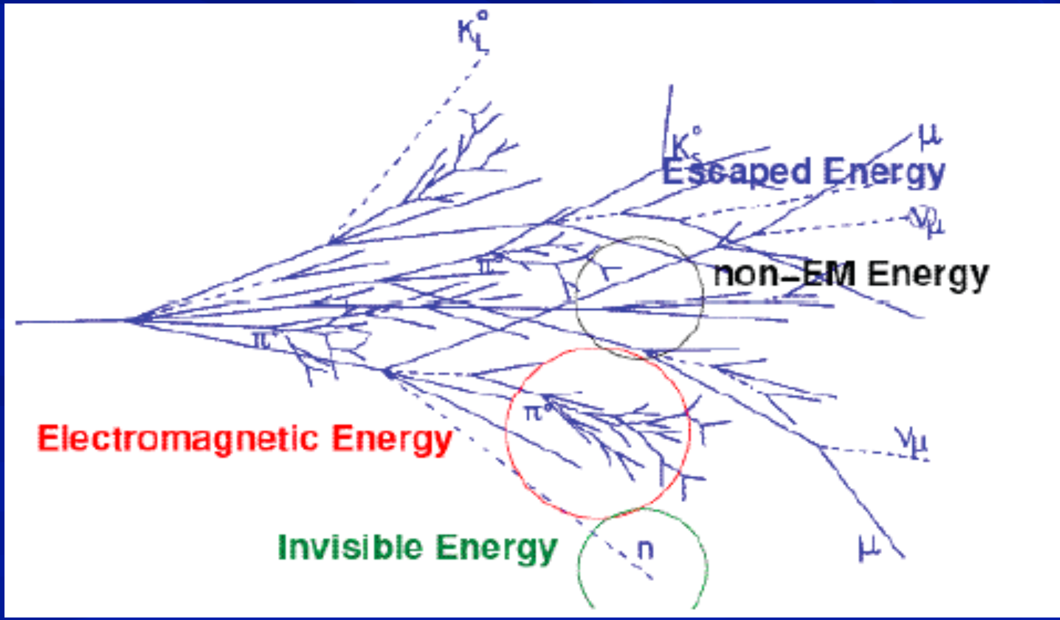
Technology (Experiment)	Depth	Energy resolution	Date
NaI(Tl) (Crystal Ball)	$20X_0$	$2.7\%/E^{1/4}$	1983
$Bi_4Ge_3O_{12}$ (BGO) (L3)	$22X_0$	$2\%/\sqrt{E} \oplus 0.7\%$	1993
CsI (KTeV)	$27X_0$	$2\%/\sqrt{E} \oplus 0.45\%$	1996
CsI(Tl) (BaBar)	$16–18X_0$	$2.3\%/E^{1/4} \oplus 1.4\%$	1999
CsI(Tl) (BELLE)	$16X_0$	1.7% for $E_\gamma > 3.5$ GeV	1998
$PbWO_4$ (PWO) (CMS)	$25X_0$	$3\%/\sqrt{E} \oplus 0.5\% \oplus 0.2/E$	1997
Lead glass (OPAL)	$20.5X_0$	$5\%/\sqrt{E}$	1990
Liquid Kr (NA48)	$27X_0$	$3.2\%/\sqrt{E} \oplus 0.42\% \oplus 0.09/E$	1998
Scintillator/depleted U (ZEUS)	$20–30X_0$	$18\%/\sqrt{E}$	1988
Scintillator/Pb (CDF)	$18X_0$	$13.5\%/\sqrt{E}$	1988
Scintillator fiber/Pb spaghetti (KLOE)	$15X_0$	$5.7\%/\sqrt{E} \oplus 0.6\%$	1995
Liquid Ar/Pb (NA31)	$27X_0$	$7.5\%/\sqrt{E} \oplus 0.5\% \oplus 0.1/E$	1988
Liquid Ar/Pb (SLD)	$21X_0$	$8\%/\sqrt{E}$	1993
Liquid Ar/Pb (H1)	$20–30X_0$	$12\%/\sqrt{E} \oplus 1\%$	1998
Liquid Ar/depl. U (DØ)	$20.5X_0$	$16\%/\sqrt{E} \oplus 0.3\% \oplus 0.3/E$	1993
Liquid Ar/Pb accordion (ATLAS)	$25X_0$	$10\%/\sqrt{E} \oplus 0.4\% \oplus 0.3/E$	1996

Homogeneous

Sampling

Hadron Showers

- Hadrons interact with detector material also through the strong interaction
- Hadron calorimeter measurement:
  - Charged hadrons: complementary to track measurement
  - Neutral hadrons: the only way to measure their energy
- In nuclear collisions many secondary particles are produced
  - Secondary, tertiary nuclear reactions ➔ hadronic cascades
  - Electromagnetically decaying particles (**π,η**) initiate EM shower
  - Energy can also be absorbed as nuclear binding energy or target recoil (Invisible energy)
- Similar to EM showers, but more complex ➔need simulation tools (MC)
- Characterized by the hadronic interaction length



Hadronic shower

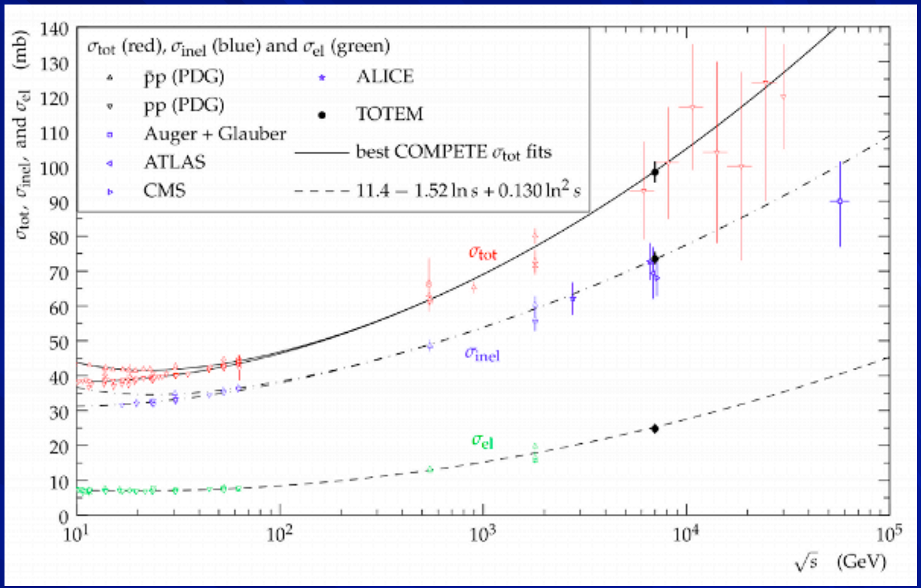
- Hadronic interaction Cross section

$$\sigma_{Tot} = \sigma_{el} + \sigma_{inel}$$

$$\sigma_{el} \approx 10mb \qquad \sigma_{inel} \approx A^{2/3}$$

$$\sigma_{Tot} = \sigma_{tot}(pp)A^{2/3}$$

where: $\sigma_{tot}(pp)$ increases with $\sqrt{s}$



- Hadronic interaction length

$$\lambda_{\text{int}} = \frac{1}{\sigma_{tot} \cdot n} = \frac{A\rho}{\sigma_{pp} A^{2/3} N_A} \approx \left(35g/cm^2\right) A^{1/3}$$

$$N(x) = N(0)e^{-x/\lambda_{\text{int}}}$$

- λ_{int} characterizes both longitudinal and transverse shower profile

Rule of thumb argument: the geometric cross section goes as the square of the size of the nucleus, a_N^2, and since the nuclear radius scales as $a_N \sim A^{1/3}$, the nuclear mean free path in gm/cm^2 units scales as $A^{1/3}$.

Hadronic vs EM showers

Hadronic vs. electromagnetic interaction length:

$$X_0 \sim \frac{A}{Z^2}$$

$$\lambda_{\text{int}} \sim A^{1/3}$$

$$\rightarrow \frac{\lambda_{\text{int}}}{X_0} \sim A^{4/3}$$

$$\lambda_{\text{int}} \gg X_0$$

[λ_{int}/X_0 > 30 possible; see below]

Typical
Longitudinal size: 6 ... 9 λ_{int} [95% containment] [EM: 15-20 X_0]

Typical
Transverse size: one λ_{int} [95% containment] [EM: 2 R_M; compact]

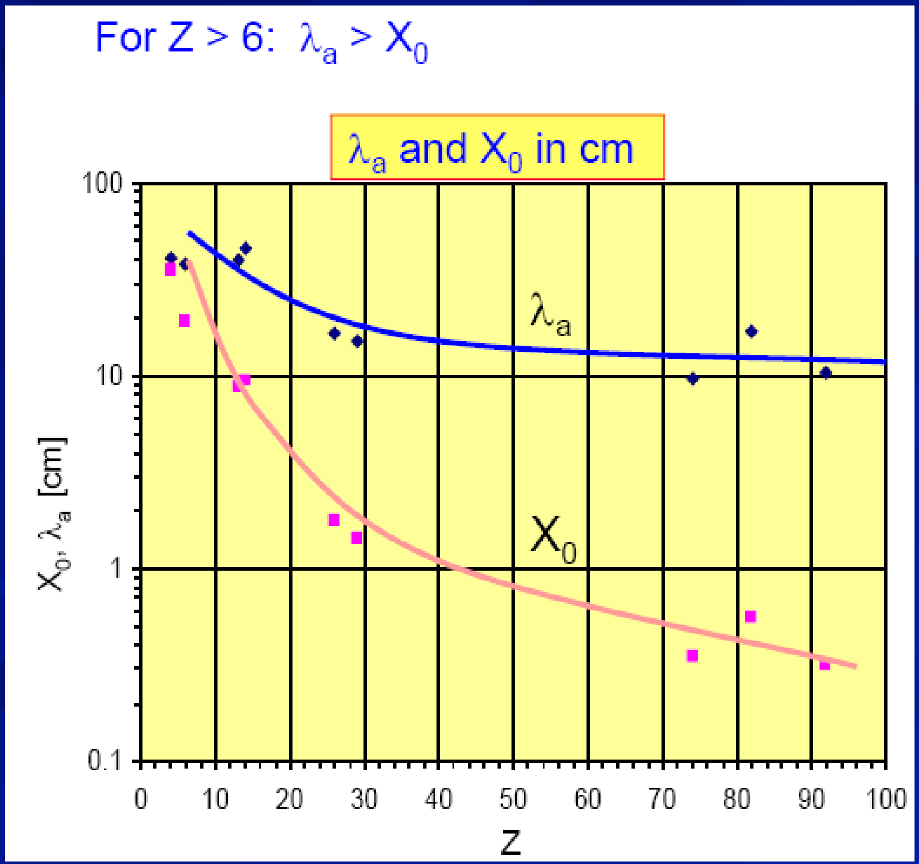
Hadronic calorimeter need more depth than electromagnetic calorimeter ...

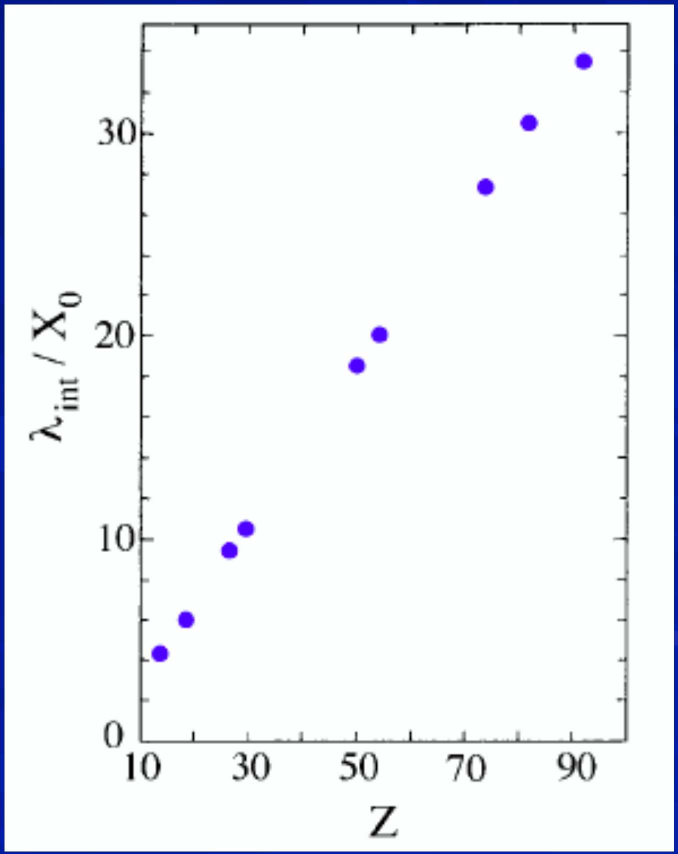
Some numerical values for materials typical used in hadron calorimeters

	λ_{int} [cm]	X_0 [cm]
Szint.	79.4	42.2
LAr	83.7	14.0
Fe	16.8	1.76
Pb	17.1	0.56
U	10.5	0.32
C	38.1	18.8

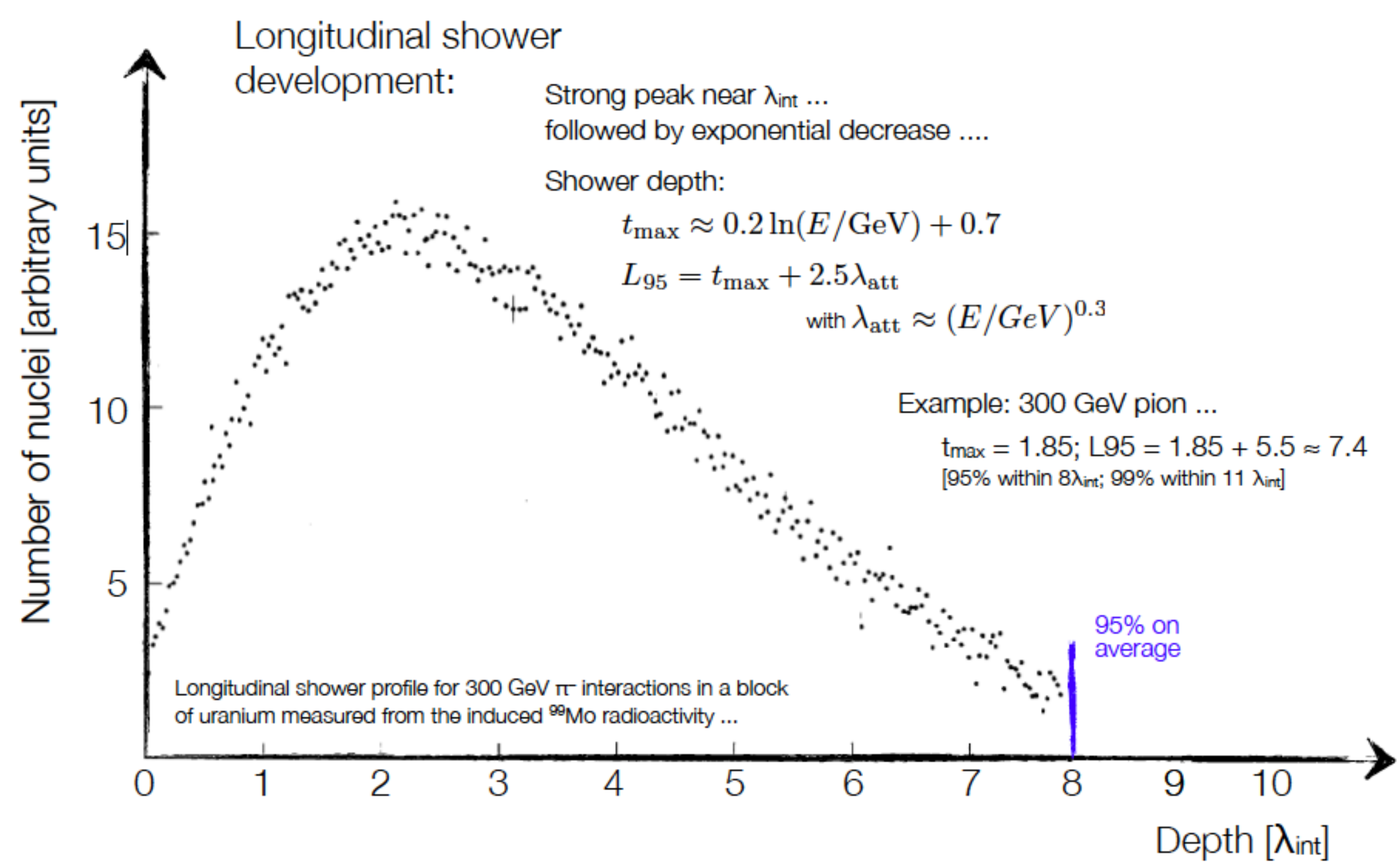
Material dependence

- λ_{int}: mean free path between nuclear collisions
- λ_{int} (g cm^{-2}) $\propto A^{1/3}$
- Hadron showers are much longer than EM ones. Length depends on Z

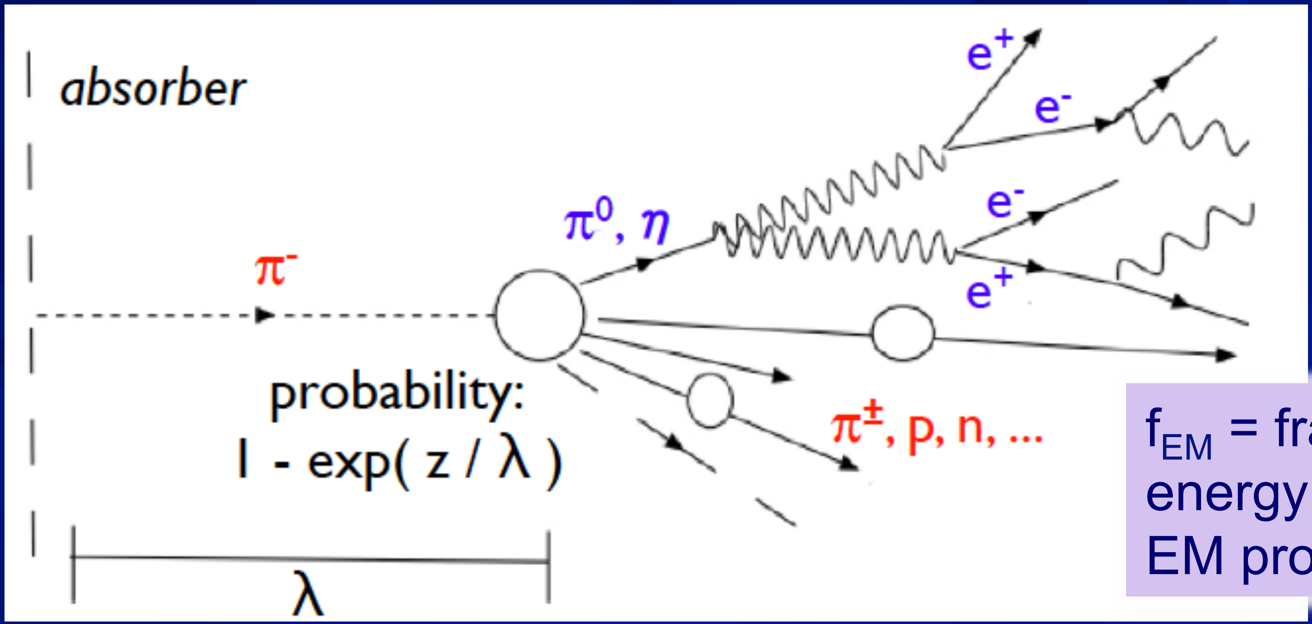




Hadronic shower: Longitudinal development



Hadronic Shower



π^0 can deposit energy via EM processes

f_{EM} = fraction of hadron energy deposited via EM processes

- Electromagnetic
  - ionization, excitation (e±)
  - photo effect, scattering (γ)
- Hadronic
  - ionization (π±, p)
  - invisible energy (binding, recoil)

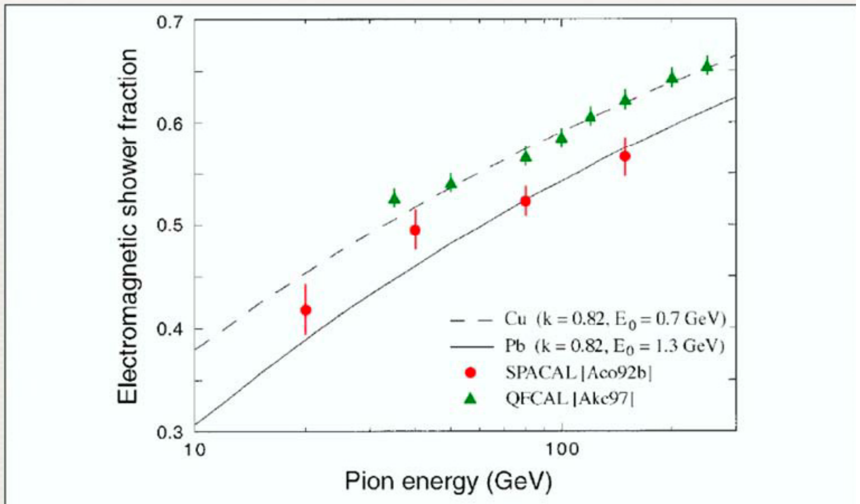


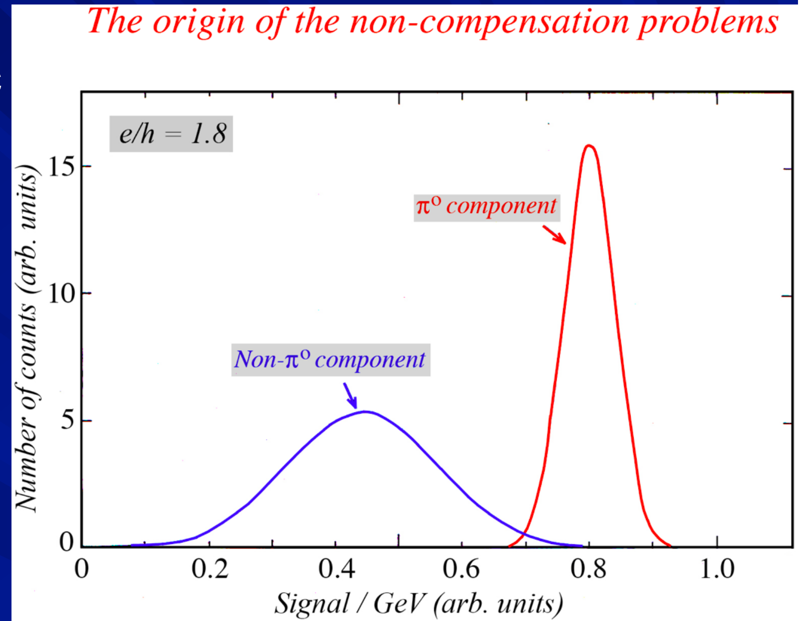
FIG. 2.22. Comparison between the experimental results on the em fraction of pion-induced showers in the (copper-based) QFCAL and (lead-based) SPACAL detectors. Data from [Akc 97] and [Aco 92b].

EM fraction in hadronic calorimeters

Charge conversion of $\pi^{+/-}$ produces electromagnetic component of hadronic shower (π^0)

- e = response to the EM shower component
- h = response to the non-EM component

$$\pi = f_{em} e + (1-f_{em}) h$$

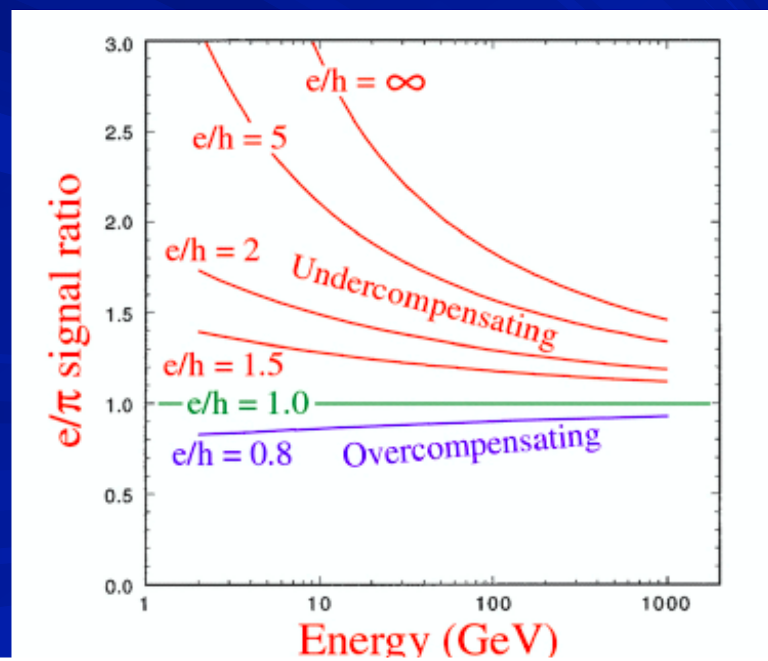


Comparing pion and electron showers:

$$\frac{e}{\pi} = \frac{e}{f_{em} e + (1-f_{em}) h} = \frac{e}{h} \cdot \frac{1}{1 + f_{em}(e/h-1)}$$

Calorimeters can be:

- Overcompensating e/h < 1
- Undercompensating e/h > 1
- Compensating e/h = 1



Compensation

- Non-linearity determined by e/h value of the calorimeter
- Measurement of non-linearity is one of the methods to determine e/h
- Assuming linearity for EM showers, e(E1)=e(E2):

$$\frac{\pi(E_1)}{\pi(E_2)} = \frac{f_{em}(E_1)+[1-f_{em}(E_1)]\cdot e/h}{f_{em}(E_2)+[1-f_{em}(E_2)]\cdot e/h}$$

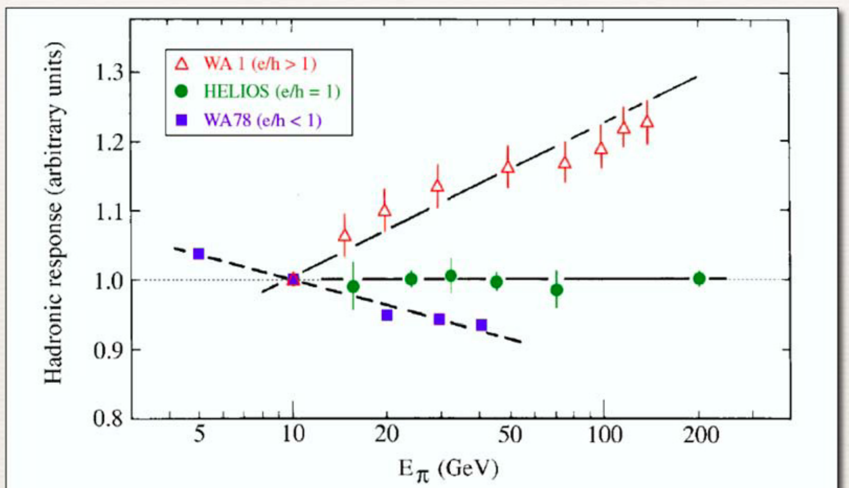
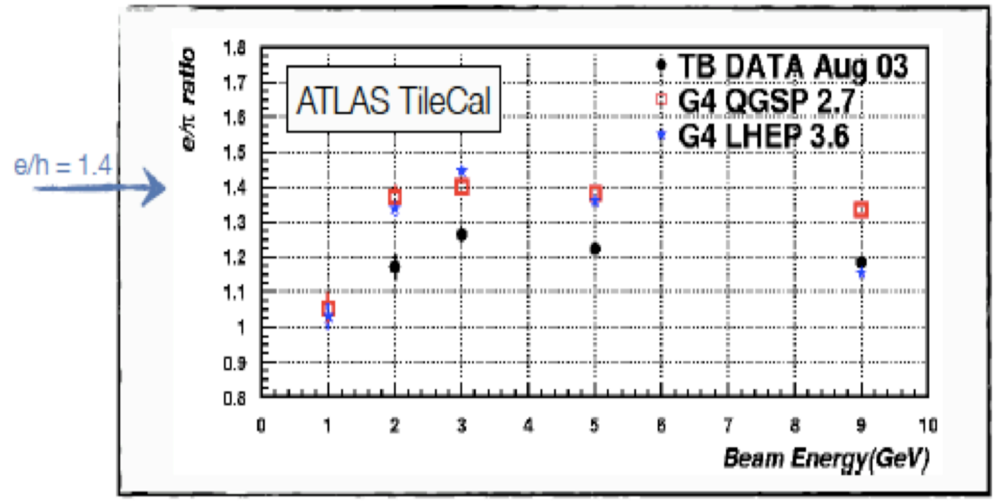


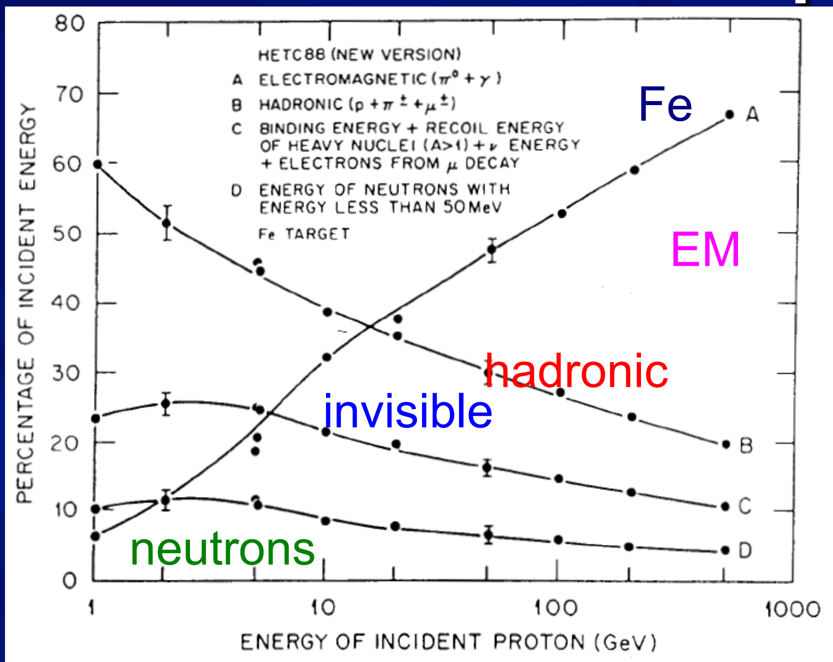
FIG. 3.14. The response to pions as a function of energy for three calorimeters with different e/h values: the WA1 calorimeter ($e/h > 1$, [Abr 81]), the HELIOS calorimeter ($e/h \approx 1$, [Ake 87]) and the WA78 calorimeter ($e/h < 1$, [Dev 86, Cat 87]). All data are normalized to the results for 10 GeV.

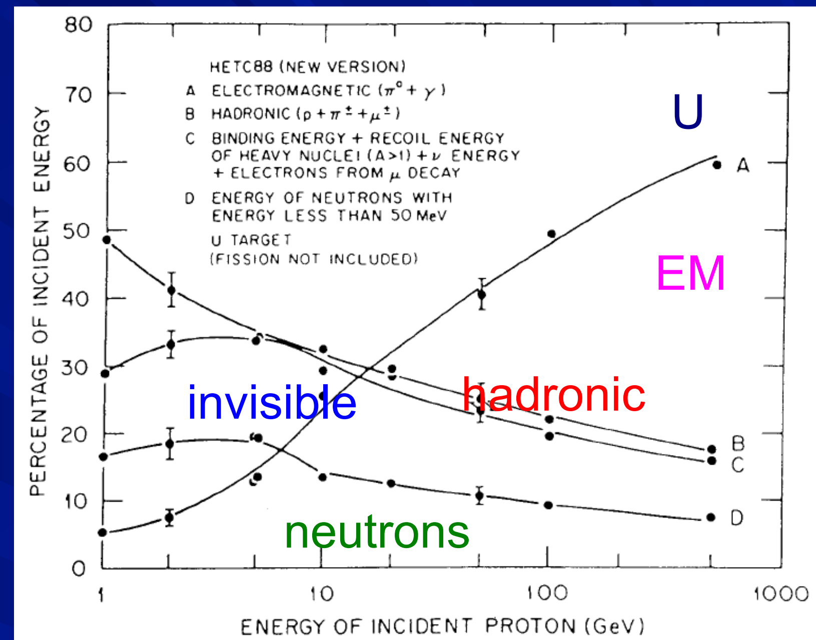
For e/h=1 ➔ $\frac{\pi(E_1)}{\pi(E_2)} = 1$



- Response of calorimeters is usually higher for electromagnetic (e) than hadronic (h) energy deposits➔e/h>1

Compensation





$$E_p = f_{em} e + (1 - f_{em}) h$$

$$h = f_{rel} \cdot rel + f_p \cdot p + f_n \cdot n + f_{inv} \cdot inv$$

Energy deposition mechanisms

- f_{rel}= Ionization by charged pions (relativistic shower component)
- f_p=spallation protons
- f_n=neutrons evaporation
- f_{inv}=invisible energy by recoil nuclei

- Compensation:
  - Tuning the neutron response using hydrogenous active material (L3 Uranium/gas calorimeter)
  - Compensation adjusting the sampling frequency

Compensation by tuning neutron response

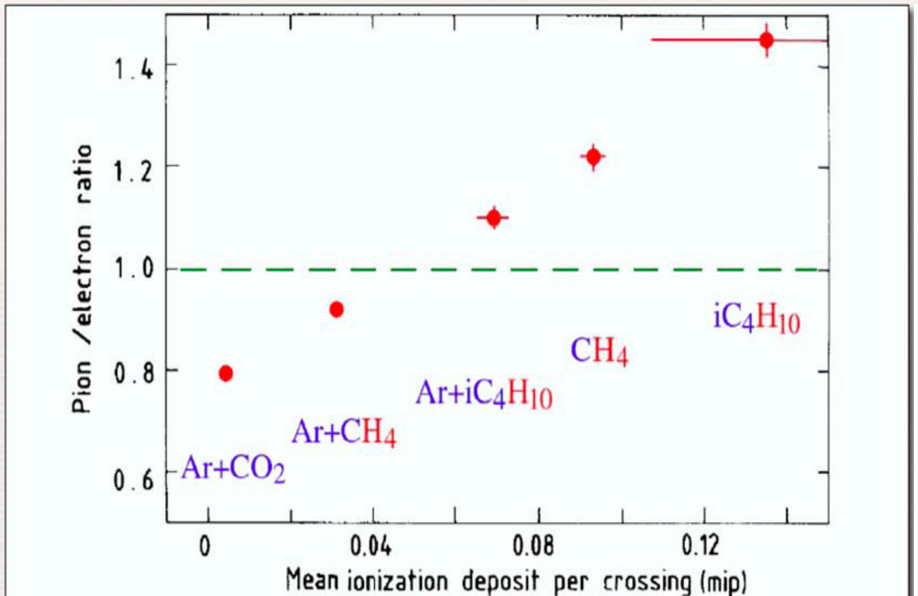
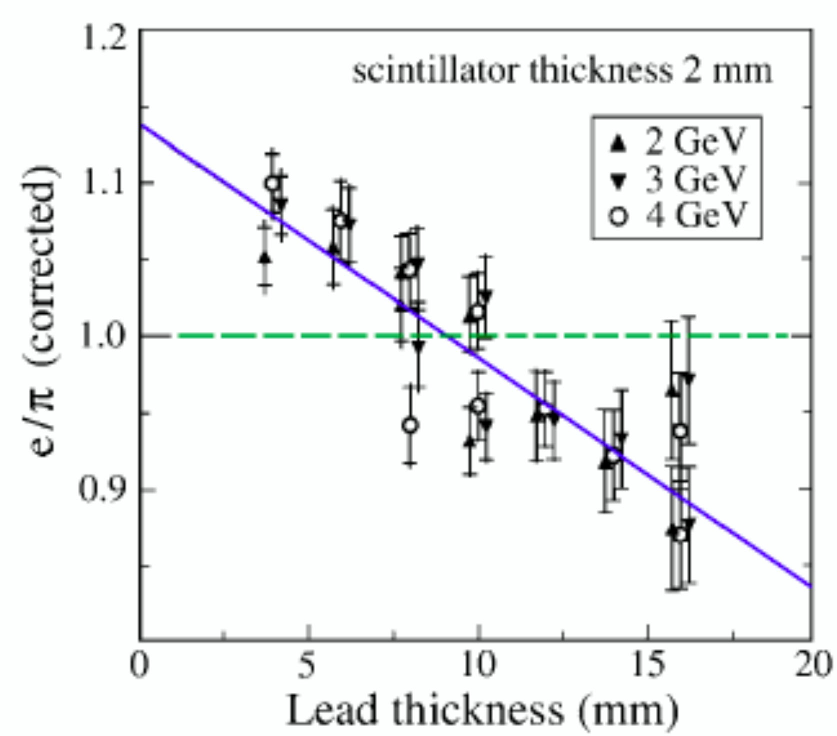
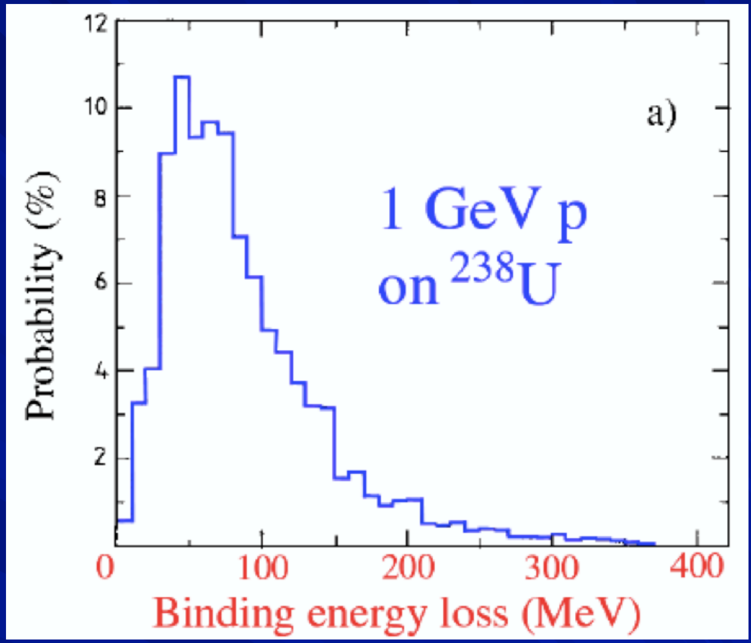


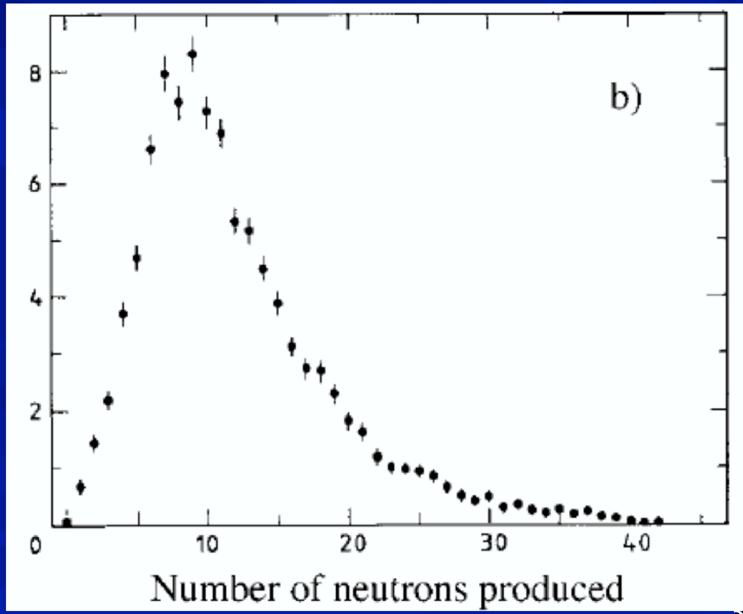
FIG. 3.32. The pion/electron signal ratio, averaged over the energy range 1 5 GeV, measured for different gas mixtures with the uranium/gas calorimeter of the L3 Collaboration. The horizontal scale gives the (calculated) average energy deposit in a chamber gap by slow neutrons [Gal 86].



Energy resolution of hadronic showers

- Fluctuations in visible energy (ultimate limit of hadronic energy resolution)
  - fluctuations of nuclear binding energy loss in high-Z materials ~15%
- Fluctuations in the EM shower fraction, f_{em}
  - Dominating effect in most hadron calorimeters (e/h >1)
  - Fluctuations are asymmetric in pion showers
  - Differences between p, π induced showers (No leading π^0 in proton showers)
- Sampling fluctuations only minor contribution to hadronic resolution in non-compensating calorimeter





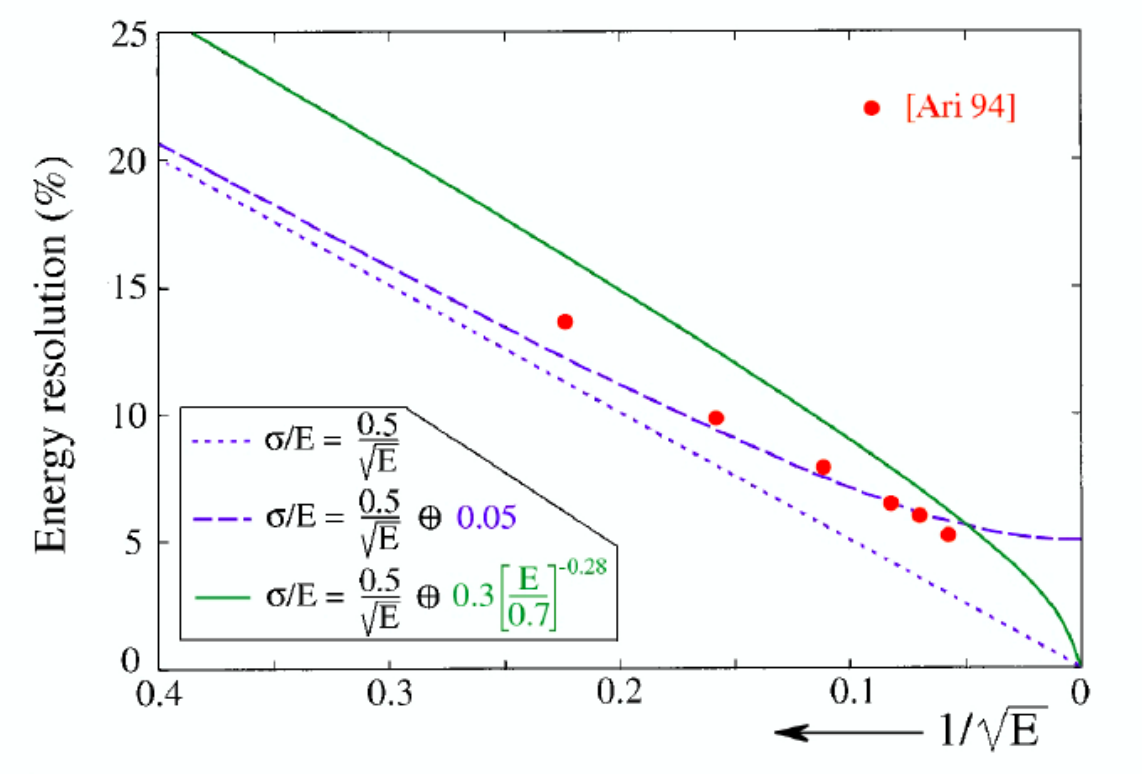
Energy resolution of hadron showers

- Hadronic energy resolution of non-compensating calorimeters does not scale with $1/\sqrt{E}$ but as

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b\left(\frac{E}{E_0}\right)$$

- But in practice we use

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus b$$



A realistic calorimetric system

Typical Calorimeter: two components ...

Electromagnetic (EM) +
Hadronic section (Had) ...

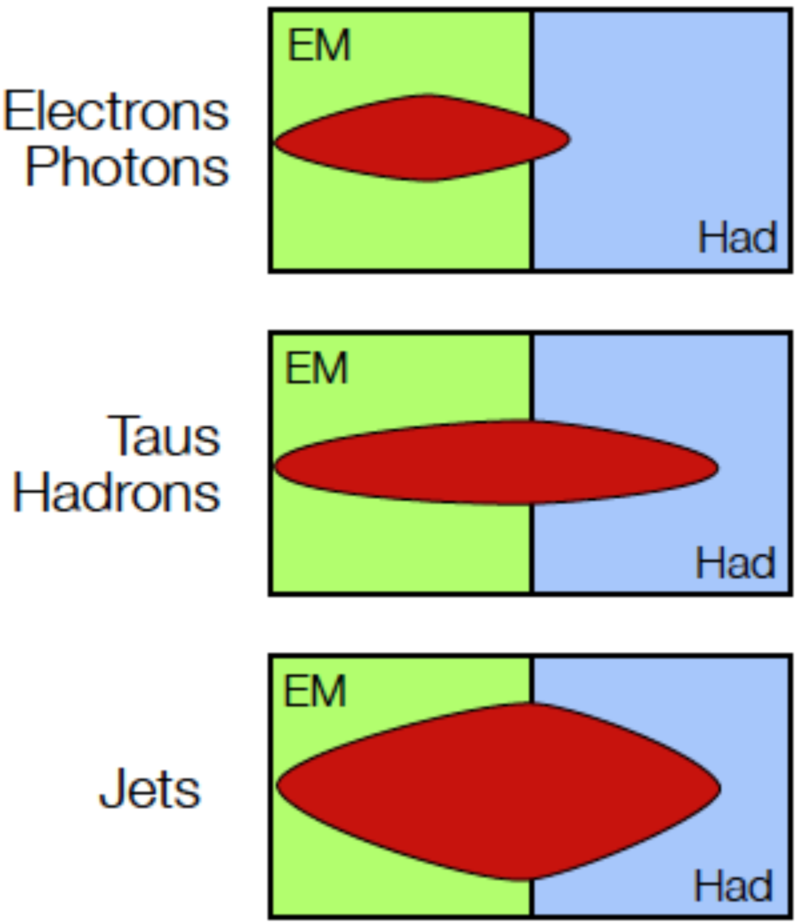
Different setups chosen for
optimal energy resolution ...

But:

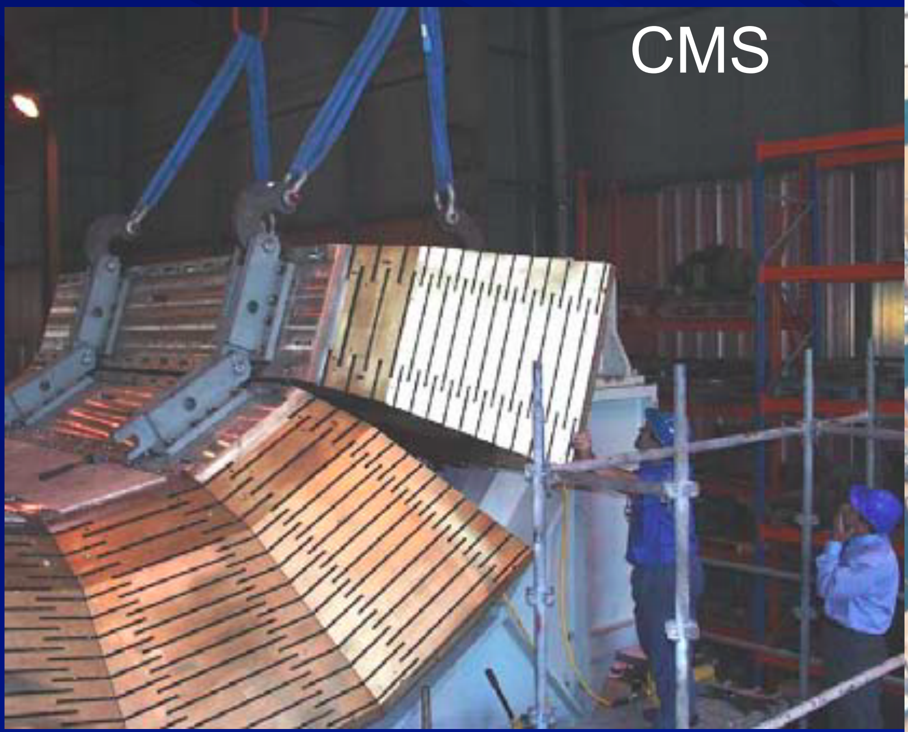
Hadronic energy measured in
both parts of calorimeter ...

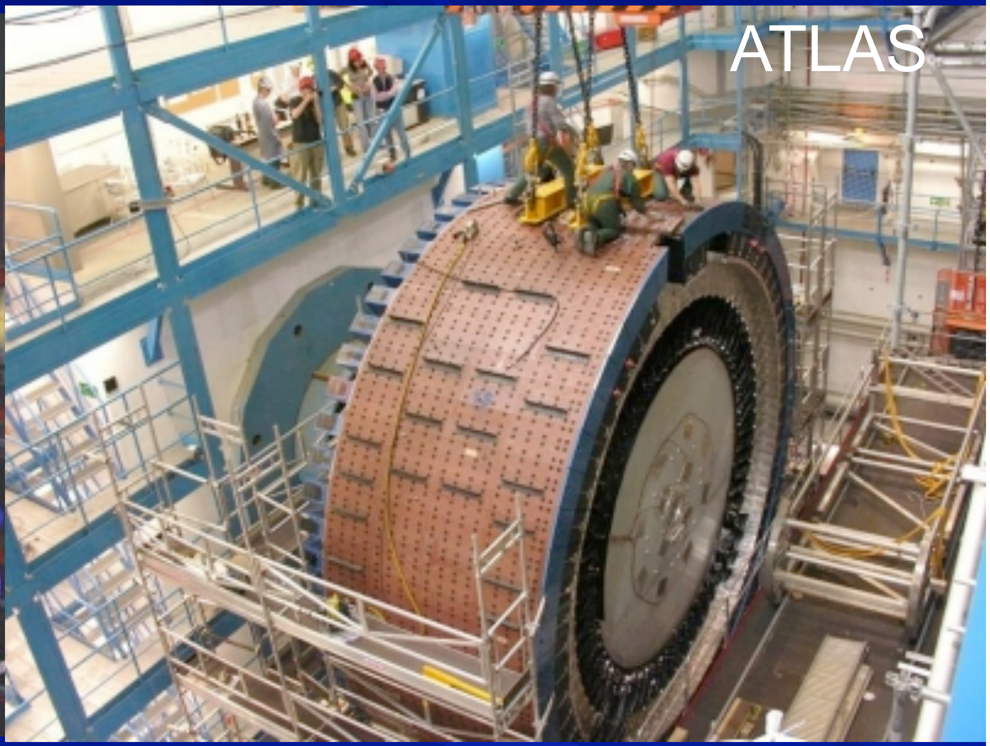
Needs careful consideration of
different response ...

Schematic of a
typical HEP calorimeter



LHC CALORIMETERS





5 cm brass / 3.7 cm scint.
Embedded fibres, HPD readout

14 mm iron / 3 mm scint.
sci. fibres, read out by phototubes

Hadronic calorimeters resolution

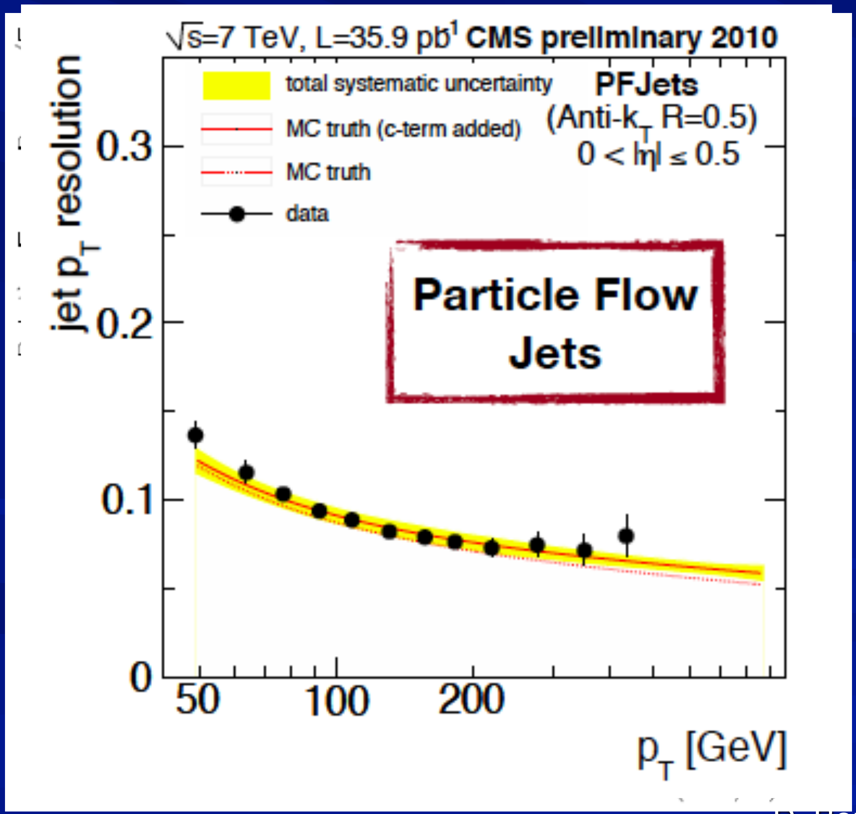
- HCAL only

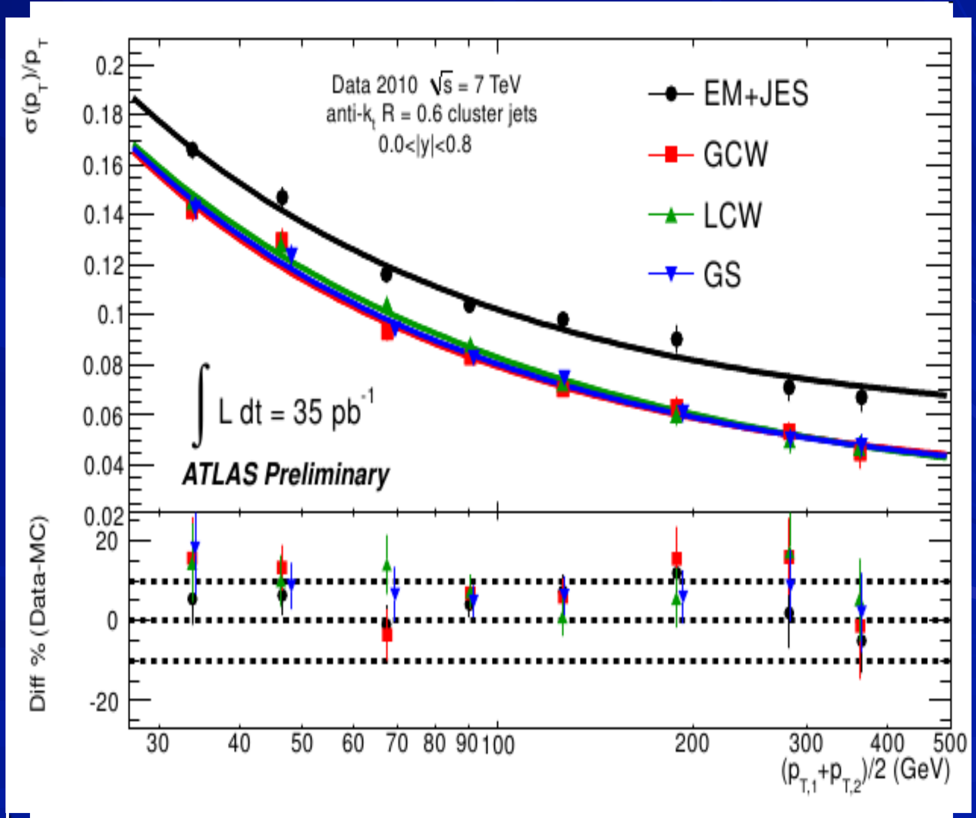
  $\sigma/E = (93.8 \pm 0.9)\%/\sqrt{E} \oplus (4.4 \pm 0.1)\%$
- ECAL+HCAL

  $\sigma/E = (82.6 \pm 0.6)\%/\sqrt{E} \oplus (4.5 \pm 0.1)\%$

- Improved resolution using full calorimetric system (ECAL+HCAL)

ATLAS LAr + Tile for pions: $\frac{\sigma(E)}{E} = \frac{42\%}{\sqrt{E}} \oplus 2\%$



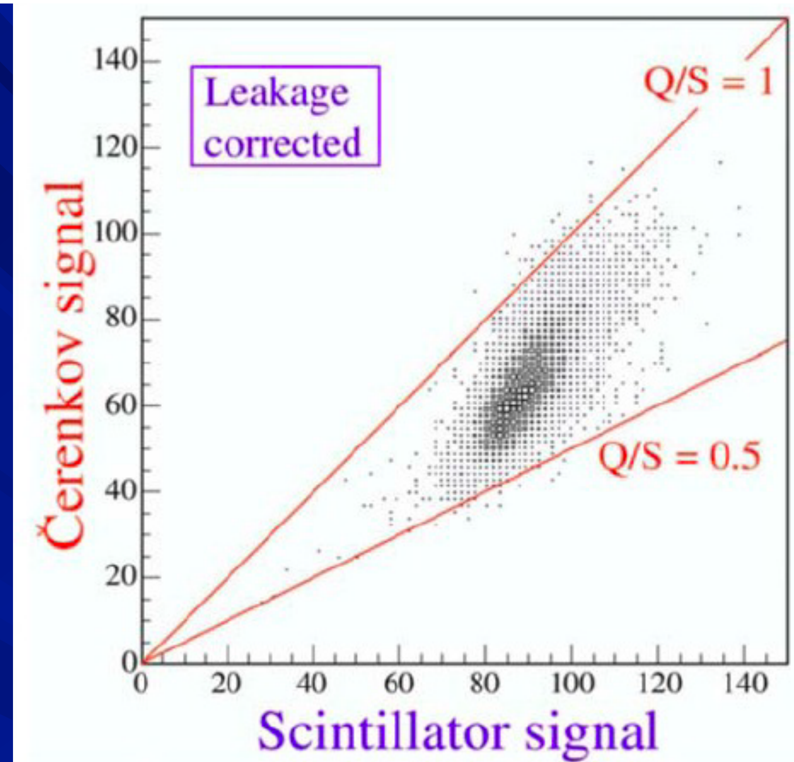


Future calorimeters

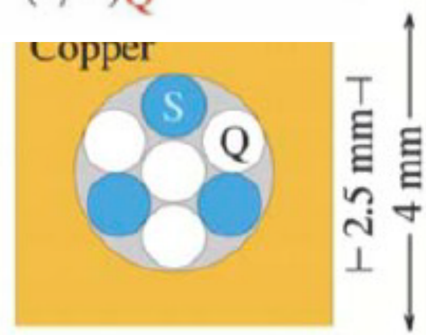
- Concentrate on improvement of jet energy resolution to match the requirement of the new physics expected in the next 30-50 years:
- Two approaches:
  - minimize the influence of the calorimeter and measure jets using the combination of all detectors ➔ Particle Flow
  - measure the shower hadronic shower components in each event & weight directly access the source of fluctuations ➔Dual (Triple) Readout

DREAM

- Measure f_{EM} cell-by-cell by comparing Cherenkov and dE/dx signals
- Densely packed SPAgetti CALorimeter with interleaved Quartz (Cherenkov) and Scintillating Fibers
- Production of Cerenkov light only by em particles (f_{EM})
- **Aim at: $\sigma_E/E \sim 15\%/\sqrt{E}$**

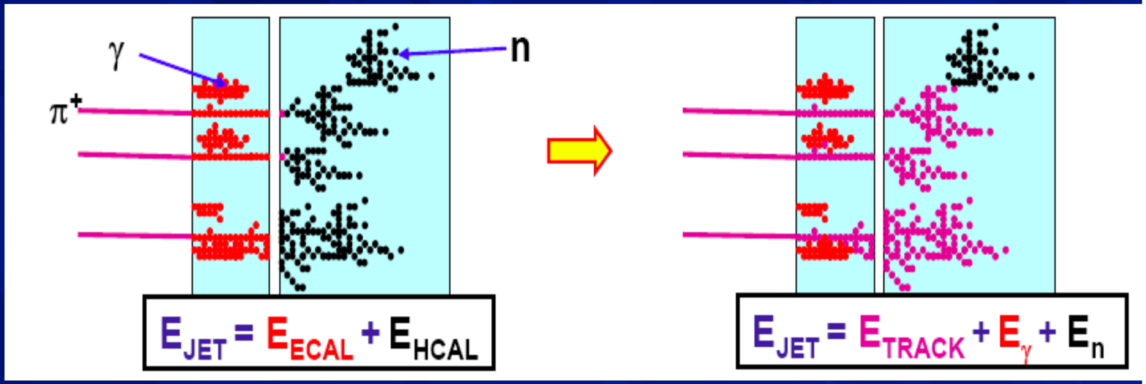


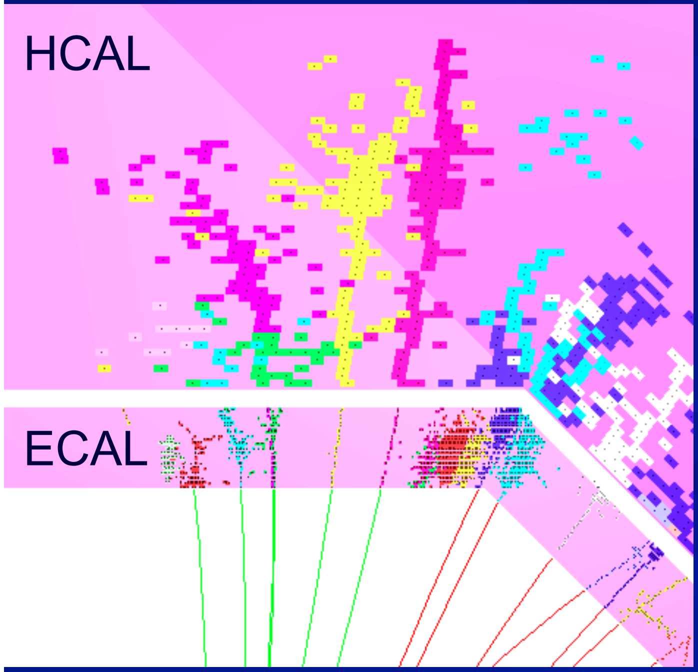
$$S = E\left[f_{em} + \frac{1}{(e/h)_S}(1 - f_{em})\right]$$
$$Q = E\left[f_{em} + \frac{1}{(e/h)_Q}(1 - f_{em})\right]$$



PF calorimetry (CALICE)

- Design detectors for Pflow
  - ECAL and HCAL: inside solenoids
  - Low mass tracker
  - High granularity for imaging calorimetry
  - It also require sophisticated software

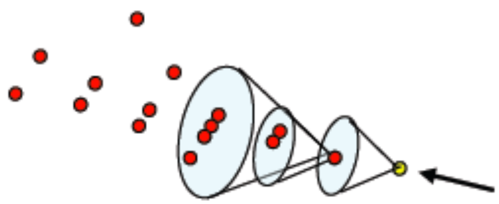




- Two proto-collaborations for ILC (ILD and SLD)
  - ECAL: Highly segmented SIW or Scintillator-W sampling calorimeters
    - Transverse segmentation: ~5 x 5 mm^2
    - ~30 longitudinal sampling layers
  - HCAL: Highly segmented sampling calorimeters Steel or W absorber+ active material (RPC, GEM)
    - Transverse segmentation: 1x1 cm^2 – 3x3 cm^2
    - ~50 Longitudinal sampling layers !
  - Aiming at $\sigma_E/E < 3.5\%$

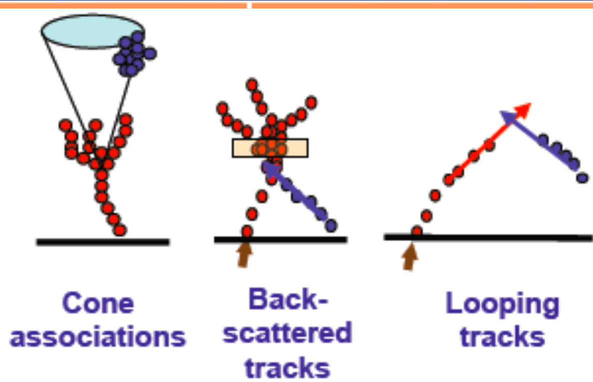
Particle flow

Mark Thomson

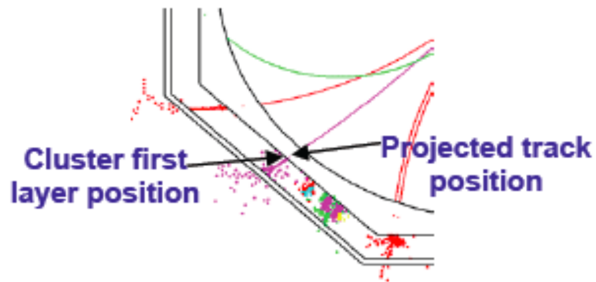


ConeClustering Algorithm

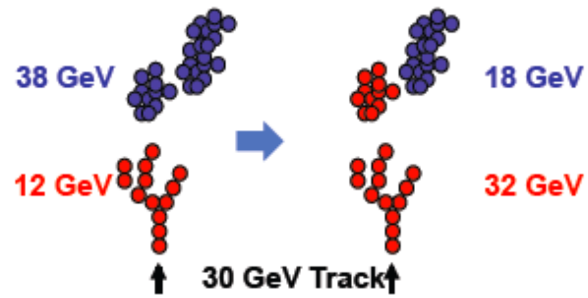
Topological Association Algorithms



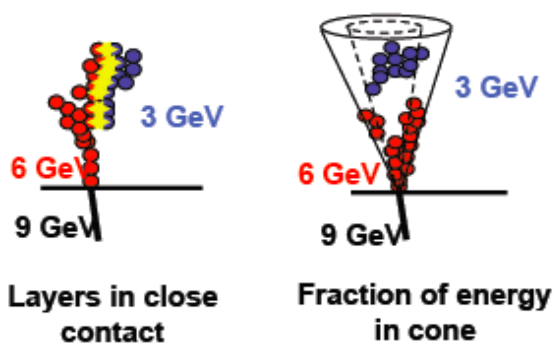
Track-Cluster Association Algorithms



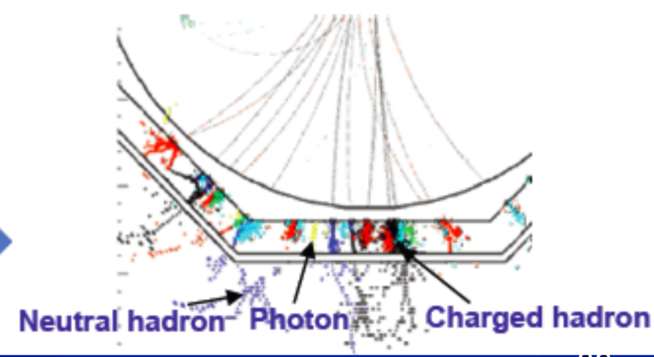
Reclustering Algorithms



Fragment Removal Algorithms



PFO Construction Algorithms

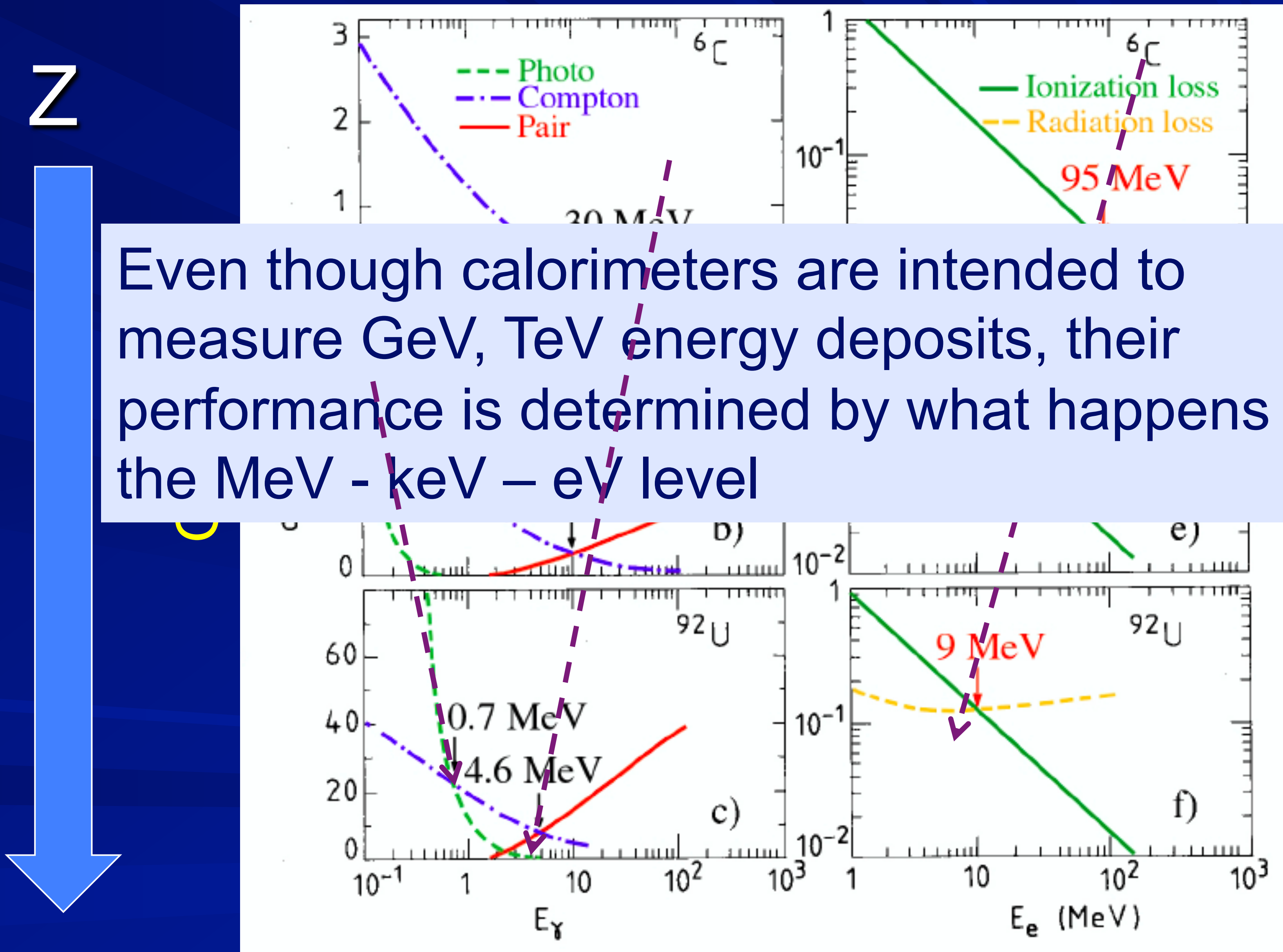


References

- Particle flow- M. Thompson
- Calorimetry for Particle Physics- C. Fabjan and F. Gianotti- CERN-EP/ 2003-075

- BACKUP

Material dependence



Even though calorimeters are intended to measure GeV, TeV energy deposits, their performance is determined by what happens at the MeV - keV – eV level

Summary

Problem:
Calculate how much Pb, Fe or Cu is needed to stop a 10 GeV electron.

Pb : Z=82 , A=207, ρ=11.34 g/cm^3
Fe : Z=26 , A=56, ρ=7.87 g/cm^3
Cu : Z=29 , A=63, ρ=8.92 g/cm^3

Radiation length:

$$X_0 = \frac{180A}{Z^2} \frac{\mathrm{g}}{\mathrm{cm}^2}$$

Critical energy:
[Attention: Definition of Rossi used]

$$E_c = \frac{550\ \mathrm{MeV}}{Z}$$

Shower maximum:

$$t_{\max} = \ln \frac{E}{E_c} - \begin{cases} 1.0 & e^- \text{ induced shower} \\ 0.5 & \gamma \text{ induced shower} \end{cases}$$

Longitudinal
energy containment:

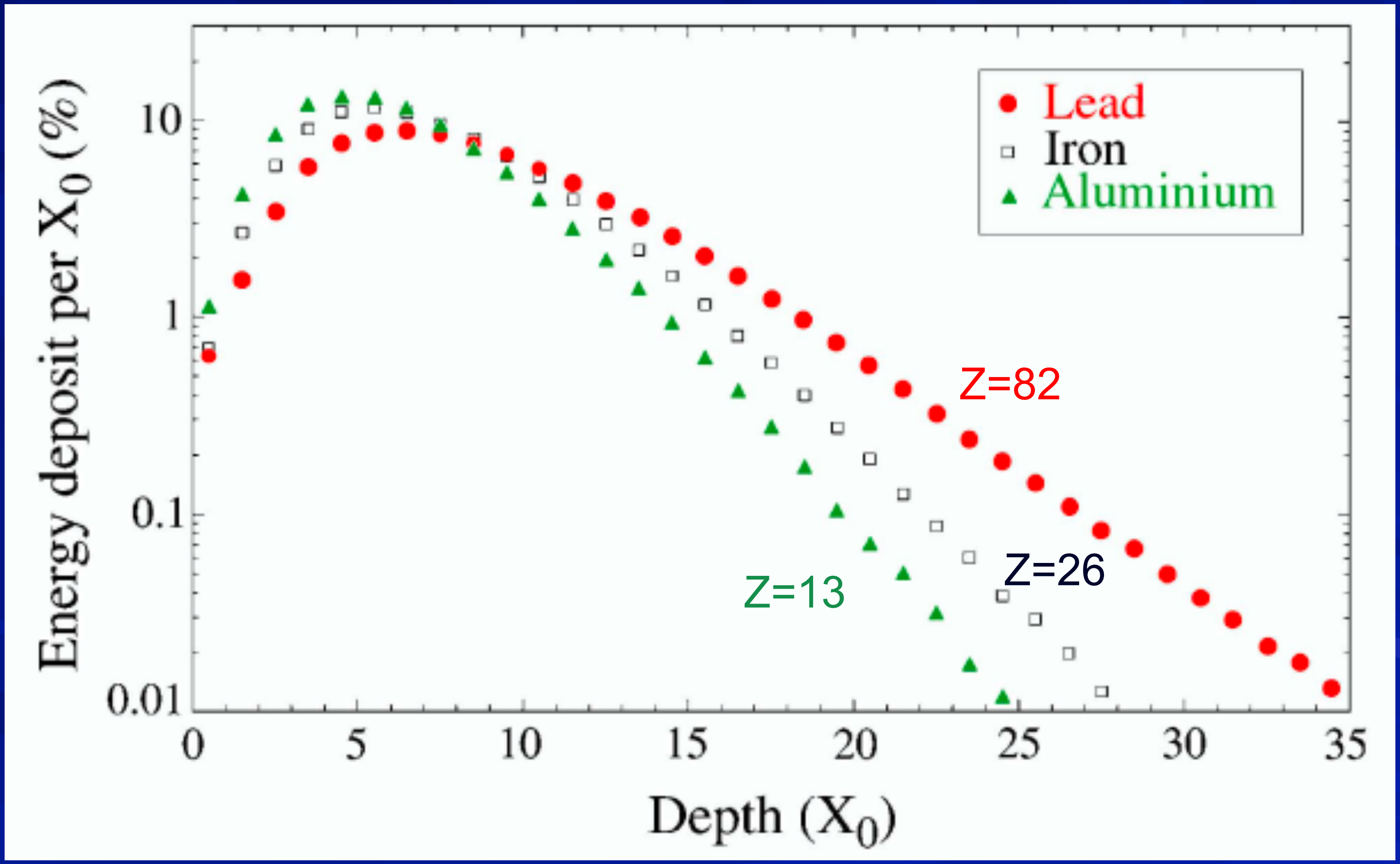
$$L(95\%) = t_{\max} + 0.08Z + 9.6\ [X_0]$$

Transverse
Energy containment:

$$R(90\%) = R_M$$
$$R(95\%) = 2R_M$$

Longitudinal development of EM shower

- Shower decay: after the shower maximum the shower decays slowly through ionization and Compton scattering → proportional to X_0



Resolution in Homogenous calorimeters

- Homogeneous calorimeters: signal = sum of all E deposited by charged particles with $E>E_{threshold}$
- If W is the mean energy required to produce a 'signal quantum' (eg an electron-ion pair in a noble liquid or a 'visible' photon in a crystal) the mean number of 'quanta' produced is $\langle n \rangle = E / W$
- The intrinsic energy resolution is given by the fluctuations on n.

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{E/W}}$$

i.e. in a semiconductor crystals W ≈ 3 eV (to produce e-hole pair)

1 MeV γ = 350000 electrons➔ $1/\sqrt{n} = 0.17\%$ stochastic term

- Fluctuations on n are reduced by correlation in the production of consecutive e-hole pairs: the Fano factor F

$$\frac{\sigma_E}{E} = \frac{1}{\sqrt{FE/W}}$$

The Fano factor depends on the material

Resolution in Sampling calorimeters

- Main contribution: sampling fluctuations, from variations in the number of charged particles crossing the active layers.
- Increases linearly with incident energy and with the finess of the sampling.
- Thus:

  $n_{ch} \propto E / t$ where (is the thickness of each absorber layer)

- For statistically independent sampling the sampling contribution to the stochastic term is:

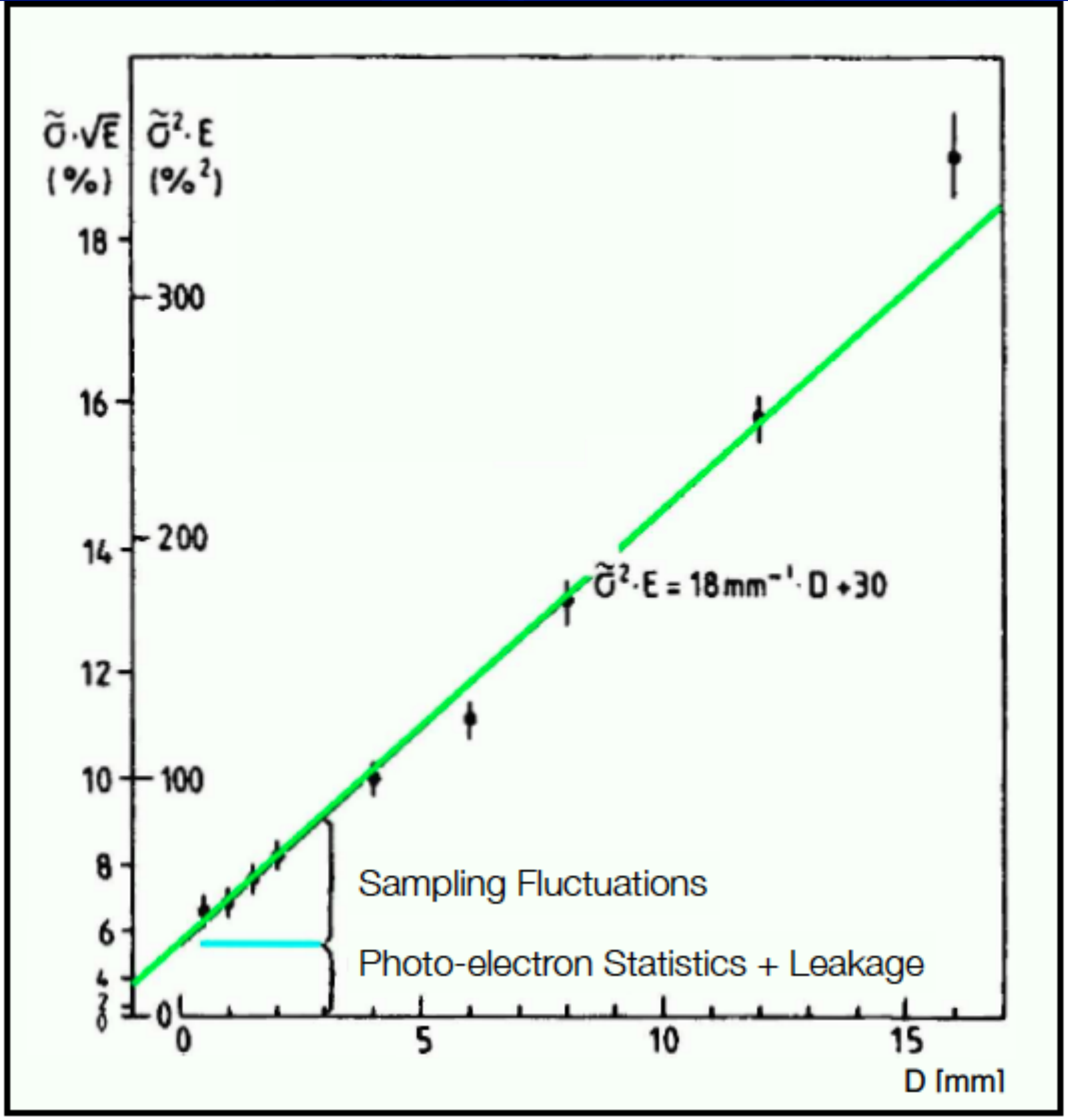
$$\frac{\sigma_{samp}}{E} = \frac{1}{\sqrt{n_{ch}}} \propto \sqrt{\frac{t}{E}}$$

- Thus the resolution improves as **t** is decreased.
- For EM calorimeters the 100 samplings required to approach the resolution of homogeneous devices is not feasible
- **Typically**

$$\frac{\sigma_{samp}}{E} = \frac{10\%}{\sqrt{E}}$$

Dependence on sampling

Measure energy resolution of a sampling calorimeter for different absorber thicknesses



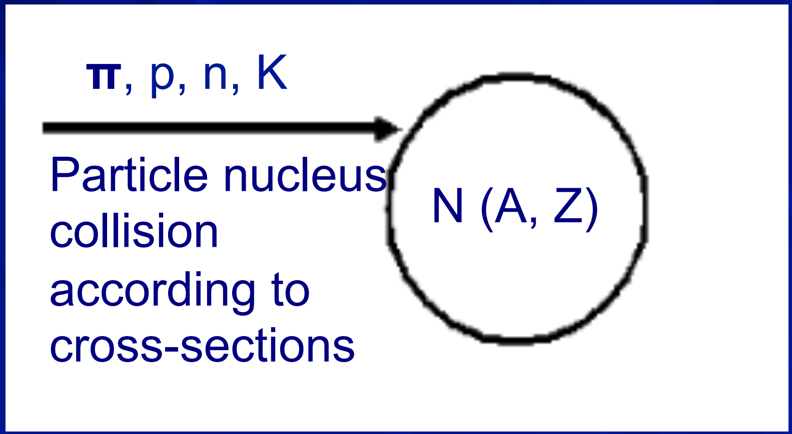
Sampling contribution:

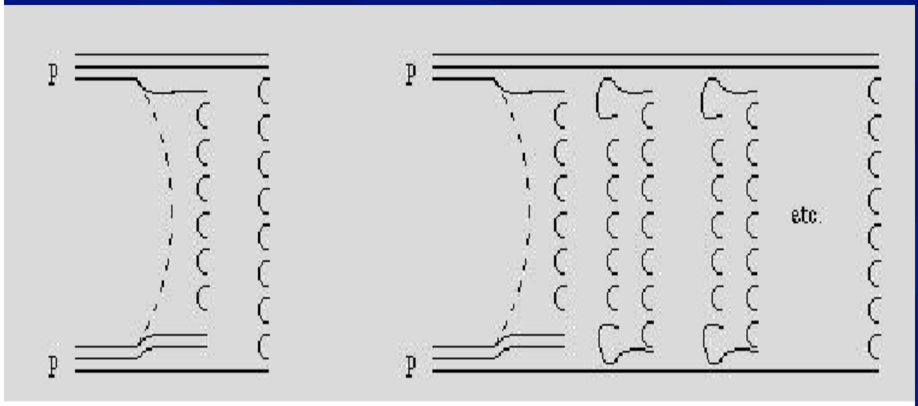
$$\frac{\sigma_E}{E} = 3.2\% \sqrt{\frac{E_c\ [\mathrm{MeV}] \cdot t_{\mathrm{abs}}}{F \cdot E\ [\mathrm{GeV}]}}$$

Hadronic interactions

1st stage: the hard collision

- pions travel 25-50% longer than protons (~2/3 smaller in size)
- a pion loses ~100-300 MeV by ionization (Z dependent)





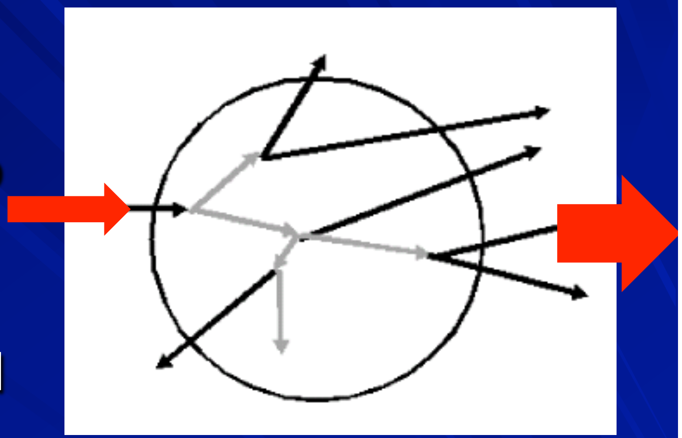
Nucleon is split in quark di-quark
Strings are formed String hadronisation (adding qqbar pair)
fragmentation of damaged nucleus

- Particle multiplication (string model)
  - average energy needed to produce a pion 0.7 (1.3) GeV in Cu (Pb)
  - Multiplicity scales with E and particle type
  - ~ 1/3 $\pi^0 \rightarrow \gamma\gamma$ produced in charge exchange processes: $\pi^+ p \rightarrow \pi^0 n$ and $\pi^- n \rightarrow \pi^0 p$
  - Leading particle effect: depends on incident hadron type e.g fewer π^0 from protons, barion number conservation

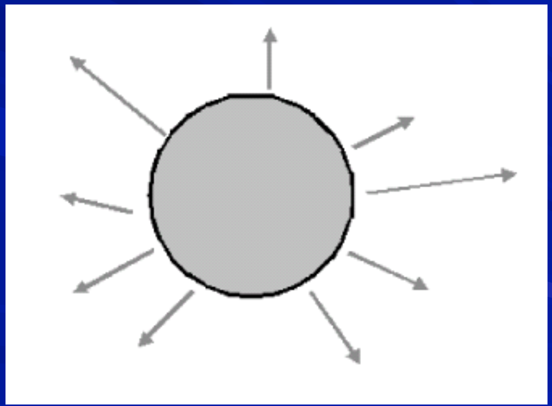
Hadronic interactions

2nd stage: spallation

- A fast hadron traversing the nucleus frees protons and neutrons in number proportional to their numerical presence in the nucleus.
- The nucleons involved in the cascade transfer energy to the nucleus which is left in an excited state
- Nuclear de-excitation
  - Evaporation of soft (~10 MeV) nucleons and α
  - fission for some materials
- The number of nucleons released depends on the binding E (7.9 MeV in Pb, 8.8 MeV in Fe)
- Mainly neutrons released by evaporation→ protons are trapped by the Coulomb barrier (12 MeV in Pb, only 5 MeV in Fe)

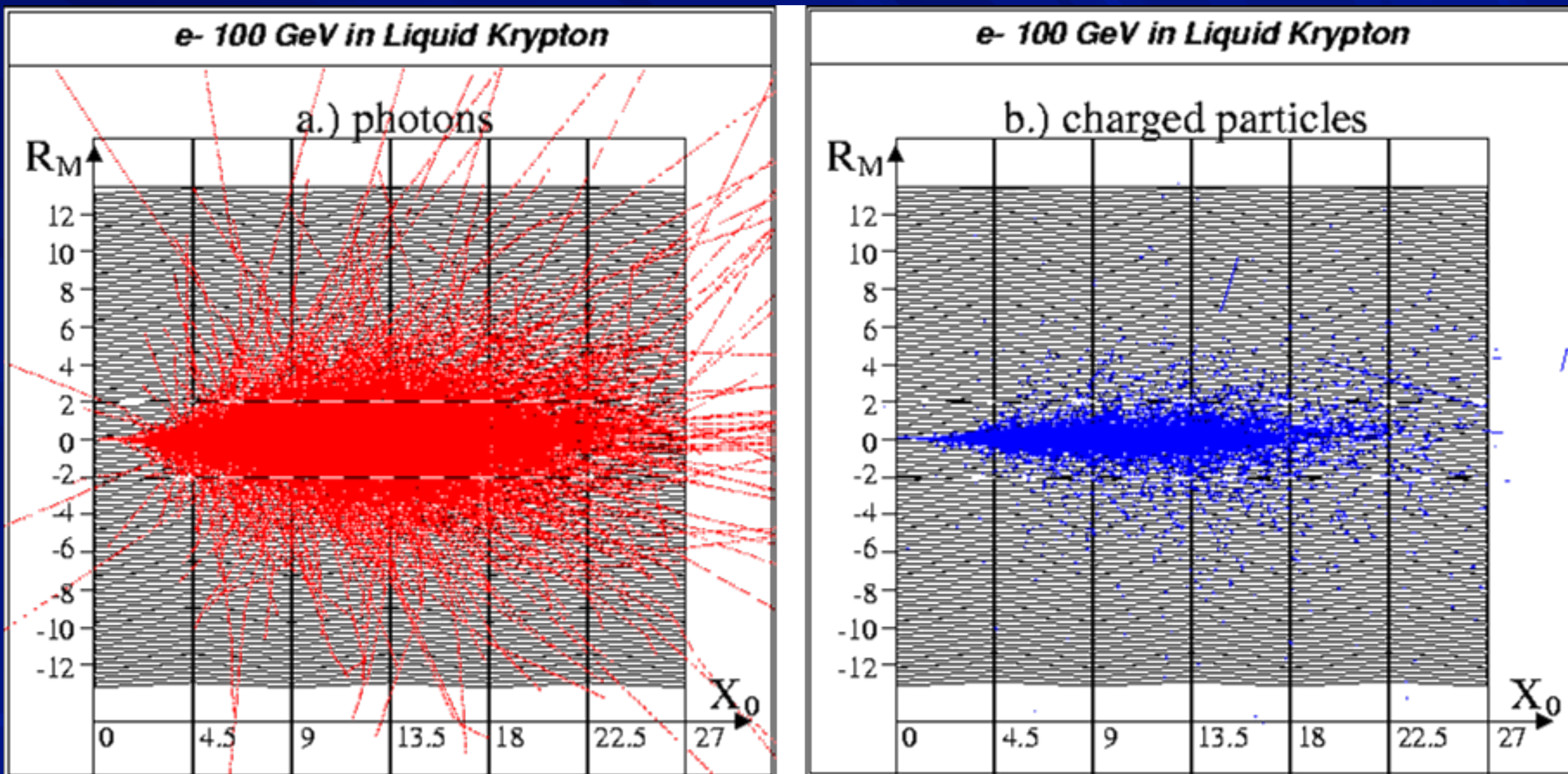


Dominating momentum component along incoming particle direction



isotropic process

EM shower development in liquid krypton (Z=36, A=84)



GEANT simulation of a 100 GeV electron shower in the NA48 liquid Krypton calorimeter (D.Schinzel)

Hadronic shower

Hadronic interaction:

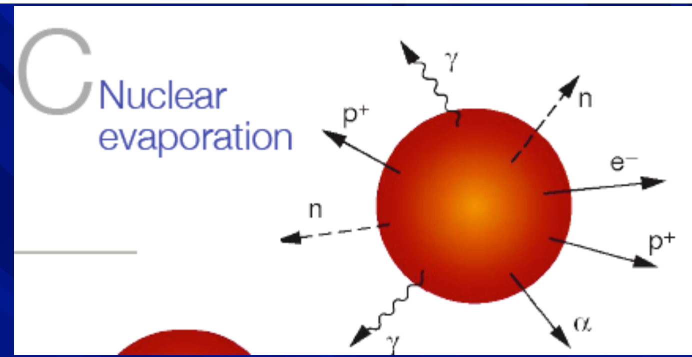
Elastic:

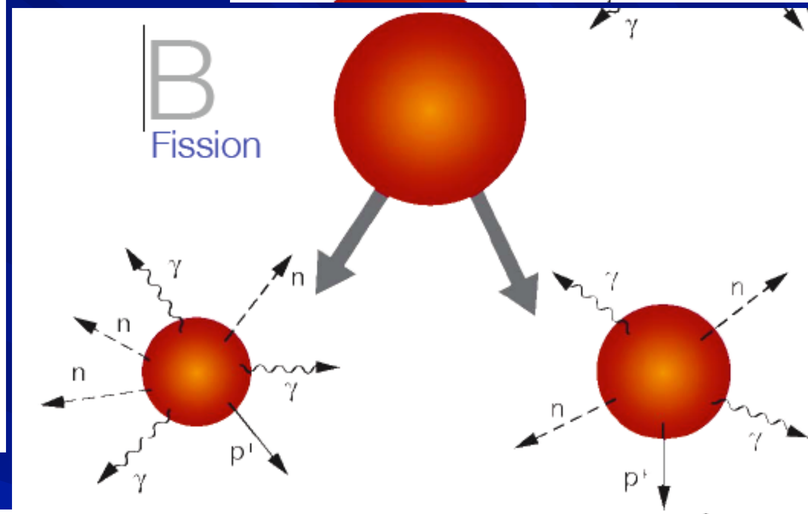
$$p + \text{Nucleus} \rightarrow p + \text{Nucleus}$$

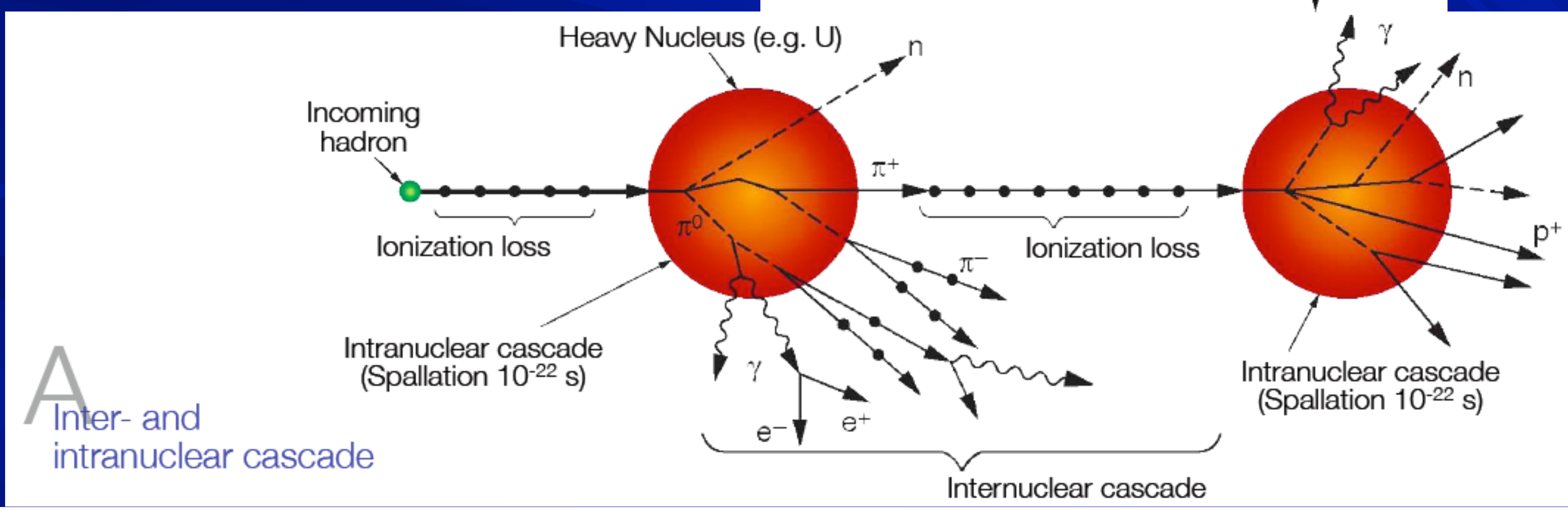
Inelastic:

$$p + \text{Nucleus} \rightarrow$$
$$\pi^+ + \pi^- + \pi^0 + \ldots + \text{Nucleus}^*$$

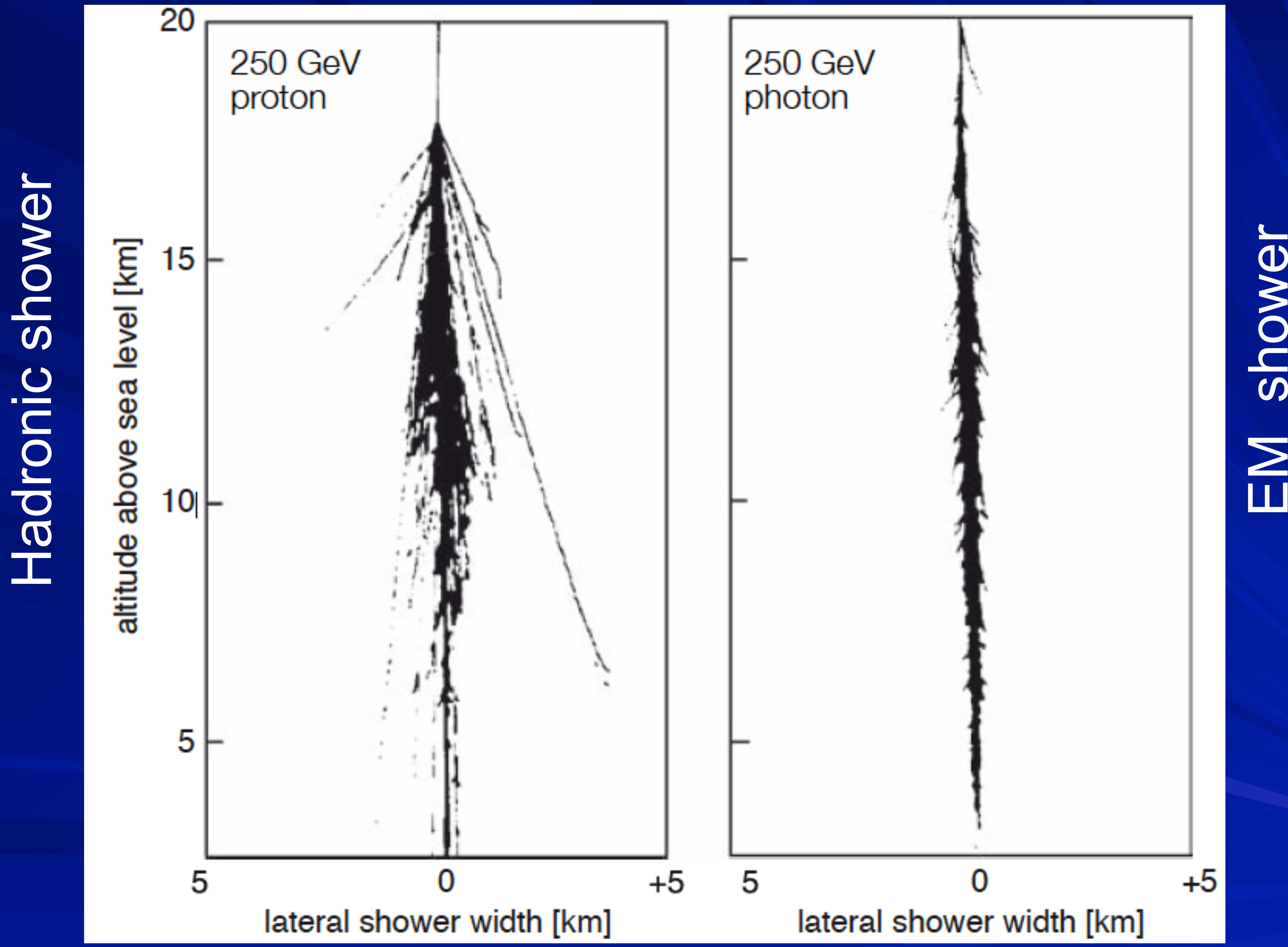
$$\left[\begin{array}{l}\text{Nucleus}^* \rightarrow \text{Nucleus A} + n, p, \alpha, \ldots \\ \qquad\quad\;\; \rightarrow \text{Nucleus B} + 5p, n, \pi, \ldots \\ \qquad\quad\;\; \rightarrow \text{Nuclear fission}\end{array}\right]$$





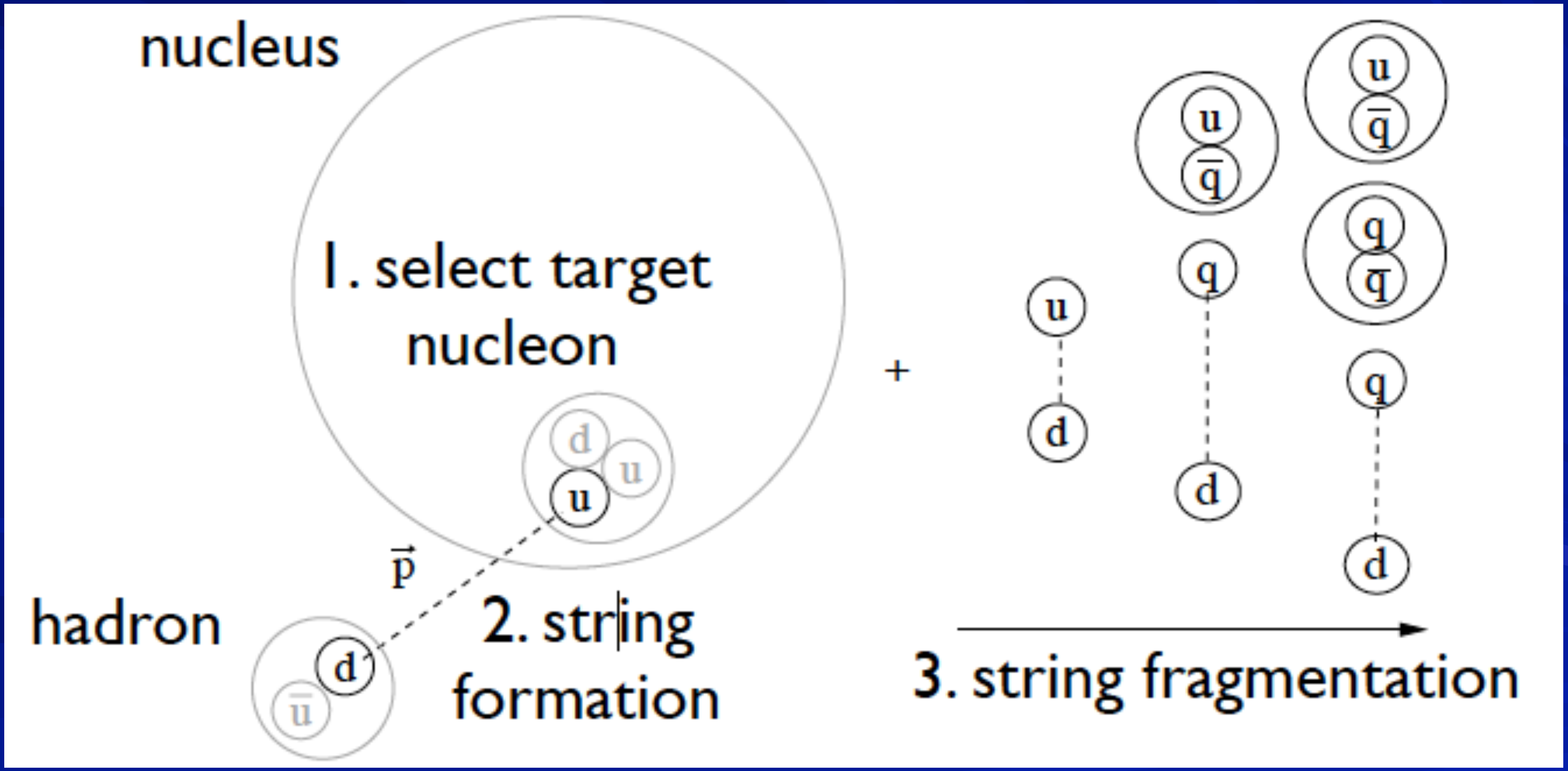


Courtesy of H. C. Schoultz Coulon


Hadronic shower
20
15
10
5
altitude above sea level [km]
250 GeV
proton
5
0
+5
lateral shower width [km]
250 GeV
photon
5
0
+5
lateral shower width [km]
EM shower

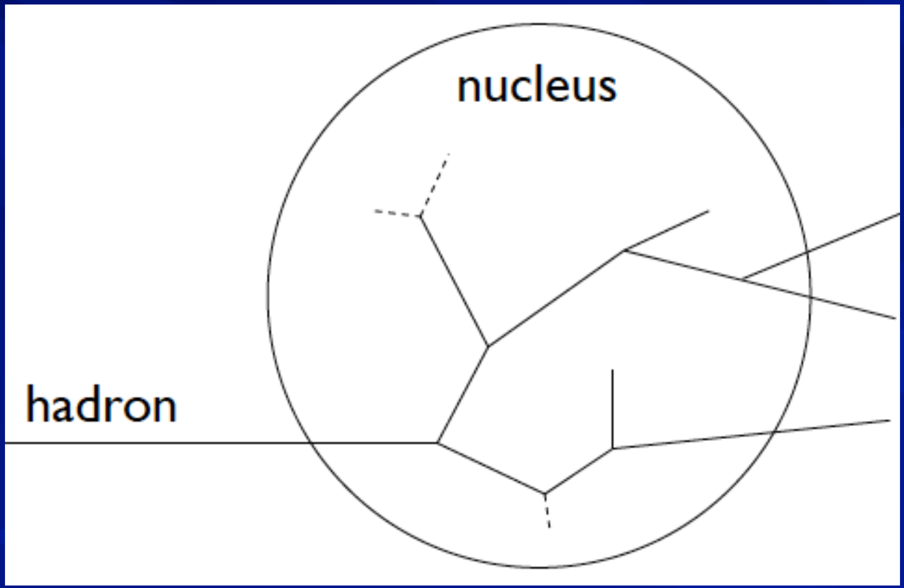
Simulation

- Interaction of hadrons with E > 10 GeV described by string models
  - projectile interacts with single nucleon (p,n)
  - a string is formed between quarks from interacting nucleons
  - the string fragmentation generates hadrons



Simulation

- Interaction of hadrons with 10 MeV < E < 10 GeV via intra-nuclear cascades
- For E < 10 MeV only relevant are fission, photon emission, evaporation, …



Approximations

- $\lambda_{deBroglie} \leq$ d nucleon
- nucleus = Fermi gas (all nucleons included)
- Pauli exclusion: allow only secondaries above Fermi energy