



MONTE CARLO'S: EVENT SIMULATION FOR THE LHC

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LECTURE I

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INDEPENDENCE DAY 2012



Clear evidence for a new resonance! Now reaching >10 σ



Even this plot actually needs theory input (and the total quoted uncertainty in the measurements does have a contribution from theory)!!!

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NO SIGN OF NEW PHYSICS (SO FAR)!

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Summer Student



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• Optimism: New Physics could be hiding there already, **I might be the one** to dig it out.

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WHY HAPPY?

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• Accuracy: accurate simulations for both SM and BSM are a must.

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- Confidence on possible excesses, evidences and eventually discoveries builds upon an intense (and often non-linear) process of description/ prediction of data via MC's.
- Both **measurements** and **exclusions** rely on accurate predictions.

NEW GENERATION (LHC) OF MC TOOLS

Theory

Lagrangian Gauge invariance QCD Partons NLO Resummation

...

Detector simulation Pions, Kaons, ... Reconstruction B-tagging efficiency Boosted decision tree Neural network



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<image>

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P, THE LHC SIMULATION CHAIN Idea Lagrangian FeynRules **ME** Generator Signal & Bkg **Events PS+Had** Detect. Sim. Data

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AIMS FOR THESE TWO LECTURES

- Basics of Monte Carlo techniques.
- Recall the basics of the necessary QCD concepts to understand what is going on in a pp event at the TeV scale.
- Critically revisit the "old" ways of making predictions for hadron colliders: either via fixed-order predictions or parton showers.
- Mention the new *predictive* techniques that are available to us.

TRY IT OUT YOURSELF

Wiki with exercises on MC integration event generation:

https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/MCSummerCERN13



- Basics : LO predictions
- Event generation
- Exclusive predictions : Parton Showers
- The simulation frontier

MASTER FORMULA FOR THE LHC



MASTER FORMULA FOR THE LHC

$\sum_{a,b} \int \frac{dx_1 dx_2 d\Phi_{FS} f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)}{P_{hase-space}} P_{arton density} P_{arton-level cross}$

integral

functions

section

Two ingredients necessary:

I. Parton distribution functions : non perturbative (fit from experiments, but evolution from theory)

2. Parton-level cross section: short distance coefficients as an expansion in $\alpha_{\rm S}$ (from theory)

$\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

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$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots \right)$$

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$\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

 The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter



 Including higher corrections improves predictions and reduces theoretical uncertainties

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PREDICTIONS AT LO

How do we calculate a LO cross section for 3 jets at the LHC?

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PHASE-SPACE INTEGRAL

• Calculations of cross section or decay widths involve integrations over phase space of very complex functions



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• Calculations of cross section or decay widths involve integrations over phase space of very complex functions

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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PHASE-SPACE INTEGRAL

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PHASE-SPACE INTEGRAL

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General and flexible method is needed: Numerical (Monte Carlo) integration



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PHASE-SPACE

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$$d\Phi_n = \left[\prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 (2E_i)}\right] (2\pi)^4 \delta^{(4)} (p_0 - \sum_{i=1}^n p_i)$$

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$$d\Phi_2(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$

PHASE-SPACE

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$$d\Phi_{n} = \left[\Pi_{i=1}^{n} \frac{d^{3}p_{i}}{(2\pi)^{3}(2E_{i})}\right] (2\pi)^{4} \delta^{(4)}(p_{0} - \sum_{i=1}^{n} p_{i})$$

$$d\Phi_{2}(M) = \frac{1}{8\pi} \frac{2p}{M} \frac{d\Omega}{4\pi}$$

$$(n) = \frac{1}{2\pi} \int_{0}^{(M-\mu)^{2}} d\mu^{2} d\Phi_{2}(M) d\Phi_{n-1}(\mu)$$

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INTEGRALS AS AVERAGES

$$I = \int_{x_1}^{x_2} f(x) dx \quad \square \quad V_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \square \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x)]^2 - I_N^2$$

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INTEGRALS AS AVERAGES

© Convergence is slow but it can be estimated easily © Error does not depend on # of dimensions! © Improvement by minimizing V_N © Optimal/Ideal case: $f(x) = Constant \Rightarrow V_N = 0$





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But... you need to know too much about f(x)!

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Idea: learn during the run and build a step-function approximation p(x) of $f(x) \longrightarrow VEGAS$



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MG101



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more bins where f(x) is large



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Idea: learn during the run and build a step-function approximation p(x) of $f(x) \longrightarrow VEGAS$



MC101

more bins where f(x) is large

$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

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can be generalized to n dimensions:

$$p(\mathbf{x}) = p(\mathbf{x}) \bullet p(\mathbf{y}) \bullet p(\mathbf{z}) \dots$$

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This is ok...

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This is not ok...

can be generalized to n dimensions:

$$p(\vec{x}) = p(x) \cdot p(y) \cdot p(z) \dots$$

but the peaks of f(x) need to be "aligned" to the axis!



but it is sufficient to make a change of variables!



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MULTI-CHANNEL

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MULTI-CHANNEL



In this case there is no unique tranformation: Vegas is bound to fail!

MULTI-CHANNEL



In this case there is no unique tranformation: Vegas is bound to fail!

Solution: use different transformations = channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time
MULTI-CHANNEL



In this case there is no unique tranformation: Vegas is bound to fail! B



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MULTI-CHANNEL



In this case there is no unique tranformation: Vegas is bound to fail!

But if you know where the peaks are (=in which variables) we can use different transformations= channels:

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \quad \text{with} \quad \sum_{i=1}^{n} \alpha_i = 1$$
$$I = \int f(x) dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

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EXERCISE: TOP DECAY



Easy but non-trivial

• Breit-Wigner peak
$$rac{1}{(q^2-m_W^2)^2+\Gamma_W^2m_W^2}$$
 to be ''flattened'':

• Choose the right "channel" for the phase space:



EXERCISE: TOP DECAY





- Every phase-space point computed in this way, can be seen as an event (=collision) in a detector
- However, they still carry the "weight" of the matrix elements:
 events with large weights where the cross section is large
 events with small weights where the cross section is small
- In nature, the events don't carry a weight:
 more events where the cross section is large
 less events where the cross section is small
- How to go from weighted events to unweighted events?



Alternative way

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Alternative way

I. (randomly) pick x

Par



Alternative way

I. (randomly) pick x2. calculate f(x)



Alternative way

I. (randomly) pick x
2. calculate f(x)
3. (randomly) pick 0<y<fmax



Alternative way

- I. (randomly) pick x
- 2. calculate f(x)
- 3. (randomly) pick 0<y<fmax

4. Compare: if f(x)>y accept event,



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- I. (randomly) pick x
- 2. calculate f(x)
- 3. (randomly) pick 0<y<fmax
- 4. Compare: if f(x)>y accept event,else reject it.



Alternative way

- I. (randomly) pick x
- 2. calculate f(x)
- 3. (randomly) pick 0<y<fmax
- 4. Compare: if f(x)>y accept event,
 - else reject it.
 - = efficiency



What's the difference?

before:

Same # of events in areas of phase space with very different probabilities:

Events must have different weights:

$$w_i = p(x_i)$$



What's the difference? after:

events is proportional to the probability of areas of phase space:

Events have all the same weight (''unweighted'')

Events distributed as in Nature



Improved

I. pick x distributed as p(x)

2. calculate f(x) and p(x)

3. pick 0<y<1

Compare:
 if f(x)>y p(x) accept event,

else reject it.

much better efficiency!!!



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MC integrator

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MC EVENT GENERATOR: DEFINITION

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

In practice it performs (a possibly large) number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

Note that, at least among theorists, the definition of a "Monte Carlo program" also includes codes which don't provide a fully exclusive information on the final state but only cross sections or distributions at the parton level, even when no unweighting can be performed (typically at NLO).

I will refer to these kind of codes as "MC integrators".



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- Having accurate and flexible simulations tools available for the LHC is a necessity (even more now!!)
- At LO event generation is technically challenging, yet conceptually straightforward.



To organize this presentation I have benefited from lectures (and actual slides), talks and discussions with many people. In particular:

- Mike Seymour (MC basics)
- Claude Duhr (FeynRules)
- Johan Alwall (ME+PS merging)
- Rikkert Frederix, Paolo Torrielli (NLO+PS)
- Stefano Frixione, Michelangelo Mangano, Paolo Nason (for QCD, PS, LO, NLO, and more...)

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Whom I all warmly thank!!